N=1 flux vacua on twisted tori

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In collaboration with

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Generalized complex geometry natural
language for flux vacua

SUPERSYMMETRIC SOLUTIONS

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- Vanishing of spinor variations

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SUPERSYMMETRIC SOLUTIONS

- Vanishing of spinor variations

$$\left.\begin{array}{r}\delta_\epsilon\psi_M = \nabla_M\epsilon + H_{Mnp}\gamma^{np}\sigma^3\epsilon + e^\phi\sum_n \not{F}_n(-1)^{[n/2]}\gamma_M\sigma^1\epsilon = 0\\ \delta\lambda = (\not\partial\phi + \frac{1}{2}\not{H}\sigma^3)\epsilon + \frac{1}{8}e^\phi\sum_n(5-n)\not{F}_{(n)}\,\sigma^1(-1)^{[n/2]}\epsilon = 0\\ \text{•Bianchi identities and EOM for H, F}_\text{n}\end{array}\right\}\ \text{Equations of motion}$$

Reduction

- Metric $ds^2_{10} = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + ds^2_6(y)$
- Fluxes $F^{10} = F + vol_4 \wedge \lambda(*F)$ (with $\lambda(F_n) = (-1)^{\mathrm{Int[n/2]}}F_n$)
- Spinors $\epsilon^1 = \theta^1_+ \otimes \eta^1_+ + c.c.$

$$\epsilon^2 = \theta^2_\pm \underbrace{\otimes\,\eta^2_+}_{\text{SU(3) x SU(3) structure on T⊕T*}} + c.c.$$

Off-shell SUSY: $\exists\eta^{1,2} \longrightarrow$ SU(3) x SU(3) structure on T⊕T*

(Louis' talk)

On-shell SUSY: $\exists\eta^{1,2}$ / $\rightarrow$ differential properties: integrability of structure

$\delta_\epsilon\psi_m = 0$, $\delta_\epsilon\lambda = 0$

Structure on T⊕T*: defined by O(6,6) pure spinors

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$$\Phi^{o}_{\pm} = \eta^{1}_{+} \otimes \eta^{2\,\dagger}_{\pm}$$ sum of forms ⟷ 1-1 correpondance with GACS

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CY: $d\Phi_+ = 0$

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GCY: $d\Phi_{+} = 0$ or $d\Phi_{-} = 0$

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What does SUSY tell us about integrability of the pure spinors?

IIA

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$$\Phi_\pm = e^B e^{2A-\phi} \eta^1_+ \otimes \eta^{2\,\dagger}_\pm$$

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$$d\Phi_+ = 0$$

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$$\Phi_\pm = e^B e^{2A-\phi} \eta^1_+ \otimes \eta^{2\,\dagger}_\pm$$

$$F_A = e^{3A}(F_0 + F_2 + F_4 + F_6)$$

IIA

$$d\Phi_+ = 0$$

$$d\Phi_- = dA \wedge \bar{\Phi}_- + i * F_A$$

$$\downarrow$$

$$\Phi_\pm = e^B e^{2A-\phi} \eta^1_+ \otimes {\eta^2_\pm}^\dagger$$

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IIA

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↓

Φ_+ is closed

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↓

Φ_+ is closed

IIB

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IIA

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↓

Φ_+ is closed

IIB

$$d\Phi_- = 0$$

$$\Phi_\pm = e^B e^{2A-\phi} \eta^1_+ \otimes \eta^{2\,\dagger}_\pm$$

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IIA

$$d\Phi_+ = 0$$

$$d\Phi_- = dA \wedge \bar{\Phi}_- + i * F_A$$

$$\downarrow$$

Φ_+ is closed

IIB

$$d\Phi_- = 0$$

$$d\Phi_+ = dA \wedge \bar{\Phi}_+ + i * F_B$$

$$\Phi_\pm = e^B e^{2A-\phi} \eta^1_+ \otimes \eta^{2\,\dagger}_\pm$$

$$F_A = e^{3A}(F_0 + F_2 + F_4 + F_6)$$

IIA

$$d\Phi_+ = 0$$

$$d\Phi_- = dA \wedge \bar{\Phi}_- + i * F_A$$

Φ_+ is closed

IIB

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IIB

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Φ_- is closed

$$\Phi_\pm = e^B e^{2A-\phi} \eta^1_+ \otimes \eta^{2\,\dagger}_\pm$$

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$$F_B = e^{3A}(F_1 + F_3 + F_5)$$

IIA

$$d\Phi_+ = 0$$

$$d\Phi_- = dA \wedge \bar{\Phi}_- + i * F_A$$

↓

Φ_+ is closed

generalized

←→

mirror symmetry

IIB

$$d\Phi_- = 0$$

$$d\Phi_+ = dA \wedge \bar{\Phi}_+ + i * F_B$$

↓

Φ_- is closed

$$\Phi_\pm = e^B e^{2A-\phi} \eta_+^1 \otimes \eta_\pm^{2\,\dagger}$$

$$F_A = e^{3A}(F_0 + F_2 + F_4 + F_6)$$

$$F_B = e^{3A}(F_1 + F_3 + F_5)$$

IIA | IIB

$$d\Phi_+ = 0 \qquad\qquad d\Phi_- = 0$$

$$\begin{tikzcd} d\Phi_- = dA \wedge \bar{\Phi}_- + i * F_A \arrow[d] & & d\Phi_+ = dA \wedge \bar{\Phi}_+ + i * F_B \arrow[d] \\ \Phi_+ \text{ is closed} \arrow[rr, leftrightarrow, "\text{generalized}", "\text{mirror symmetry}"'] & & \Phi_- \text{ is closed} \end{tikzcd}$$

Susy vacua are all generalized Calabi-Yau's !

$$\Phi_\pm = e^B e^{2A-\phi} \eta^1_+ \otimes \eta^{2\,\dagger}_\pm$$

$$F_A = e^{3A}(F_0 + F_2 + F_4 + F_6)$$

$$F_B = e^{3A}(F_1 + F_3 + F_5)$$

IIA

$$d\Phi_+ = 0$$

$$d\Phi_- = dA \wedge \bar{\Phi}_- + i * F_A$$

$$\downarrow$$

Φ_+ is closed

IIB

$$d\Phi_- = 0$$

$$d\Phi_+ = dA \wedge \bar{\Phi}_+ + i * F_B$$

$$\downarrow$$

Φ_- is closed

$$\Phi_+ \text{ is closed} \xleftrightarrow[\text{mirror symmetry}]{\text{generalized}} \Phi_- \text{ is closed}$$

Susy vacua are all generalized Calabi-Yau's !

$\mathcal{M}$ is symplectic ($\mathbb{R}^{3x2}$, J) in SU(3)

$$\Phi_\pm = e^B e^{2A-\phi} \eta^1_+ \otimes \eta^{2\,\dagger}_\pm$$

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$$F_B = e^{3A}(F_1 + F_3 + F_5)$$

IIA

$$d\Phi_+ = 0$$

$$d\Phi_- = dA \wedge \bar{\Phi}_- + i * F_A$$

$$\downarrow$$

Φ_+ is closed

IIB

$$d\Phi_- = 0$$

$$d\Phi_+ = dA \wedge \bar{\Phi}_+ + i * F_B$$

$$\downarrow$$

Φ_- is closed

$$\xleftrightarrow[\text{mirror symmetry}]{\text{generalized}}$$

Susy vacua are all generalized Calabi-Yau's !

$\mathcal{M}$ is symplectic ($\mathbb{R}^{3x2}$, J) in SU(3)

is hybrid $\mathbb{C}^2 \otimes$ ($\mathbb{R}^{1x2}$, J) in SU(2)

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$$F_A = e^{3A}(F_0 + F_2 + F_4 + F_6)$$

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IIA

$$d\Phi_+ = 0$$

$$d\Phi_- = dA \wedge \bar{\Phi}_- + i * F_A$$

$$\downarrow$$

Φ_+ is closed

IIB

$$d\Phi_- = 0$$

$$d\Phi_+ = dA \wedge \bar{\Phi}_+ + i * F_B$$

$$\downarrow$$

Φ_- is closed

generalized
↔
mirror symmetry

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$$\Phi_\pm = e^B e^{2A-\phi} \eta^1_+ \otimes \eta^{2\,\dagger}_\pm$$

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IIA

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↓

Φ_+ is closed

IIB

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$$d\Phi_+ = dA \wedge \bar{\Phi}_+ + i * F_B$$

↓

Φ_- is closed

generalized ↔ mirror symmetry

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IIA

$$d\Phi_+ = 0$$

$$d\Phi_- = dA \wedge \bar{\Phi}_- + i * F_A$$

$$\downarrow$$

IIB

$$d\Phi_- = 0$$

$$d\Phi_+ = dA \wedge \bar{\Phi}_+ + i * F_B$$

$$\downarrow$$

Φ_+ is closed $\xleftrightarrow[\text{mirror symmetry}]{\text{generalized}}$ Φ_- is closed

