A simple, yet subtle, invariance of the two-body decay kinematics

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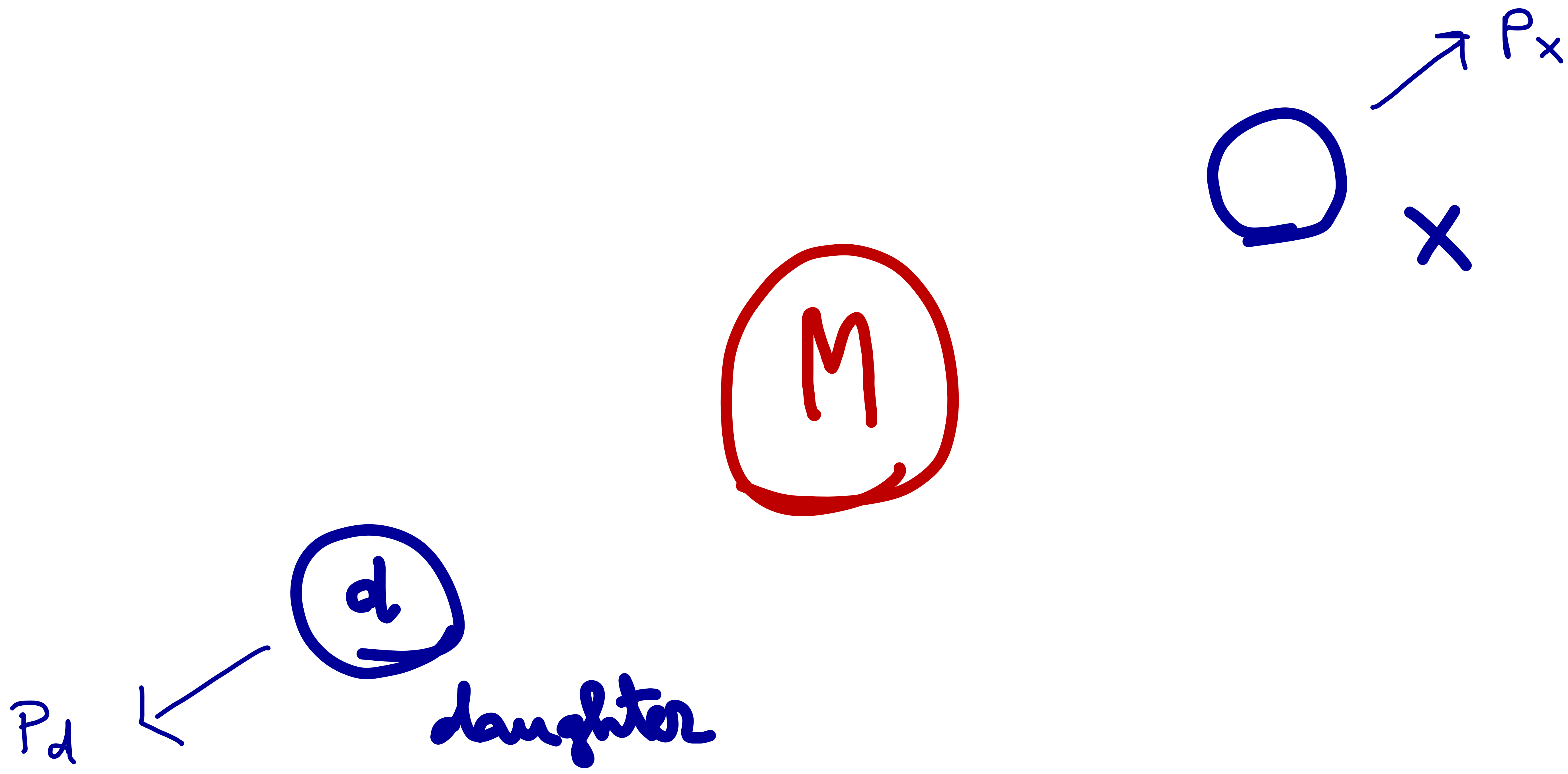
arXiv:1209.0772
with K.Agashe and D.Kim
arXiv:1212.5230
with K.Agashe, D.Kim, K. Wardlow

TWO-BODY DECAY : $M \to d\,X$

$$P_M^\mu = P_d^\mu + P_X^\mu$$

KINEMATICS FULLY FIXED BY THE MASSES



$$E_d =: E_d^* = \frac{m_M^2 + m_d^2 - m_X^2}{2 m_M} \qquad \bar{p}_d + \bar{p}_x = 0$$

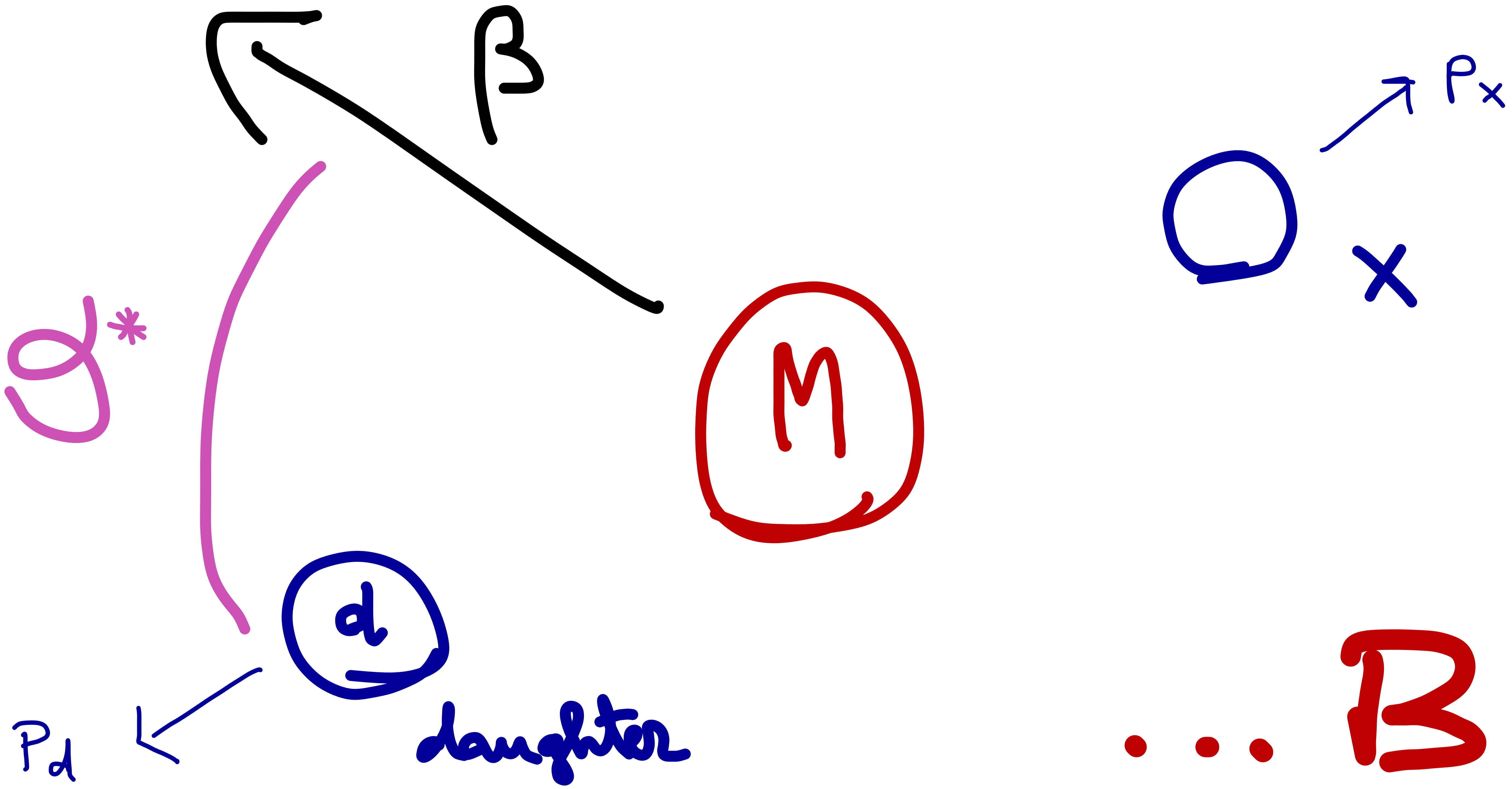
IN THE REST FRAME OF M

WHAT DOES IT LOOK LIKE IN ANOTHER FRAME?

IN GENERAL WE KNOW THE ANSWER

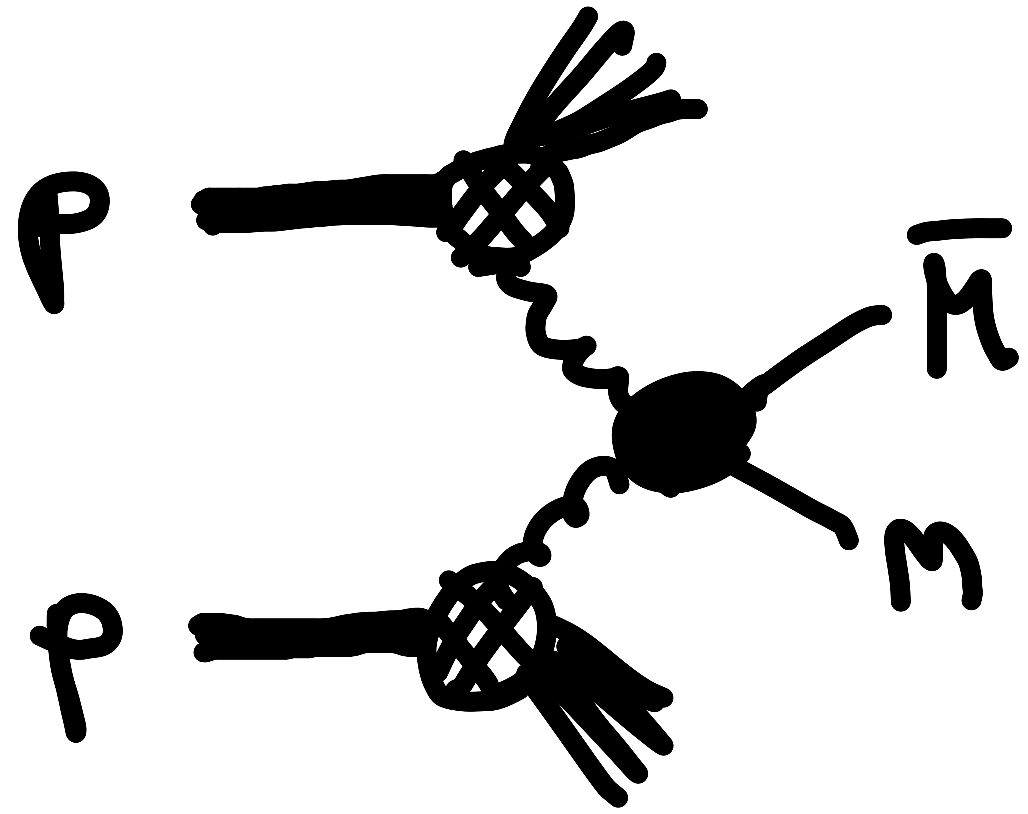
IF THE FRAME OF THE OBSERVER AND THAT OF REST OF THE MOTHER ARE CONNECTED BY A BOOST β

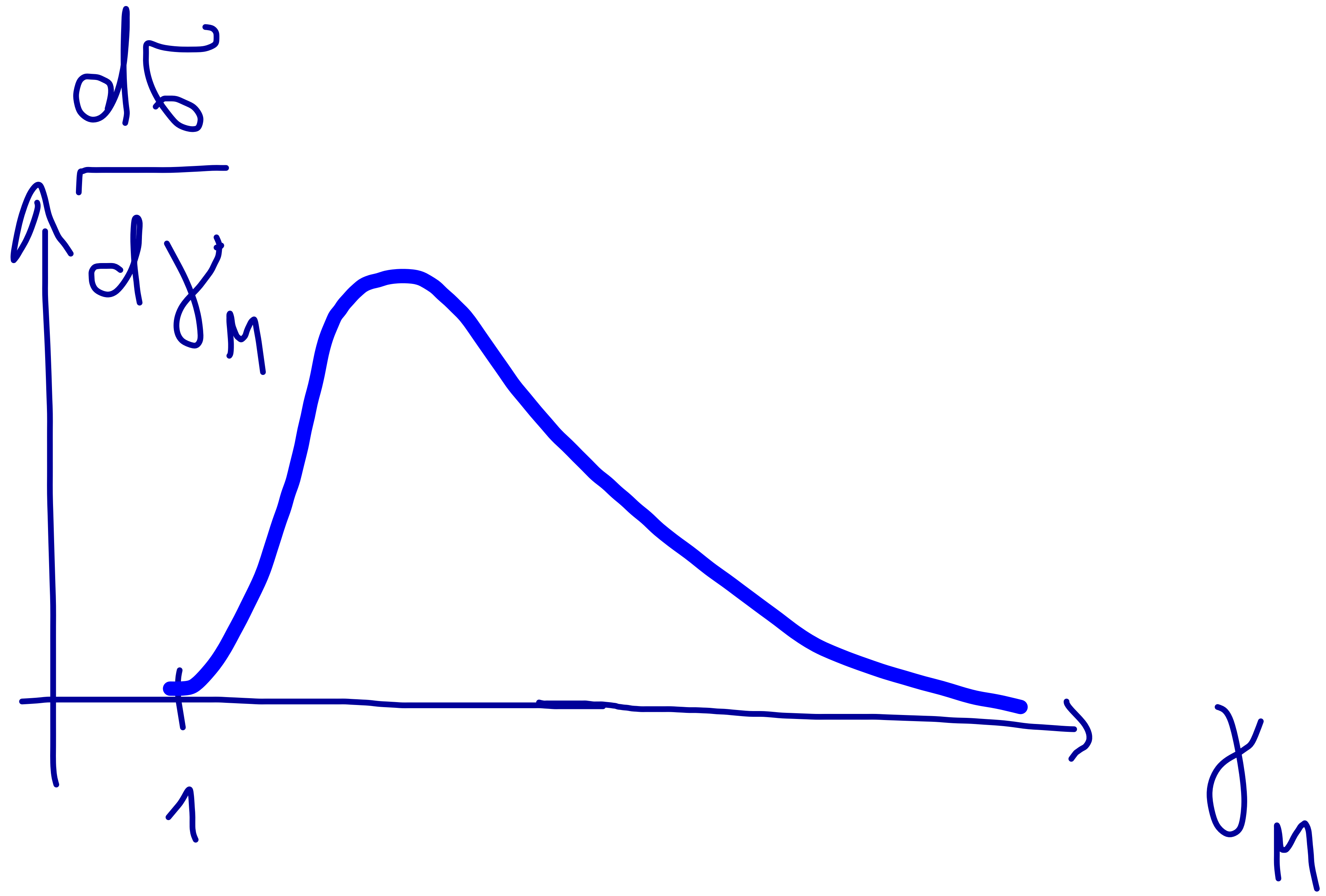
$$E'_d = E^*_d \gamma + p^*_d \gamma\beta \cos\vartheta^*$$

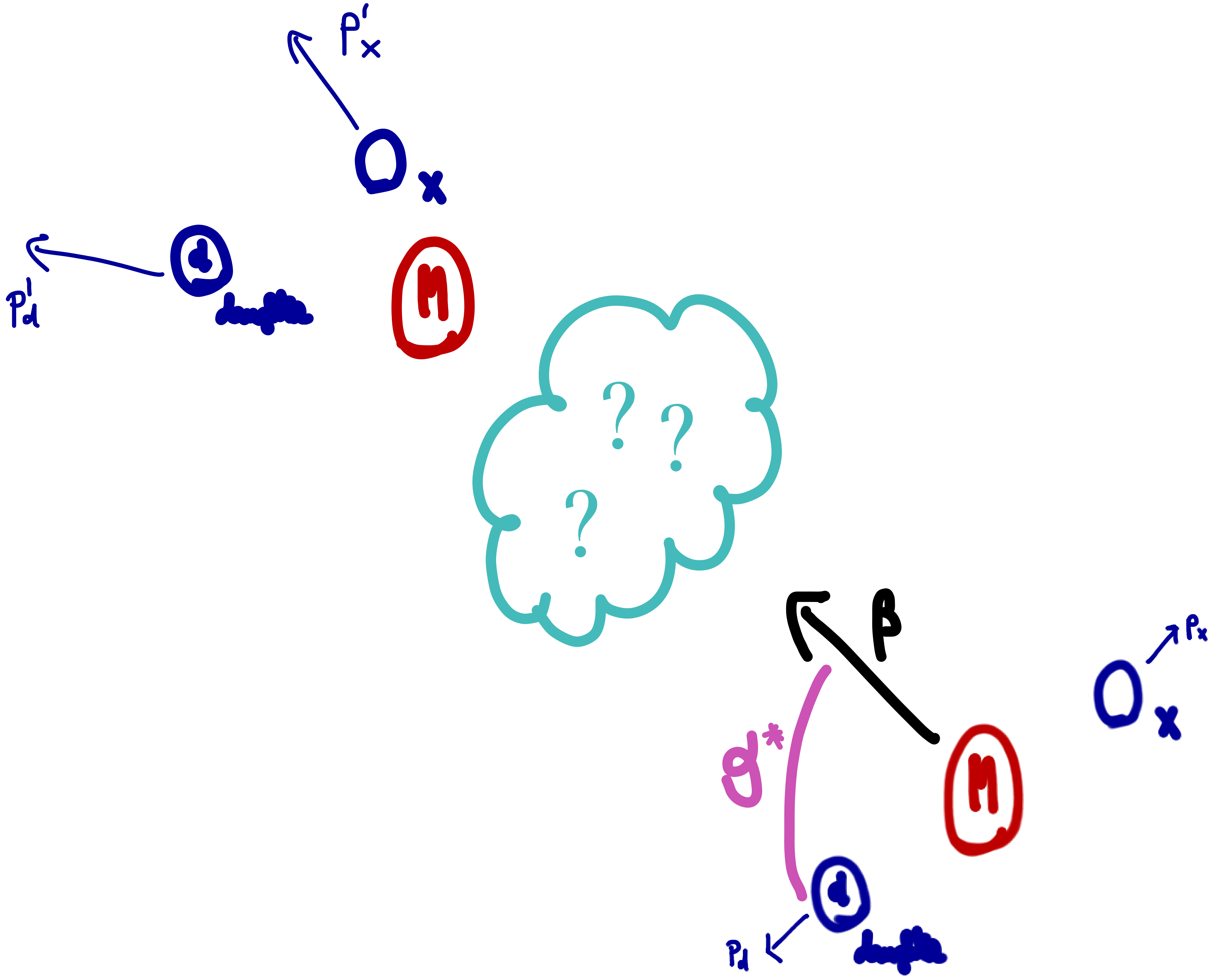


...BUT

BUT IN MOST CASES WE DO NOT KNOW THE BOOST OF THE MOTHER







SOLUTION TO OVERCOME THE UNKNOWN BOOST

USE BOOST INVARIANT QUANTITIES

- CONSERVED EVENT BY EVENT
- SIMPLE TO UNDERSTAND
- UNIVERSAL (SPECIAL RELATIVITY IS THE SAME FOR ALL PARTICLES)

- DEMANDING
    - GENERICALLY THEY ARE FUNCTION OF SEVERAL QUANTITIES
    TO MAKE AN INVARIANT MASS YOU NEED <u>TWO FOUR-VECTORS</u>
    WITH BOTH <u>ENERGY</u> <u>AND</u> <u>ANGLES</u>

IN THIS TALK :

LORENTZ VARIANT QUANTITIES
WITH SOME KIND OF "PHENOMENOLOGICAL INVARIANCE"
TO ACCESS INVARIANTS OF THE DECAY

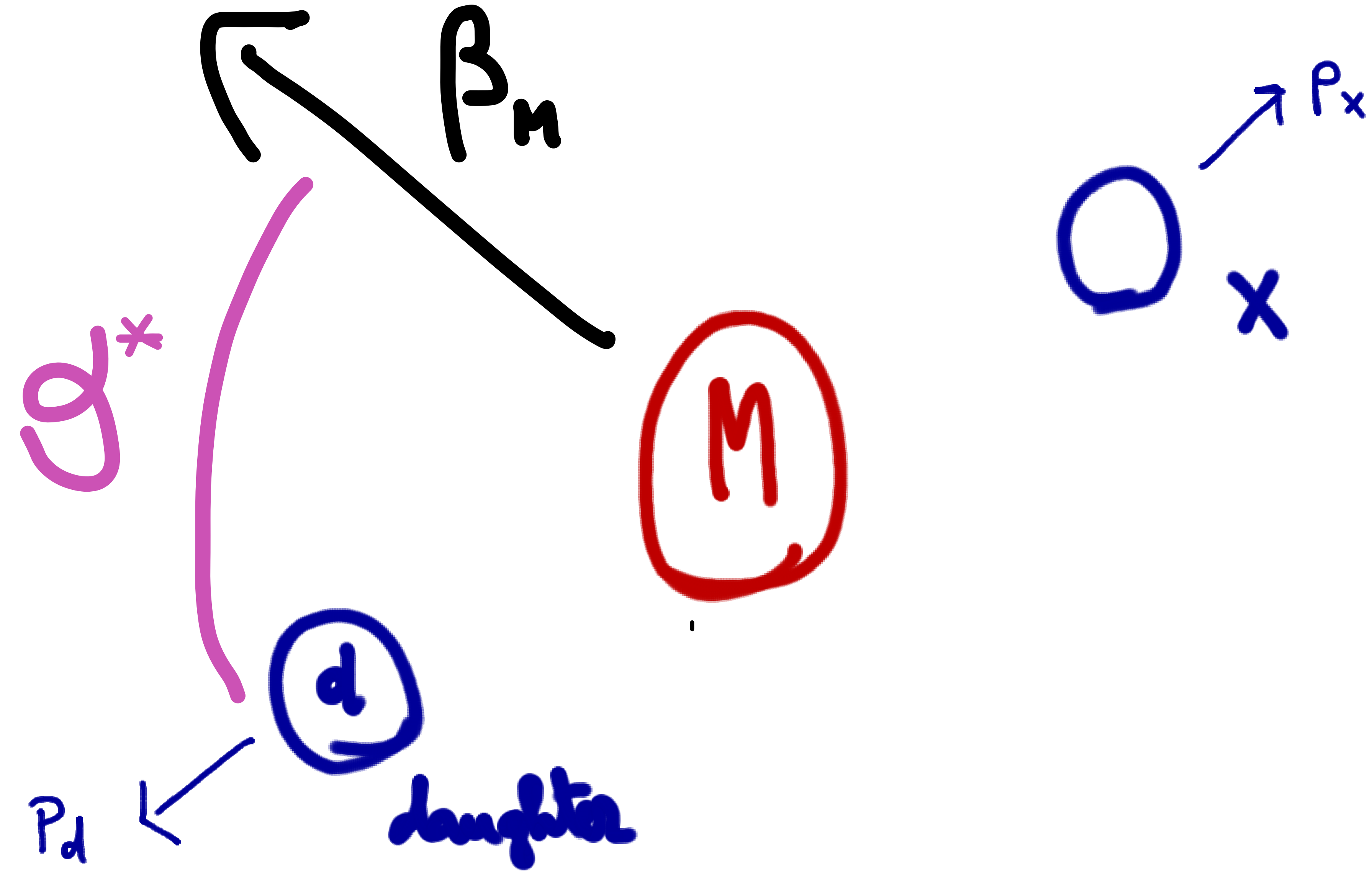
FOR INSTANCE:

THE OBSERVED ENERGY DEPENDS ON THE FRAME

THE <u>ENERGY</u> <u>DISTRIBUTION</u> IN PHENOMENOLOGICALLY RELEVANT
SITUATIONS HAS SOME INVARIANCE

- DAUGHTER d IS MASSLESS (for now)

- IMAGINE THE MOTHER HAS A BOOST β_M IN THE LAB FRAME



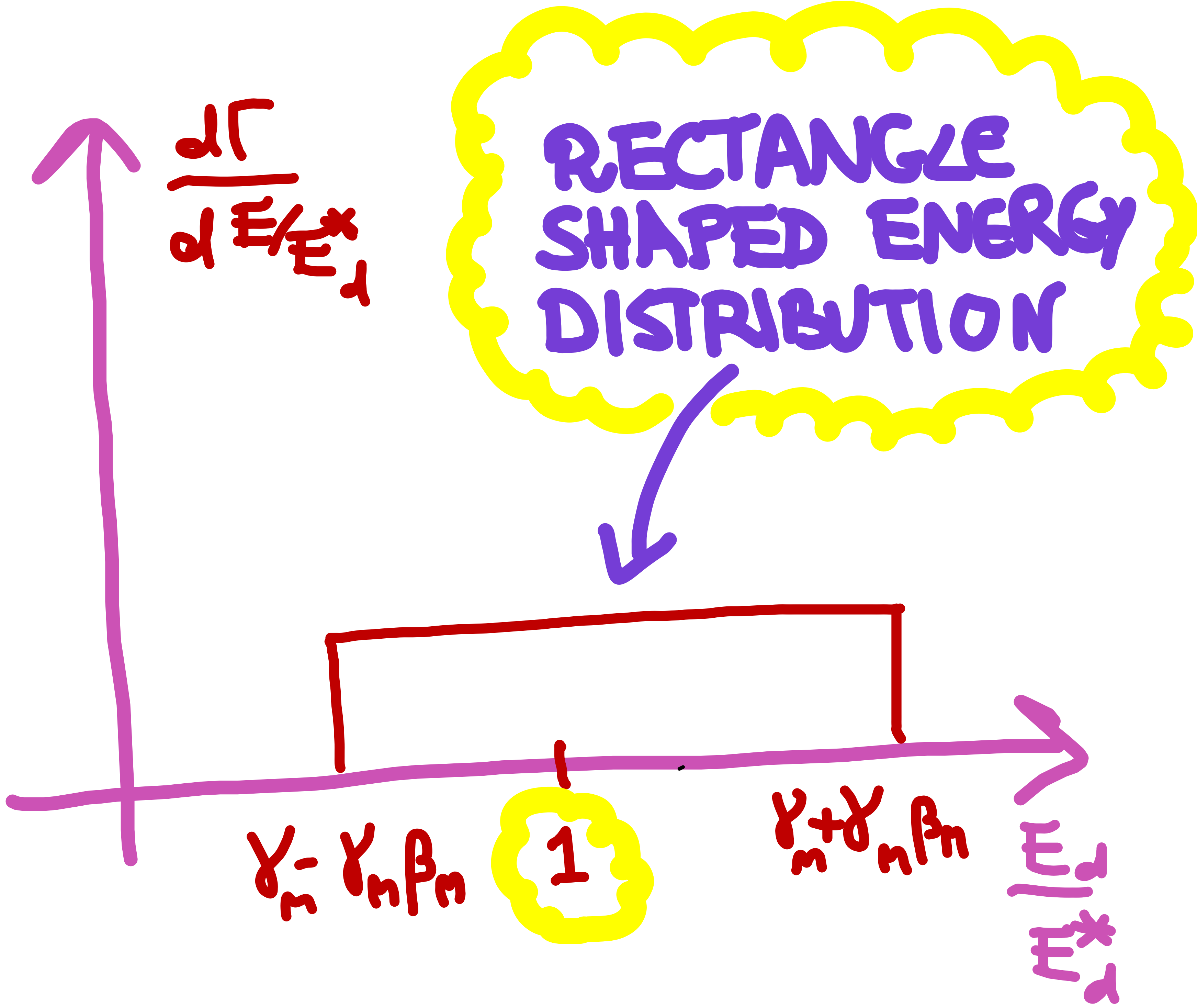
- THE DAUGHTER MOMENTUM IS AT AN ANGLE ϑ W.R.T. β_M

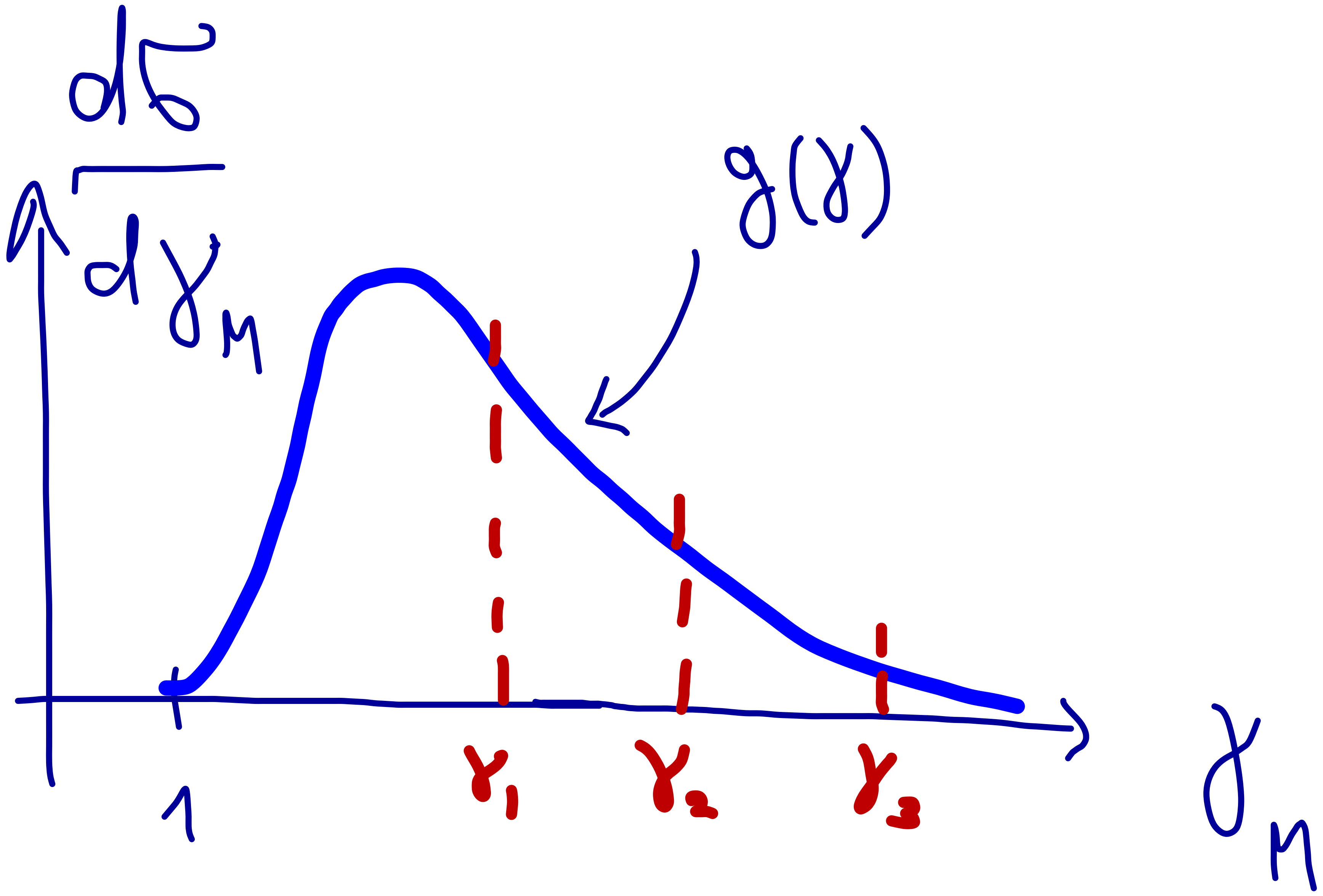
IN THE LAB

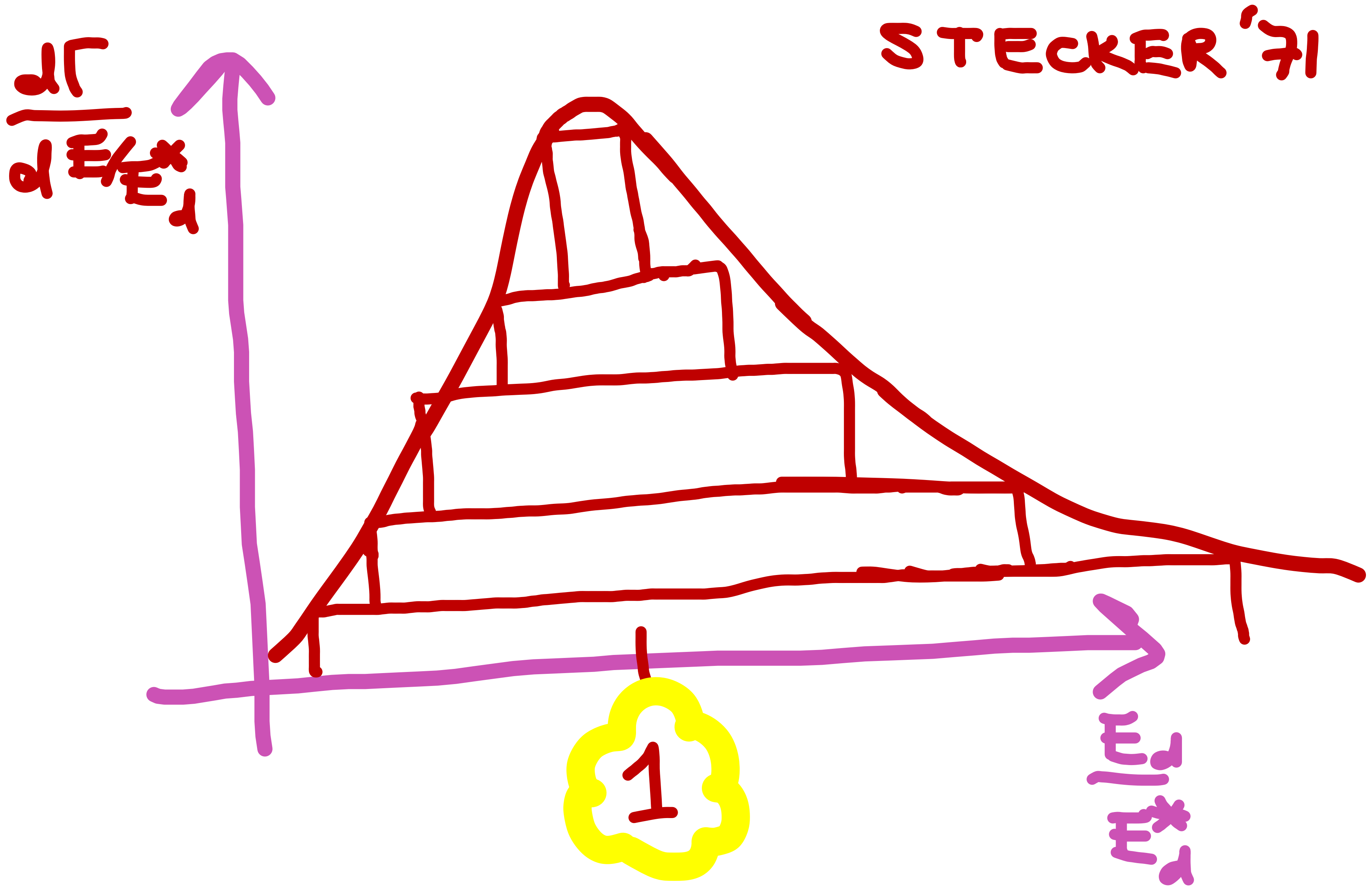
$$E_d = E_d^* \left(\gamma_M + \cos\vartheta^* \beta_M \gamma_M \right)$$

IF THE MOTHER IS A SCALAR

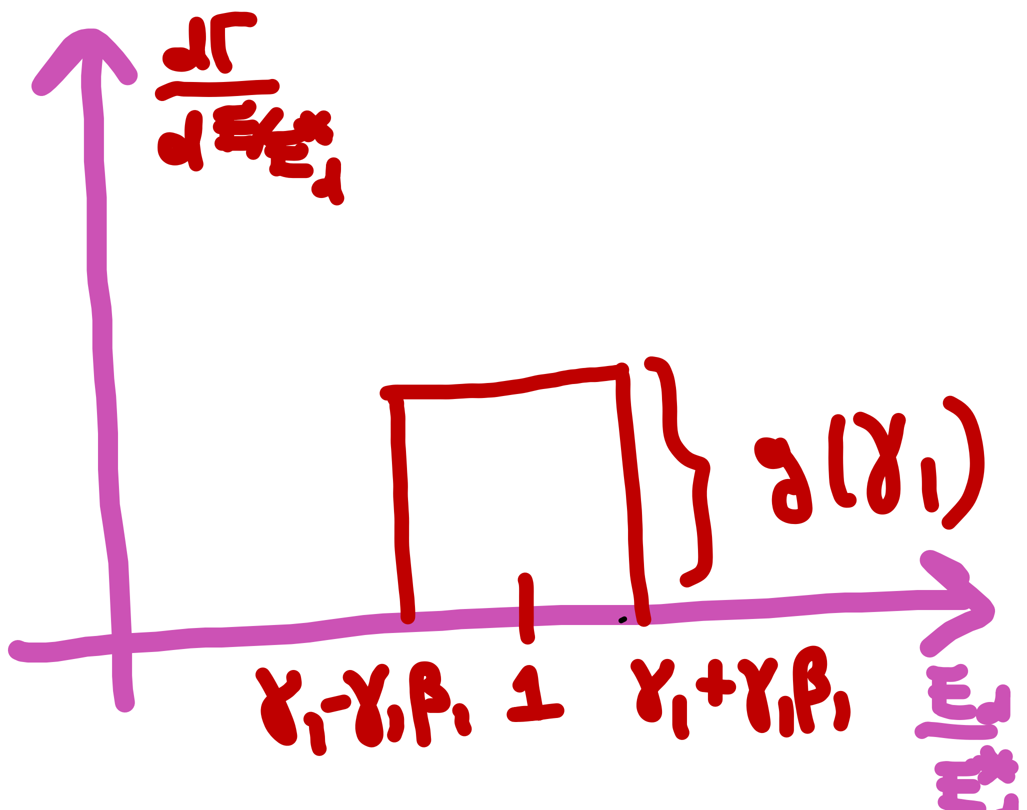
$\cos\vartheta$ IS FLAT FROM -1 TO 1

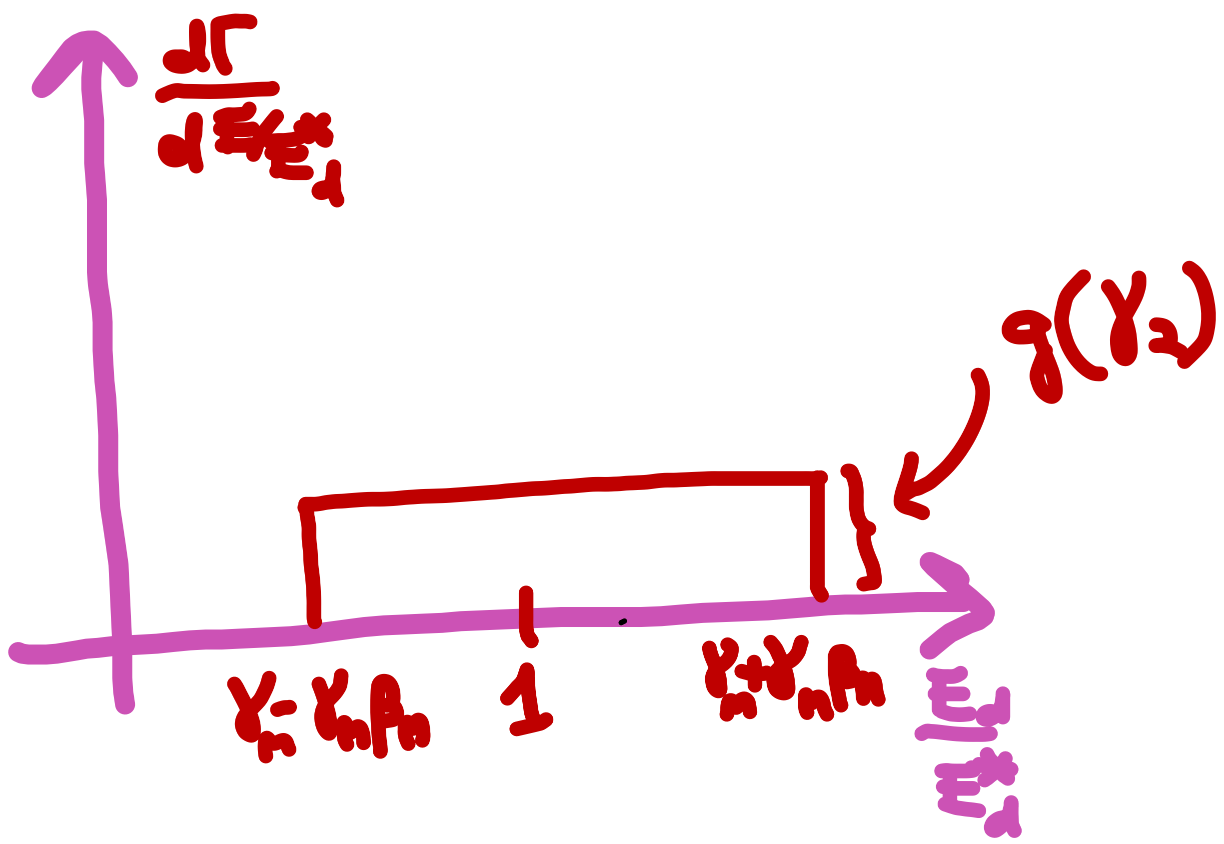


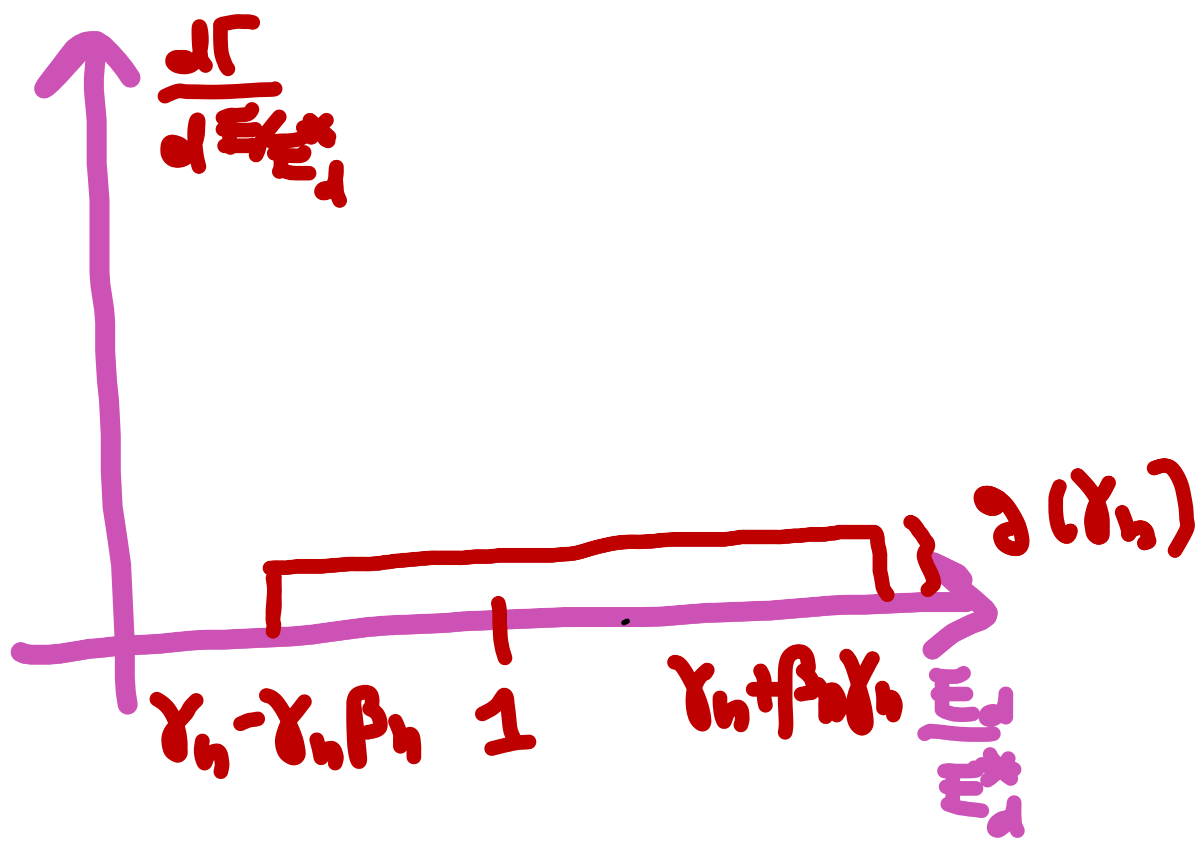




THE ENERGY DISTRIBUTION IN THE LAB IS THE SUM OF ALL THE RECTANGLES





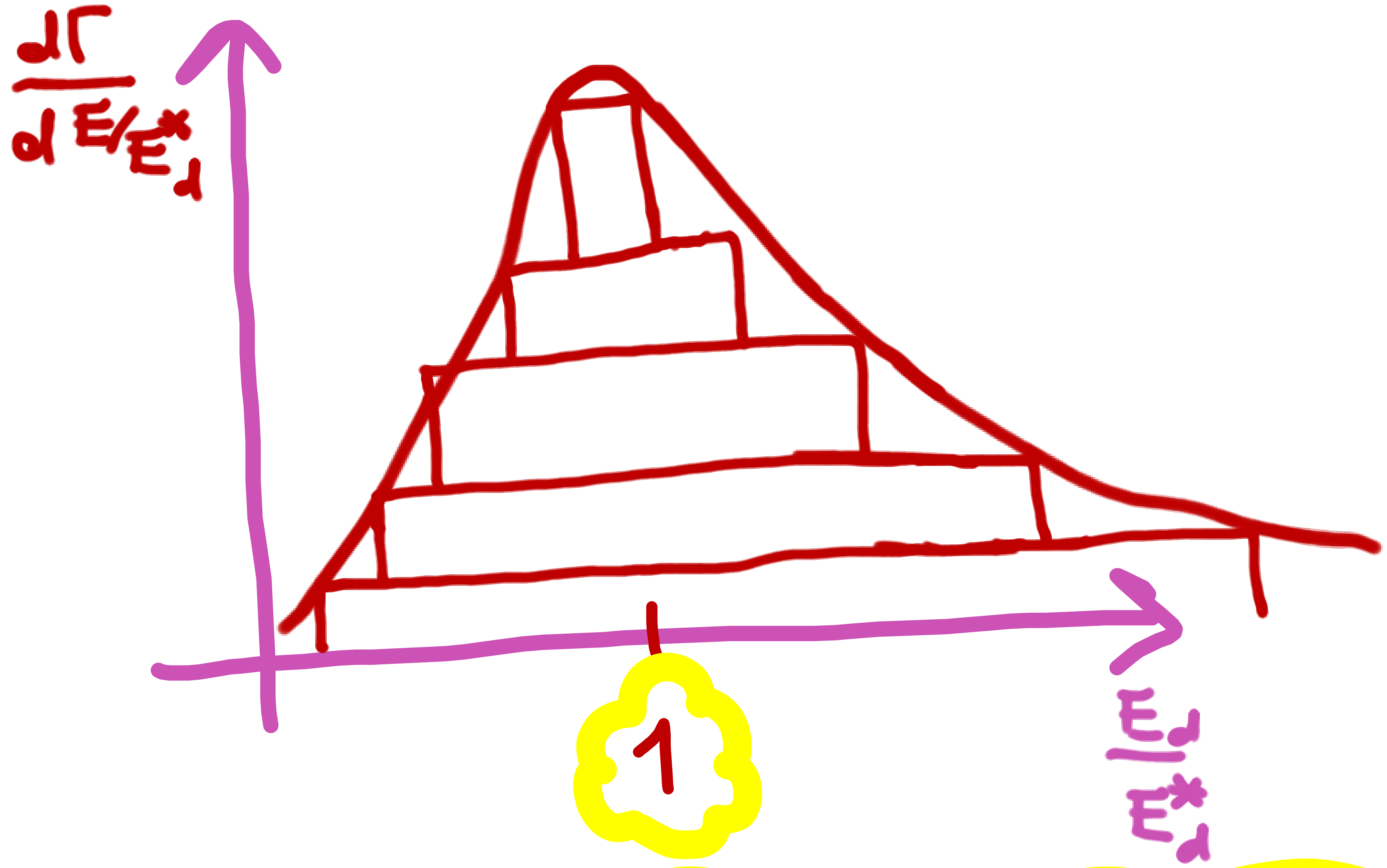


GENERALIZATIONS:

- INSTEAD OF A SCALAR MOTHER ONE CAN TAKE AN UNPOLARIZED ENSEMBLE OF PARTICLES WITH SPIN

- THE DAUGHTER CAN BE MASSIVE IF $g(\gamma) = 0$ for $\gamma \geq 2\gamma^* - 1$ WHERE $\gamma^* = \frac{E_d^*}{m_d}$

$$E_d^* = \frac{m_M^2 + m_d^2 - m_X^2}{2m_M}$$



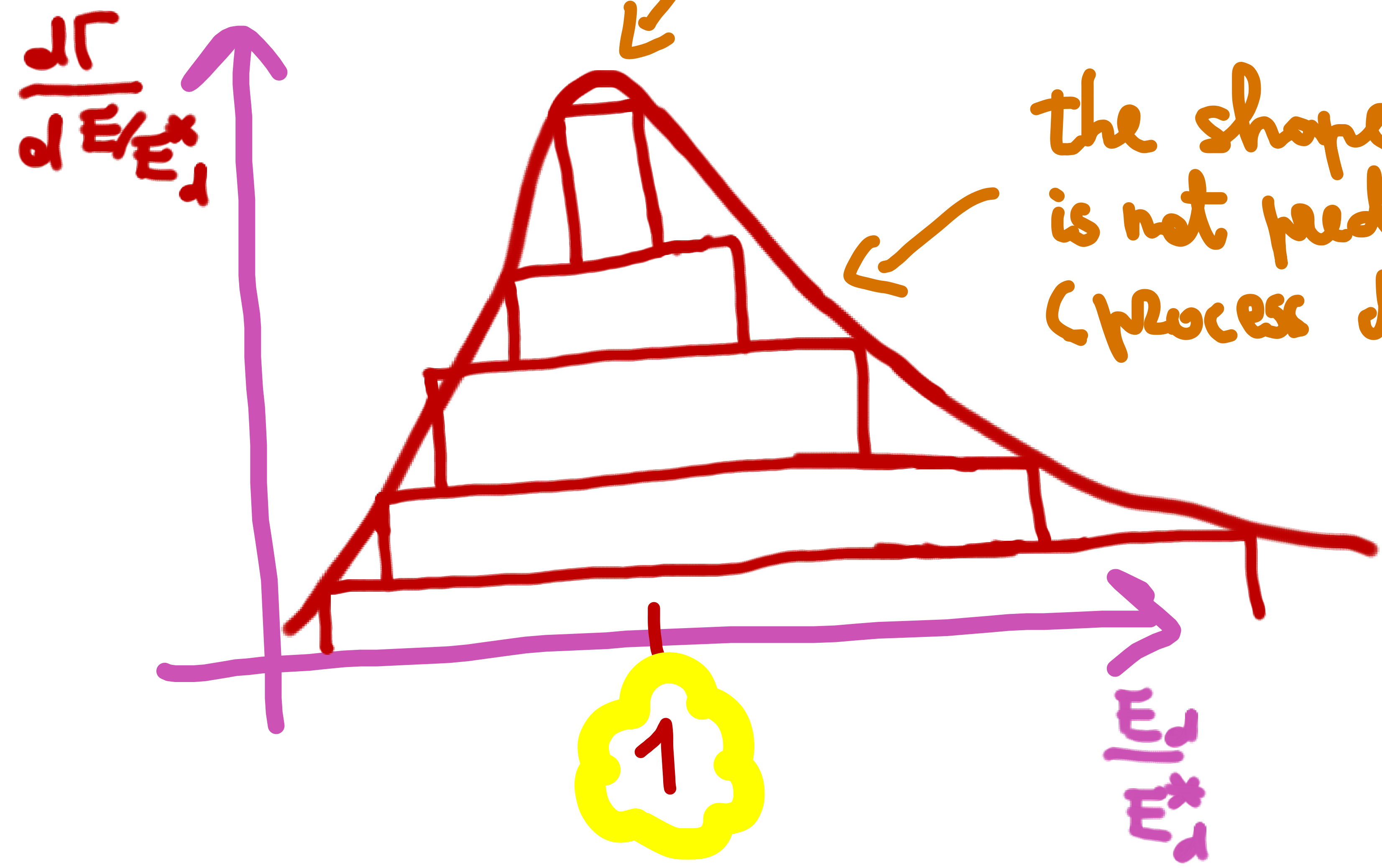
$E = E_d^*$ IS THE PEAK

THE FRAME-DEPENDENT ENERGY DISTRIBUTION ENCODES THE INVARIANT E_d^* IN A VERY SIMPLE WAY

ADVANTAGES (GENERAL: ALMOST ONLY KINEMATICS)

SAME PEAK AS IN THE REST FRAME

the shape is not predicted (process dependent)

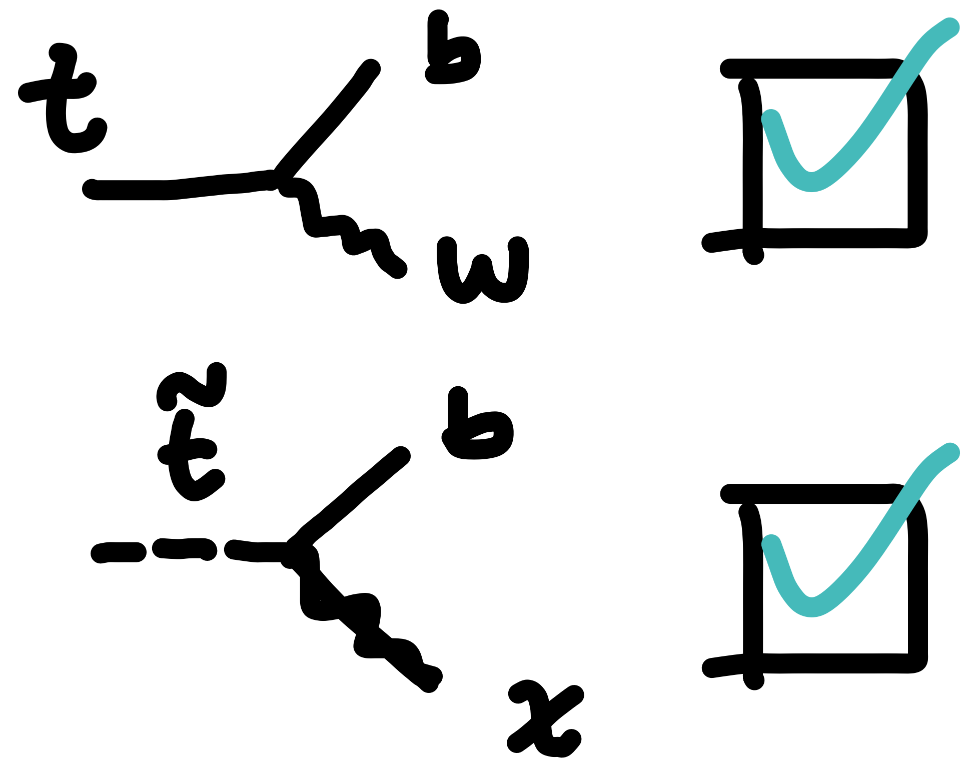


THE FRAME-DEPENDENT ENERGY DISTRIBUTION ENCODES THE INVARIANT E_d^* IN A VERY SIMPLE WAY

THE ONLY DYNAMICAL ASSUMPTION WAS THE MOTHER TO BE NOT POLARIZED

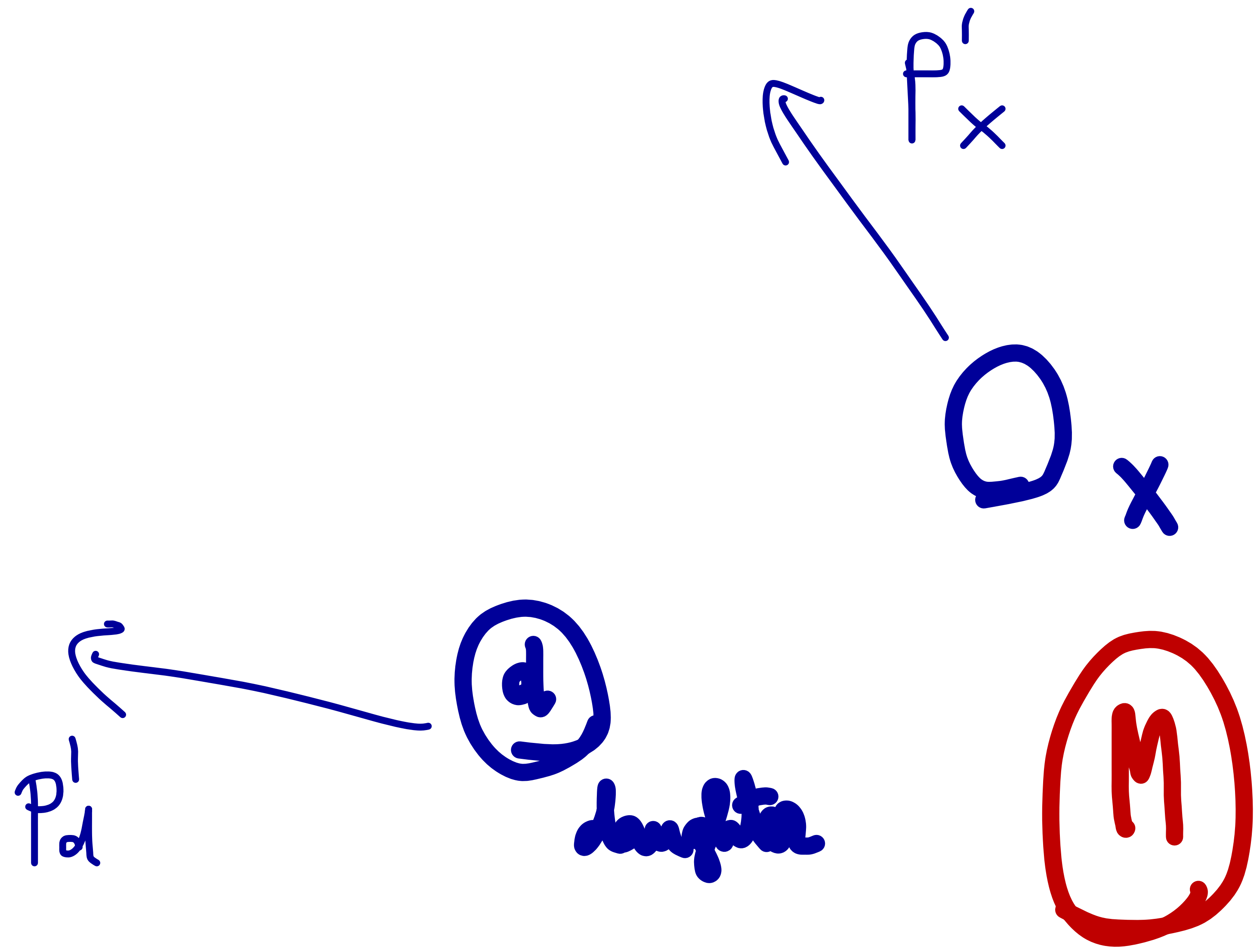
⇓

THE RESULT APPLIES FOR BOTH KNOW PARTICLES OF THE SM AND FOR NEW PHYSICS

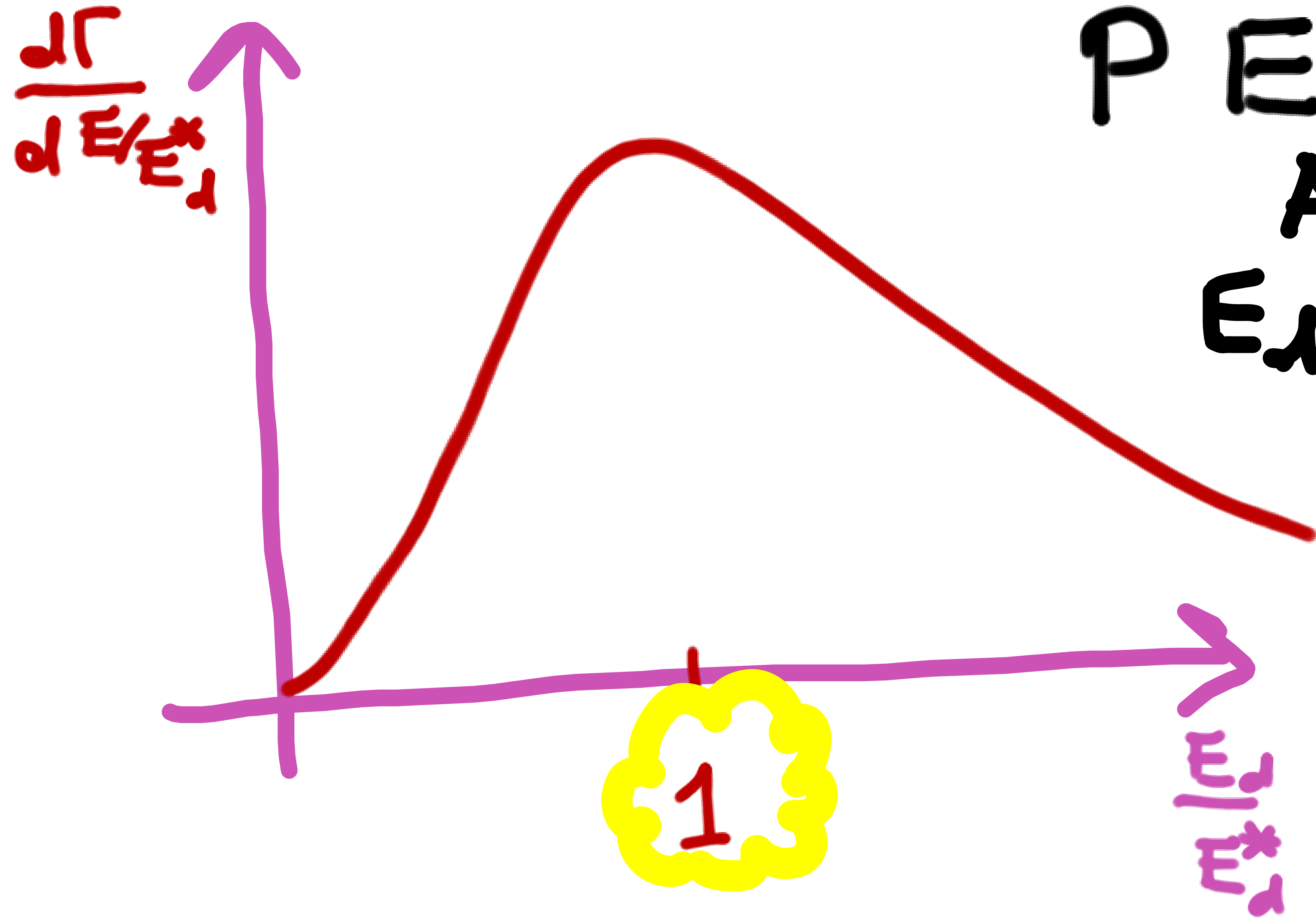


ADVANTAGES

(OVER INVARIANT MASS FOR INSTANCE)



$$E_d^* = \frac{m_M^2 + m_d^2 - m_x^2}{2 m_M}$$



PEAK AT $E_d = E_d^*$

- NO NEED TO MEASURE THE OTHER DECAY PRODUCT

$\Downarrow$

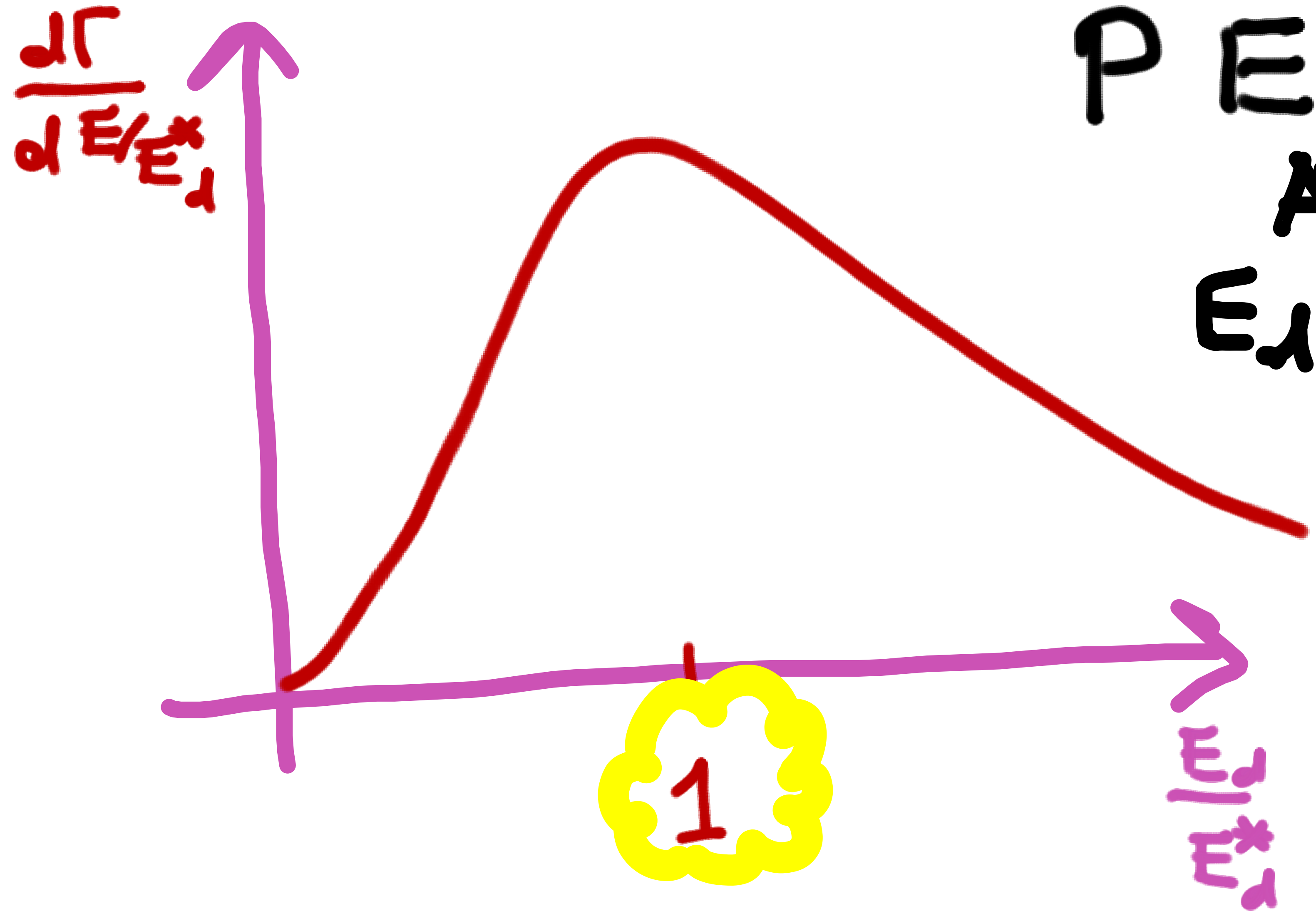
$\tilde{b} \rightarrow b\, \chi^0$ ← DARK MATTER

$W \rightarrow \ell\, \nu$ ← INVISIBLE

$t \rightarrow b\, W \rightarrow b\, \ell\, \nu$

ADVANTAGES

OVER MANY TRANSVERSE KINEMATICAL VARIABLES IN USE IN COLLIDER PHYSICS (m_{T2}, ...)