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is hybrid $\mathbb{C}^1 \otimes$ ($\mathbb{R}^{2x2}$, J) in SU(2)

$$\Phi_\pm = e^B \overbrace{e^{2A-\phi}\eta^1_+ \otimes \eta^{2\,\dagger}_\pm}^{\Phi^0}$$

$$F_A = e^{3A}(F_0 + F_2 + F_4 + F_6)$$

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IIA

$$d\Phi_+ = 0$$

$$d\Phi_- = dA \wedge \bar{\Phi}_- + i * F_A$$

↓

Φ_+ is closed

IIB

$$d\Phi_- = 0$$

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↓

Φ_- is closed

generalized ↔ mirror symmetry

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$$\Phi_\pm = e^B \overbrace{e^{2A-\phi}\eta^1_+ \otimes \eta^{2\,\dagger}_\pm}^{\Phi^0}$$

$$(d - H\wedge)\Phi_\pm = e^B d\Phi^0_\pm$$

$$F_A = e^{3A}(F_0 + F_2 + F_4 + F_6)$$

$$F_B = e^{3A}(F_1 + F_3 + F_5)$$

IIA

$$d\Phi_+ = 0$$

$$d\Phi_- = dA \wedge \bar{\Phi}_- + i * F_A$$

$\downarrow$

Φ_+ is closed

IIB

$$d\Phi_- = 0$$

$$d\Phi_+ = dA \wedge \bar{\Phi}_+ + i * F_B$$

$\downarrow$

Φ_- is closed

Φ_+ is closed $\xleftrightarrow[\text{mirror symmetry}]{\text{generalized}}$ Φ_- is closed

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$$\Phi_\pm = e^B \overbrace{e^{2A-\phi}\eta^1_+ \otimes \eta^{2\,\dagger}_\pm}^{\Phi^0}$$

$$\underbrace{(d - H\wedge)}_{d_H}\Phi_\pm = e^B d\Phi^0_\pm$$

$$F_A = e^{3A}(F_0 + F_2 + F_4 + F_6)$$

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Type B solution: SU(3) structure, F_3, H_3 and F_5 , ϕ constant

Type B solution: SU(3) structure, F_3, H_3 and F_5 , ϕ constant

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$$\Phi_+ = ie^{3A}e^{iJ}$$

Type B solution: SU(3) structure, F_3, H_3 and F_5 , ϕ constant

$$(d - H\wedge)\Phi_- = 0 \qquad\qquad \rightarrow \;\; d(e^{3A}\Omega_3) = 0$$

$$(d - H\wedge)\Phi_+ = dA \wedge \bar{\Phi}_+ + ie^{\phi} * F_B$$

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$$(d - H\wedge)\Phi_+ = dA \wedge \bar{\Phi}_+ + ie^{\phi} * F_B \nearrow d(e^{2A}J) = 0$$

$$\Phi_- = e^{3A}\Omega$$

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Type B solution: SU(3) structure, F3, H3 and F5 ,ϕ constant

$$(d - H\wedge)\Phi_- = 0 \qquad \longrightarrow \quad d(e^{3A}\Omega_3) = 0$$

$$\boxed{ds^2 = e^{2A}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A}\widetilde{ds}^2_{CY}}$$

$$(d - H\wedge)\Phi_+ = dA \wedge \bar{\Phi}_+ + ie^{\phi} * F_B \nearrow d(e^{2A}J) = 0$$

$$\Phi_- = e^{3A}\Omega$$

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Type B solution: SU(3) structure, F_3, H_3 and F_5 , ϕ constant

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$$\begin{tikzcd}
 & d(e^{2A}J) = 0 \\
(d - H\wedge)\Phi_+ = dA \wedge \bar{\Phi}_+ + ie^{\phi} * F_B \arrow[ur] \arrow[dr] & \\
 & d(e^{4A}) = e^{4A} * F_5
\end{tikzcd}$$

$$\Phi_- = e^{3A}\Omega$$

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Type B solution: SU(3) structure, F_3, H_3 and F_5 , ϕ constant

$$(d - H\wedge)\Phi_- = 0 \qquad \longrightarrow \quad d(e^{3A}\Omega_3) = 0$$

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$$(d - H\wedge)\Phi_+ = dA \wedge \bar{\Phi}_+ + ie^{\phi} * F_B \begin{cases} d(e^{2A}J) = 0 \\ d(e^{4A}) = e^{4A} * F_5 \end{cases}$$

$$-H = e^{\phi} * F_3$$

$$\Phi_- = e^{3A}\Omega$$

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Type B solution: SU(3) structure, F_3, H_3 and F_5 , ϕ constant

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$$-H = e^{\phi} * F_3$$

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$$\boxed{*G_3 \equiv *(F_3 + ie^{-\phi}H_3) = iG_3}$$

Type B solution: SU(3) structure, F_3, H_3 and F_5 , ϕ constant

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Beyond type B solutions... beyond CY

Type B solution: SU(3) structure, F_3, H_3 and F_5 , ϕ constant

$$(d - H\wedge)\Phi_- = 0 \qquad \longrightarrow \quad d(e^{3A}\Omega_3) = 0$$

$$\boxed{ds^2 = e^{2A}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A}\widetilde{ds}^2_{CY}}$$

$$(d - H\wedge)\Phi_+ = dA \wedge \bar{\Phi}_+ + ie^{\phi} * F_B \quad \nearrow \quad d(e^{2A}J) = 0$$

$$\searrow \quad d(e^{4A}) = e^{4A} * F_5$$

$$-H = e^{\phi} * F_3$$

$$\Phi_- = e^{3A}\Omega$$

$$\Phi_+ = ie^{3A}e^{iJ}$$

$$\boxed{*G_3 \equiv *(F_3 + ie^{-\phi}H_3) = iG_3}$$

Beyond type B solutions... beyond CY

but GCY!

Simplest case: torus

Simplest case: torus

$(e^1, e^2, e^3, e^4, e^5, e^6)$

Simplest case: torus

$(e^1, e^2, e^3, e^4, e^5, e^6)$

$\Phi^- = (e^1+ie^2) \wedge (e^3+ie^4) \wedge (e^5+ie^6)$

Simplest case: torus

$(e^1, e^2, e^3, e^4, e^5, e^6)$

$\Phi^- = (e^1+ie^2) \wedge (e^3+ie^4) \wedge (e^5+ie^6)$ $\Phi^+ = \exp[i(e^1\wedge e^2 + e^3\wedge e^4 + e^5\wedge e^6)]$

Simplest case: torus

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$\Phi^+ = \exp[i(e^1\wedge e^2 + e^3\wedge e^4 + e^5\wedge e^6)]$

type 0

Simplest case: torus

$(e^1, e^2, e^3, e^4, e^5, e^6)$

$\Phi^- = (e^1+ie^2) \wedge (e^3+ie^4) \wedge (e^5+ie^6)$

type 3

$\Phi^+ = \exp[i(e^1\wedge e^2 + e^3\wedge e^4 + e^5\wedge e^6)]$

type 0

Simplest case: torus

$(e^1, e^2, e^3, e^4, e^5, e^6)$

$\Phi^- = (e^1+ie^2) \wedge (e^3+ie^4) \wedge (e^5+ie^6)$

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type 3 type 0

$\Phi^- = (e^1+ie^2) \exp[i(e^3\wedge e^4+ e^5 \wedge e^6)])$

Simplest case: torus

$(e^1, e^2, e^3, e^4, e^5, e^6)$

$\Phi^- = (e^1+ie^2) \wedge (e^3+ie^4) \wedge (e^5+ie^6)$

type 3

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type 0

- -

$\Phi^- = (e^1+ie^2) \exp[i(e^3\wedge e^4+ e^5 \wedge e^6)])$

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Simplest case: torus

$(e^1, e^2, e^3, e^4, e^5, e^6)$

$\Phi^- = (e^1+ie^2) \wedge (e^3+ie^4) \wedge (e^5+ie^6)$

type 3

$\Phi^+ = \exp[i(e^1\wedge e^2 + e^3\wedge e^4 + e^5\wedge e^6)]$

type 0

$\Phi^- = (e^1+ie^2) \exp[i(e^3\wedge e^4 + e^5 \wedge e^6)]$)

$\Phi^+ = (e^3+ie^4) \wedge (e^5+ie^6) \exp[i\, e^1\wedge e^2]]$

type 2

Simplest case: torus

$(e^1, e^2, e^3, e^4, e^5, e^6)$

$\Phi^- = (e^1+ie^2) \wedge (e^3+ie^4) \wedge (e^5+ie^6)$

type 3

$\Phi^+ = \exp[i(e^1\wedge e^2 + e^3\wedge e^4 + e^5\wedge e^6)]$

type 0

$\Phi^- = (e^1+ie^2) \exp[i(e^3\wedge e^4+ e^5 \wedge e^6)]])$

type 1

$\Phi^+ = (e^3+ie^4) \wedge (e^5+ie^6) \exp[i\, e^1\wedge e^2]]$

type 2

Simplest case: torus

$(e^1, e^2, e^3, e^4, e^5, e^6)$

$\Phi^- = (e^1+ie^2) \wedge (e^3+ie^4) \wedge (e^5+ie^6)$

type 3

$\Phi^+ = \exp[i(e^1\wedge e^2 + e^3\wedge e^4 + e^5\wedge e^6)]$

type 0

$\Phi^- = (e^1+ie^2) \exp[i(e^3\wedge e^4+ e^5 \wedge e^6)]$)