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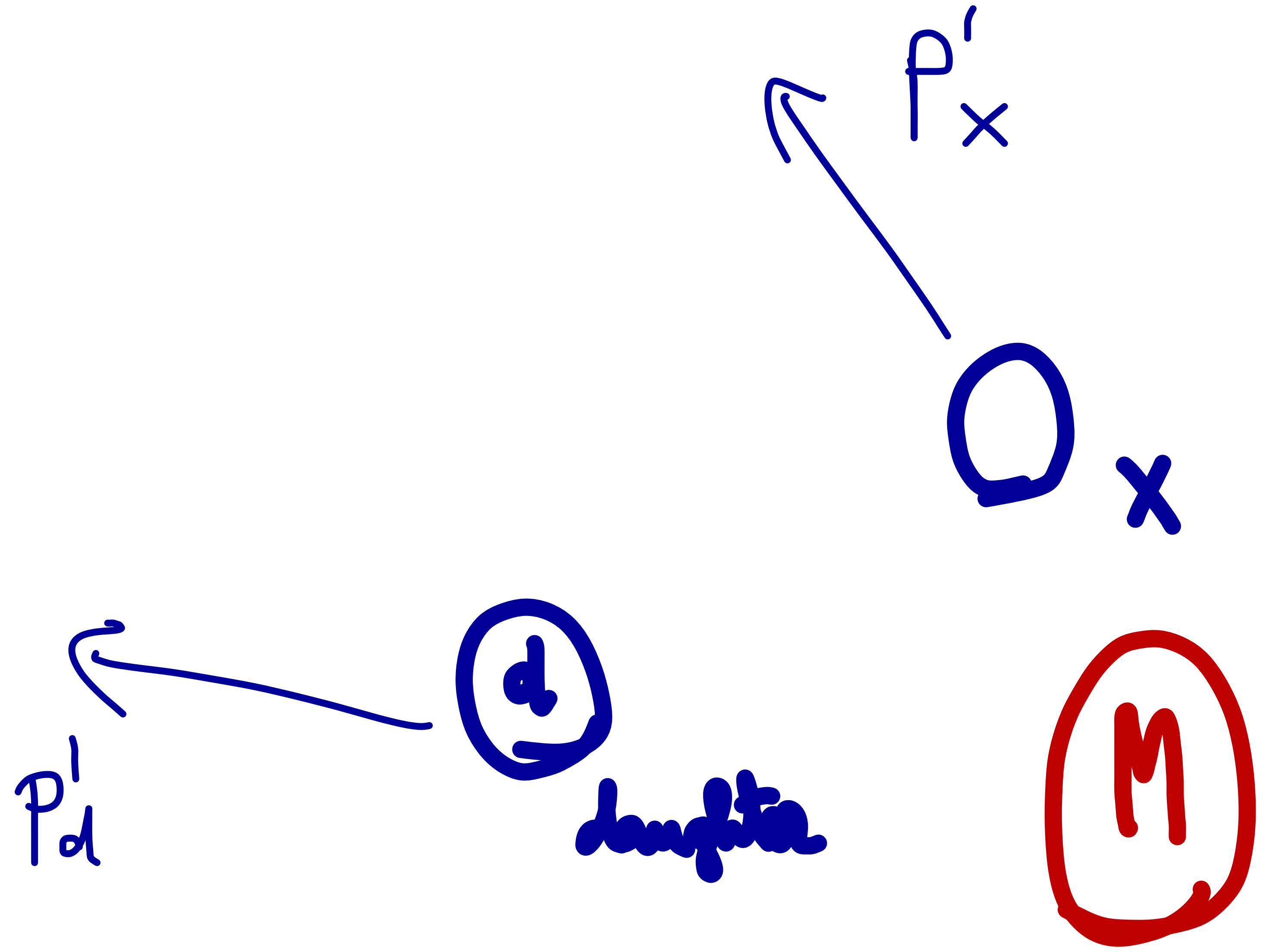
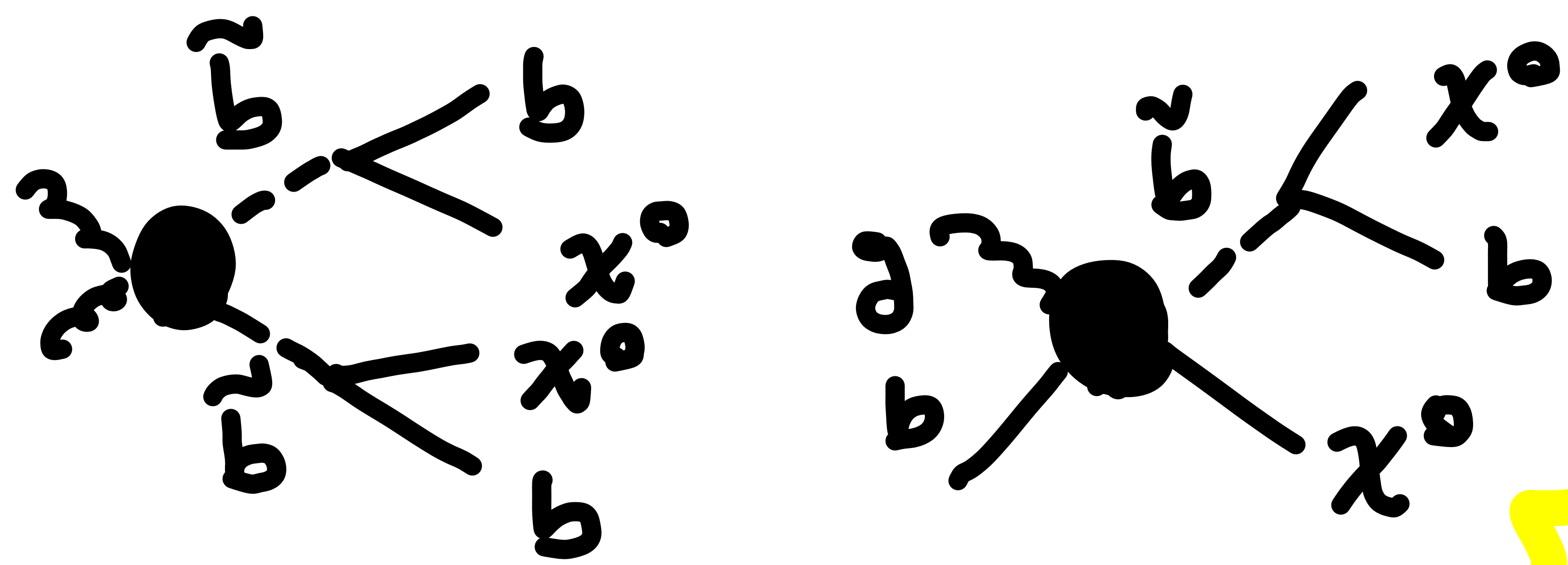


$$E_d^* = \frac{m_M^2 + m_d^2 - m_X^2}{2 m_M}$$

- NO NEED TO KNOW ANYTHING ABOUT THE REST OF THE EVENT



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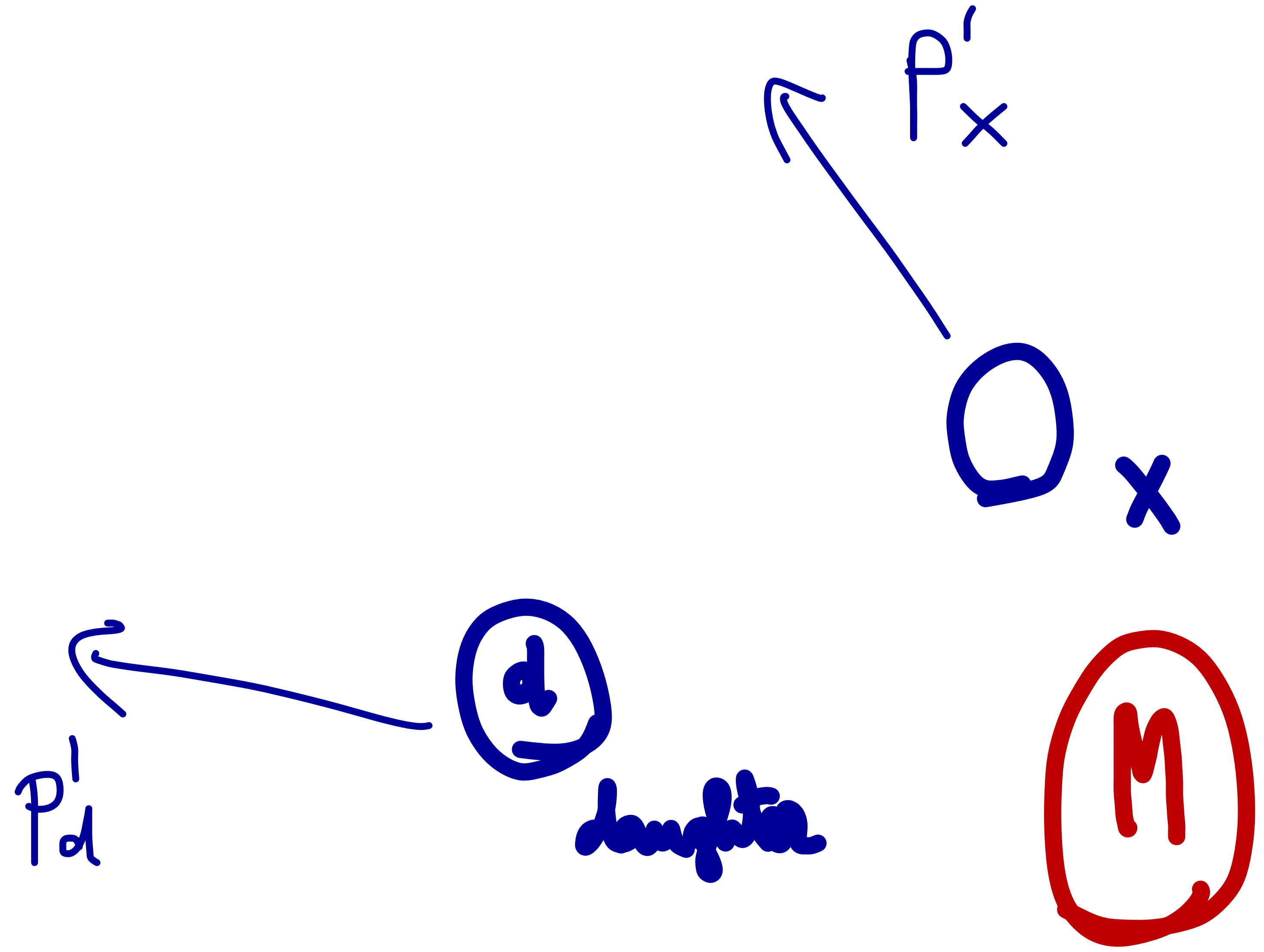

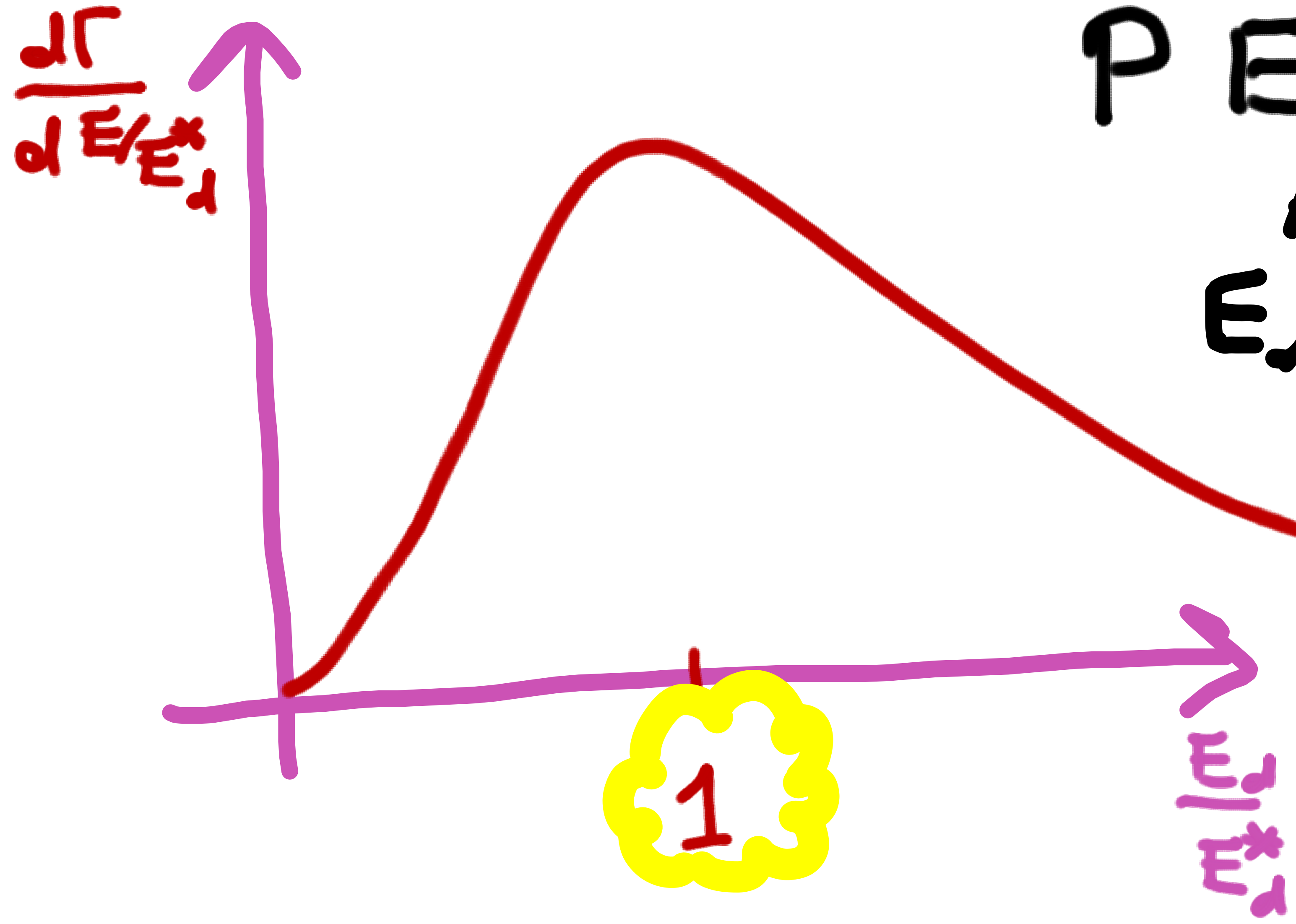

PEAK AT $E_d = E_d^*$

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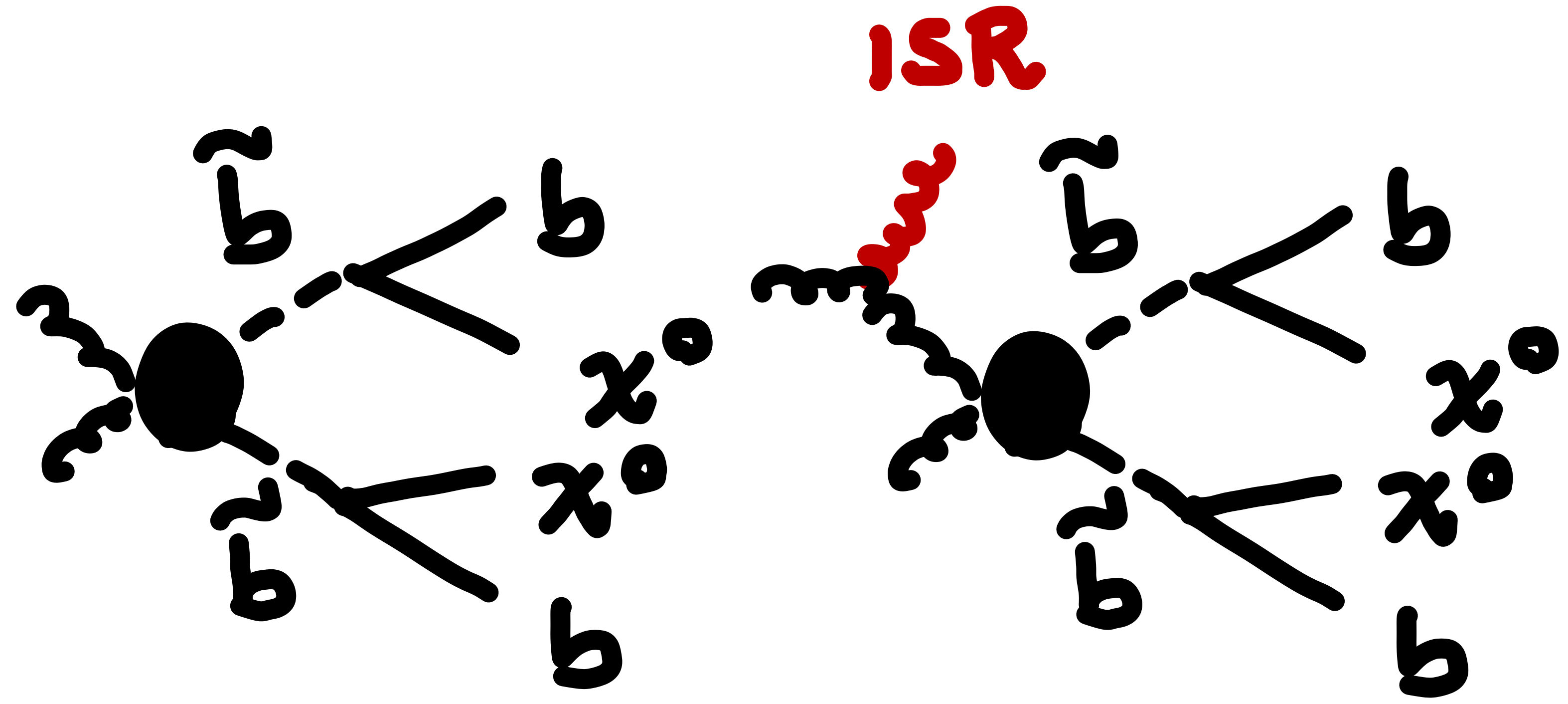
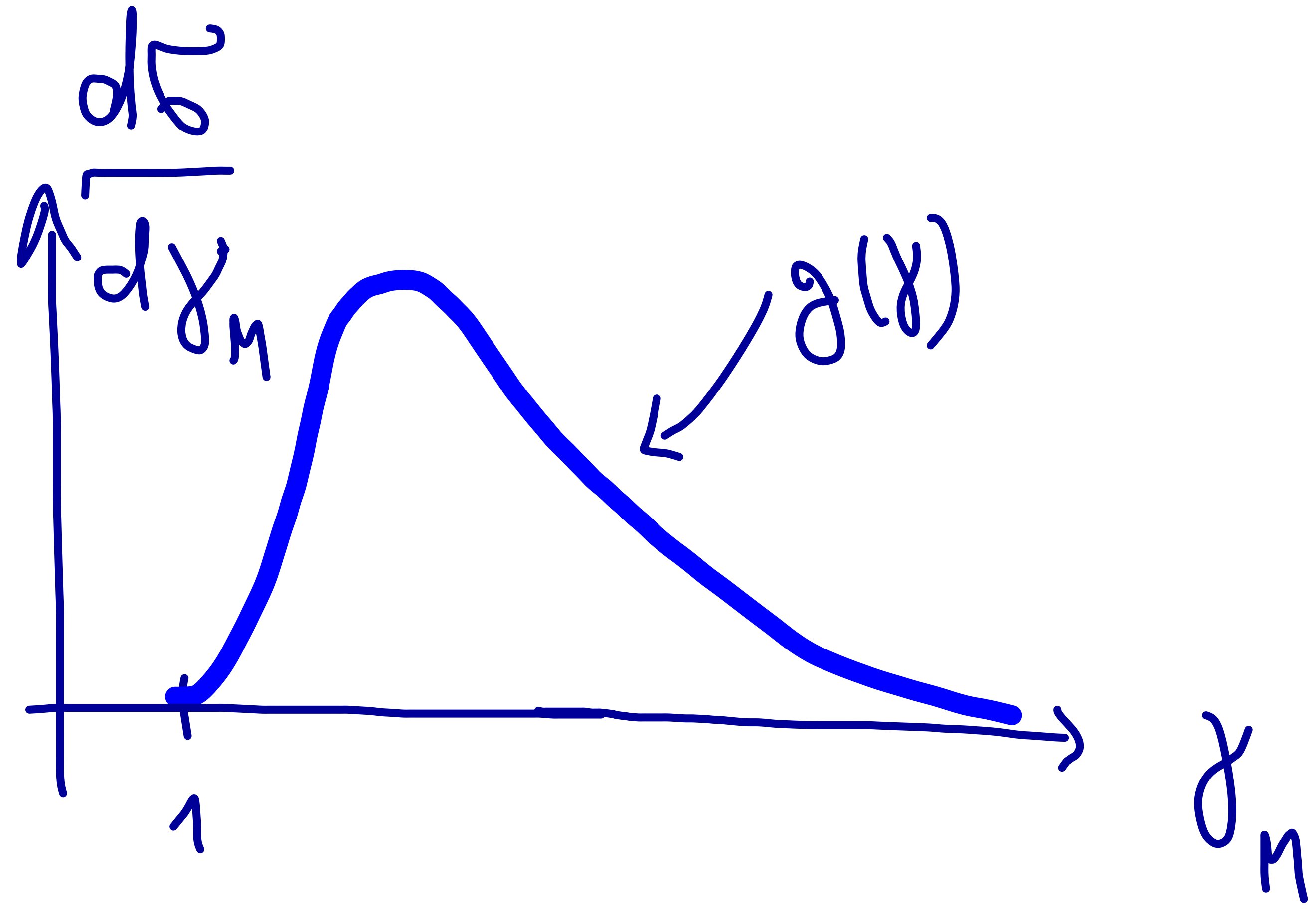


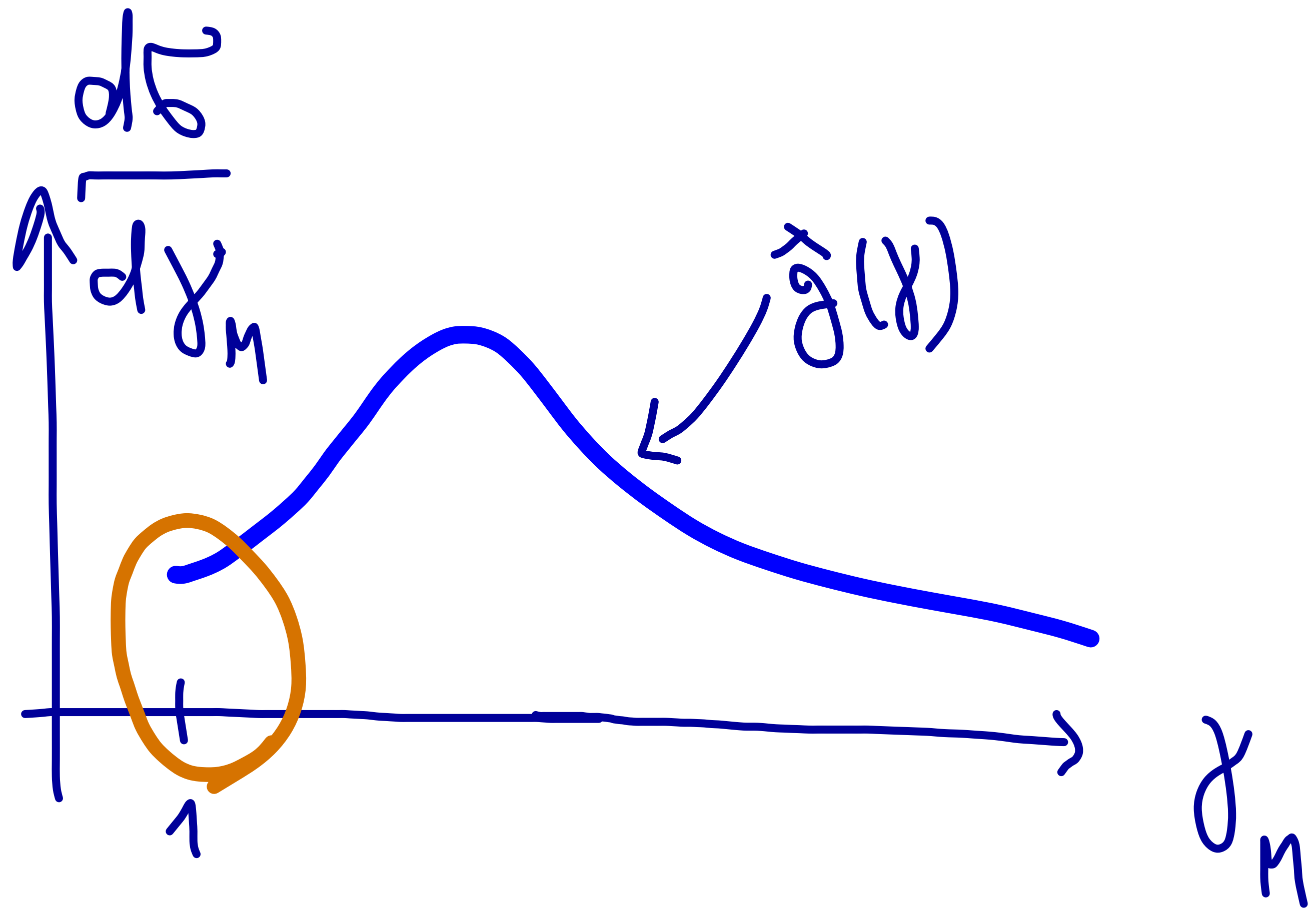
SOME MORE INSIGHTS BY GOING THROUGH AN ANALYTIC PROOF:



$$x := \frac{E_d}{E_d^*}$$

(MASSLESS DAUGHTER)

$$f(x) = \frac{1}{\Gamma}\frac{d\Gamma}{dx} = \int_{\frac{1}{2}\left(x+\frac{1}{x}\right)}^{\infty} d\gamma \, \frac{g(\gamma)}{2\sqrt{\gamma^2-1}}$$

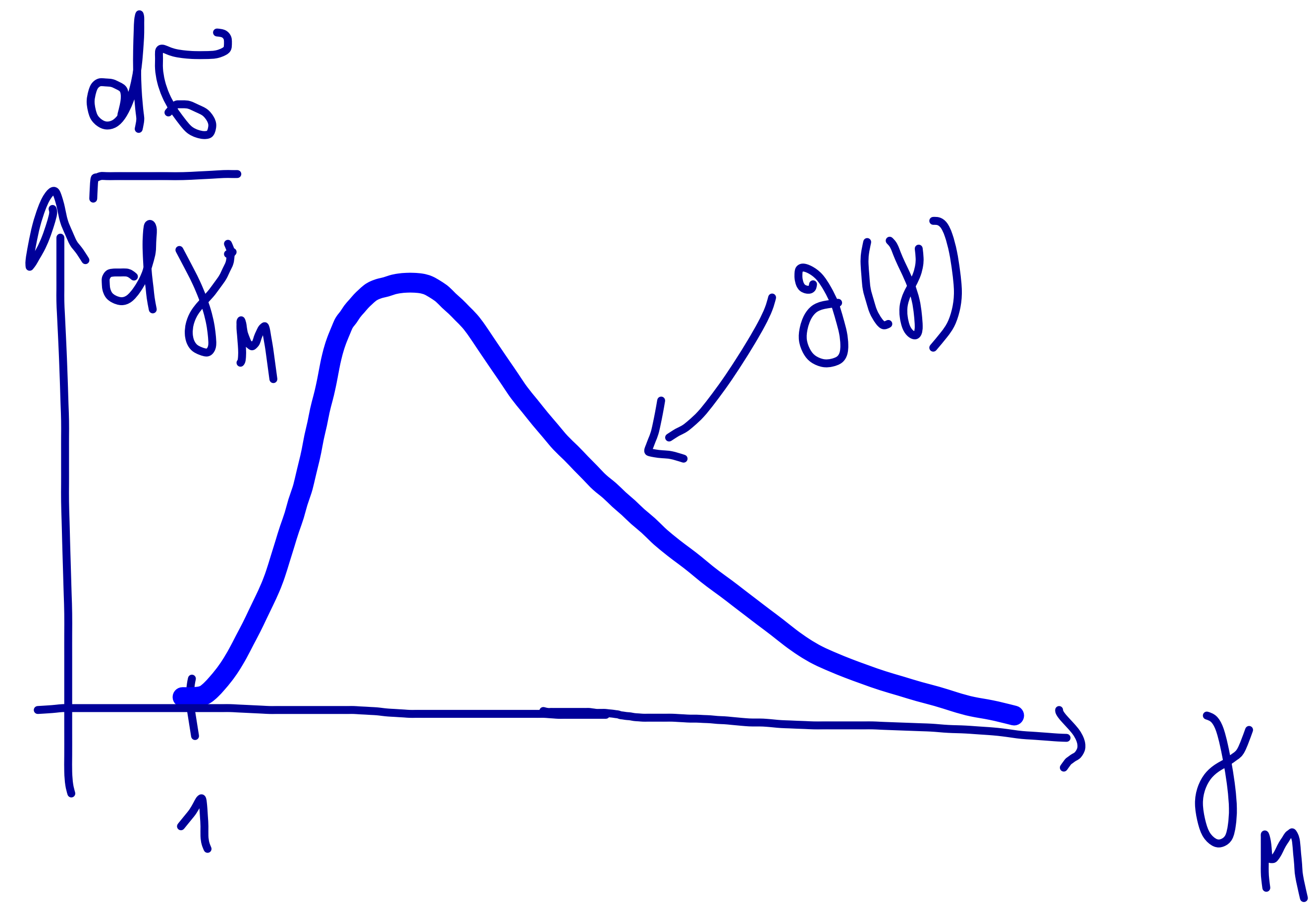


$$f'(x) = \frac{\text{sign}(1-x)}{2x}\, g\left(\frac{1}{2}\left(x+\frac{1}{x}\right)\right)$$

$g(1) = 0$

$g(1) \neq 0$ the derivative changes sign $\Rightarrow$ KINK IN THE OBSERVED ENERGY DISTRIBUTION

SOME MORE INSIGHTS BY GOING THROUGH AN ANALYTIC PROOF:



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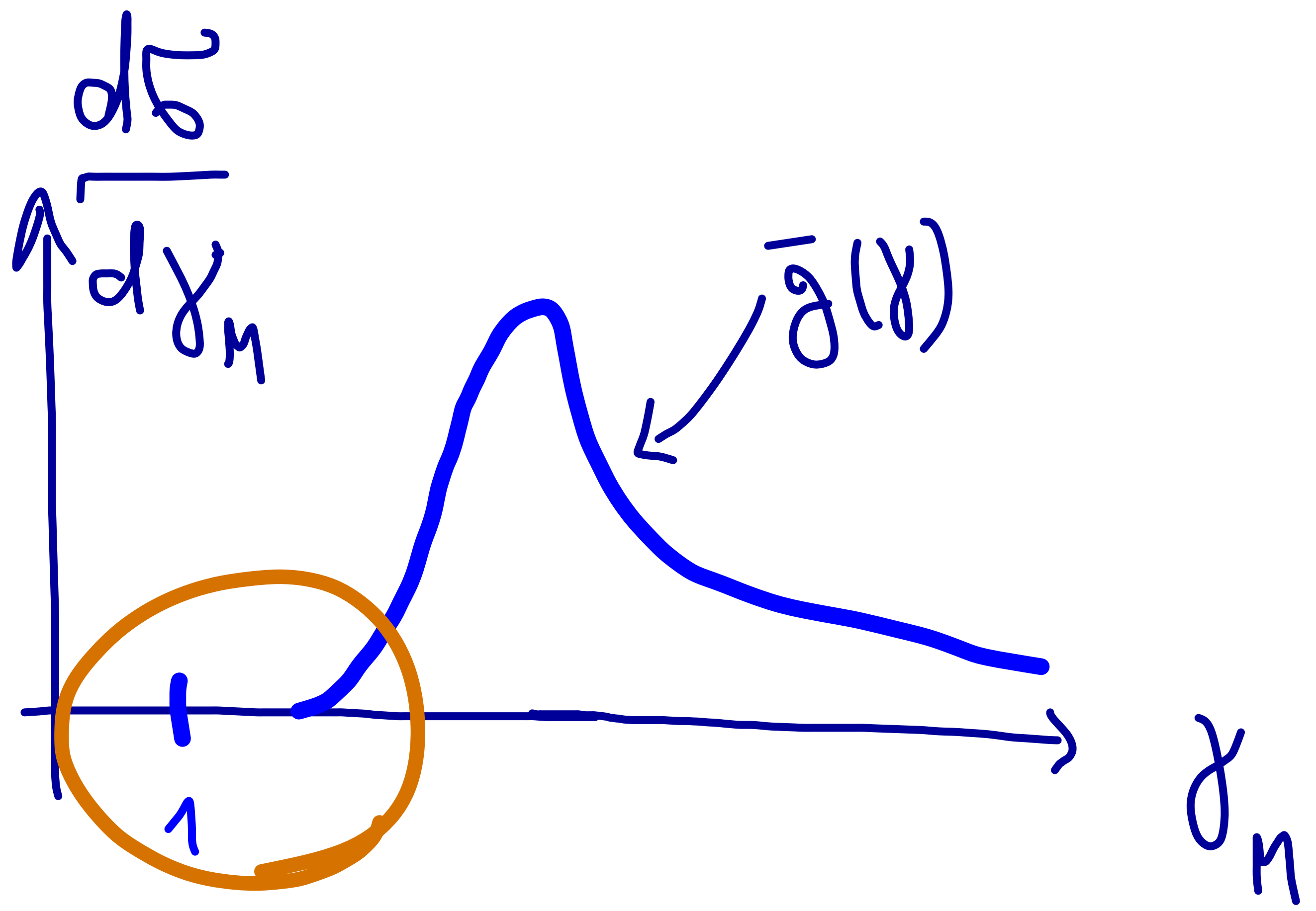
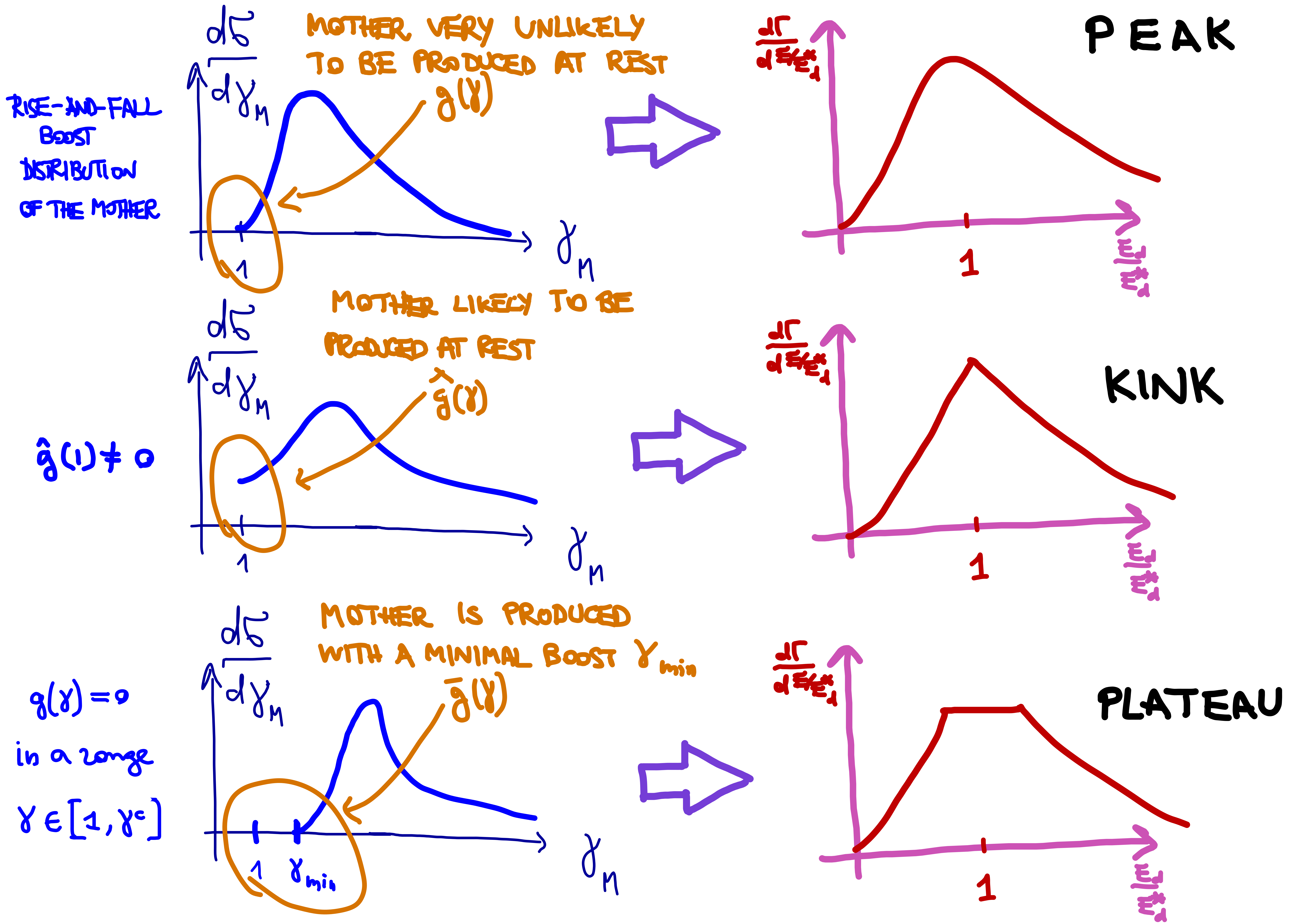


$$f'(x) = \frac{\mathrm{sign}(1-x)}{2x} \, g\left(\frac{1}{2}\left(x+\frac{1}{x}\right)\right)$$

$g(\gamma) = 0$

IN A RANGE $[1, \gamma^c]$ $\Rightarrow$ **PLATEAU IN THE OBSERVED ENERGY DISTRIBUTION**



RISE-AND-FALL BOOST DISTRIBUTION OF THE MOTHER
dσ/dγ_M
MOTHER VERY UNLIKELY TO BE PRODUCED AT REST
g(γ)
1
γ_M
dΓ/dE_d/E*_d
1
E_d/E*_d
PEAK
ĝ(1) ≠ 0
dσ/dγ_M
MOTHER LIKELY TO BE PRODUCED AT REST
ĝ(γ)
1
γ_M
dΓ/dE_d/E*_d
1
E_d/E*_d
KINK
g(γ) = 0
in a range
γ ∈ [1, γ^c]
dσ/dγ_M
MOTHER IS PRODUCED WITH A MINIMAL BOOST γ_min
ḡ(γ)
1
γ_min
γ_M
dΓ/dE_d/E*_d
1
E_d/E*_d
PLATEAU

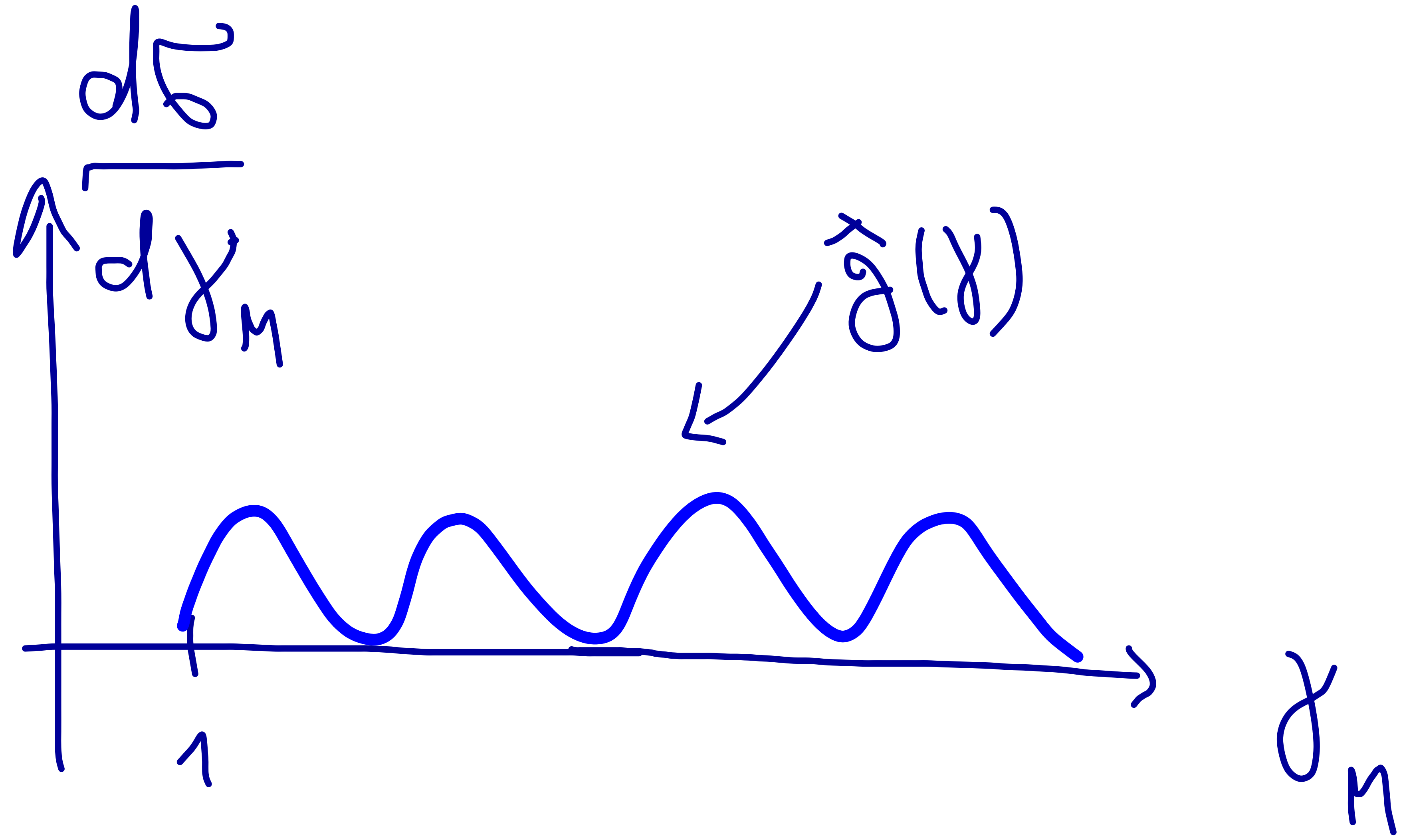
WHEN AND WHY THIS BREAKS DOWN ?

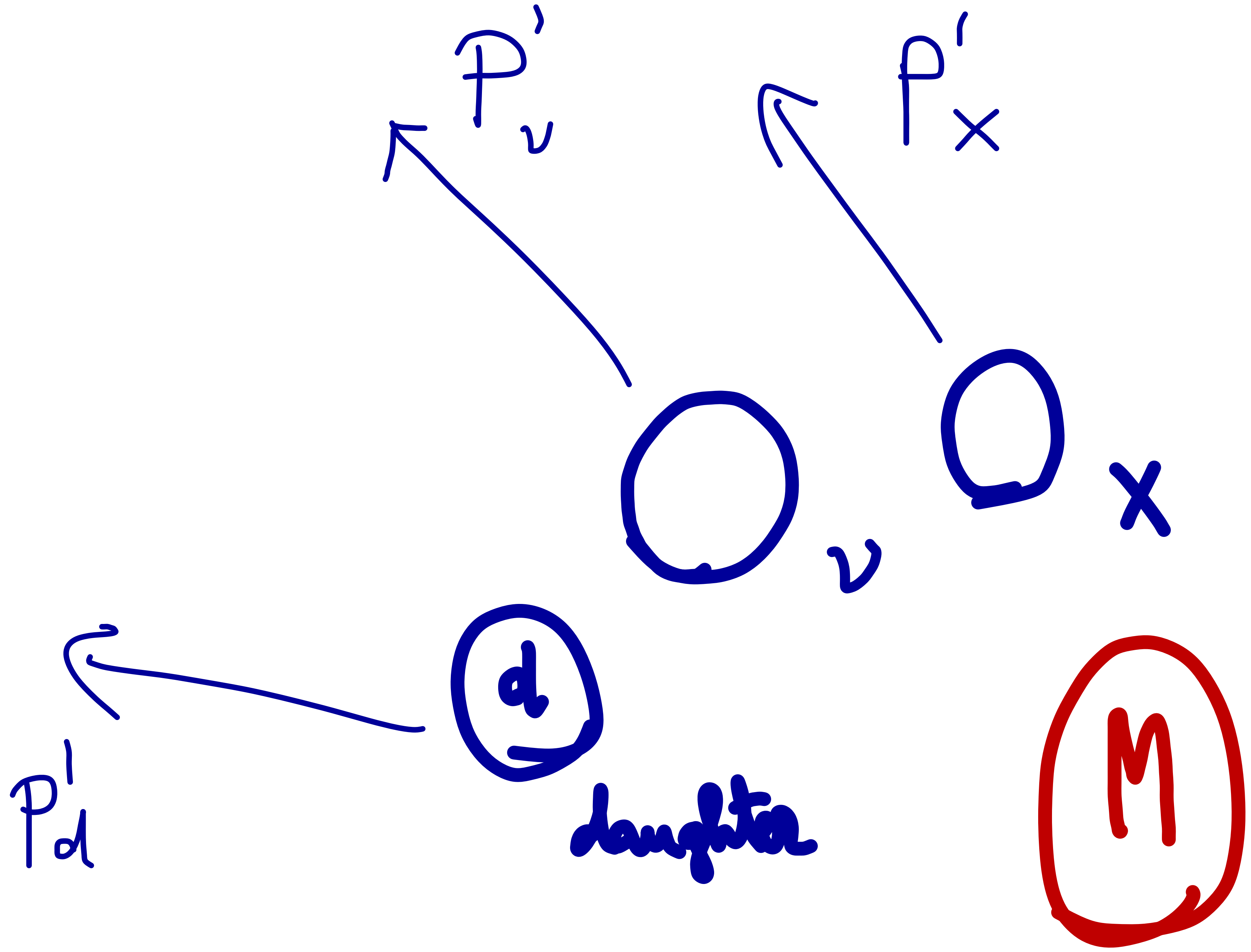
① BOOST DISTRIBUTION OF THE MOTHER WITH SPECIAL FEATURES

(MANY MINIMA, LARGE FLAT PORTIONS, ...)

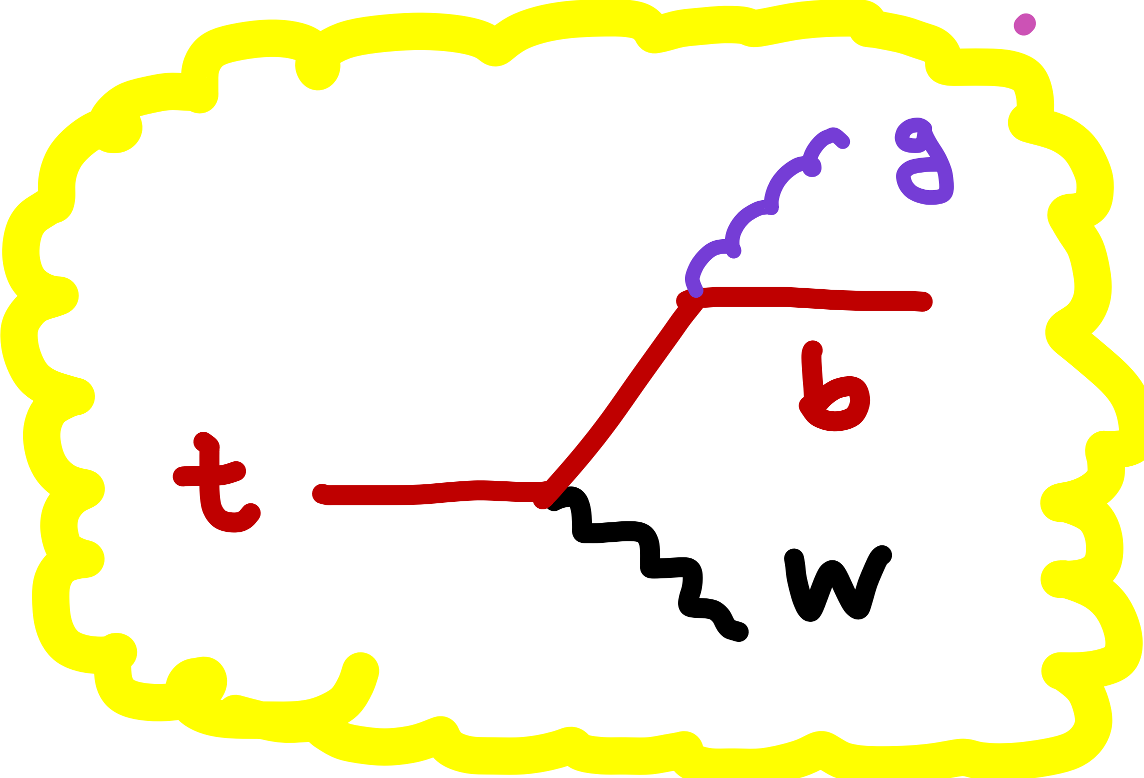
② THE DECAY WAS NOT TWO-BODY

EXTRA INVISIBLE/UNDETECTED PARTICLES IN THE DECAY





WHEN AND WHY THIS BREAKS DOWN?



CAVEAT:

$M \to d X$

IS TWO BODY ONLY UP TO EXTRA RADIATION

IF THE FINAL STATES ARE COLORED $M \to d X + \text{gluons}$

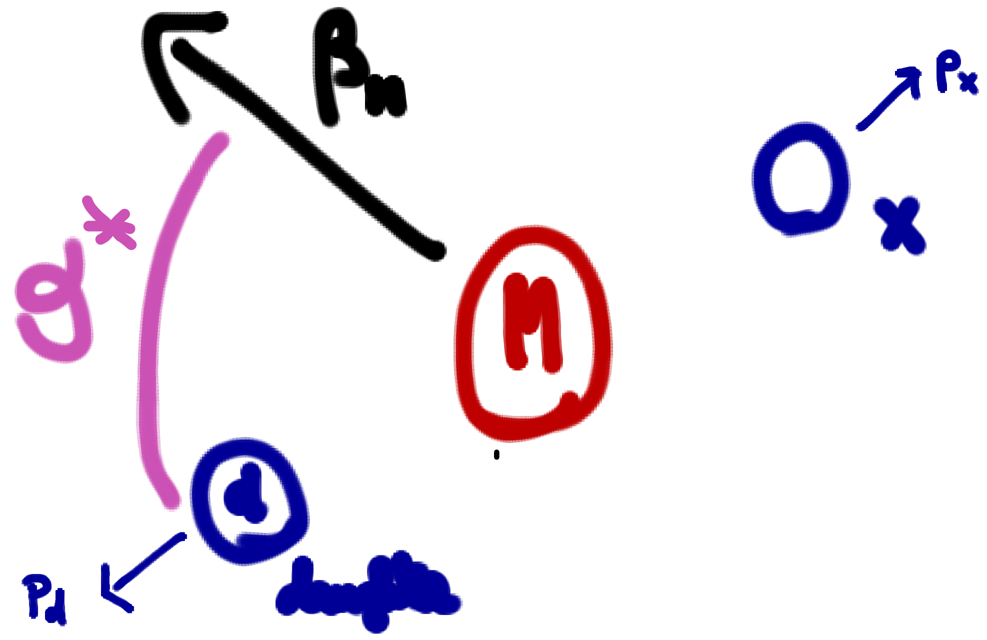
JET CLUSTERING SOLVES THIS ISSUE TO SOME EXTENT

HARD RADIATION MAY BE RESOLVABLE AND EFFECTIVELY GIVE RISE TO A THREE-BODY DECAY

RESOLVABLE RADIATION CAN BE VETOED

WHEN AND WHY THIS BREAKS DOWN ?

③ THE DAUGHTER'S MASS

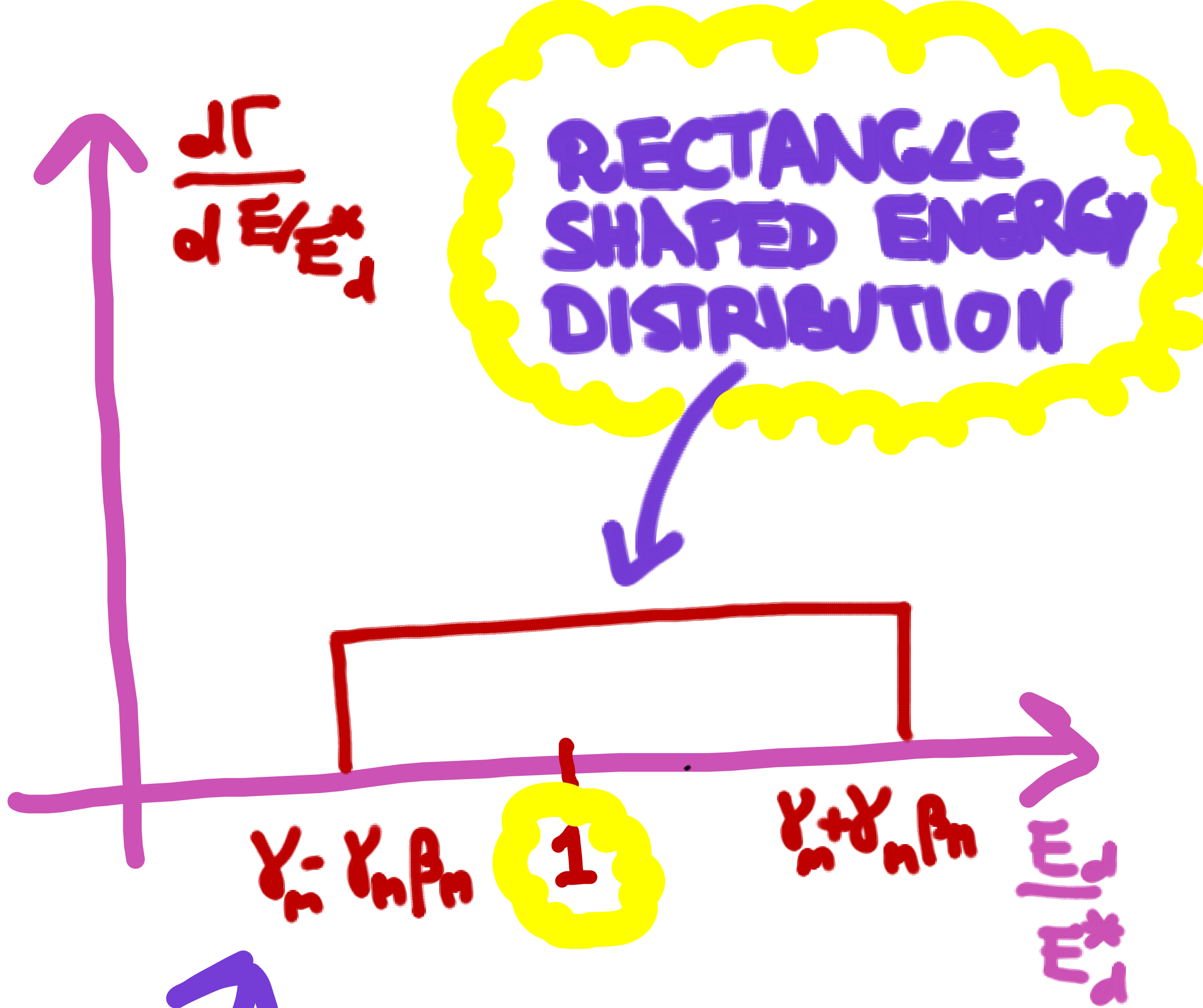


$$E'_d = E^*_d \gamma_M + \cos\theta^* \gamma_M \beta_M p^*_d$$

THE MINIMUM OF THIS QUANTITY AT $\theta^* = \pi$
(BOOST ANTI-PARALLEL TO THE MOMENTUM OF d)

IF $p^*_d = E^*_d$ (MASSLESS DAUGHTER)

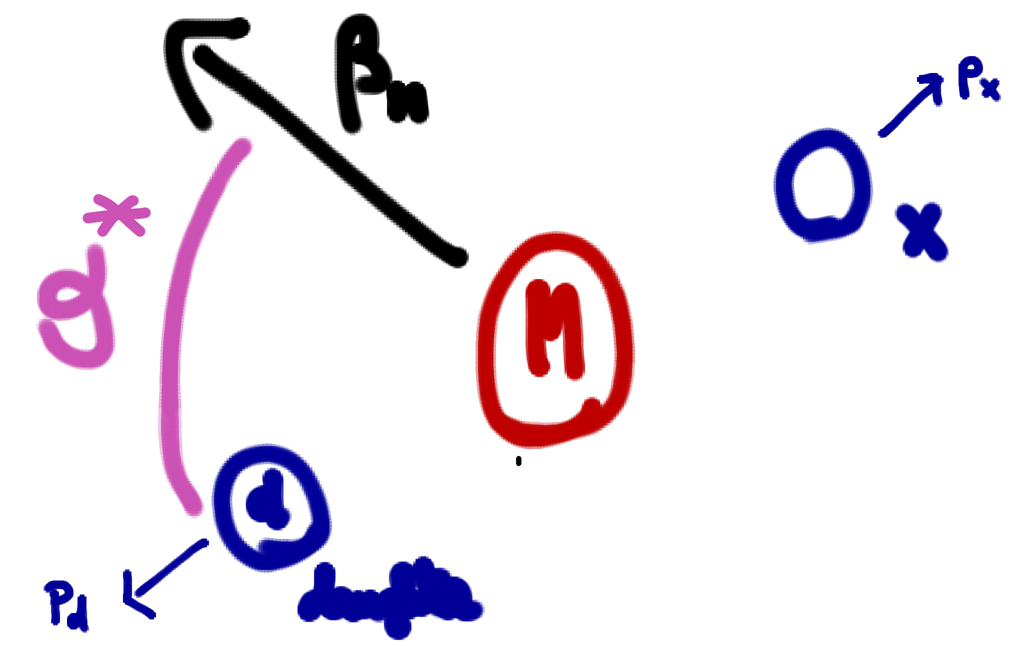
$$E'_{d,min} = E^*_d \left(\gamma_M - \sqrt{\gamma_M^2 - 1} \right) < E^*_d$$



LOWER EDGE OF EACH RECTANGLE BELOW E^*

WHEN AND WHY THIS BREAKS DOWN ?

- THE DAUGHTER'S MASS



$$E'_d = E^*_d \gamma_M + \cos\theta^* \gamma_M \beta_M p^*_d$$

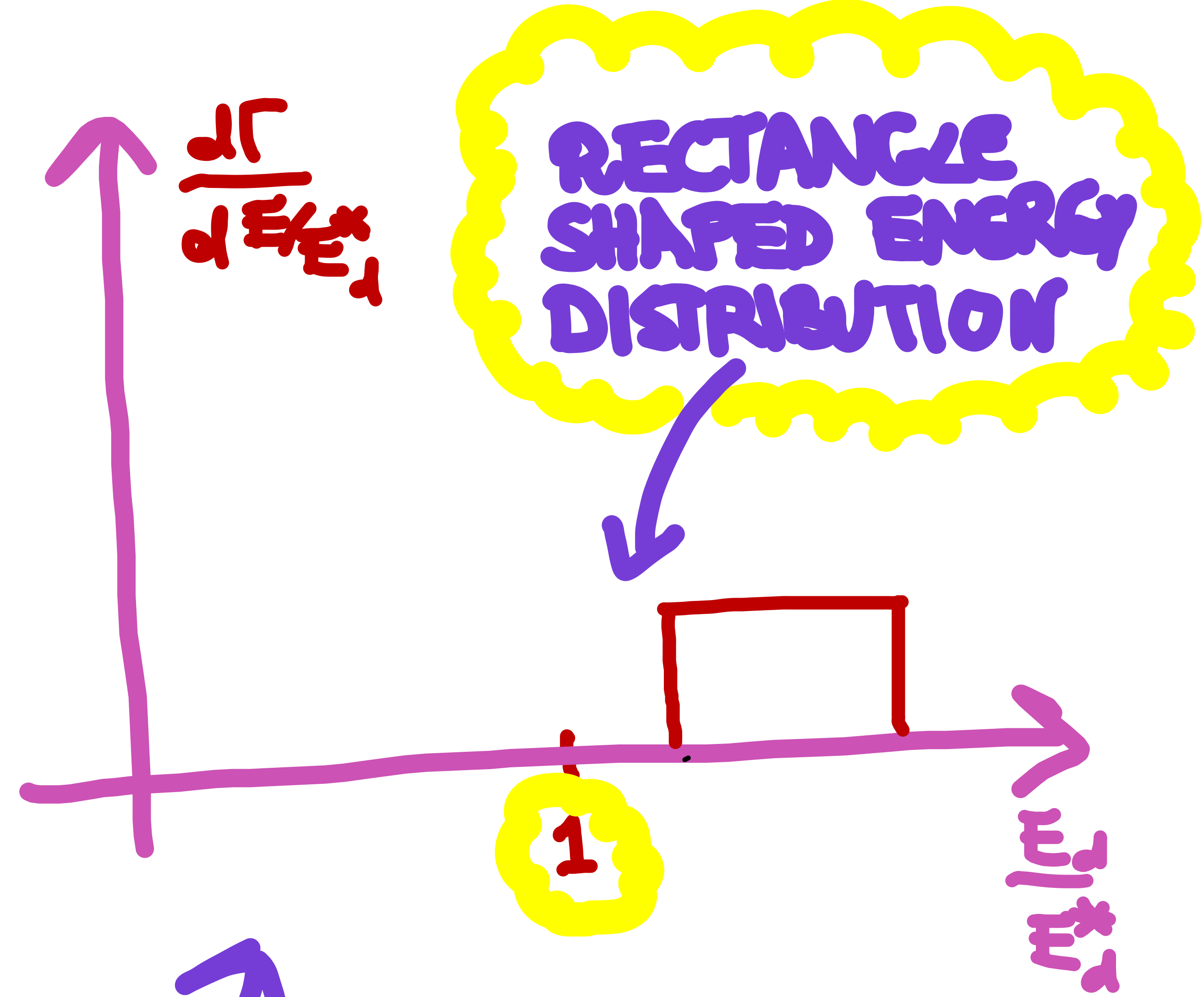
THE MINIMUM OF THIS QUANTITY AT $\theta^* = \pi$
(BOOST ANTI-PARALLEL TO THE MOMENTUM OF d)

IF $p^*_d \leq E^*_d$ (MASSIVE DAUGHTER)

$$p^*_d \to 0 \quad E^*_d \to m_d$$

$$E'_d = m_d \gamma_M + \dots$$

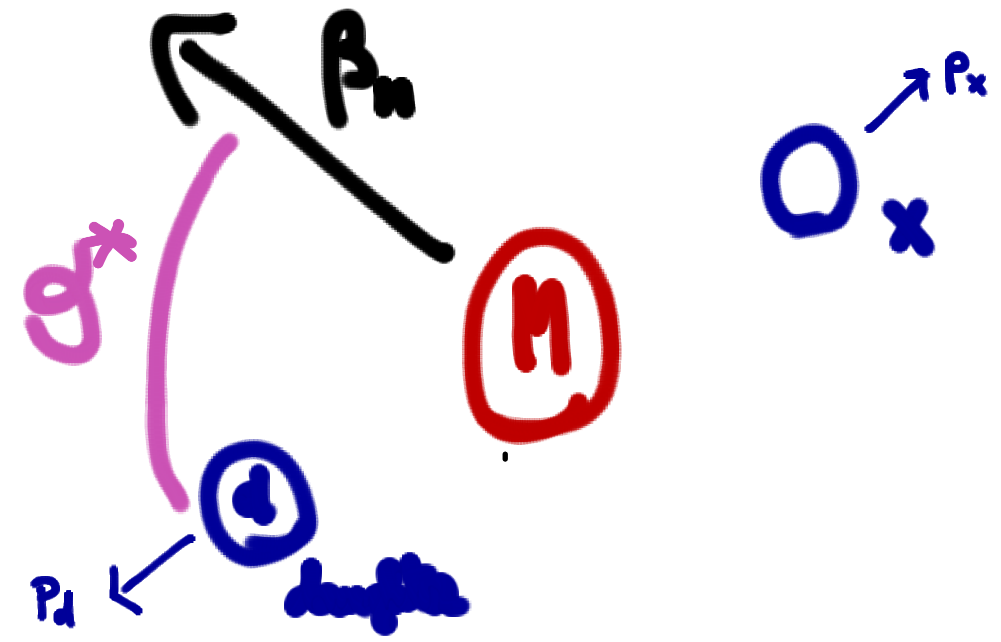
FOR γ_M LARGE GIVES RECTANGLES $E_{d,min} > E^*_d$



LOWER EDGE OF EACH RECTANGLE ABOVE E^*

WHEN AND WHY THIS BREAKS DOWN ?