type 1

$\Phi^+ = (e^3+ie^4) \wedge (e^5+ie^6) \exp[i\, e^1\wedge e^2]]$

type 2

On a torus, all Φ 's are closed

Simplest case: twisted torus

$(e^1, e^2, e^3, e^4, e^5, e^6)$

$\Phi^- = (e^1+ie^2) \wedge (e^3+ie^4) \wedge (e^5+ie^6)$

type 3

$\Phi^+ = \exp[i(e^1\wedge e^2 + e^3\wedge e^4 + e^5\wedge e^6)]$

type 0

- -

$\Phi^- = (e^1+ie^2) \exp[i(e^3\wedge e^4+ e^5 \wedge e^6)])$

type 1

$\Phi^+ = (e^3+ie^4) \wedge (e^5+ie^6) \exp[i\, e^1\wedge e^2]]$

type 2

On a torus, all Φ 's are closed

Simplest case: twisted torus

$(e^1, e^2, e^3, e^4, e^5, e^6)$

$$de^a = \frac{1}{2} f^a_{bc}\, e^b \wedge e^c$$

$\Phi^- = (e^1+ie^2) \wedge (e^3+ie^4) \wedge (e^5+ie^6)$

type 3

$\Phi^+ = \exp[i(e^1\wedge e^2 + e^3\wedge e^4 + e^5\wedge e^6)]$

type 0

$\Phi^- = (e^1+ie^2) \exp[i(e^3\wedge e^4+ e^5 \wedge e^6)])$

type 1

$\Phi^+ = (e^3+ie^4) \wedge (e^5+ie^6) \exp[i\, e^1\wedge e^2]]$

type 2

On a torus, all Φ 's are closed

Simplest case: twisted torus

structure constants of Lie algebra

↓

$(e^1, e^2, e^3, e^4, e^5, e^6)$

$$de^a = \frac{1}{2} f^a_{bc}\, e^b \wedge e^c$$

$\Phi^- = (e^1+ie^2) \wedge (e^3+ie^4) \wedge (e^5+ie^6)$

type 3

$\Phi^+ = \exp[i(e^1\wedge e^2 + e^3\wedge e^4 + e^5\wedge e^6)]$

type 0

- -

$\Phi^- = (e^1+ie^2) \exp[i(e^3\wedge e^4+ e^5 \wedge e^6)])$

type 1

$\Phi^+ = (e^3+ie^4) \wedge (e^5+ie^6) \exp[i\, e^1\wedge e^2]]$

type 2

On a torus, all Φ 's are closed

Simplest case: twisted torus

structure constants of Lie algebra

$(e^1, e^2, e^3, e^4, e^5, e^6)$

$$de^a = \frac{1}{2} f^a_{bc}\, e^b \wedge e^c$$

algebra nilpotent:
34 classes of 6D nilmanifolds

$\Phi^- = (e^1+ie^2) \wedge (e^3+ie^4) \wedge (e^5+ie^6)$

type 3

$\Phi^+ = \exp[i(e^1\wedge e^2 + e^3\wedge e^4 + e^5\wedge e^6)]$

type 0

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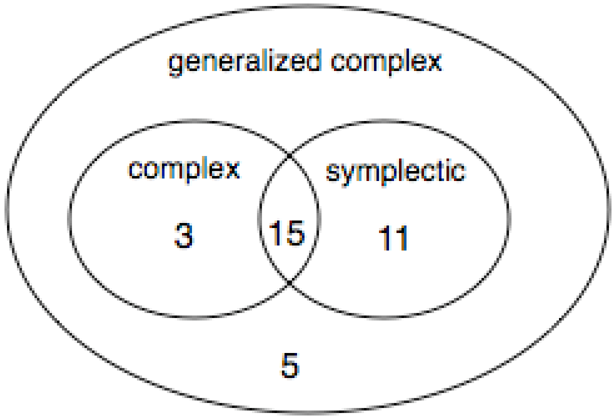

Calvacanti and Gualtieri 04

Simplest case: twisted torus

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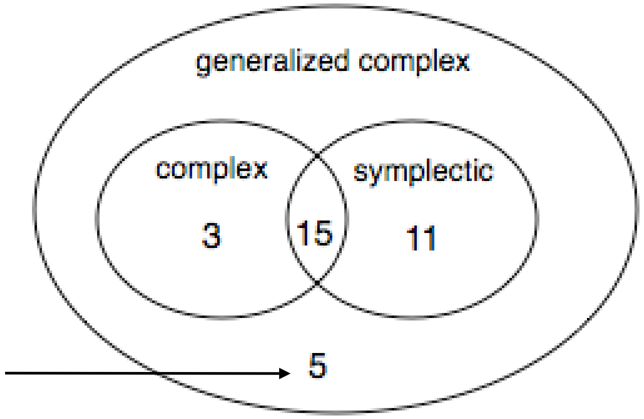

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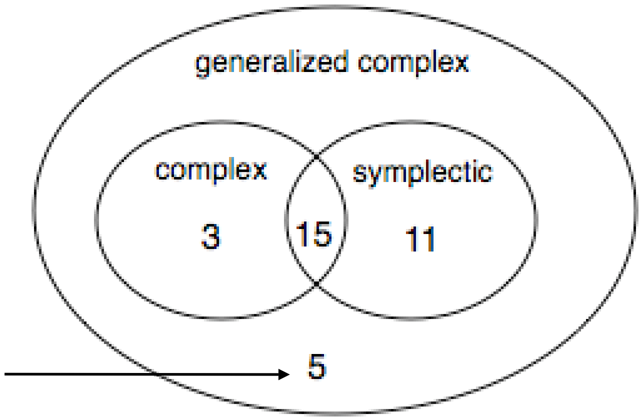

type 1 →

Calvacanti and Gualtieri 04

All generalized Calabi-Yau's

Twisted tori

Twisted tori

- ∃ d globally defined 1-forms e^a

Twisted tori

- ∃ d globally defined 1-forms $e^a \rightarrow$ d-dimensional parallelizable manifolds

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d=3 Example: $de^1 = 0\,,\; de^2 = 0\,,\; de^3 = N\, e^1 \wedge e^2$ ⇔ (0, 0, N x 12)

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$$(x^1, x^2, x^3) \sim (x^1, x^2 + a, x^3)$$

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$$\downarrow$$

$$e^1 = dx^1\,, e^2 = dx^2\,, e^3 = dx^3 + N\, x^1 dx^2$$

$$(x^1, x^2, x^3) \sim (x^1, x^2 + a, x^3) \sim (x^1, x^2, x^3 + b)$$

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twisted

Twisted tori -- Nil (solv) manifolds

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$$M = \frac{G}{\Gamma}$$

Twisted tori -- Nil (solv) manifolds

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GCY...!		

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$$M = \frac{G}{\Gamma} = \frac{\text{Lie Group}}{\text{discrete maximal subgroup}}$$

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Saito, 61

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negative sign!

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Step 2

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← Step 3 →

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Nilmanifolds

Nilmanifolds

n	Nilmanifold class
2.1	$(0,0,12,13,14,15)$
2.2	$(0,0,12,13,14,34+52)$
2.3	$(0,0,12,13,14,23+15)$
2.4	$(0,0,12,13,23,14)$
2.5	$(0,0,12,13,23,14-25)$
2.6	$(0,0,12,13,23,14+25)$
2.7	$(0,0,12,13,14+23,34+52)$
2.8	$(0,0,12,13,14+23,24+15)$
3.1	$(0,0,0,12,13,14+35)$
3.2	$(0,0,0,12,13,14+23)$
3.3	$(0,0,0,12,13,24)$
3.4	$(0,0,0,12,13,14)$
3.5	$(0,0,0,12,13,23)$
3.6	$(0,0,0,12,14,15+23)$
3.7	$(0,0,0,12,14,15+23+24)$
3.8	$(0,0,0,12,14,15+24)$
3.9	$(0,0,0,12,14,15)$
3.10	$(0,0,0,12,14,24)$
3.11	$(0,0,0,12,14,13+42)$
3.12	$(0,0,0,12,14,23+24)$
3.13	$(0,0,0,12,23,14+35)$
3.14	$(0,0,0,12,23,14-35)$
3.15	$(0,0,0,12,14-23,15+34)$
3.16	$(0,0,0,12,14+23,13+42)$
4.1	$(0,0,0,0,12,15+34)$
4.2	$(0,0,0,0,12,15)$
4.3	$(0,0,0,0,12,14+25)$
4.4	$(0,0,0,0,12,14+23)$
4.5	$(0,0,0,0,12,34)$
4.6	$(0,0,0,0,12,13)$
4.7	$(0,0,0,0,13+42,14+23)$
5.1	$(0,0,0,0,0,12+34)$
5.2	$(0,0,0,0,0,12)$
6.1	$(0,0,0,0,0,0)$

Nilmanifolds

n	Nilmanifold class
2.1	$(0,0,12,13,14,15)$
2.2	$(0,0,12,13,14,34+52)$
2.3	$(0,0,12,13,14,23+15)$
2.4	$(0,0,12,13,23,14)$
2.5	$(0,0,12,13,23,14-25)$
2.6	$(0,0,12,13,23,14+25)$
2.7	$(0,0,12,13,14+23,34+52)$
2.8	$(0,0,12,13,14+23,24+15)$
3.1	$(0,0,0,12,13,14+35)$
3.2	$(0,0,0,12,13,14+23)$
3.3	$(0,0,0,12,13,24)$
3.4	$(0,0,0,12,13,14)$
3.5	$(0,0,0,12,13,23)$
3.6	$(0,0,0,12,14,15+23)$
3.7	$(0,0,0,12,14,15+23+24)$
3.8	$(0,0,0,12,14,15+24)$
3.9	$(0,0,0,12,14,15)$
3.10	$(0,0,0,12,14,24)$
3.11	$(0,0,0,12,14,13+42)$
3.12	$(0,0,0,12,14,23+24)$
3.13	$(0,0,0,12,23,14+35)$
3.14	$(0,0,0,12,23,14-35)$
3.15	$(0,0,0,12,14-23,15+34)$
3.16	$(0,0,0,12,14+23,13+42)$
4.1	$(0,0,0,0,12,15+34)$
4.2	$(0,0,0,0,12,15)$
4.3	$(0,0,0,0,12,14+25)$
4.4	$(0,0,0,0,12,14+23)$
4.5	$(0,0,0,0,12,34)$
4.6	$(0,0,0,0,12,13)$
4.7	$(0,0,0,0,13+42,14+23)$
5.1	$(0,0,0,0,0,12+34)$
5.2	$(0,0,0,0,0,12)$
6.1	$(0,0,0,0,0,0)$

vacua (but T-dual to T^6)

Nilmanifolds

n	Nilmanifold class
2.1	$(0,0,12,13,14,15)$
2.2	$(0,0,12,13,14,34+52)$
2.3	$(0,0,12,13,14,23+15)$
2.4	$(0,0,12,13,23,14)$
2.5	$(0,0,12,13,23,14-25)$
2.6	$(0,0,12,13,23,14+25)$
2.7	$(0,0,12,13,14+23,34+52)$
2.8	$(0,0,12,13,14+23,24+15)$
3.1	$(0,0,0,12,13,14+35)$
3.2	$(0,0,0,12,13,14+23)$
3.3	$(0,0,0,12,13,24)$
3.4	$(0,0,0,12,13,14)$
3.5	$(0,0,0,12,13,23)$
3.6	$(0,0,0,12,14,15+23)$
3.7	$(0,0,0,12,14,15+23+24)$
3.8	$(0,0,0,12,14,15+24)$
3.9	$(0,0,0,12,14,15)$
3.10	$(0,0,0,12,14,24)$
3.11	$(0,0,0,12,14,13+42)$
3.12	$(0,0,0,12,14,23+24)$
3.13	$(0,0,0,12,23,14+35)$
3.14	$(0,0,0,12,23,14-35)$
3.15	$(0,0,0,12,14-23,15+34)$
3.16	$(0,0,0,12,14+23,13+42)$
4.1	$(0,0,0,0,12,15+34)$
4.2	$(0,0,0,0,12,15)$
4.3	$(0,0,0,0,12,14+25)$
4.4	$(0,0,0,0,12,14+23)$
4.5	$(0,0,0,0,12,34)$
4.6	$(0,0,0,0,12,13)$
4.7	$(0,0,0,0,13+42,14+23)$
5.1	$(0,0,0,0,0,12+34)$
5.2	$(0,0,0,0,0,12)$
6.1	$(0,0,0,0,0,0)$

vacua (but T-dual to T^6)

Nilmanifolds

Only non T-dual vacuum
(0,0,0,12,23,14-35)

n	Nilmanifold class
2.1	$(0,0,12,13,14,15)$
2.2	$(0,0,12,13,14,34+52)$
2.3	$(0,0,12,13,14,23+15)$
2.4	$(0,0,12,13,23,14)$
2.5	$(0,0,12,13,23,14-25)$
2.6	$(0,0,12,13,23,14+25)$
2.7	$(0,0,12,13,14+23,34+52)$
2.8	$(0,0,12,13,14+23,24+15)$
3.1	$(0,0,0,12,13,14+35)$
3.2	$(0,0,0,12,13,14+23)$
3.3	$(0,0,0,12,13,24)$
3.4	$(0,0,0,12,13,14)$
3.5	$(0,0,0,12,13,23)$
3.6	$(0,0,0,12,14,15+23)$
3.7	$(0,0,0,12,14,15+23+24)$
3.8	$(0,0,0,12,14,15+24)$
3.9	$(0,0,0,12,14,15)$
3.10	$(0,0,0,12,14,24)$
3.11	$(0,0,0,12,14,13+42)$
3.12	$(0,0,0,12,14,23+24)$
3.13	$(0,0,0,12,23,14+35)$
3.14	$(0,0,0,12,23,14-35)$
3.15	$(0,0,0,12,14-23,15+34)$
3.16	$(0,0,0,12,14+23,13+42)$
4.1	$(0,0,0,0,12,15+34)$
4.2	$(0,0,0,0,12,15)$
4.3	$(0,0,0,0,12,14+25)$
4.4	$(0,0,0,0,12,14+23)$
4.5	$(0,0,0,0,12,34)$
4.6	$(0,0,0,0,12,13)$
4.7	$(0,0,0,0,13+42,14+23)$
5.1	$(0,0,0,0,0,12+34)$
5.2	$(0,0,0,0,0,12)$
6.1	$(0,0,0,0,0,0)$

vacua (but T-dual to T^6)

Nilmanifolds

n	Nilmanifold class
2.1	$(0,0,12,13,14,15)$
2.2	$(0,0,12,13,14,34+52)$
2.3	$(0,0,12,13,14,23+15)$
2.4	$(0,0,12,13,23,14)$
2.5	$(0,0,12,13,23,14-25)$
2.6	$(0,0,12,13,23,14+25)$
2.7	$(0,0,12,13,14+23,34+52)$
2.8	$(0,0,12,13,14+23,24+15)$
3.1	$(0,0,0,12,13,14+35)$
3.2	$(0,0,0,12,13,14+23)$
3.3	$(0,0,0,12,13,24)$
3.4	$(0,0,0,12,13,14)$
3.5	$(0,0,0,12,13,23)$
3.6	$(0,0,0,12,14,15+23)$
3.7	$(0,0,0,12,14,15+23+24)$
3.8	$(0,0,0,12,14,15+24)$
3.9	$(0,0,0,12,14,15)$
3.10	$(0,0,0,12,14,24)$
3.11	$(0,0,0,12,14,13+42)$
3.12	$(0,0,0,12,14,23+24)$
3.13	$(0,0,0,12,23,14+35)$
3.14	$(0,0,0,12,23,14-35)$
3.15	$(0,0,0,12,14-23,15+34)$
3.16	$(0,0,0,12,14+23,13+42)$
4.1	$(0,0,0,0,12,15+34)$
4.2	$(0,0,0,0,12,15)$
4.3	$(0,0,0,0,12,14+25)$
4.4	$(0,0,0,0,12,14+23)$
4.5	$(0,0,0,0,12,34)$
4.6	$(0,0,0,0,12,13)$
4.7	$(0,0,0,0,13+42,14+23)$
5.1	$(0,0,0,0,0,12+34)$
5.2	$(0,0,0,0,0,12)$
6.1	$(0,0,0,0,0,0)$

vacua (but T-dual to T^6)

Only non T-dual vacuum

(0,0,0,12,23,14-35)

$$\begin{tikzcd}
S^1_{\{6\}} \arrow[r, hook] & M_6 \arrow[d] \\
T^2_{\{4,5\}} \arrow[r, hook] & M_5 \arrow[d] \\
 & T^3_{\{1,2,3\}}
\end{tikzcd}$$

Nilmanifolds

n	Nilmanifold class
2.1	$(0,0,12,13,14,15)$
2.2	$(0,0,12,13,14,34+52)$
2.3	$(0,0,12,13,14,23+15)$
2.4	$(0,0,12,13,23,14)$
2.5	$(0,0,12,13,23,14-25)$
2.6	$(0,0,12,13,23,14+25)$
2.7	$(0,0,12,13,14+23,34+52)$
2.8	$(0,0,12,13,14+23,24+15)$
3.1	$(0,0,0,12,13,14+35)$
3.2	$(0,0,0,12,13,14+23)$
3.3	$(0,0,0,12,13,24)$
3.4	$(0,0,0,12,13,14)$
3.5	$(0,0,0,12,13,23)$
3.6	$(0,0,0,12,14,15+23)$
3.7	$(0,0,0,12,14,15+23+24)$
3.8	$(0,0,0,12,14,15+24)$
3.9	$(0,0,0,12,14,15)$
3.10	$(0,0,0,12,14,24)$
3.11	$(0,0,0,12,14,13+42)$
3.12	$(0,0,0,12,14,23+24)$
3.13	$(0,0,0,12,23,14+35)$
3.14	$(0,0,0,12,23,14-35)$
3.15	$(0,0,0,12,14-23,15+34)$
3.16	$(0,0,0,12,14+23,13+42)$
4.1	$(0,0,0,0,12,15+34)$
4.2	$(0,0,0,0,12,15)$
4.3	$(0,0,0,0,12,14+25)$
4.4	$(0,0,0,0,12,14+23)$
4.5	$(0,0,0,0,12,34)$
4.6	$(0,0,0,0,12,13)$
4.7	$(0,0,0,0,13+42,14+23)$
5.1	$(0,0,0,0,0,12+34)$
5.2	$(0,0,0,0,0,12)$
6.1	$(0,0,0,0,0,0)$

vacua (but T-dual to T^6)