- THE DAUGHTER'S MASS



$$E'_d = E^*_d \gamma_M + \cos\theta^* \gamma_M \beta_M p^*_d$$

THE MINIMUM OF THIS QUANTITY AT $\theta^* = \pi$

(BOOST ANTI-PARALLEL TO THE MOMENTUM OF d)

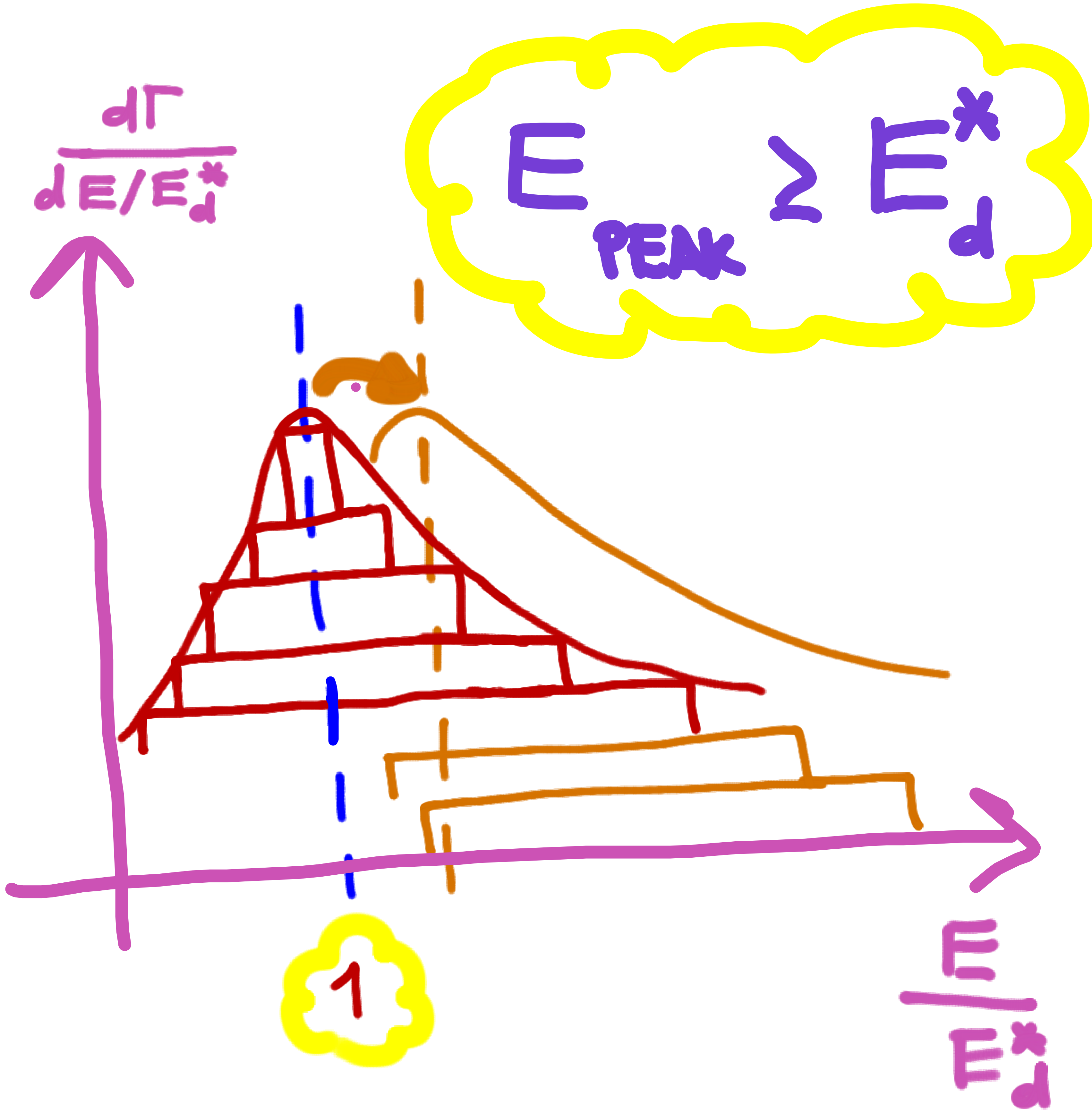
IF $p^*_d \leq E^*_d$ (MASSIVE DAUGHTER)

$$p^*_d \to 0 \qquad E^*_d \to m_d$$

$$E'_d = m_d \gamma_M + \dots$$

FOR γ_M LARGE GIVES

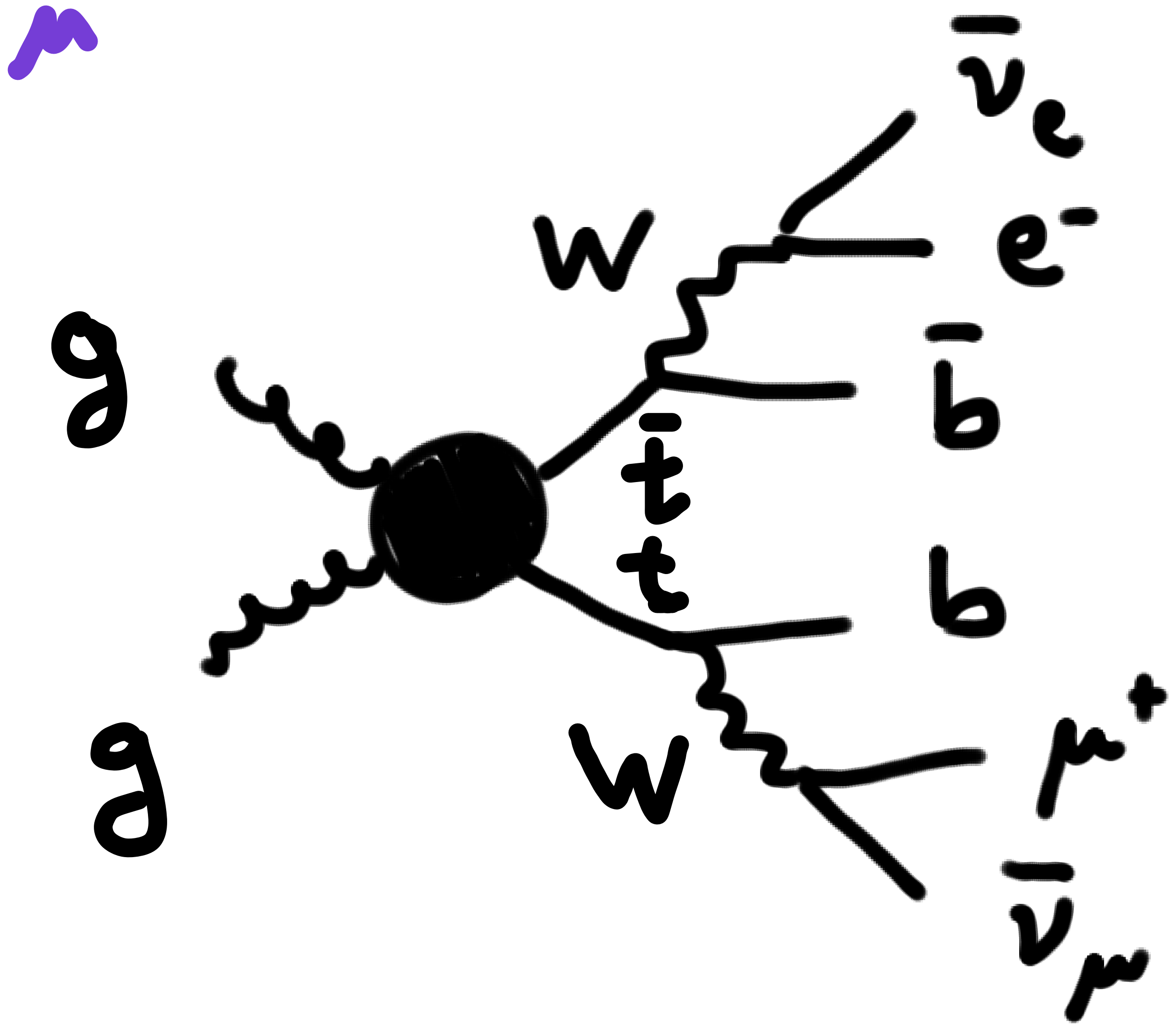
RECTANGLES $E_{d,min} \geq E^*_d$



APPLICATIONS (for mass measurements)

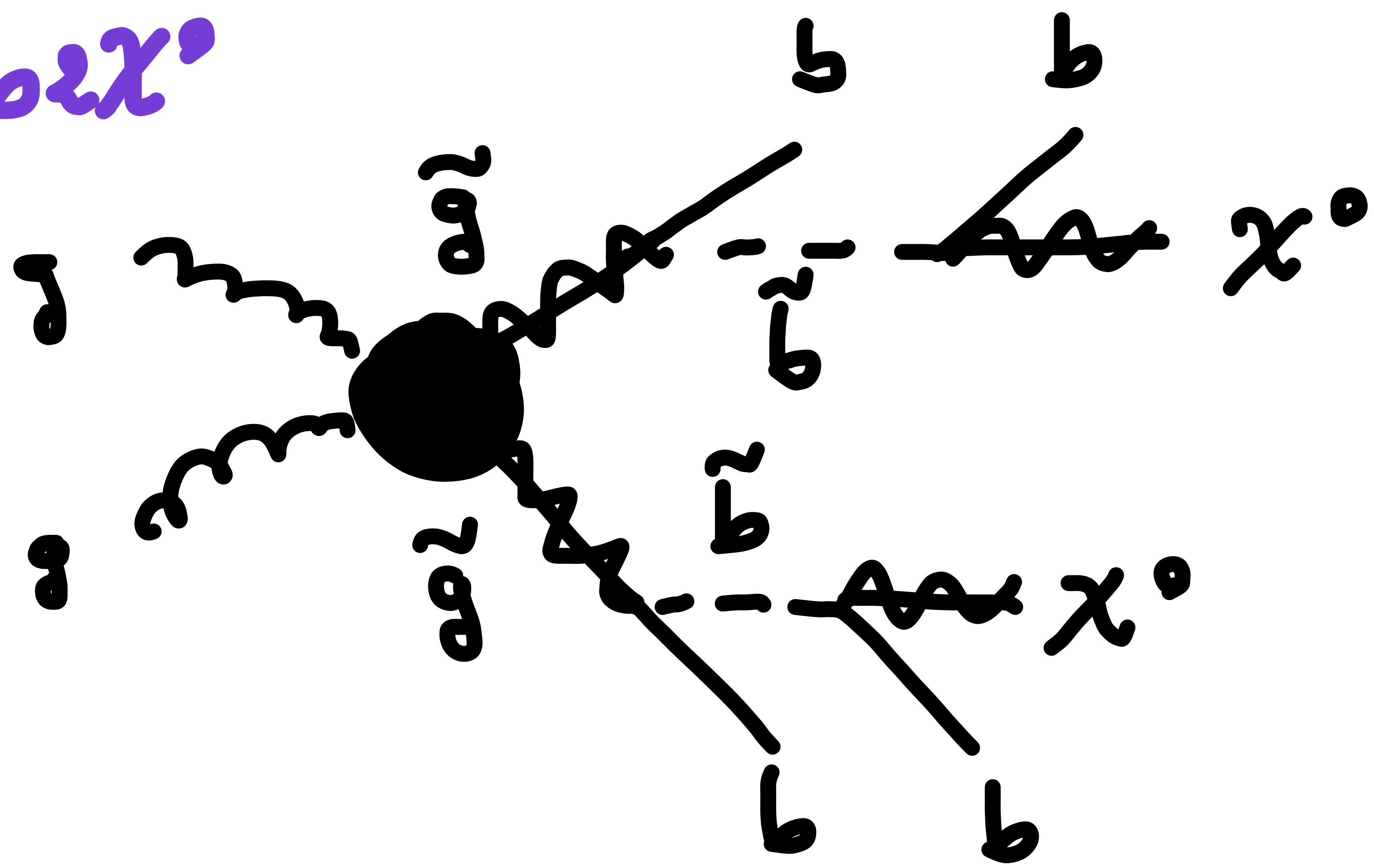
1) $pp \to t\bar{t} \to b\bar{b}\,\mu^+ e^- \bar{\nu}_e \nu_\mu$

m_{top} AS PROOF OF PRINCIPLE



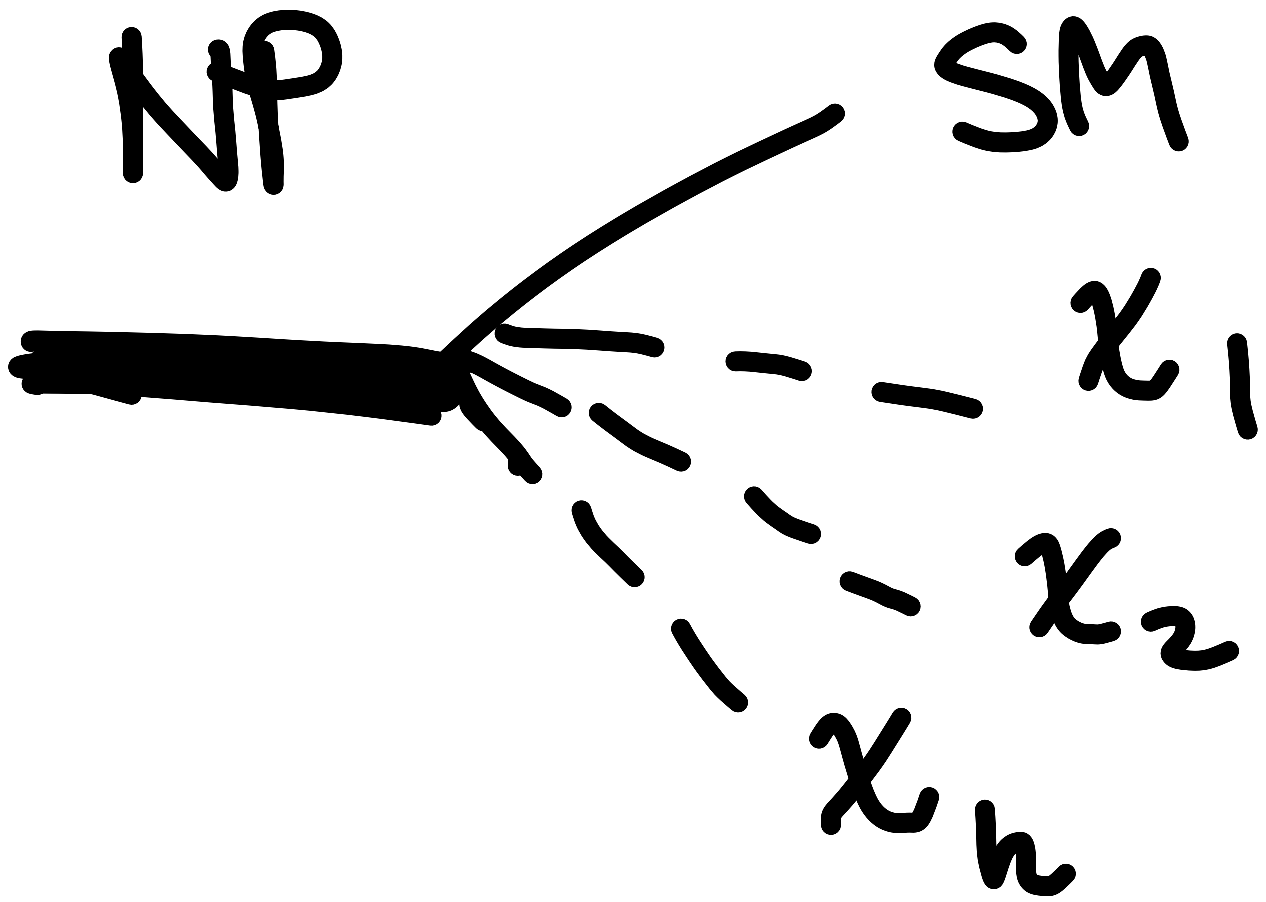
2) $pp \to \tilde{g}\tilde{g} \to b\tilde{b}\,b\tilde{b} \to 4b\,2\chi^0$

$m_{\tilde{g}}$, $m_{\tilde{b}}$, m_{χ}



APPLICATIONS

DISTINGUISHING BETWEEN 2-bodies AND 3-bodies



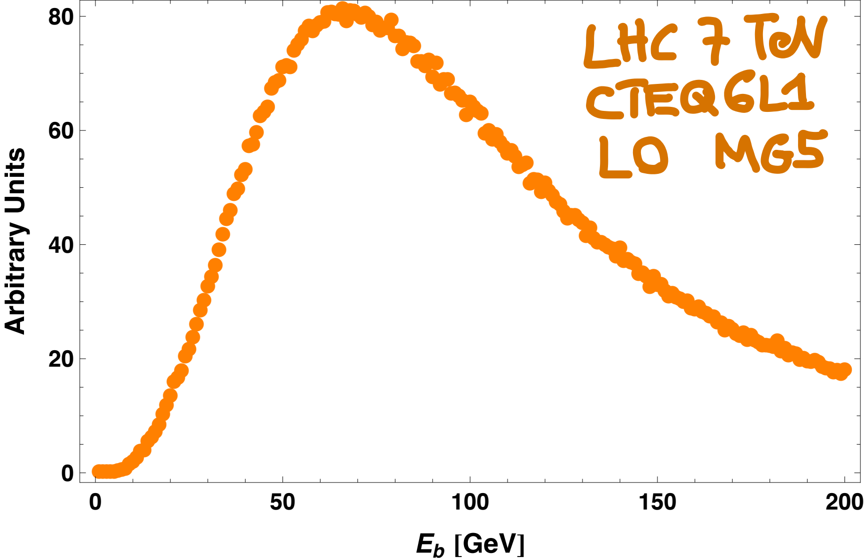
COUNT INVISIBLE PARTICLES IN SEMI-INVISIBLE DECAYS

$$pp \to t\bar{t} \to b\bar{b}\, e\bar{\mu}\, \not{E}_T$$

- QCD PAIR PRODUCTION OF $t\bar{t}$ ENSURES THAT THE OVERALL SAMPLE OF TOP DECAYS IS UNPOLARIZED

- $E_b^* = \dfrac{m_t^2 - m_W^2 + m_b^2}{2m_t} \simeq 67\ \text{GeV}$ $\qquad E_b^* \gg m_b$

THE b QUARK CAN BE TAKEN AS MASSLESS



LHC 7 TeV
CTEQ6L1
LO MG5
Arbitrary Units
E_b [GeV]
0
20
40
60
80
0
50
100
150
200

FINDING THE PEAK :

- LOOK BY EYE

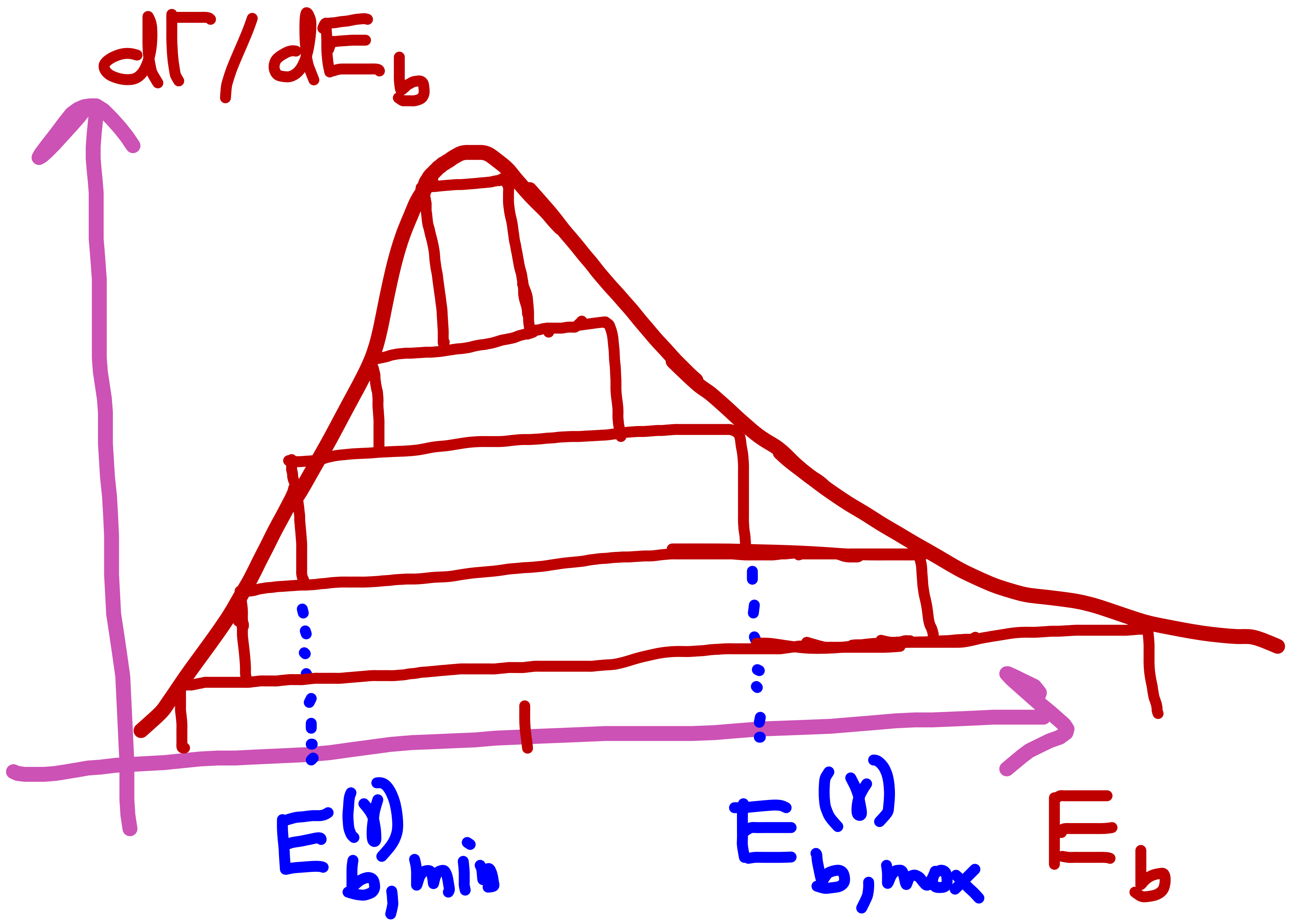
- $\boxed{\text{FIND A TEMPLATE AND USE IT TO FIT DATA}}$

  - A TEMPLATE MOTIVATED FROM PRIME PRINCIPLES SEEMS UNATTAINABLE BECAUSE IT DEPENDS ON PARTON DISTRIBUTION FUNCTIONS AND ON THE MATRIX ELEMENT FOR THE PRODUCTION PROCESS

FIND A TEMPLATE AND USE IT TO FIT DATA

$$E_b = E_b^* \left(\gamma_n + \cos\vartheta^* \sqrt{\gamma_M^2 - 1} \right)$$

$$E_{b,\substack{min \\ max}}^{(\gamma_n)} = E_b^* \left(\gamma_M \mp \sqrt{\gamma^2 - 1} \right)$$



$$\sqrt{E_{b,min} E_{b,max}} = E_b^*$$

$d\Gamma/dE_b$

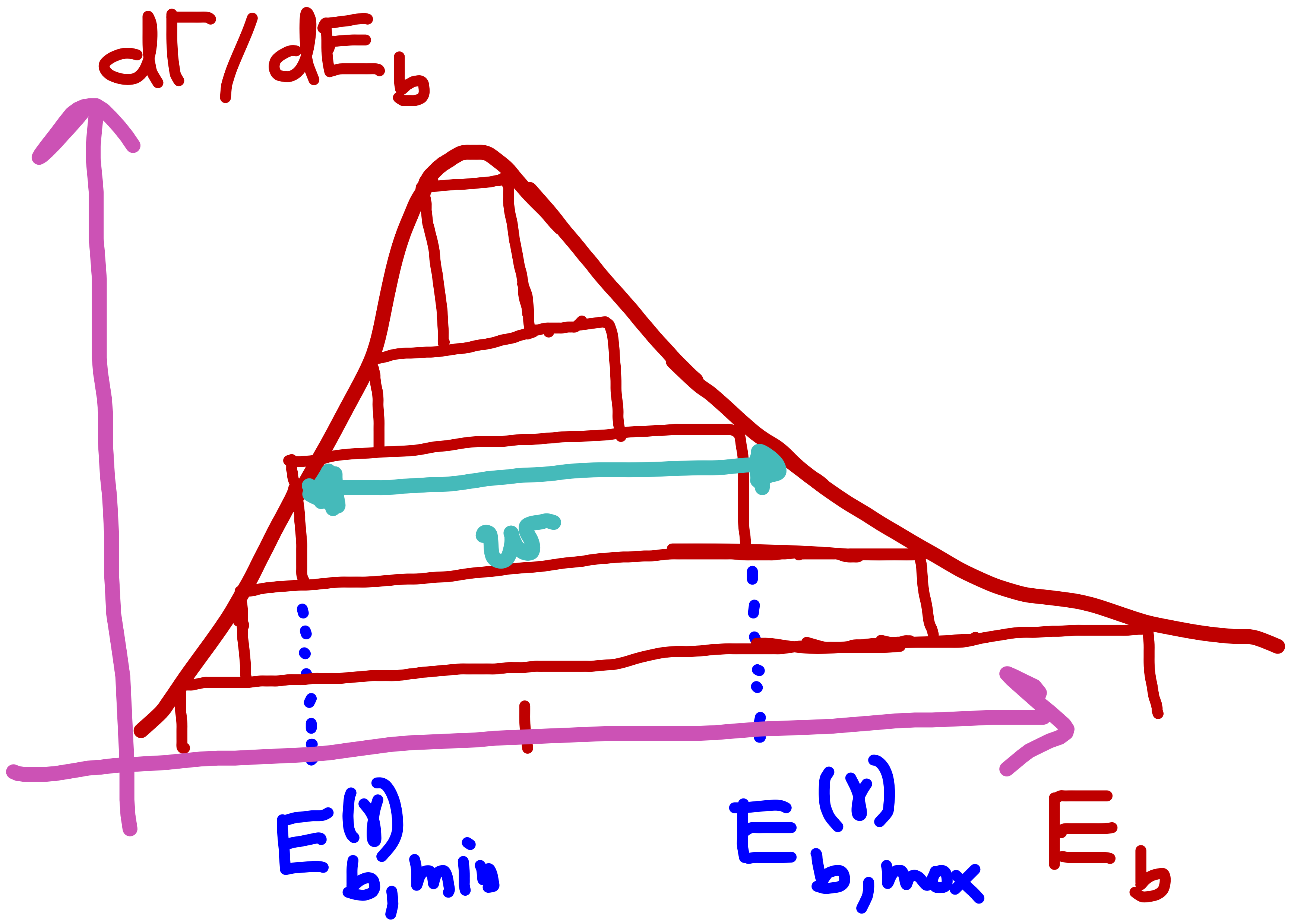
MUST BE A FUNCTION OF

$$\frac{E_b}{E_b^*} + \frac{E_b^*}{E_b}$$

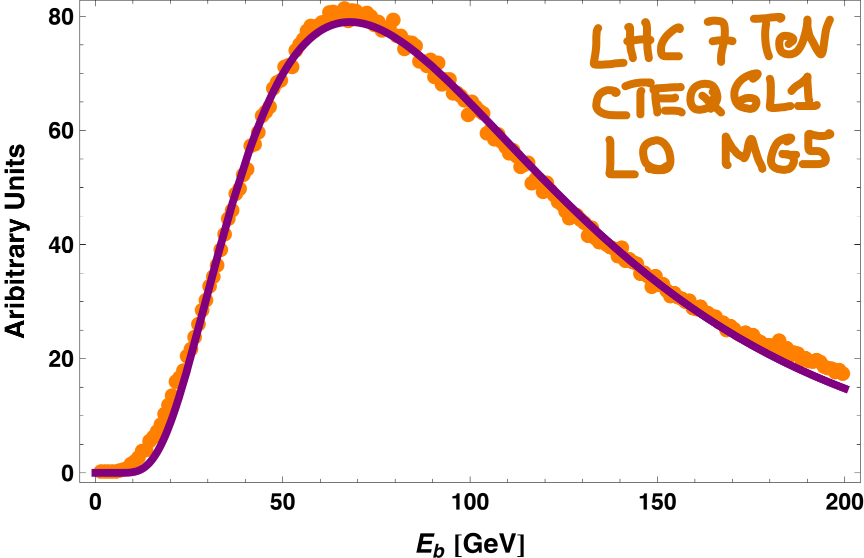
FIND A TEMPLATE AND USE IT TO FIT DATA

- $d\Gamma/dE_b \xrightarrow{E_b \to 0,\infty} 0$ (at least)
- $d\Gamma/dE_b$ max at $E_b = E_b^*$
- IN SOME LIMIT SHOULD BE A δ-FUNCTION (MOTHER AT REST)
- $d\Gamma/dE_b$ MUST BE A FUNCTION OF

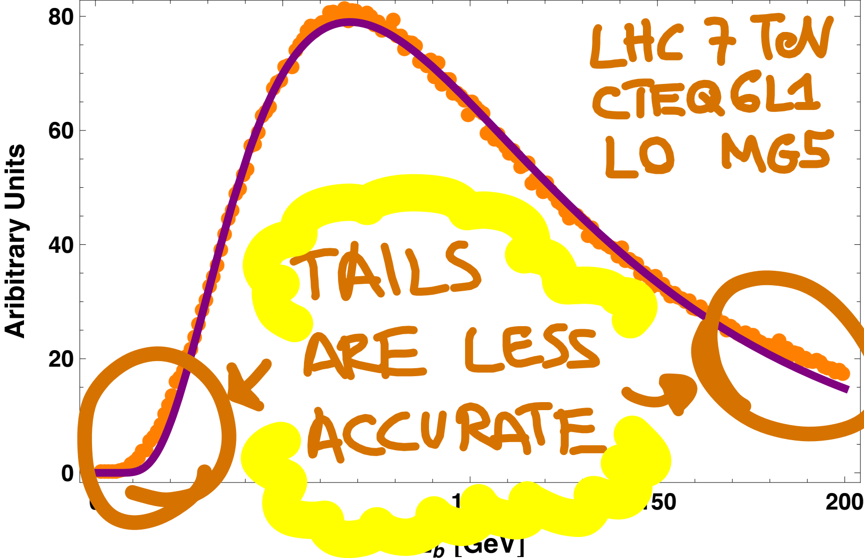
$$\frac{E_b}{E_b^*} + \frac{E_b^*}{E_b}$$



$$d\Gamma/dE \sim \exp\left(-w\left(\frac{E}{E^*} + \frac{E^*}{E}\right)\right)$$