Only non T-dual vacuum
(0,0,0,12,23,14-35)

$$\begin{tikzcd}
S^1_{\{6\}} \arrow[r, hook] & M_6 \arrow[d] \\
T^2_{\{4,5\}} \arrow[r, hook] & M_5 \arrow[d] \\
& T^3_{\{1,2,3\}}
\end{tikzcd}$$

Large volume limit (A ➔ 0)

Nilmanifolds

n	Nilmanifold class
2.1	$(0,0,12,13,14,15)$
2.2	$(0,0,12,13,14,34+52)$
2.3	$(0,0,12,13,14,23+15)$
2.4	$(0,0,12,13,23,14)$
2.5	$(0,0,12,13,23,14-25)$
2.6	$(0,0,12,13,23,14+25)$
2.7	$(0,0,12,13,14+23,34+52)$
2.8	$(0,0,12,13,14+23,24+15)$
3.1	$(0,0,0,12,13,14+35)$
3.2	$(0,0,0,12,13,14+23)$
3.3	$(0,0,0,12,13,24)$
3.4	$(0,0,0,12,13,14)$
3.5	$(0,0,0,12,13,23)$
3.6	$(0,0,0,12,14,15+23)$
3.7	$(0,0,0,12,14,15+23+24)$
3.8	$(0,0,0,12,14,15+24)$
3.9	$(0,0,0,12,14,15)$
3.10	$(0,0,0,12,14,24)$
3.11	$(0,0,0,12,14,13+42)$
3.12	$(0,0,0,12,14,23+24)$
3.13	$(0,0,0,12,23,14+35)$
3.14	$(0,0,0,12,23,14-35)$
3.15	$(0,0,0,12,14-23,15+34)$
3.16	$(0,0,0,12,14+23,13+42)$
4.1	$(0,0,0,0,12,15+34)$
4.2	$(0,0,0,0,12,15)$
4.3	$(0,0,0,0,12,14+25)$
4.4	$(0,0,0,0,12,14+23)$
4.5	$(0,0,0,0,12,34)$
4.6	$(0,0,0,0,12,13)$
4.7	$(0,0,0,0,13+42,14+23)$
5.1	$(0,0,0,0,0,12+34)$
5.2	$(0,0,0,0,0,12)$
6.1	$(0,0,0,0,0,0)$

vacua (but T-dual to T^6)

Only non T-dual vacuum
(0,0,0,12,23,14-35)

$$\begin{tikzcd} S^1_{\{6\}} \arrow[r, hook] & M_6 \arrow[d] \\ T^2_{\{4,5\}} \arrow[r, hook] & M_5 \arrow[d] \\ & T^3_{\{1,2,3\}} \end{tikzcd}$$

Large volume limit (A ➔ 0)

$$\Omega_3 = (e^1 - ie^3) \wedge (e^2 + i\tau e^6) \wedge (e^4 + ie^5)$$

Nilmanifolds

n	Nilmanifold class
2.1	$(0,0,12,13,14,15)$
2.2	$(0,0,12,13,14,34+52)$
2.3	$(0,0,12,13,14,23+15)$
2.4	$(0,0,12,13,23,14)$
2.5	$(0,0,12,13,23,14-25)$
2.6	$(0,0,12,13,23,14+25)$
2.7	$(0,0,12,13,14+23,34+52)$
2.8	$(0,0,12,13,14+23,24+15)$
3.1	$(0,0,0,12,13,14+35)$
3.2	$(0,0,0,12,13,14+23)$
3.3	$(0,0,0,12,13,24)$
3.4	$(0,0,0,12,13,14)$
3.5	$(0,0,0,12,13,23)$
3.6	$(0,0,0,12,14,15+23)$
3.7	$(0,0,0,12,14,15+23+24)$
3.8	$(0,0,0,12,14,15+24)$
3.9	$(0,0,0,12,14,15)$
3.10	$(0,0,0,12,14,24)$
3.11	$(0,0,0,12,14,13+42)$
3.12	$(0,0,0,12,14,23+24)$
3.13	$(0,0,0,12,23,14+35)$
3.14	$(0,0,0,12,23,14-35)$
3.15	$(0,0,0,12,14-23,15+34)$
3.16	$(0,0,0,12,14+23,13+42)$
4.1	$(0,0,0,0,12,15+34)$
4.2	$(0,0,0,0,12,15)$
4.3	$(0,0,0,0,12,14+25)$
4.4	$(0,0,0,0,12,14+23)$
4.5	$(0,0,0,0,12,34)$
4.6	$(0,0,0,0,12,13)$
4.7	$(0,0,0,0,13+42,14+23)$
5.1	$(0,0,0,0,0,12+34)$
5.2	$(0,0,0,0,0,12)$
6.1	$(0,0,0,0,0,0)$

vacua (but T-dual to T^6)

Only non T-dual vacuum
(0,0,0,12,23,14-35)

$$\begin{tikzcd}
S^1_{\{6\}} \arrow[r, hook] & M_6 \arrow[d] \\
T^2_{\{4,5\}} \arrow[r, hook] & M_5 \arrow[d] \\
 & T^3_{\{1,2,3\}}
\end{tikzcd}$$

Large volume limit (A ➔ 0)

$$\Omega_3 = (e^1 - ie^3) \wedge (e^2 + i\underset{\uparrow}{\tau} e^6) \wedge (e^4 + ie^5)$$

complex structure modulus

Nilmanifolds

n	Nilmanifold class
2.1	$(0,0,12,13,14,15)$
2.2	$(0,0,12,13,14,34+52)$
2.3	$(0,0,12,13,14,23+15)$
2.4	$(0,0,12,13,23,14)$
2.5	$(0,0,12,13,23,14-25)$
2.6	$(0,0,12,13,23,14+25)$
2.7	$(0,0,12,13,14+23,34+52)$
2.8	$(0,0,12,13,14+23,24+15)$
3.1	$(0,0,0,12,13,14+35)$
3.2	$(0,0,0,12,13,14+23)$
3.3	$(0,0,0,12,13,24)$
3.4	$(0,0,0,12,13,14)$
3.5	$(0,0,0,12,13,23)$
3.6	$(0,0,0,12,14,15+23)$
3.7	$(0,0,0,12,14,15+23+24)$
3.8	$(0,0,0,12,14,15+24)$
3.9	$(0,0,0,12,14,15)$
3.10	$(0,0,0,12,14,24)$
3.11	$(0,0,0,12,14,13+42)$
3.12	$(0,0,0,12,14,23+24)$
3.13	$(0,0,0,12,23,14+35)$
3.14	$(0,0,0,12,23,14-35)$
3.15	$(0,0,0,12,14-23,15+34)$
3.16	$(0,0,0,12,14+23,13+42)$
4.1	$(0,0,0,0,12,15+34)$
4.2	$(0,0,0,0,12,15)$
4.3	$(0,0,0,0,12,14+25)$
4.4	$(0,0,0,0,12,14+23)$
4.5	$(0,0,0,0,12,34)$
4.6	$(0,0,0,0,12,13)$
4.7	$(0,0,0,0,13+42,14+23)$
5.1	$(0,0,0,0,0,12+34)$
5.2	$(0,0,0,0,0,12)$
6.1	$(0,0,0,0,0,0)$

vacua (but T-dual to T^6)

Only non T-dual vacuum
(0,0,0,12,23,14-35)

$$\begin{tikzcd} S^1_{\{6\}} \arrow[r, hook] & M_6 \arrow[d] \\ T^2_{\{4,5\}} \arrow[r, hook] & M_5 \arrow[d] \\ & T^3_{\{1,2,3\}} \end{tikzcd}$$

Large volume limit (A ➔ 0)

$$\Omega_3 = (e^1 - ie^3) \wedge (e^2 + \underset{\substack{\uparrow \\ \text{complex structure modulus}}}{i\tau} e^6) \wedge (e^4 + ie^5)$$

$$J = -t_1 e^1 e^3 + t_2 \tau_r e^2 e^6 + t_3 e^4 e^5$$

Nilmanifolds

n	Nilmanifold class
2.1	$(0,0,12,13,14,15)$
2.2	$(0,0,12,13,14,34+52)$
2.3	$(0,0,12,13,14,23+15)$
2.4	$(0,0,12,13,23,14)$
2.5	$(0,0,12,13,23,14-25)$
2.6	$(0,0,12,13,23,14+25)$
2.7	$(0,0,12,13,14+23,34+52)$
2.8	$(0,0,12,13,14+23,24+15)$
3.1	$(0,0,0,12,13,14+35)$
3.2	$(0,0,0,12,13,14+23)$
3.3	$(0,0,0,12,13,24)$
3.4	$(0,0,0,12,13,14)$
3.5	$(0,0,0,12,13,23)$
3.6	$(0,0,0,12,14,15+23)$
3.7	$(0,0,0,12,14,15+23+24)$
3.8	$(0,0,0,12,14,15+24)$
3.9	$(0,0,0,12,14,15)$
3.10	$(0,0,0,12,14,24)$
3.11	$(0,0,0,12,14,13+42)$
3.12	$(0,0,0,12,14,23+24)$
3.13	$(0,0,0,12,23,14+35)$
3.14	$(0,0,0,12,23,14-35)$
3.15	$(0,0,0,12,14-23,15+34)$
3.16	$(0,0,0,12,14+23,13+42)$
4.1	$(0,0,0,0,12,15+34)$
4.2	$(0,0,0,0,12,15)$
4.3	$(0,0,0,0,12,14+25)$
4.4	$(0,0,0,0,12,14+23)$
4.5	$(0,0,0,0,12,34)$
4.6	$(0,0,0,0,12,13)$
4.7	$(0,0,0,0,13+42,14+23)$
5.1	$(0,0,0,0,0,12+34)$
5.2	$(0,0,0,0,0,12)$
6.1	$(0,0,0,0,0,0)$

vacua (but T-dual to T^6)

Only non T-dual vacuum
(0,0,0,12,23,14-35)

$$\begin{tikzcd}
S^1_{\{6\}} \arrow[r, hook] & M_6 \arrow[d] \\
T^2_{\{4,5\}} \arrow[r, hook] & M_5 \arrow[d] \\
 & T^3_{\{1,2,3\}}
\end{tikzcd}$$

Large volume limit (A ➔ 0)

$$\Omega_3 = (e^1 - ie^3)\wedge(e^2 + i\underset{\uparrow}{\tau} e^6)\wedge(e^4 + ie^5)$$

complex structure modulus

$$J = -\underset{\uparrow}{t_1}e^1e^3 + \underset{\uparrow}{t_2}\tau_r e^2e^6 + \underset{\uparrow}{t_3}e^4e^5$$

Kahler moduli

Nilmanifolds

n	Nilmanifold class
2.1	$(0,0,12,13,14,15)$
2.2	$(0,0,12,13,14,34+52)$
2.3	$(0,0,12,13,14,23+15)$
2.4	$(0,0,12,13,23,14)$
2.5	$(0,0,12,13,23,14-25)$
2.6	$(0,0,12,13,23,14+25)$
2.7	$(0,0,12,13,14+23,34+52)$
2.8	$(0,0,12,13,14+23,24+15)$
3.1	$(0,0,0,12,13,14+35)$
3.2	$(0,0,0,12,13,14+23)$
3.3	$(0,0,0,12,13,24)$
3.4	$(0,0,0,12,13,14)$
3.5	$(0,0,0,12,13,23)$
3.6	$(0,0,0,12,14,15+23)$
3.7	$(0,0,0,12,14,15+23+24)$
3.8	$(0,0,0,12,14,15+24)$
3.9	$(0,0,0,12,14,15)$
3.10	$(0,0,0,12,14,24)$
3.11	$(0,0,0,12,14,13+42)$
3.12	$(0,0,0,12,14,23+24)$
3.13	$(0,0,0,12,23,14+35)$
3.14	$(0,0,0,12,23,14-35)$
3.15	$(0,0,0,12,14-23,15+34)$
3.16	$(0,0,0,12,14+23,13+42)$
4.1	$(0,0,0,0,12,15+34)$
4.2	$(0,0,0,0,12,15)$
4.3	$(0,0,0,0,12,14+25)$
4.4	$(0,0,0,0,12,14+23)$
4.5	$(0,0,0,0,12,34)$
4.6	$(0,0,0,0,12,13)$
4.7	$(0,0,0,0,13+42,14+23)$
5.1	$(0,0,0,0,0,12+34)$
5.2	$(0,0,0,0,0,12)$
6.1	$(0,0,0,0,0,0)$

vacua (but T-dual to T^6)

Only non T-dual vacuum
(0,0,0,12,23,14-35)

$$\begin{tikzcd}
S^1_{\{6\}} \arrow[r, hook] & M_6 \arrow[d] \\
T^2_{\{4,5\}} \arrow[r, hook] & M_5 \arrow[d] \\
 & T^3_{\{1,2,3\}}
\end{tikzcd}$$

Large volume limit (A ➔ 0)

$$\Omega_3 = (e^1 - ie^3) \wedge (e^2 + i\tau e^6) \wedge (e^4 + ie^5)$$

complex structure modulus

$$J = -t_1 e^1 e^3 + t_2 \tau_r e^2 e^6 + t_3 e^4 e^5$$

Kahler moduli

$$F_3 = -(\tau_i e^2 - |\tau|^2 e^6) \wedge \left(t_2 (e^1 e^4 - e^3 e^5) + \frac{t_3}{\tau_r} (e^1 e^5 + e^3 e^4) \right)$$

Nilmanifolds

n	Nilmanifold class
2.1	$(0,0,12,13,14,15)$
2.2	$(0,0,12,13,14,34+52)$
2.3	$(0,0,12,13,14,23+15)$
2.4	$(0,0,12,13,23,14)$
2.5	$(0,0,12,13,23,14-25)$
2.6	$(0,0,12,13,23,14+25)$
2.7	$(0,0,12,13,14+23,34+52)$
2.8	$(0,0,12,13,14+23,24+15)$
3.1	$(0,0,0,12,13,14+35)$
3.2	$(0,0,0,12,13,14+23)$
3.3	$(0,0,0,12,13,24)$
3.4	$(0,0,0,12,13,14)$
3.5	$(0,0,0,12,13,23)$
3.6	$(0,0,0,12,14,15+23)$
3.7	$(0,0,0,12,14,15+23+24)$
3.8	$(0,0,0,12,14,15+24)$
3.9	$(0,0,0,12,14,15)$
3.10	$(0,0,0,12,14,24)$
3.11	$(0,0,0,12,14,13+42)$
3.12	$(0,0,0,12,14,23+24)$
3.13	$(0,0,0,12,23,14+35)$
3.14	$(0,0,0,12,23,14-35)$
3.15	$(0,0,0,12,14-23,15+34)$
3.16	$(0,0,0,12,14+23,13+42)$
4.1	$(0,0,0,0,12,15+34)$
4.2	$(0,0,0,0,12,15)$
4.3	$(0,0,0,0,12,14+25)$
4.4	$(0,0,0,0,12,14+23)$
4.5	$(0,0,0,0,12,34)$
4.6	$(0,0,0,0,12,13)$
4.7	$(0,0,0,0,13+42,14+23)$
5.1	$(0,0,0,0,0,12+34)$
5.2	$(0,0,0,0,0,12)$
6.1	$(0,0,0,0,0,0)$

vacua (but T-dual to T^6)

Only non T-dual vacuum
(0,0,0,12,23,14-35)

$$\begin{tikzcd}
S^1_{\{6\}} \arrow[r, hook] & M_6 \arrow[d] \\
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 & T^3_{\{1,2,3\}}
\end{tikzcd}$$

Large volume limit (A ➔ 0)

$$\Omega_3 = (e^1 - ie^3) \wedge (e^2 + i\underset{\uparrow}{\tau} e^6) \wedge (e^4 + ie^5)$$

complex structure modulus

$$J = -\underset{\uparrow}{t_1} e^1 e^3 + \underset{\uparrow}{t_2} \tau_r e^2 e^6 + \underset{\uparrow}{t_3} e^4 e^5$$

Kahler moduli

$$F_3 = -(\tau_i e^2 - |\tau|^2 e^6) \wedge \left(t_2(e^1 e^4 - e^3 e^5) + \frac{t_3}{\tau_r}(e^1 e^5 + e^3 e^4) \right)$$

Bianchi

Nilmanifolds

n	Nilmanifold class
2.1	(0, 0, 12, 13, 14, 15)
2.2	(0, 0, 12, 13, 14, 34 + 52)
2.3	(0, 0, 12, 13, 14, 23 + 15)
2.4	(0, 0, 12, 13, 23, 14)
2.5	(0, 0, 12, 13, 23, 14 − 25)
2.6	(0, 0, 12, 13, 23, 14 + 25)
2.7	(0, 0, 12, 13, 14 + 23, 34 + 52)
2.8	(0, 0, 12, 13, 14 + 23, 24 + 15)
3.1	(0, 0, 0, 12, 13, 14 + 35)
3.2	(0, 0, 0, 12, 13, 14 + 23)
3.3	(0, 0, 0, 12, 13, 24)
3.4	(0, 0, 0, 12, 13, 14)
3.5	(0, 0, 0, 12, 13, 23)
3.6	(0, 0, 0, 12, 14, 15 + 23)
3.7	(0, 0, 0, 12, 14, 15 + 23 + 24)
3.8	(0, 0, 0, 12, 14, 15 + 24)
3.9	(0, 0, 0, 12, 14, 15)
3.10	(0, 0, 0, 12, 14, 24)
3.11	(0, 0, 0, 12, 14, 13 + 42)
3.12	(0, 0, 0, 12, 14, 23 + 24)
3.13	(0, 0, 0, 12, 23, 14 + 35)
3.14	(0, 0, 0, 12, 23, 14 − 35)
3.15	(0, 0, 0, 12, 14 − 23, 15 + 34)
3.16	(0, 0, 0, 12, 14 + 23, 13 + 42)
4.1	(0, 0, 0, 0, 12, 15 + 34)
4.2	(0, 0, 0, 0, 12, 15)
4.3	(0, 0, 0, 0, 12, 14 + 25)
4.4	(0, 0, 0, 0, 12, 14 + 23)
4.5	(0, 0, 0, 0, 12, 34)
4.6	(0, 0, 0, 0, 12, 13)
4.7	(0, 0, 0, 0, 13 + 42, 14 + 23)
5.1	(0, 0, 0, 0, 0, 12 + 34)
5.2	(0, 0, 0, 0, 0, 12)
6.1	(0, 0, 0, 0, 0, 0)

vacua (but T-dual to T^6)