LHC 7 TeV
CTEQ6L1
LO MG5
Aribitrary Units
E_b [GeV]
0
20
40
60
80
0
50
100
150
200

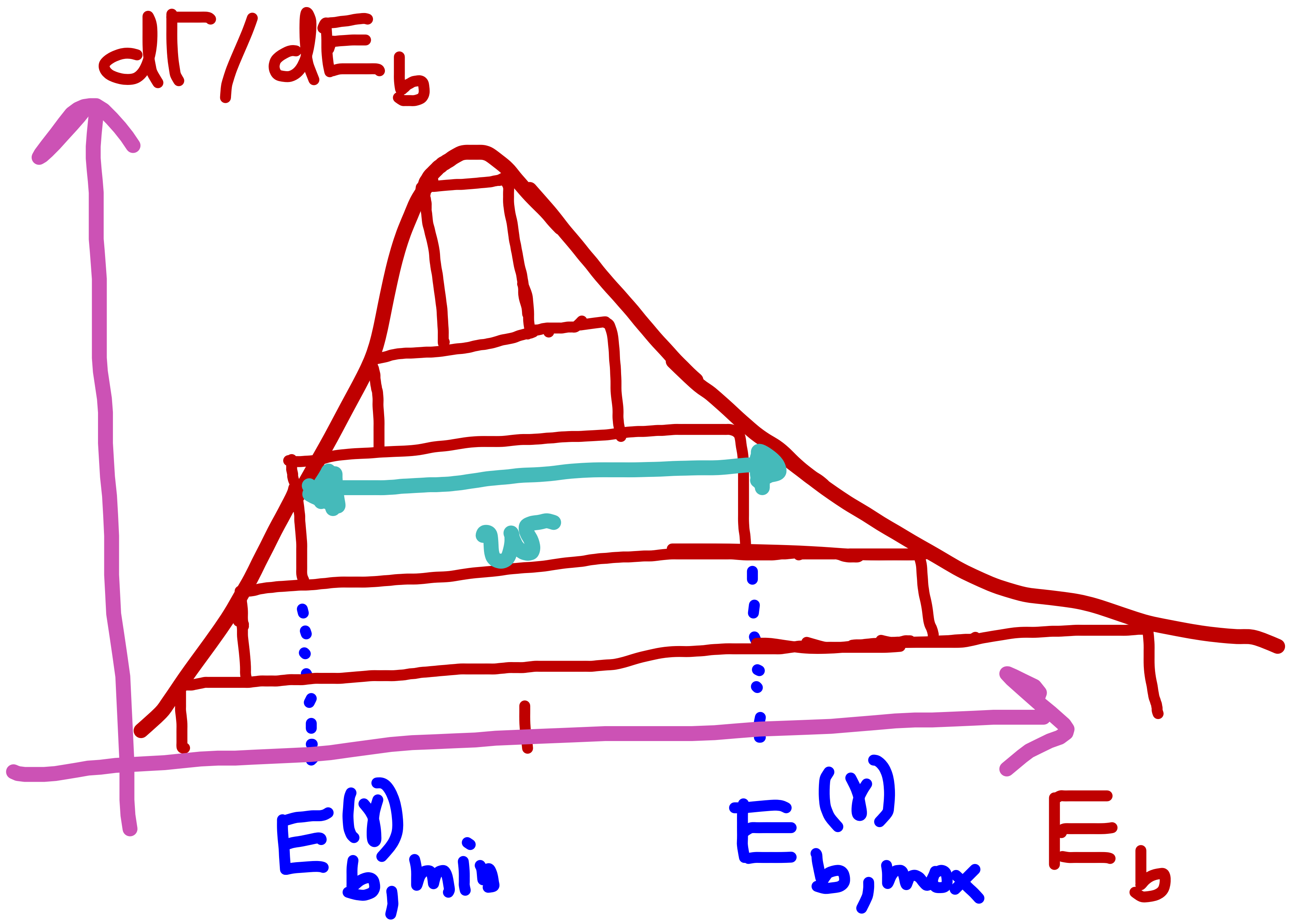


Aribitrary Units
80
60
40
20
0
150
200
[GeV]
LHC 7 TeV
CTEQ6L1
LO MG5
TAILS
ARE LESS
ACCURATE

FIND A TEMPLATE AND USE IT TO FIT DATA

- $d\Gamma/dE_b \xrightarrow{E_b \to 0,\infty} 0$
- $d\Gamma/dE_b$ max at $E_b = E_b^*$
- IN SOME LIMIT SHOULD BE A δ-FUNCTION (MOTHER AT REST)
- $d\Gamma/dE_b$ MUST BE A FUNCTION OF

$$\frac{E_b}{E_b^*} + \frac{E_b^*}{E_b}$$



$$d\Gamma/dE \sim \exp\left(-w\left(\frac{E}{E^*} + \frac{E^*}{E}\right)\right)$$

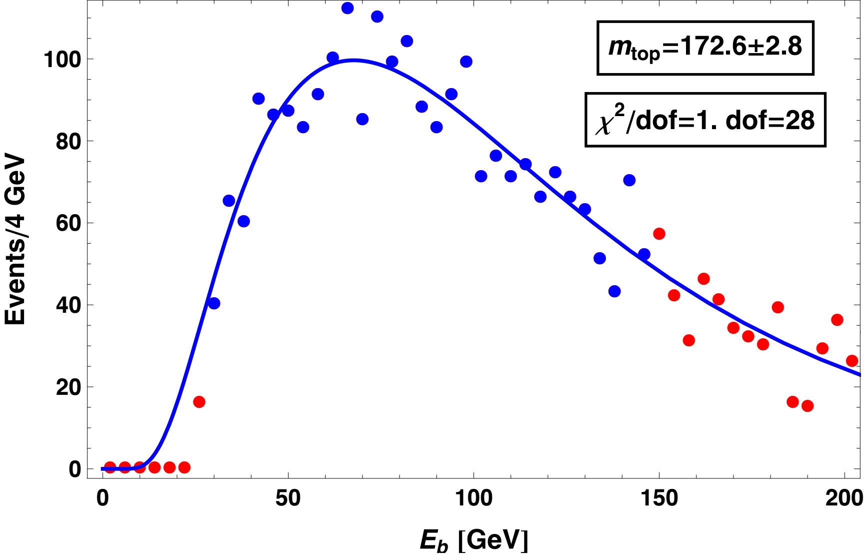
CAN WE MEASURE PARTICLE MASSES ?

- $E_b^* = \dfrac{m_t^2 - m_W^2 + m_b^2}{2m_t} \simeq 67\ \mathrm{GeV}$

FROM THE RESULT OF THE FIT TO THE LEADING ORDER MATRIX ELEMENT WE HAVE AT LEAST A CHANCE

NEED TO EVALUATE :

- DETECTOR EFFECTS ⟶ DELPHES 1.9
- EXTRA QCD RADIATION ⟶ SOFT QCD PYTHIA 6.4
- BIAS FROM EVENT SELECTION ⟶ ATLAS-CONF-2012-097



m_{top}=172.6±2.8
χ^2/dof=1. dof=28
Events/4 GeV
E_b [GeV]
0
20
40
60
80
100
0
50
100
150
200

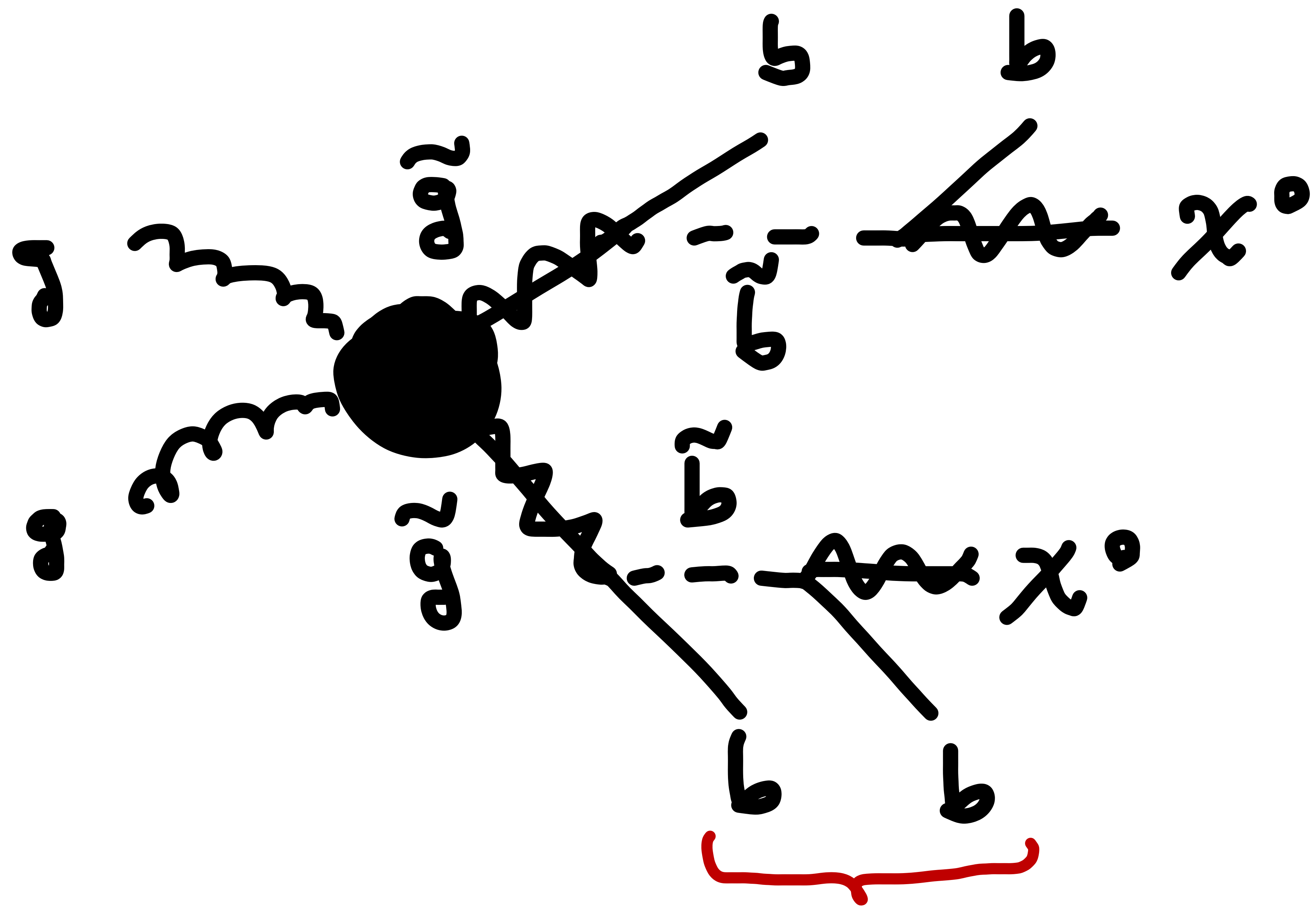
CAN WE MEASURE PARTICLE MASSES ?

FROM 100 PSEUDO EXPERIMENTS
FOR LHC $\sqrt{s} = 7$ TeV AND $\mathcal{L} = 5/\text{fb}$
WE GET

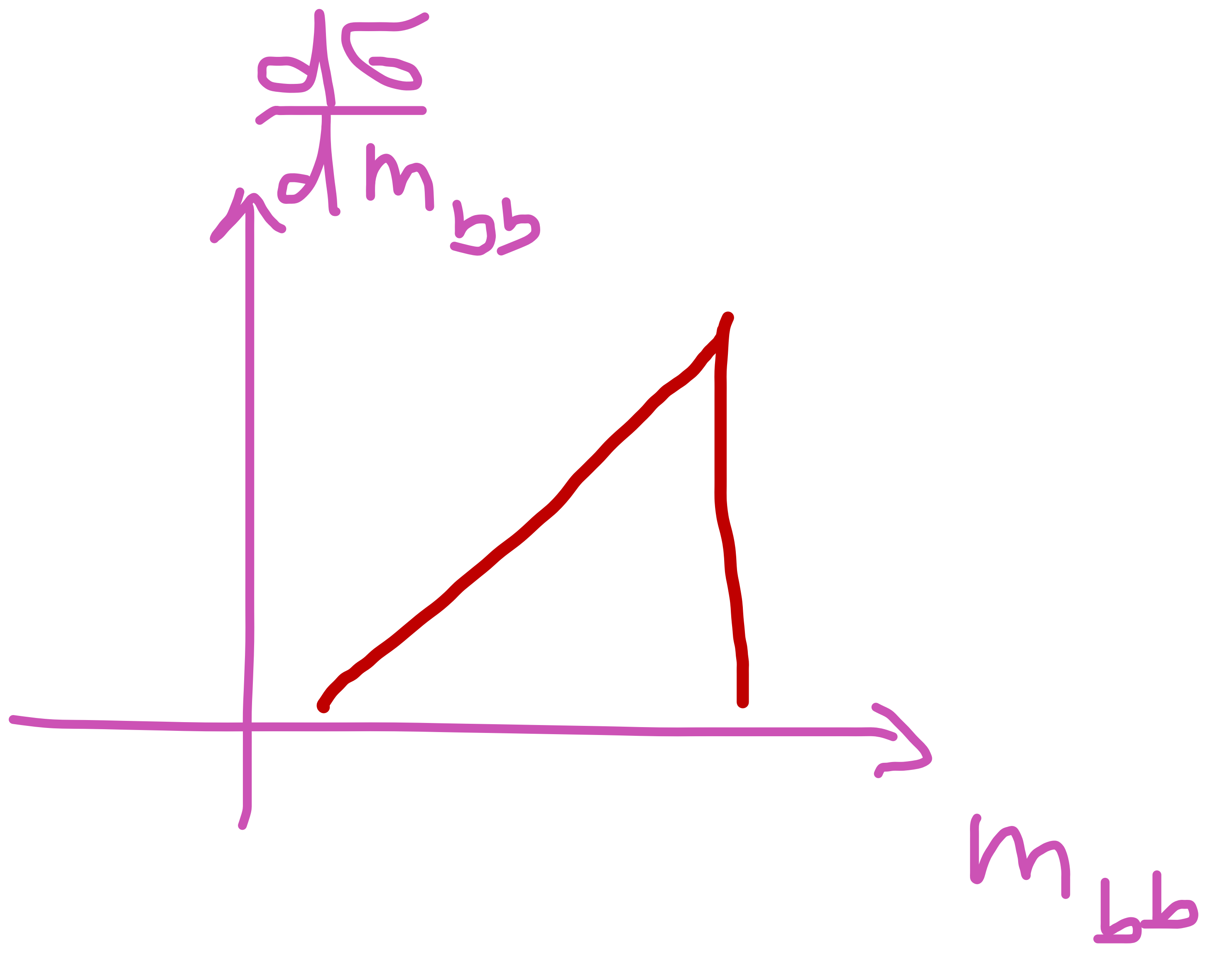
$$m_{top} = 173.1 \pm 2.5 \text{ GeV}$$

- ALL THE EFFECTS AT LEADING ORDER ARE WELL UNDER CONTROL
- HIGHER ORDER QCD WAS NOT INCLUDED ($\lesssim 10\%$)

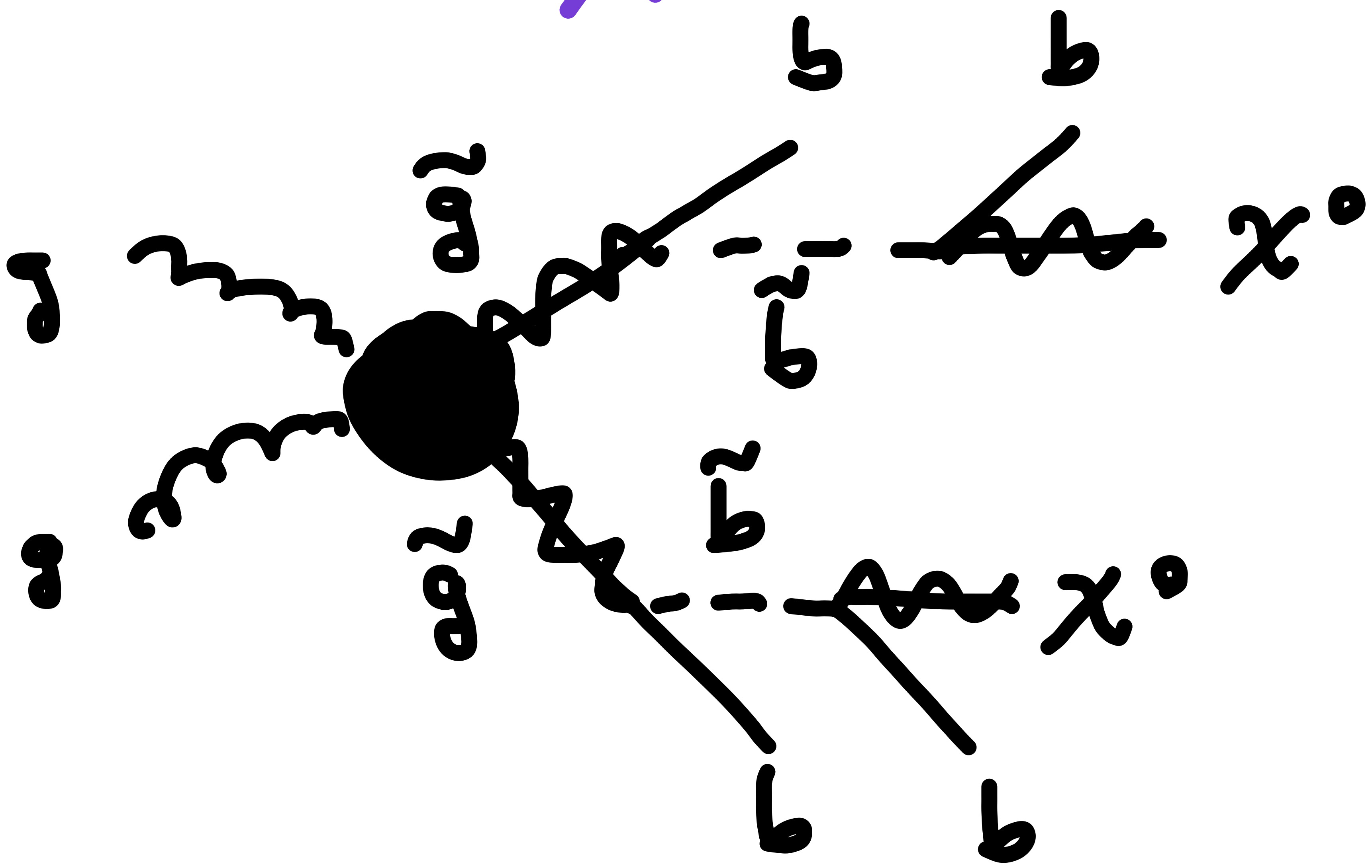
$$pp \to \tilde{g}\tilde{g} \to b\tilde{b}\, b\tilde{b} \to 4b\, \cancel{E}_T$$



$$m_{bb}^{max} = \sqrt{\frac{m_{\tilde{g}}^2 - m_{\tilde{b}}^2}{m_{\tilde{b}}} \cdot \frac{m_{\tilde{b}}^2 - m_{\chi}^2}{m_{\tilde{b}}}}$$



$$pp \to \tilde{g}\tilde{g} \to b\tilde{b}\,b\tilde{b} \to 4b\,\not{E}_T$$



TWO-STEPS DECAY $\Rightarrow$

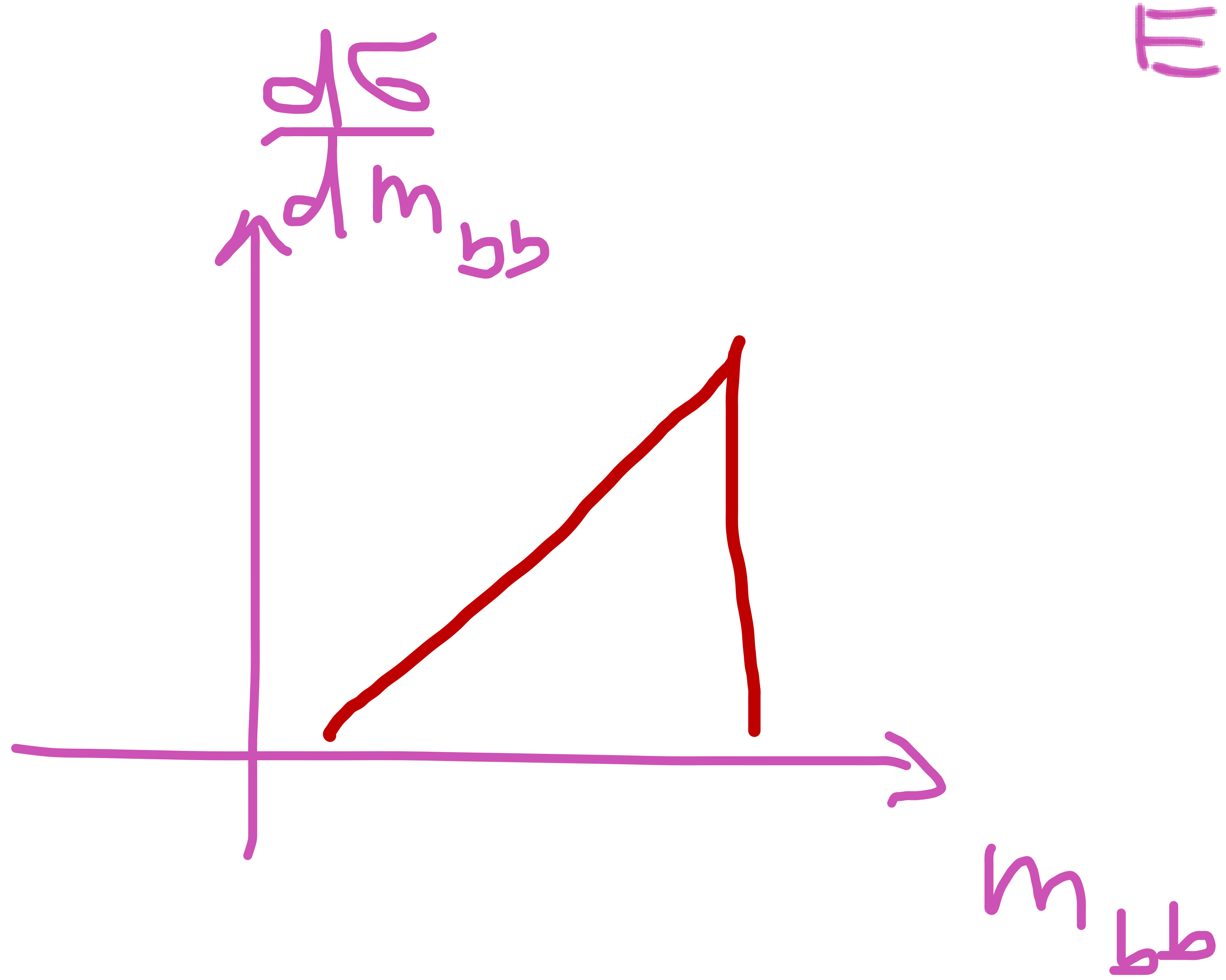
$$E_b^{peak} = \frac{m_{\tilde{g}}^2 - m_{\tilde{b}}^2}{2m_{\tilde{g}}} \qquad E_b^{peak} = \frac{m_{\tilde{b}}^2 - m_{\chi}^2}{2m_{\tilde{b}}}$$

NO COMBINATORICAL ISSUES

JUST LOOK AT $\frac{d\sigma}{dE_b}$

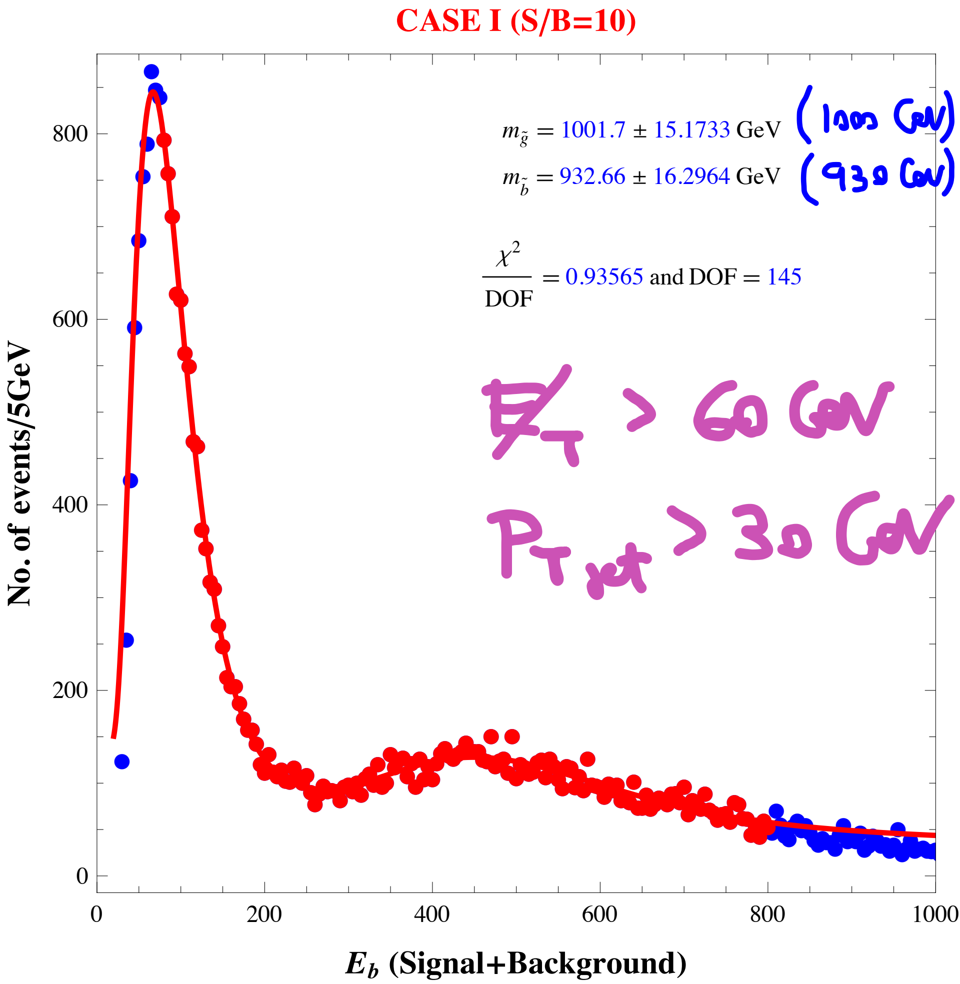
$$pp \to \tilde{g}\tilde{g} \to b\tilde{b}\, b\tilde{b} \to 4b\, \cancel{E}_T$$

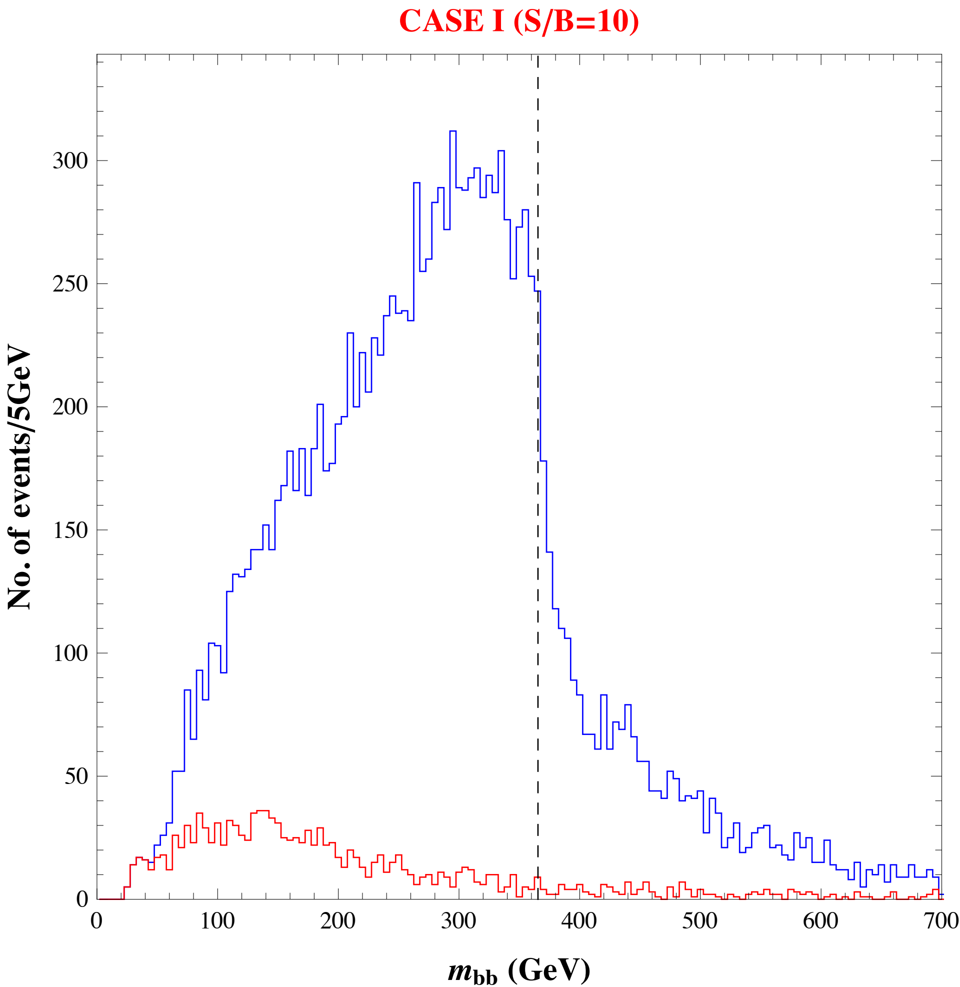
$$\boxed{\begin{aligned} E_{\tilde{b}_H} &= \frac{m_{\tilde{b}}^2 - m_{\chi}^2}{2m_{\tilde{b}}} \\ E_{\tilde{b}_L} &= \frac{m_{\tilde{g}}^2 - m_{\tilde{b}}^2}{2m_{\tilde{g}}} \\ m_{bb}^{max} &= \sqrt{4\frac{m_\chi}{m_{\tilde{b}}} E_{\tilde{b}_H} E_{\tilde{b}_L}} \end{aligned}}$$



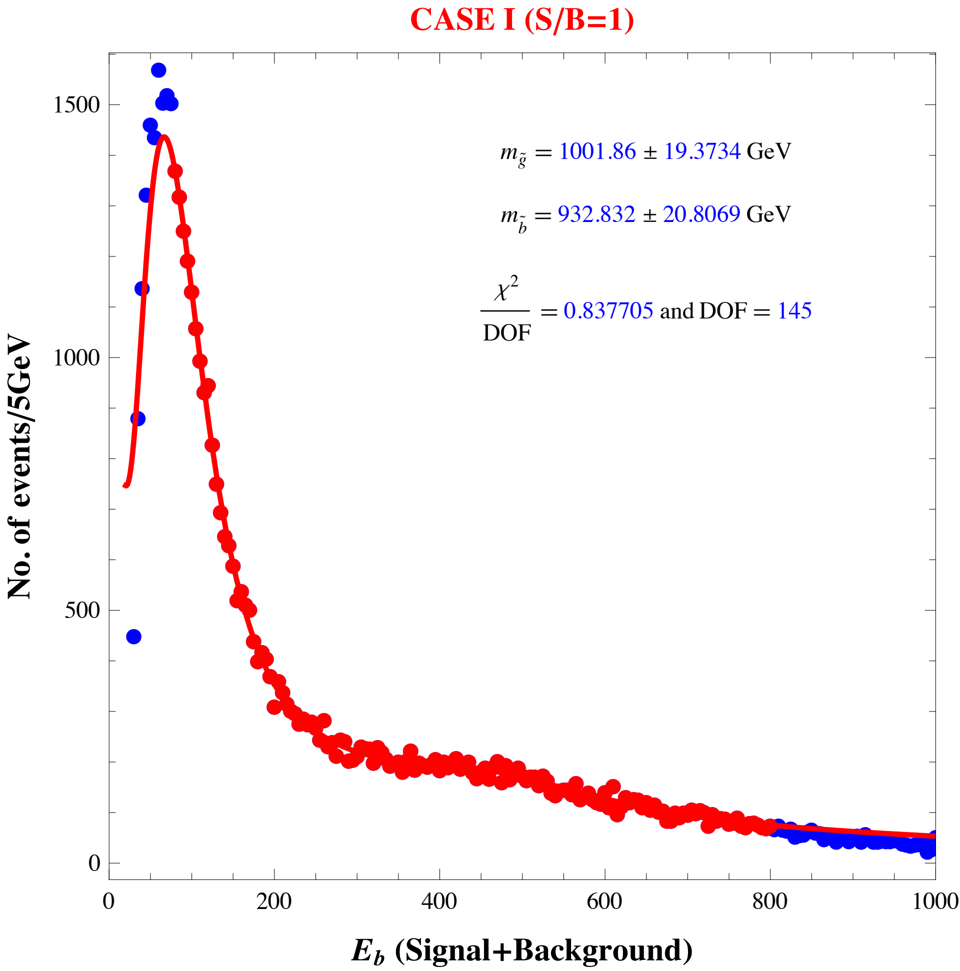
- BACKGROUNDS Z+jets (mostly Z bbbb) & $t\bar{t}b\bar{b}$ (subdom)
- CUTS MAY AFFECT THE ENERGY DISTRIBUTION