Only non T-dual vacuum
(0,0,0,12,23,14-35)

$$\begin{tikzcd}
S^1_{\{6\}} \arrow[r, hook] & M_6 \arrow[d] \\
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Large volume limit (A ➔ 0)

$$\Omega_3 = (e^1 - ie^3) \wedge (e^2 + i\tau e^6) \wedge (e^4 + ie^5)$$

complex structure modulus

$$J = -t_1 e^1 e^3 + t_2 \tau_r e^2 e^6 + t_3 e^4 e^5$$

Kahler moduli

$$F_3 = -(\tau_i e^2 - |\tau|^2 e^6) \wedge \left(t_2(e^1e^4 - e^3e^5) + \frac{t_3}{\tau_r}(e^1e^5 + e^3e^4) \right)$$

Bianchi $dF_3 = -2\frac{|\tau|^2}{g_s}\left(\frac{t_3}{\tau_r} e^1e^2e^3e^6 - t_2 e^1e^3e^4e^5 \right)$

Nilmanifolds

n	Nilmanifold class
2.1	$(0,0,12,13,14,15)$
2.2	$(0,0,12,13,14,34+52)$
2.3	$(0,0,12,13,14,23+15)$
2.4	$(0,0,12,13,23,14)$
2.5	$(0,0,12,13,23,14-25)$
2.6	$(0,0,12,13,23,14+25)$
2.7	$(0,0,12,13,14+23,34+52)$
2.8	$(0,0,12,13,14+23,24+15)$
3.1	$(0,0,0,12,13,14+35)$
3.2	$(0,0,0,12,13,14+23)$
3.3	$(0,0,0,12,13,24)$
3.4	$(0,0,0,12,13,14)$
3.5	$(0,0,0,12,13,23)$
3.6	$(0,0,0,12,14,15+23)$
3.7	$(0,0,0,12,14,15+23+24)$
3.8	$(0,0,0,12,14,15+24)$
3.9	$(0,0,0,12,14,15)$
3.10	$(0,0,0,12,14,24)$
3.11	$(0,0,0,12,14,13+42)$
3.12	$(0,0,0,12,14,23+24)$
3.13	$(0,0,0,12,23,14+35)$
3.14	$(0,0,0,12,23,14-35)$
3.15	$(0,0,0,12,14-23,15+34)$
3.16	$(0,0,0,12,14+23,13+42)$
4.1	$(0,0,0,0,12,15+34)$
4.2	$(0,0,0,0,12,15)$
4.3	$(0,0,0,0,12,14+25)$
4.4	$(0,0,0,0,12,14+23)$
4.5	$(0,0,0,0,12,34)$
4.6	$(0,0,0,0,12,13)$
4.7	$(0,0,0,0,13+42,14+23)$
5.1	$(0,0,0,0,0,12+34)$
5.2	$(0,0,0,0,0,12)$
6.1	$(0,0,0,0,0,0)$

vacua (but T-dual to T^6)

Only non T-dual vacuum
(0,0,0,12,23,14-35)

$$\begin{tikzcd}
S^1_{\{6\}} \arrow[r, hook] & M_6 \arrow[d] \\
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Large volume limit (A ➔ 0)

$$\Omega_3 = (e^1 - ie^3) \wedge (e^2 + i\tau e^6) \wedge (e^4 + ie^5)$$

complex structure modulus

$$J = -t_1 e^1 e^3 + t_2 \tau_r e^2 e^6 + t_3 e^4 e^5$$

Kahler moduli

$$F_3 = -(\tau_i e^2 - |\tau|^2 e^6) \wedge \left(t_2 (e^1 e^4 - e^3 e^5) + \frac{t_3}{\tau_r} (e^1 e^5 + e^3 e^4) \right)$$

Bianchi $dF_3 = -2 \frac{|\tau|^2}{g_s} \left(\frac{t_3}{\tau_r} e^1 e^2 e^3 e^6 - t_2 e^1 e^3 e^4 e^5 \right)$

O5 along 45 and 26

Nilmanifolds

n	Nilmanifold class
2.1	$(0,0,12,13,14,15)$
2.2	$(0,0,12,13,14,34+52)$
2.3	$(0,0,12,13,14,23+15)$
2.4	$(0,0,12,13,23,14)$
2.5	$(0,0,12,13,23,14-25)$
2.6	$(0,0,12,13,23,14+25)$
2.7	$(0,0,12,13,14+23,34+52)$
2.8	$(0,0,12,13,14+23,24+15)$
3.1	$(0,0,0,12,13,14+35)$
3.2	$(0,0,0,12,13,14+23)$
3.3	$(0,0,0,12,13,24)$
3.4	$(0,0,0,12,13,14)$
3.5	$(0,0,0,12,13,23)$
3.6	$(0,0,0,12,14,15+23)$
3.7	$(0,0,0,12,14,15+23+24)$
3.8	$(0,0,0,12,14,15+24)$
3.9	$(0,0,0,12,14,15)$
3.10	$(0,0,0,12,14,24)$
3.11	$(0,0,0,12,14,13+42)$
3.12	$(0,0,0,12,14,23+24)$
3.13	$(0,0,0,12,23,14+35)$
3.14	$(0,0,0,12,23,14-35)$
3.15	$(0,0,0,12,14-23,15+34)$
3.16	$(0,0,0,12,14+23,13+42)$
4.1	$(0,0,0,0,12,15+34)$
4.2	$(0,0,0,0,12,15)$
4.3	$(0,0,0,0,12,14+25)$
4.4	$(0,0,0,0,12,14+23)$
4.5	$(0,0,0,0,12,34)$
4.6	$(0,0,0,0,12,13)$
4.7	$(0,0,0,0,13+42,14+23)$
5.1	$(0,0,0,0,0,12+34)$
5.2	$(0,0,0,0,0,12)$
6.1	$(0,0,0,0,0,0)$

vacua (but T-dual to T^6)

Only non T-dual vacuum
(0,0,0,12,23,14-35)

$$\begin{tikzcd}
S^1_{\{6\}} \arrow[r, hook] & M_6 \arrow[d] \\
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\end{tikzcd}$$

Large volume limit (A ➔ 0)

$$\Omega_3 = (e^1 - ie^3) \wedge (e^2 + i\tau e^6) \wedge (e^4 + ie^5)$$

complex structure modulus

$$J = -t_1 e^1 e^3 + t_2 \tau_r e^2 e^6 + t_3 e^4 e^5$$

Kahler moduli

$$F_3 = -(\tau_i e^2 - |\tau|^2 e^6) \wedge \left(t_2(e^1e^4 - e^3e^5) + \frac{t_3}{\tau_r}(e^1e^5 + e^3e^4) \right)$$

Bianchi $dF_3 = -2\frac{|\tau|^2}{g_s}\left(\frac{t_3}{\tau_r} e^1e^2e^3e^6 - t_2 e^1e^3e^4e^5\right)$

O5 along 45 and 26

Generated by {$\Omega_{WS}\,\sigma^1$, $\sigma^1\sigma^2$}

Nilmanifolds

n	Nilmanifold class
2.1	$(0,0,12,13,14,15)$
2.2	$(0,0,12,13,14,34+52)$
2.3	$(0,0,12,13,14,23+15)$
2.4	$(0,0,12,13,23,14)$
2.5	$(0,0,12,13,23,14-25)$
2.6	$(0,0,12,13,23,14+25)$
2.7	$(0,0,12,13,14+23,34+52)$
2.8	$(0,0,12,13,14+23,24+15)$
3.1	$(0,0,0,12,13,14+35)$
3.2	$(0,0,0,12,13,14+23)$
3.3	$(0,0,0,12,13,24)$
3.4	$(0,0,0,12,13,14)$
3.5	$(0,0,0,12,13,23)$
3.6	$(0,0,0,12,14,15+23)$
3.7	$(0,0,0,12,14,15+23+24)$
3.8	$(0,0,0,12,14,15+24)$
3.9	$(0,0,0,12,14,15)$
3.10	$(0,0,0,12,14,24)$
3.11	$(0,0,0,12,14,13+42)$
3.12	$(0,0,0,12,14,23+24)$
3.13	$(0,0,0,12,23,14+35)$
3.14	$(0,0,0,12,23,14-35)$
3.15	$(0,0,0,12,14-23,15+34)$
3.16	$(0,0,0,12,14+23,13+42)$
4.1	$(0,0,0,0,12,15+34)$
4.2	$(0,0,0,0,12,15)$
4.3	$(0,0,0,0,12,14+25)$
4.4	$(0,0,0,0,12,14+23)$
4.5	$(0,0,0,0,12,34)$
4.6	$(0,0,0,0,12,13)$
4.7	$(0,0,0,0,13+42,14+23)$
5.1	$(0,0,0,0,0,12+34)$
5.2	$(0,0,0,0,0,12)$
6.1	$(0,0,0,0,0,0)$

vacua (but T-dual to T^6)