$p_{T_{jet}} > x \Rightarrow E_{jet} > x$

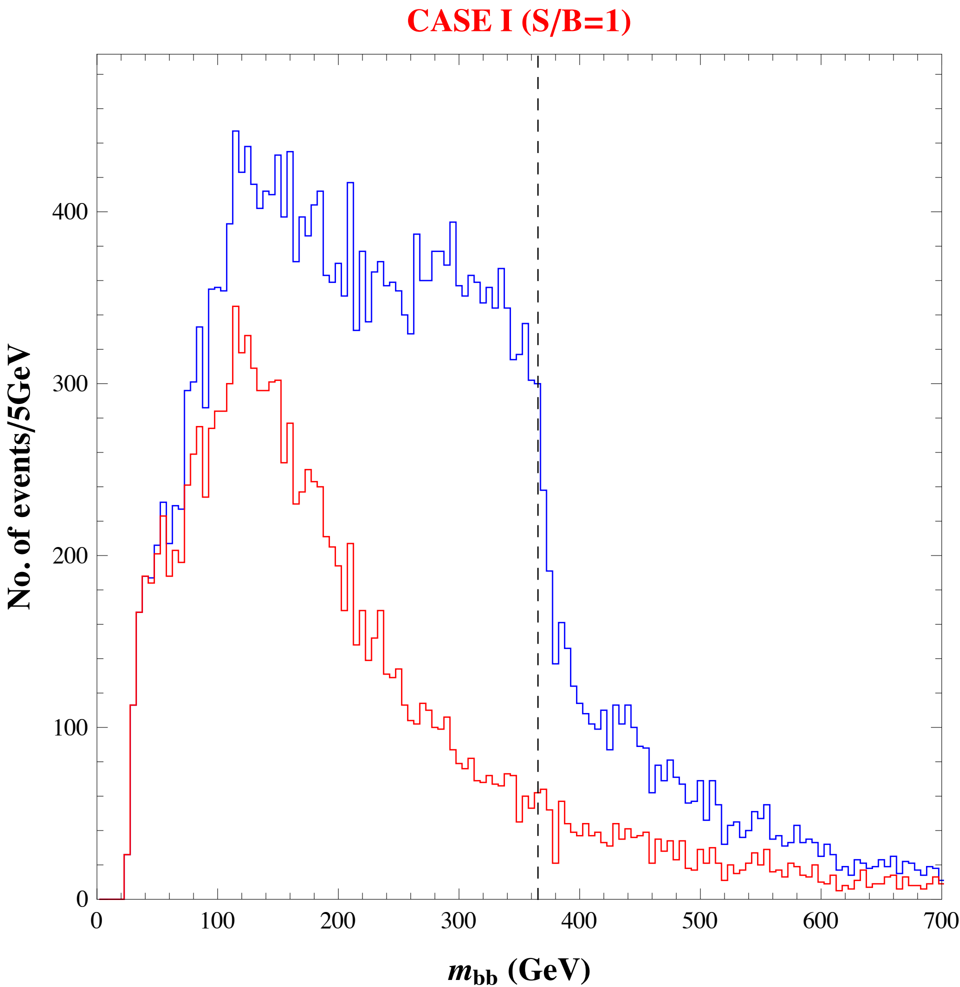

CASE I (S/B=10)
$m_{\tilde{g}} = 1001.7 \pm 15.1733$ GeV
(1000 GeV)
$m_{\tilde{b}} = 932.66 \pm 16.2964$ GeV
(930 GeV)
$\frac{\chi^2}{\mathrm{DOF}} = 0.93565$ and DOF = 145
$\not{E}_T > 60$ GeV
$P_{T\,jet} > 30$ GeV
No. of events/5GeV
E_b (Signal+Background)
0
200
400
600
800
1000



CASE I (S/B=10)
No. of events/5GeV
m_{bb} (GeV)
0
50
100
150
200
250
300
0
100
200
300
400
500
600
700



CASE I (S/B=1)
$m_{\tilde{g}} = 1001.86 \pm 19.3734$ GeV
$m_{\tilde{b}} = 932.832 \pm 20.8069$ GeV
$\frac{\chi^2}{\text{DOF}} = 0.837705$ and DOF = 145
No. of events/5GeV
E_b (Signal+Background)
0
500
1000
1500
200
400
600
800


CASE I (S/B=1)
No. of events/5GeV
0
100
200
300
400
0
100
200
300
400
500
600
700
m_{bb} (GeV)

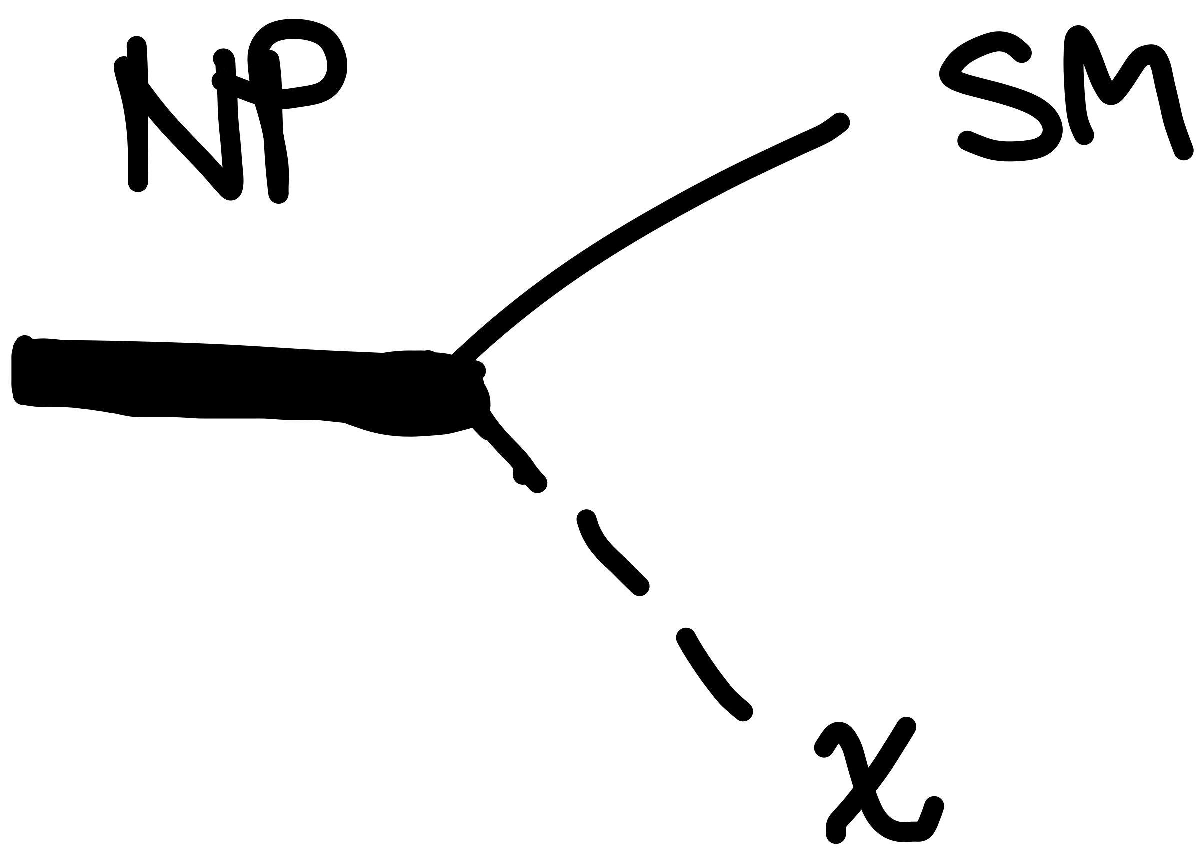
COUNTING INVISIBLE PARTICLES

1212.5230

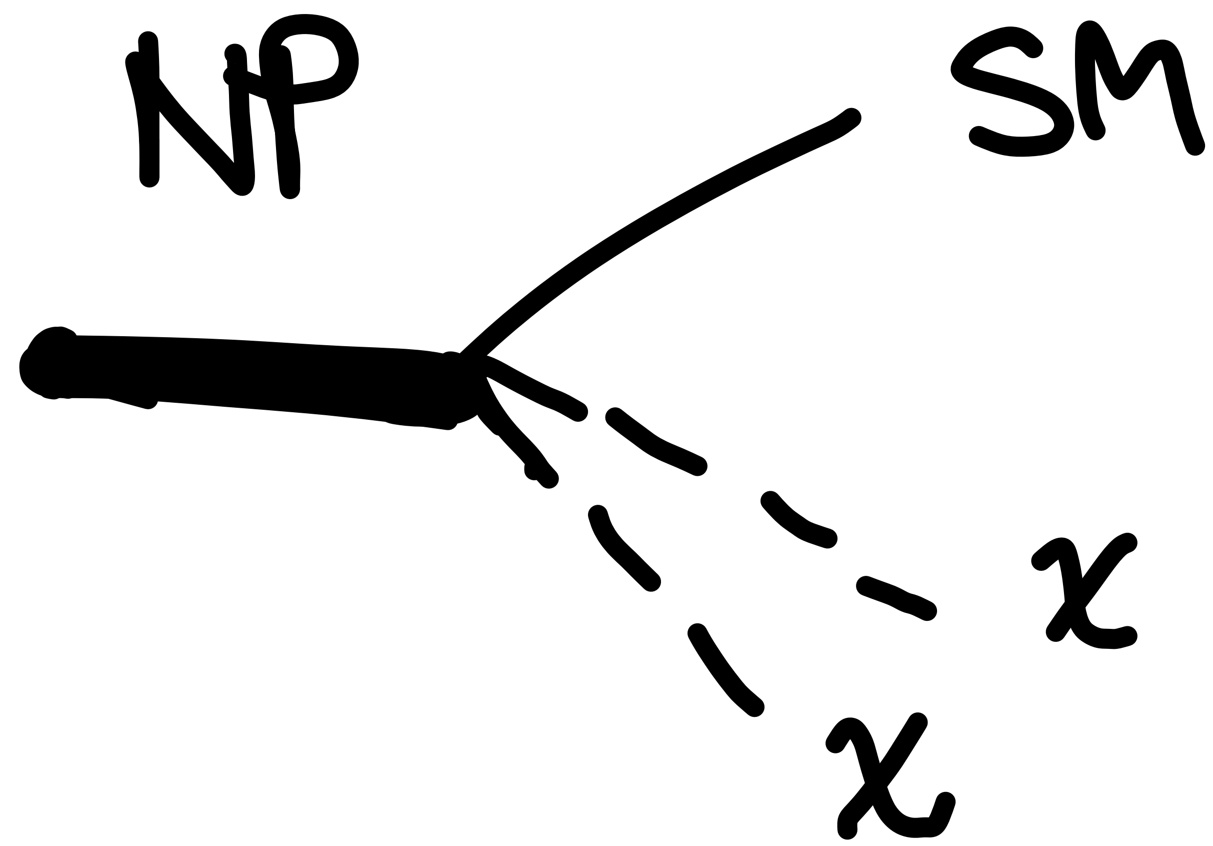
- A LOT OF LHC PHENOMENOLOGY INVOLVES INVISIBLE PARTICLES
- DARK MATTER IS AN INVISIBLE PARTICLE

WHAT SYMMETRY PREVENTS THE DM FROM DECAYING

2-BODIES

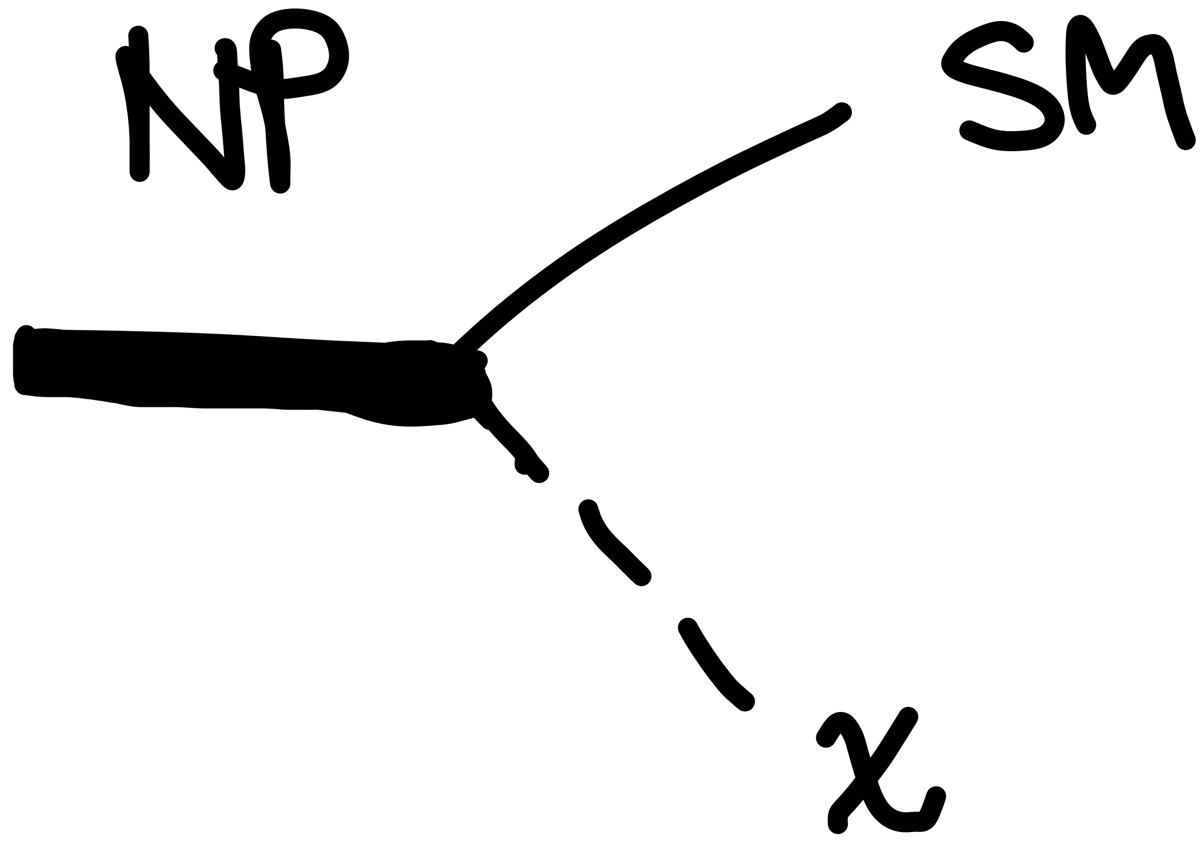


3-BODIES



ONE VISIBLE PARTICLE IN EACH DECAY : VERY LITTLE INFORMATION!

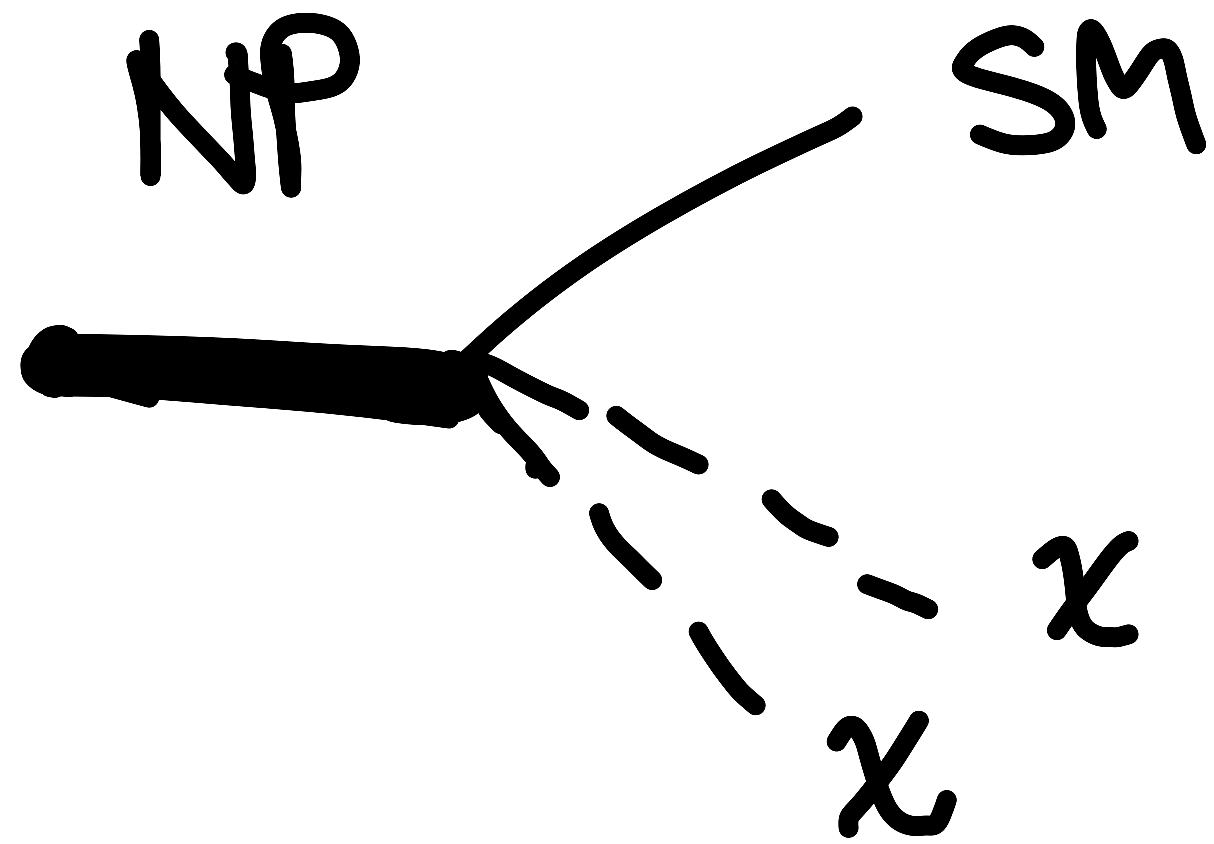
2-BODIES



E_{SM} HAS A PEAK AT

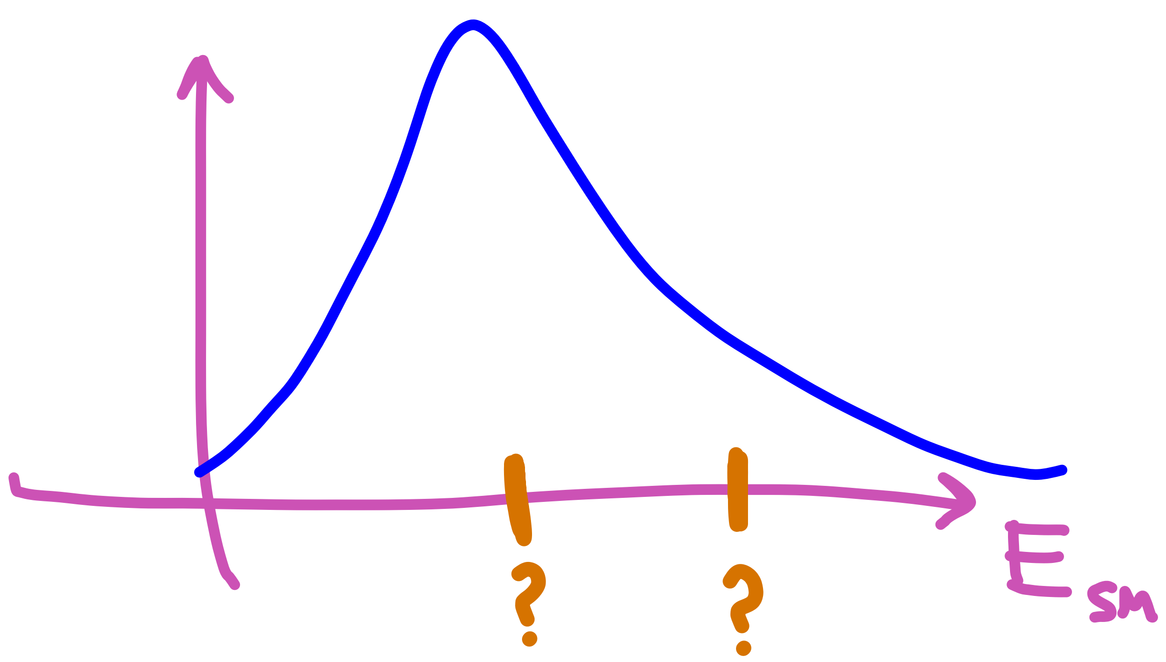
$$E_{peak} = \bar{E} = \frac{m_{NP}^2 - m_\chi^2 + m_{SM}^2}{2m_{NP}}$$

3-BODIES



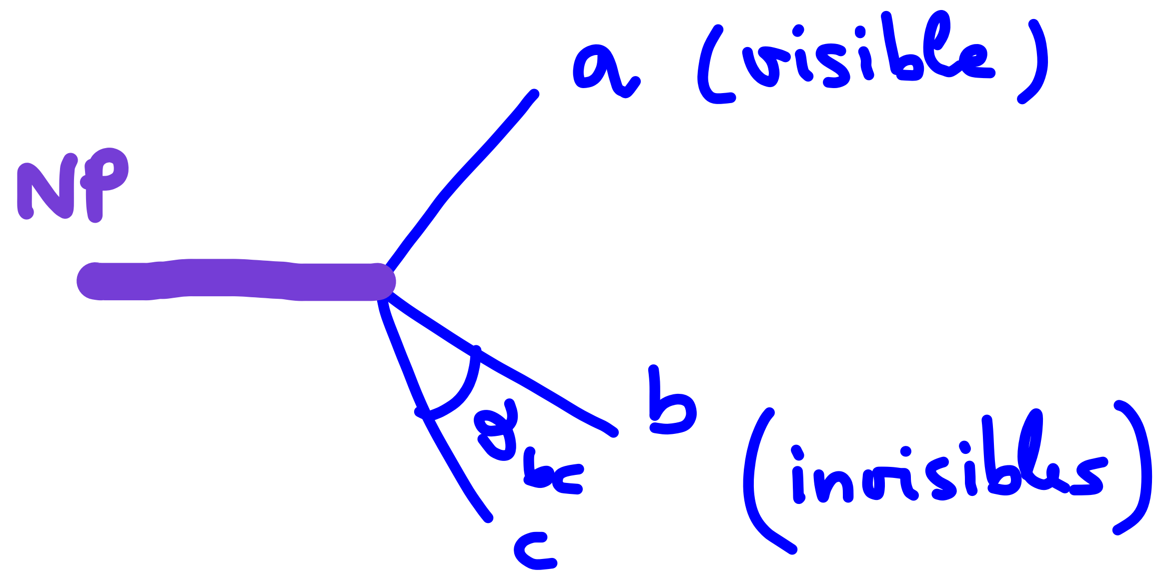
E_{SM} HAS A PEAK BELOW

$$E_{peak} \ll \bar{E}' = \frac{m_{NP}^2 + m_{SM}^2 - 2m_\chi^2}{2m_{NP}}$$



OBSERVING E_{SM} ONLY WE CANNOT TELL

n-BODIES REDUCE TO FEWER WHEN BODIES ARE ALIGNED

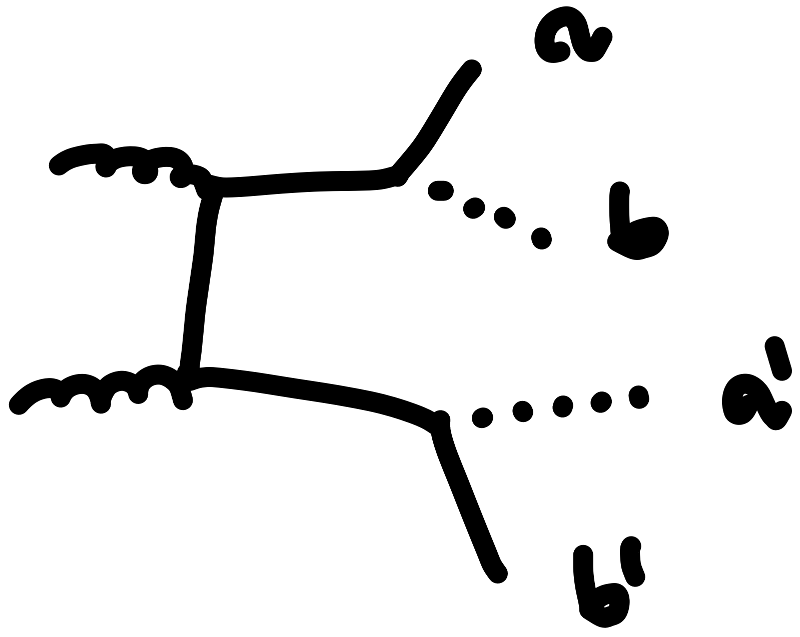


WHEN $\theta_{bc} \to 0$

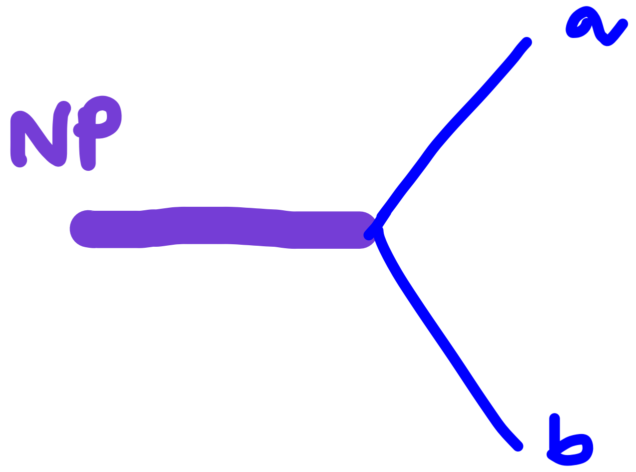
MEASURE SOME QUANTITY THAT SINGLES OUT THIS COLLINEAR CONFIGURATION

- b AND c ARE INVISIBLE
- MEASURE ONLY THE SUM OF THE INVISIBLES

m_T AND m_{T_2}



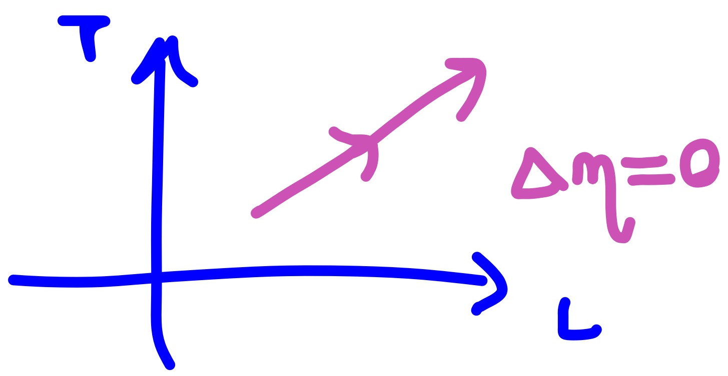
m_T IS A PROJECTION OF THE INVARIANT MASS

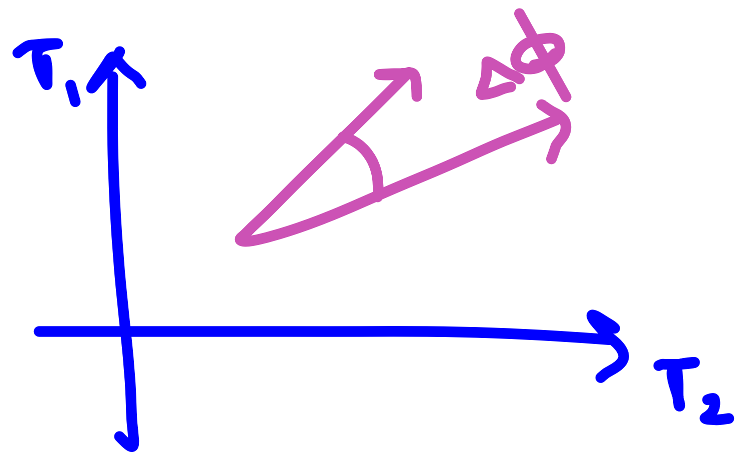


$$m_{NP}^2 = m_a^2 + m_b^2 + 2\left(E_{T_a} E_{T_b} \cosh \Delta\eta_{ab} - p_{T_a} p_{T_b} \cos \Delta\phi_{ab}\right)$$

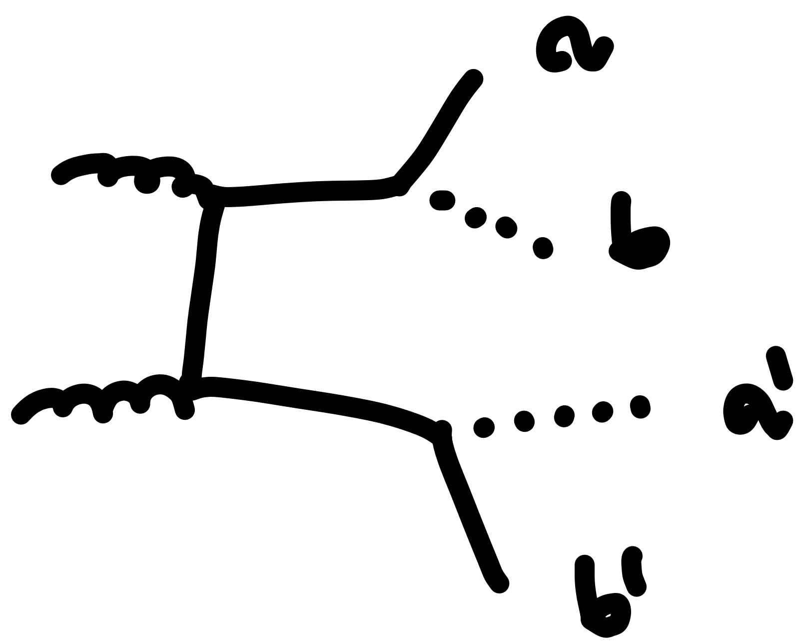
$$m_T^2 = m_a^2 + m_b^2 + 2\left(E_{T_a} E_{T_b} - p_{T_a} p_{T_b} \cos \Delta\phi_{ab}\right)$$