Only non T-dual vacuum
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$$\begin{tikzcd}
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Large volume limit (A ➔ 0)

$$\Omega_3 = (e^1 - ie^3) \wedge (e^2 + i\underset{\uparrow}{\tau} e^6) \wedge (e^4 + ie^5)$$

complex structure modulus

$$J = -t_1 e^1 e^3 + t_2 \tau_r e^2 e^6 + t_3 e^4 e^5$$

Kahler moduli

$$F_3 = -(\tau_i e^2 - |\tau|^2 e^6) \wedge \left(t_2(e^1e^4 - e^3e^5) + \frac{t_3}{\tau_r}(e^1e^5 + e^3e^4) \right)$$

Bianchi $dF_3 = -2\frac{|\tau|^2}{g_s} \left(\frac{t_3}{\tau_r} e^1e^2e^3e^6 - t_2 e^1e^3e^4e^5 \right)$

O5 along 45 and 26

Generated by {$\Omega_{WS}\sigma^1$, $\sigma^1\sigma^2$}

refl in 1236

Nilmanifolds

n	Nilmanifold class
2.1	$(0,0,12,13,14,15)$
2.2	$(0,0,12,13,14,34+52)$
2.3	$(0,0,12,13,14,23+15)$
2.4	$(0,0,12,13,23,14)$
2.5	$(0,0,12,13,23,14-25)$
2.6	$(0,0,12,13,23,14+25)$
2.7	$(0,0,12,13,14+23,34+52)$
2.8	$(0,0,12,13,14+23,24+15)$
3.1	$(0,0,0,12,13,14+35)$
3.2	$(0,0,0,12,13,14+23)$
3.3	$(0,0,0,12,13,24)$
3.4	$(0,0,0,12,13,14)$
3.5	$(0,0,0,12,13,23)$
3.6	$(0,0,0,12,14,15+23)$
3.7	$(0,0,0,12,14,15+23+24)$
3.8	$(0,0,0,12,14,15+24)$
3.9	$(0,0,0,12,14,15)$
3.10	$(0,0,0,12,14,24)$
3.11	$(0,0,0,12,14,13+42)$
3.12	$(0,0,0,12,14,23+24)$
3.13	$(0,0,0,12,23,14+35)$
3.14	$(0,0,0,12,23,14-35)$
3.15	$(0,0,0,12,14-23,15+34)$
3.16	$(0,0,0,12,14+23,13+42)$
4.1	$(0,0,0,0,12,15+34)$
4.2	$(0,0,0,0,12,15)$
4.3	$(0,0,0,0,12,14+25)$
4.4	$(0,0,0,0,12,14+23)$
4.5	$(0,0,0,0,12,34)$
4.6	$(0,0,0,0,12,13)$
4.7	$(0,0,0,0,13+42,14+23)$
5.1	$(0,0,0,0,0,12+34)$
5.2	$(0,0,0,0,0,12)$
6.1	$(0,0,0,0,0,0)$

vacua (but T-dual to T^6)

Only non T-dual vacuum
(0,0,0,12,23,14-35)

$$\begin{tikzcd} S^1_{\{6\}} \arrow[r, hook] & M_6 \arrow[d] \\ T^2_{\{4,5\}} \arrow[r, hook] & M_5 \arrow[d] \\ & T^3_{\{1,2,3\}} \end{tikzcd}$$

Large volume limit (A ➜ 0)

$$\Omega_3 = (e^1 - ie^3) \wedge (e^2 + i\underset{\uparrow}{\tau} e^6) \wedge (e^4 + ie^5)$$

complex structure modulus

$$J = -\underset{\uparrow}{t_1} e^1 e^3 + \underset{\uparrow}{t_2} \tau_r e^2 e^6 + \underset{\uparrow}{t_3} e^4 e^5$$

Kahler moduli

$$F_3 = -(\tau_i e^2 - |\tau|^2 e^6) \wedge \left(t_2 (e^1 e^4 - e^3 e^5) + \frac{t_3}{\tau_r} (e^1 e^5 + e^3 e^4) \right)$$

Bianchi $dF_3 = -2\dfrac{|\tau|^2}{g_s} \left(\dfrac{t_3}{\tau_r} e^1 e^2 e^3 e^6 - t_2 e^1 e^3 e^4 e^5 \right)$

O5 along 45 and 26

Generated by {$\underset{\uparrow}{\Omega_{WS}\sigma^1}$, $\underset{\uparrow}{\sigma^1\sigma^2}$}

refl in 1236 refl in 1345

Bianchi $dF_3 = -2\frac{|\tau|^2}{g_s}\left(\frac{t_3}{\tau_r}e^1e^2e^3e^6 - t_2e^1e^3e^4e^5\right)$

O5 along 45 and 26

Bianchi $dF_3 = -2\frac{|\tau|^2}{g_s}\left(\frac{t_3}{\tau_r}e^1e^2e^3e^6 - t_2e^1e^3e^4e^5\right)$

O5 along 45 and 26 ➞ Don’t know the “local” solution

Bianchi $dF_3 = -2\frac{|\tau|^2}{g_s}\left(\frac{t_3}{\tau_r}e^1e^2e^3e^6 - t_2e^1e^3e^4e^5\right)$

O5 along 45 and 26 ➞ Don’t know the “local” solution

in solutions T-dual to type B

Bianchi $dF_3 = -2\frac{|\tau|^2}{g_s}\left(\frac{t_3}{\tau_r}e^1e^2e^3e^6 - t_2e^1e^3e^4e^5\right)$

O5 along 45 and 26 → Don't know the "local" solution

in solutions T-dual to type B $dF_3 = ke^1e^2e^3e^4$

Bianchi $dF_3 = -2\frac{|\tau|^2}{g_s}\left(\frac{t_3}{\tau_r}e^1e^2e^3e^6 - t_2e^1e^3e^4e^5\right)$

O5 along 45 and 26 ➔ Don’t know the “local” solution

in solutions T-dual to type B $dF_3 = ke^1e^2e^3e^4$

inserting warp factor

Bianchi $dF_3 = -2\frac{|\tau|^2}{g_s}\left(\frac{t_3}{\tau_r}e^1e^2e^3e^6 - t_2e^1e^3e^4e^5\right)$

O5 along 45 and 26 ➜ Don’t know the “local” solution

in solutions T-dual to type B $dF_3 = ke^1e^2e^3e^4$

inserting warp factor $dF_3 = \nabla^2_\perp(e^{-4A})vol_\perp + ke^1e^2e^3e^4 = Q\delta(x_\perp - x^i_\perp)vol_\perp$

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Can have $g_s << 1$, V >> 1 and all cycles large

Solvmanifolds: 182 classes

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all “algebraic” compact solvmanifolds:

Solvmanifold class
$(23, -36, 26, -\alpha 56, \alpha 46, 0)$
$(24 + 35, -36, 26, -56, 46, 0)$
$(23, 0, 26, -56, 46, 0)$
$(35 - 26, 45 + 16, -46, 36, 0, 0)$
$(24 + 35, 0, -56, 0, 36, 0)$
(23,-13,0,56,-46,0)
$(25, -15, \alpha 45, -\alpha 35, 0, 0)$
(25+35,-15+45,45,-35,0,0)
(23,-13,0,56,0,0)
(23,34,-24,0,0,0)
(25,0,45,-35,0,0)
$(23 + 45, -35, 25, 0, 0, 0)$
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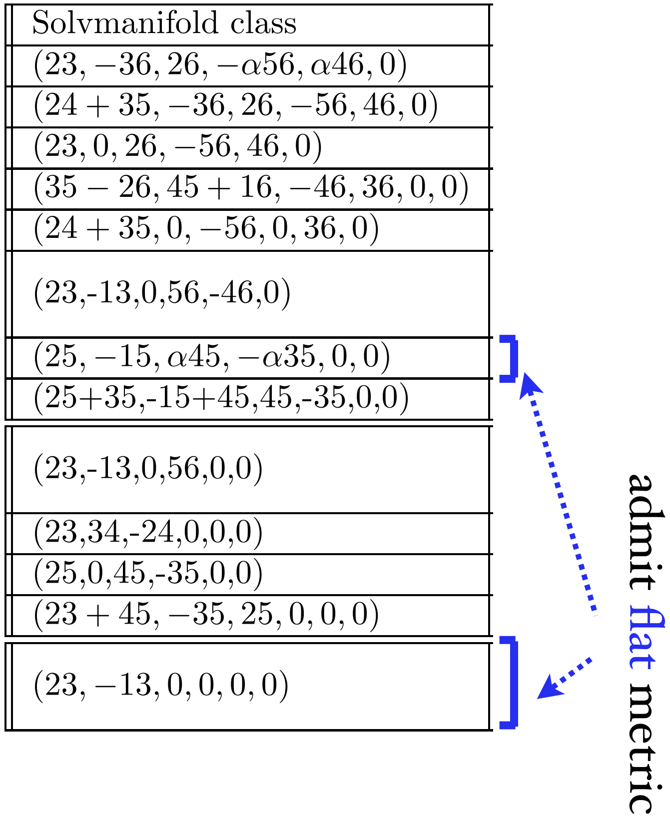

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T_6

- O5-D5 (along 13,24)		$SU(2)$
- O6-D6 (136,246)		$SU(3)$
- O6-D6 (136,246), D4 (along 5)	$H \neq 0$	$SU(2)$

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• All non T-dual solutions need intersecting sources!

Conclusions

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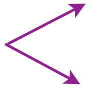

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- “Valleys of landscape” consisting of susy Minkowski geometric vacua with moduli stabilized at tree level by bulk fluxes seem very rare