$m_T \leq m_{NP}$ AND $m_T = m_{NP}$ $\Delta\eta_{ab} = 0$





THE MAX OF m_T SINGLES OUT A KIND OF COLLINEAR CONFIGURATION



- BOTH a AND a' ARE INVISIBLE
- OBSERVE ONLY $\not{E}_T$ (THE SUM OF THE INVISIBLES)
- MAKE AN ANSATZ $\bar{P}_{T_a} + \bar{P}_{T_{a'}} = \not{E}_T$

$$m_{T_2} = \min_{\text{ansatz}} \{ \max (m_T , m_T) \}$$

- min OVER ALL POSSIBLE WAYS TO SPLIT THE MISSING MOMENTUM

m_{T_2} IS MAXIMIZED WHEN PARTICLES ARE ALIGNED

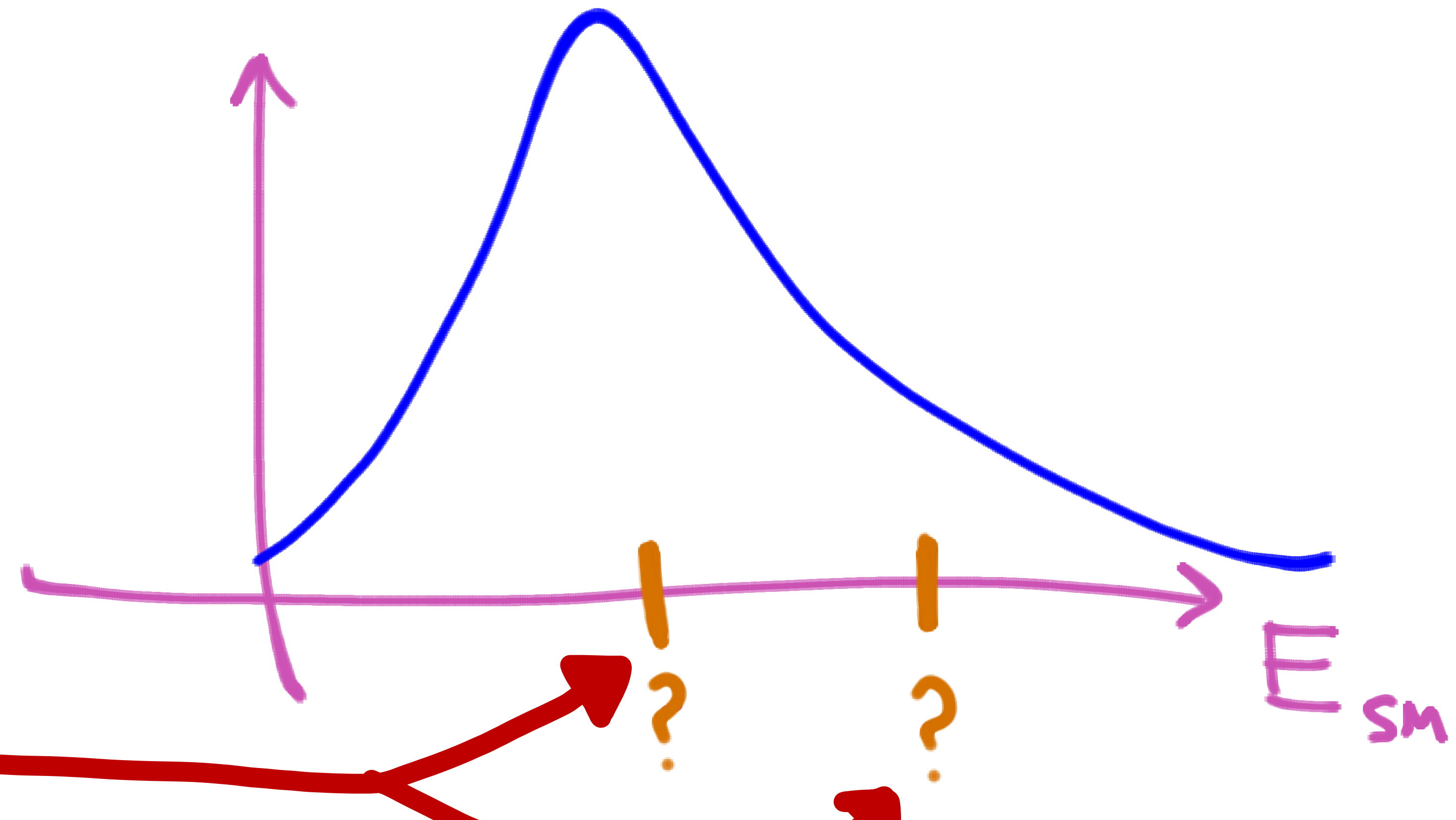
CLOSED FORM: "BALANCED CASE"

$$m_{T_2}^{max} = C + \sqrt{C^2 + \tilde{m}^2}$$

$$C_2 = \frac{m_{NP}^2 - m_a^2}{2 m_{NP}} = \boxed{E_{PEAK}}$$

$$C_n = \frac{m_{NP}^2 - n^2 m_a^2}{2 m_{NP}}$$

$m_a \to n\, m_a$



APPLICATION TO BOTTOM QUARK PARTNERS

$pp \to B'B'$ FOLLOWED BY $B' \to b\chi$ $\mathbb{Z}_2$-model

$B' \to b\chi\chi$ non-$\mathbb{Z}_2$ model

POST-DISCOVERY

$\Downarrow$

LARGE S/B

0 leptons with $|\eta_l| < 2.5$ and $p_{T\,l} > 20$ GeV for $l = e,\ \mu,\ \tau$,

2 b-tagged jets with $|\eta_b| < 2.5$ and $p_{T\,b_1} > 100$ GeV, $p_{T\,b_2} > 40$ GeV,

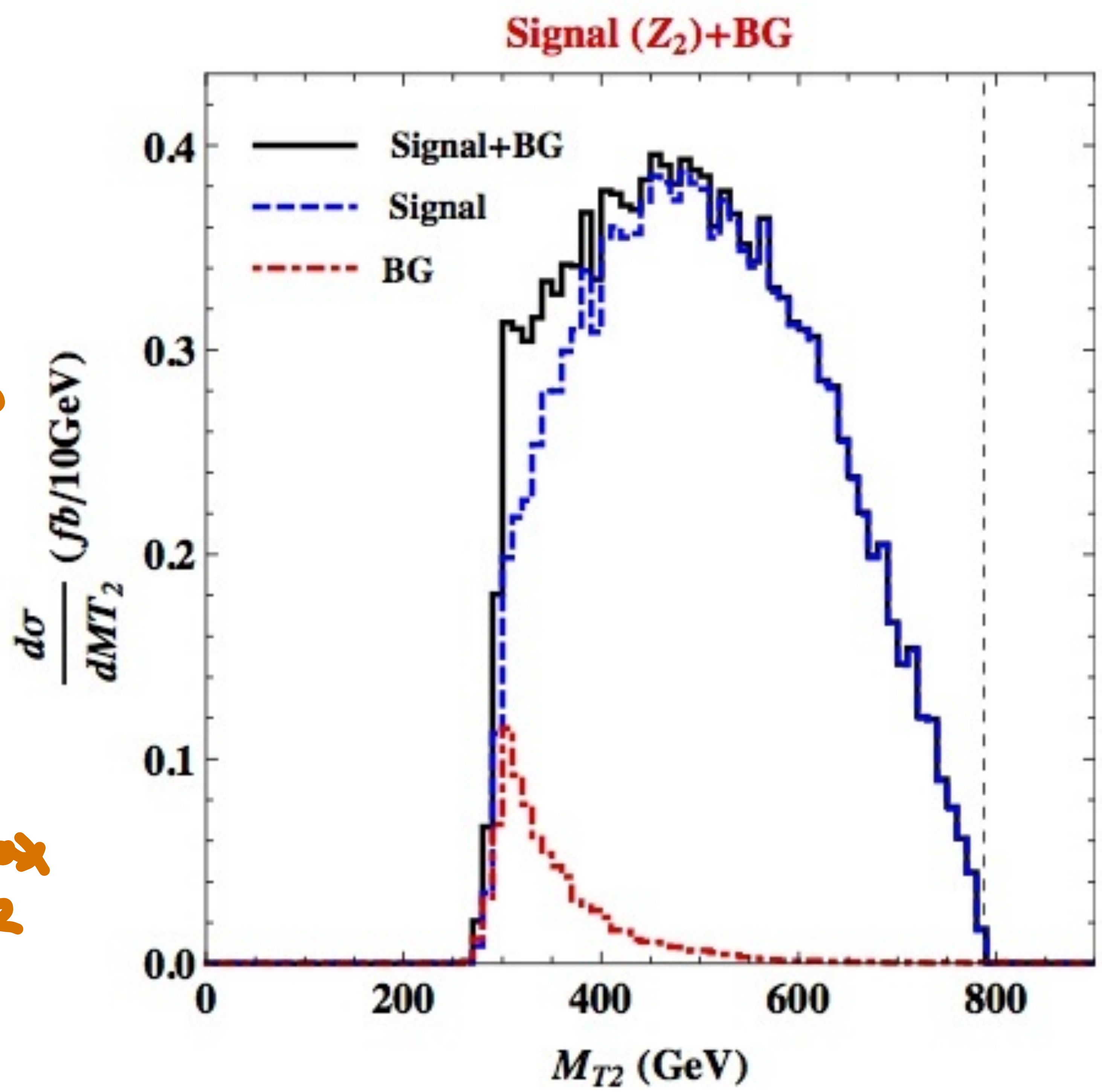
$\not{E}_T > 300$ GeV,

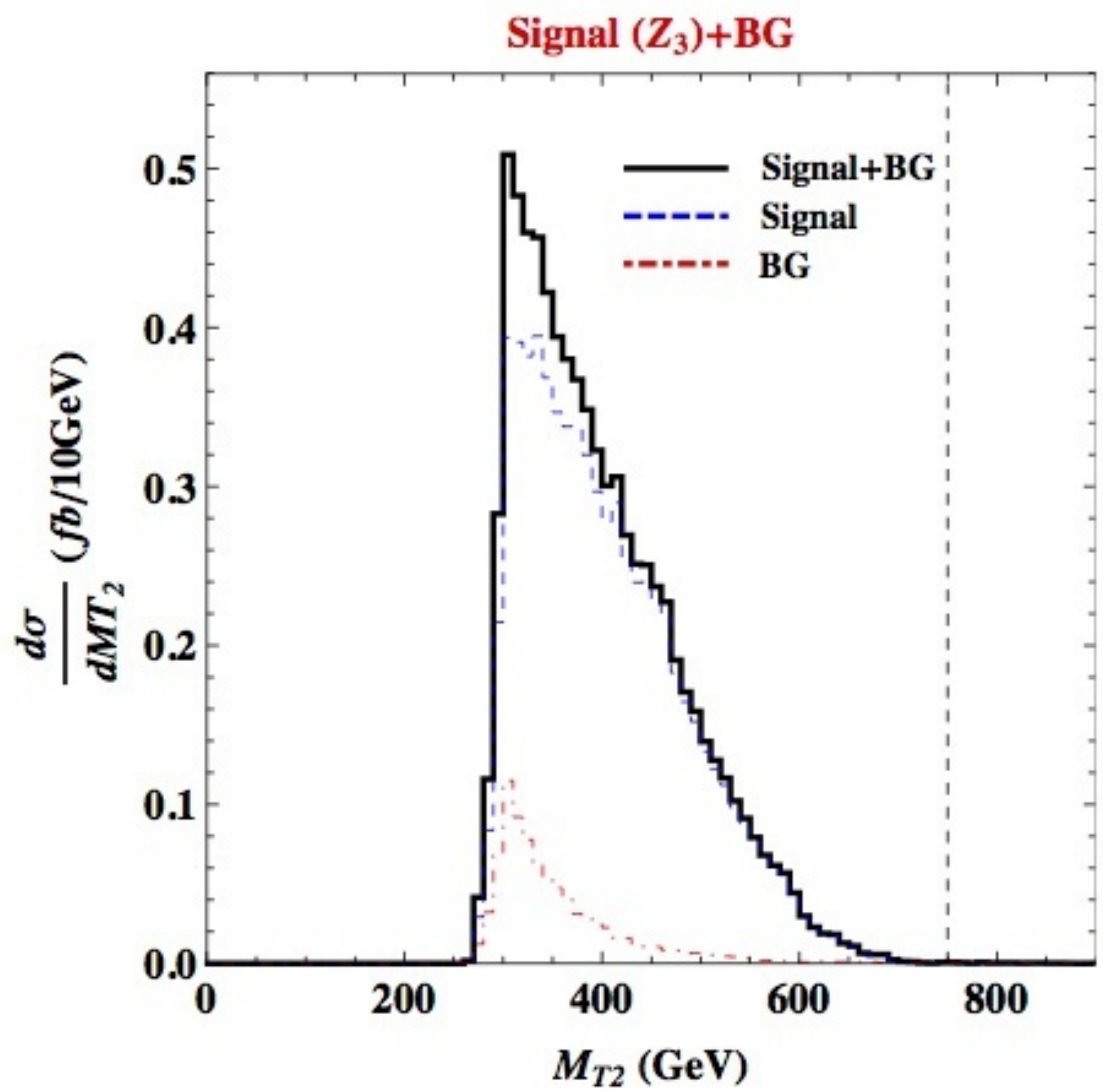
$S_T > 0.4$,

$f > 0.3$,

STEP 1:

COMPUTE M_{T2} FOR ALL EVENTS FOR A TRIAL MASS OF THE INVISIBLE





STEP 2:

$$C + \sqrt{C^2 + m_{TRIAL}^2} = m_{T2}^{max}$$

TO GET C

STEP 1:

COMPUTE M_{T2} FOR ALL EVENTS FOR A TRIAL MASS OF THE INVISIBLE

STEP 2:

$$C + \sqrt{C^2 + m^2_{TRIAL}} = m^{max}_{T2}$$

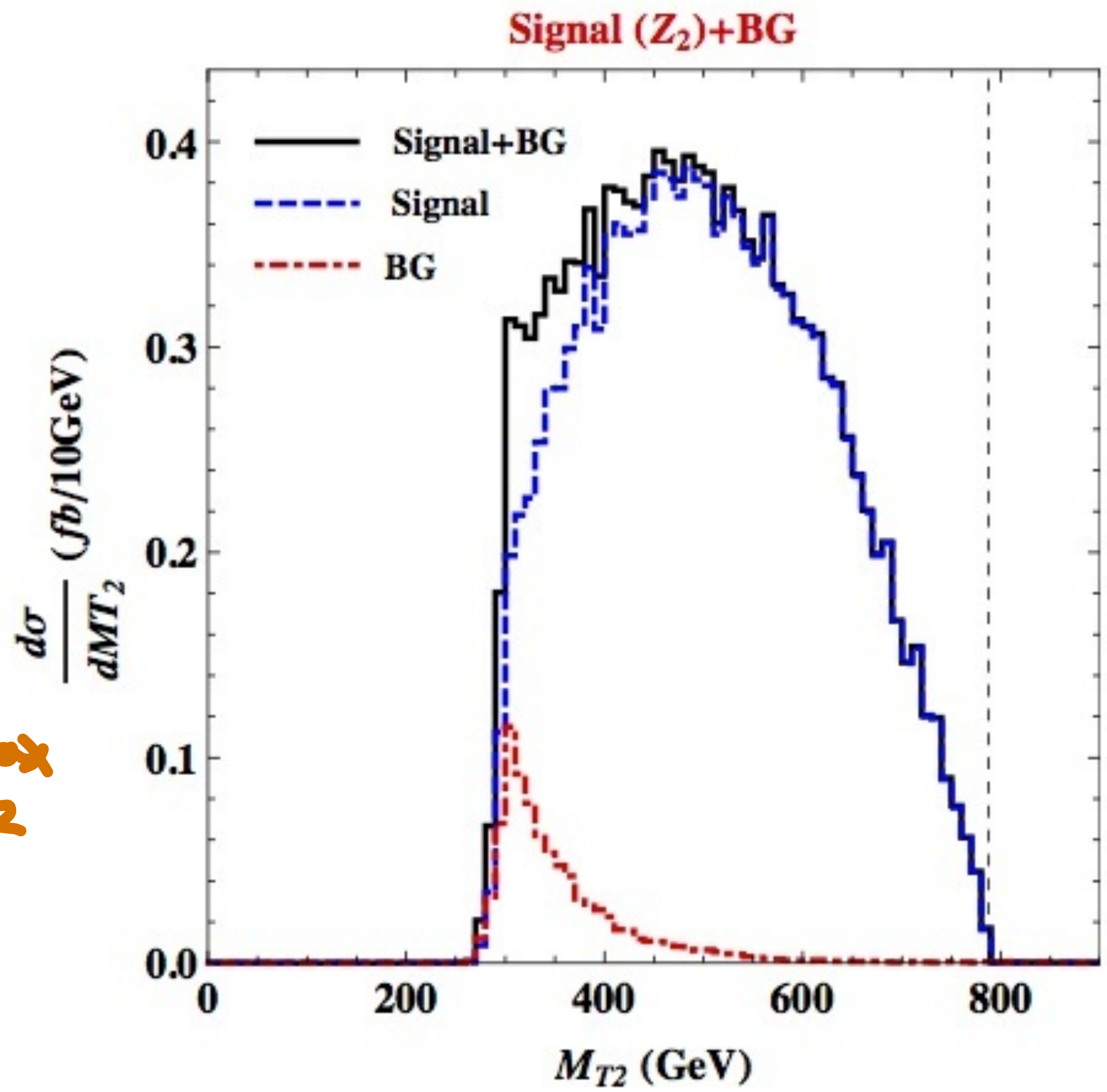
TO GET C

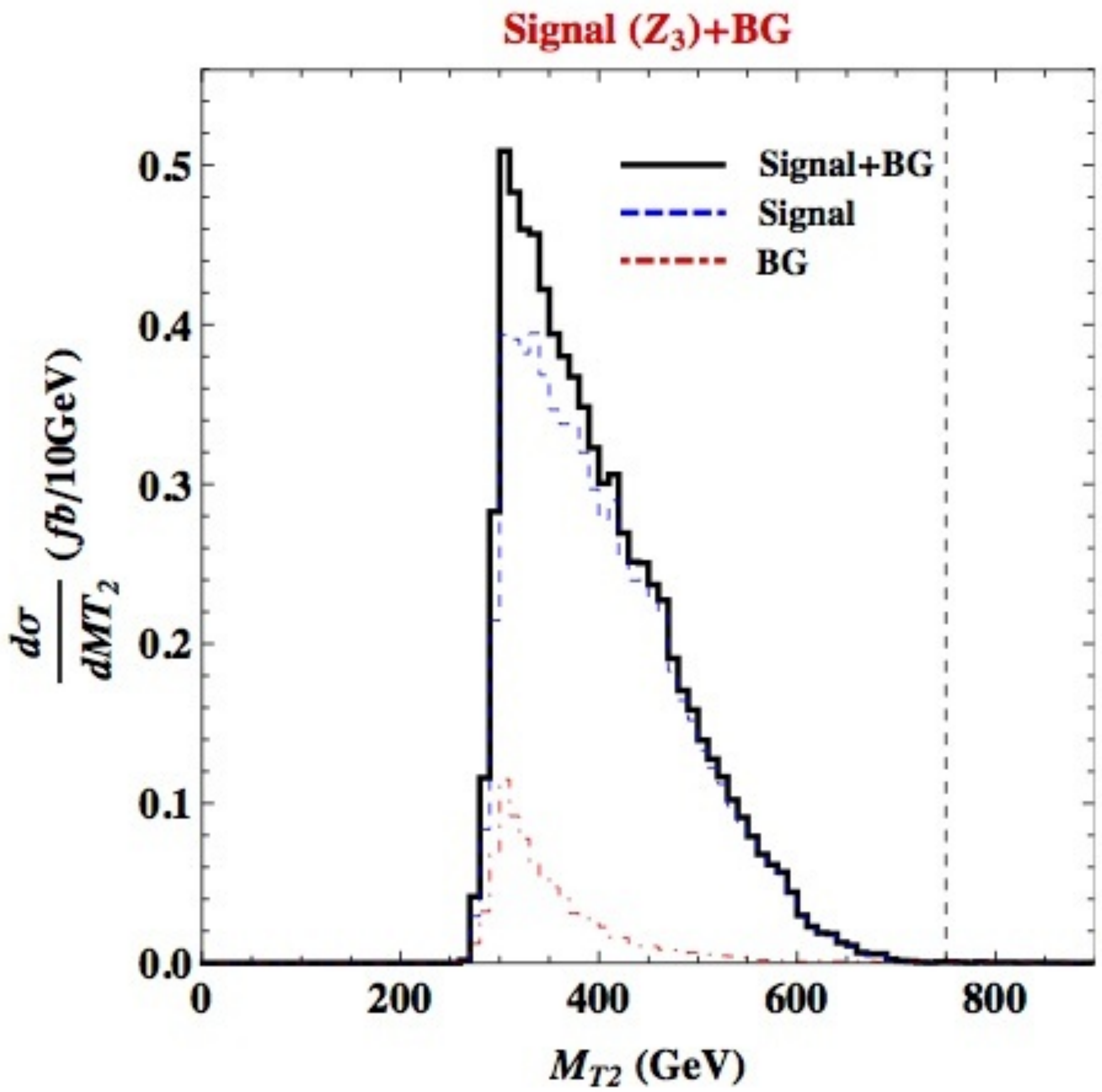
STEP 3:

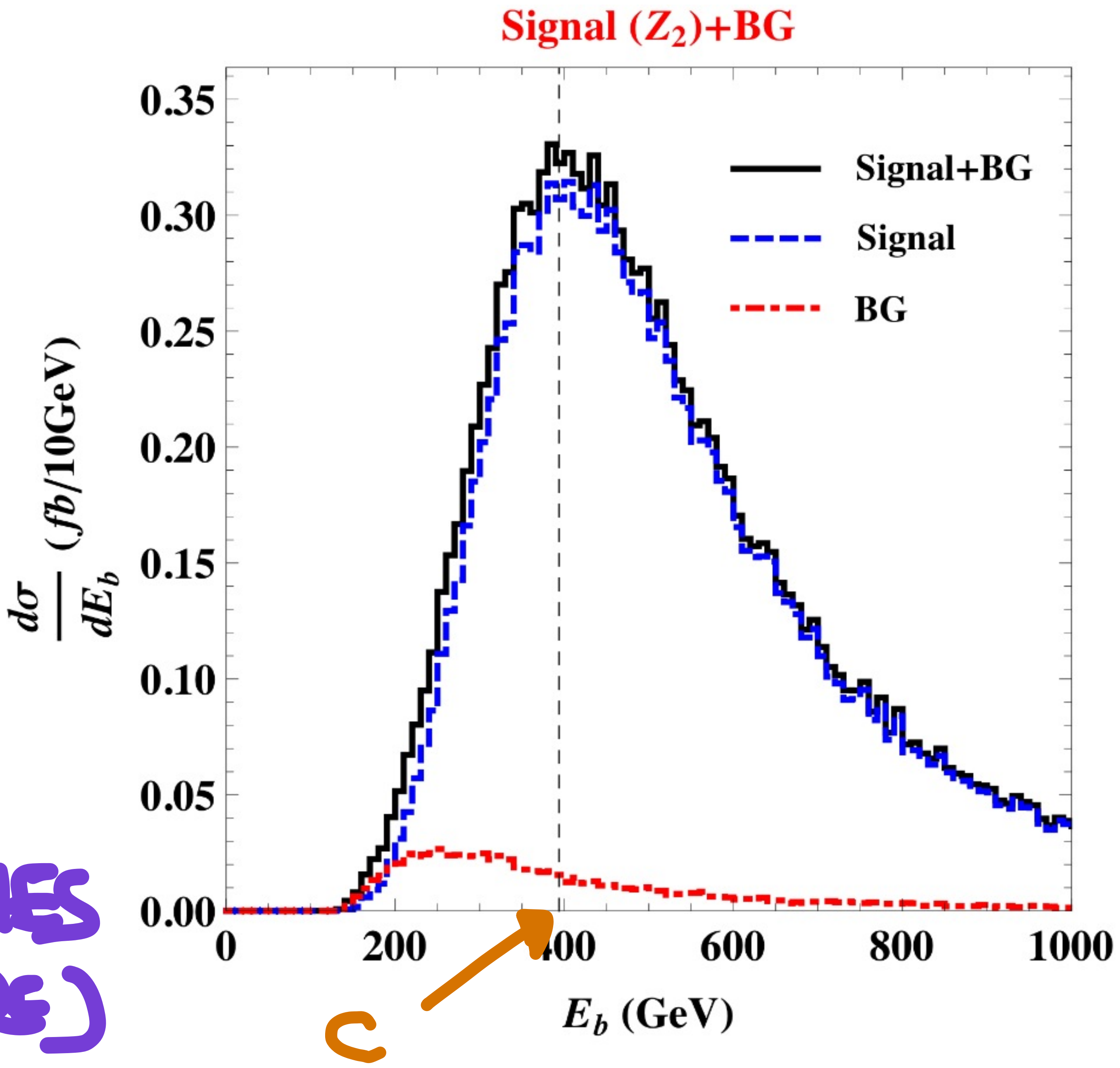
FIND THE PEAK OF THE ENERGY DISTRIBUTION (POSSIBLY REMOVING BG)

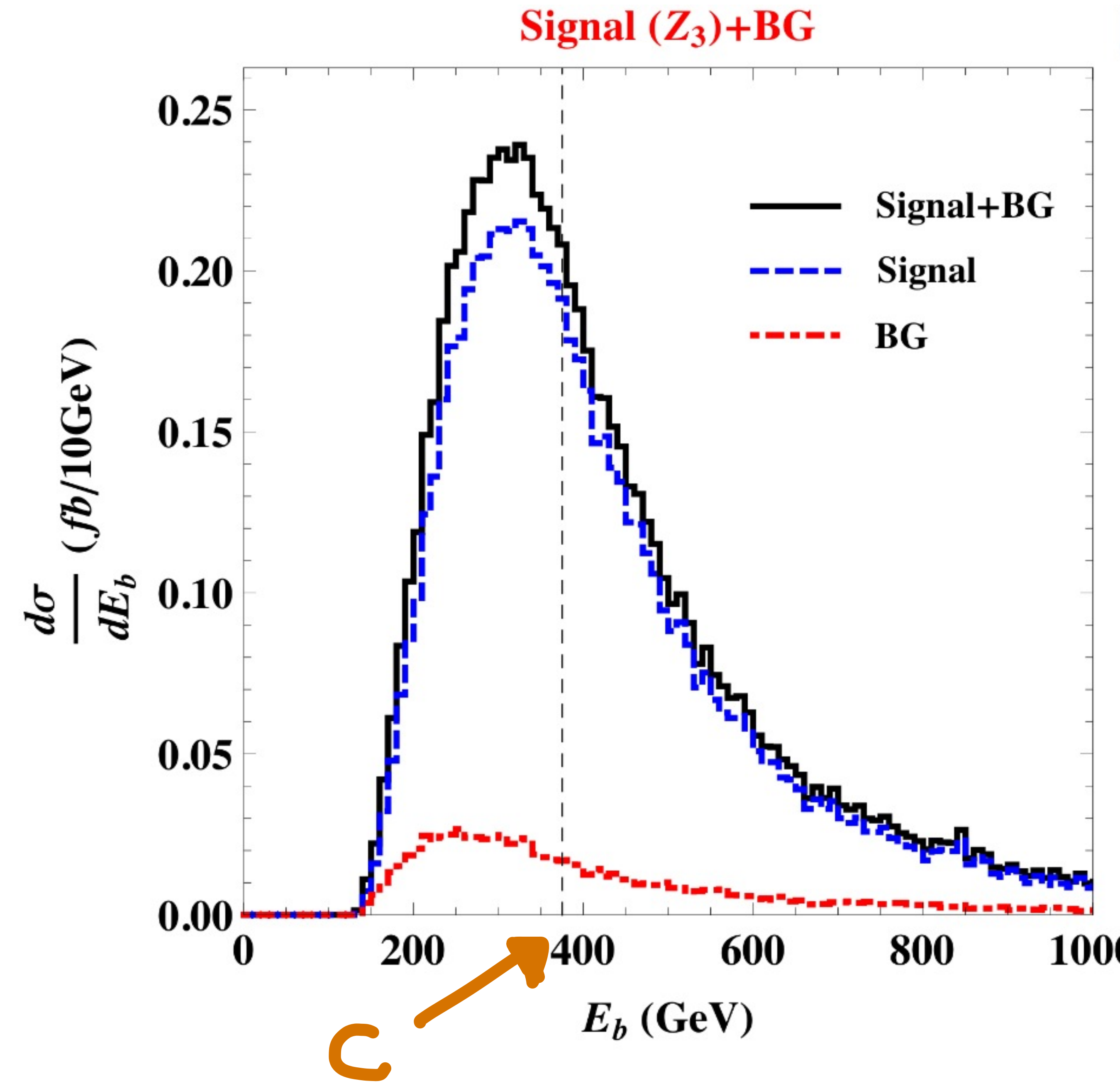
$E_{PEAK} = C \rightarrow$ 2 BODIES

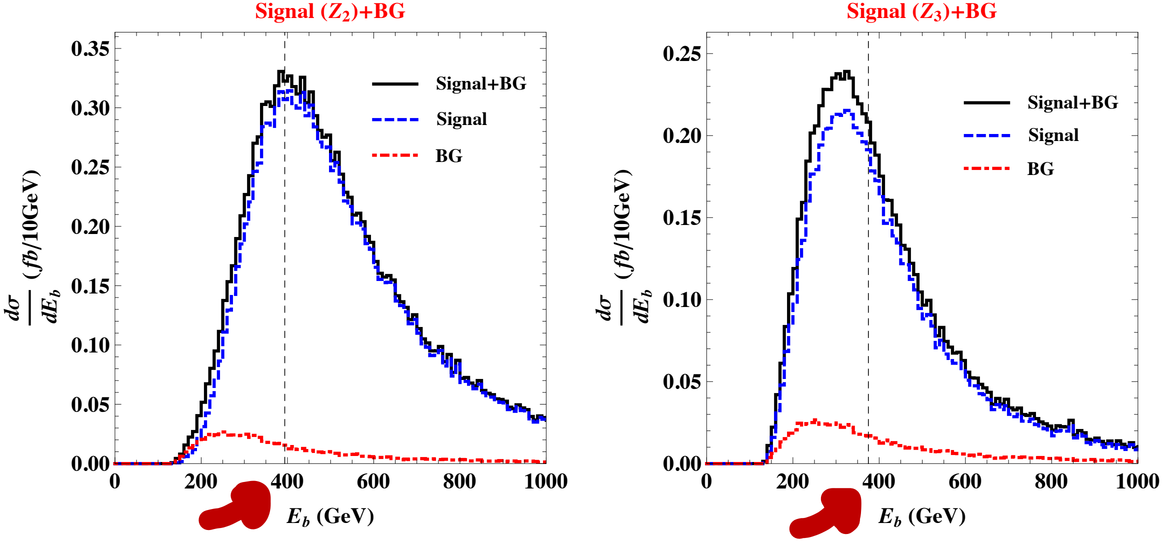
$E_{PEAK} < C \rightarrow$ 3 BODIES (OR MORE)









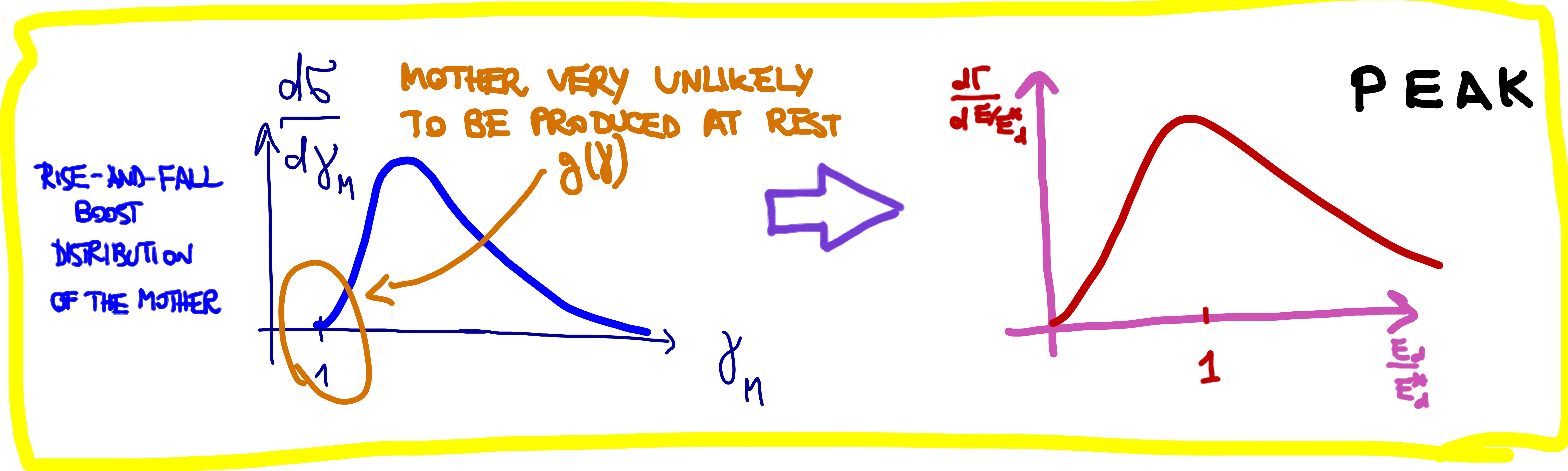


Signal (Z_2)+BG
Signal+BG
Signal
BG
$\frac{d\sigma}{dE_b}$ (fb/10GeV)
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0
200
400
600
800
1000
E_b (GeV)
Signal (Z_3)+BG
Signal+BG
Signal
BG
$\frac{d\sigma}{dE_b}$ (fb/10GeV)
0.00
0.05
0.10
0.15
0.20
0.25
0
200
400
600
800
1000
E_b (GeV)

Conclusions

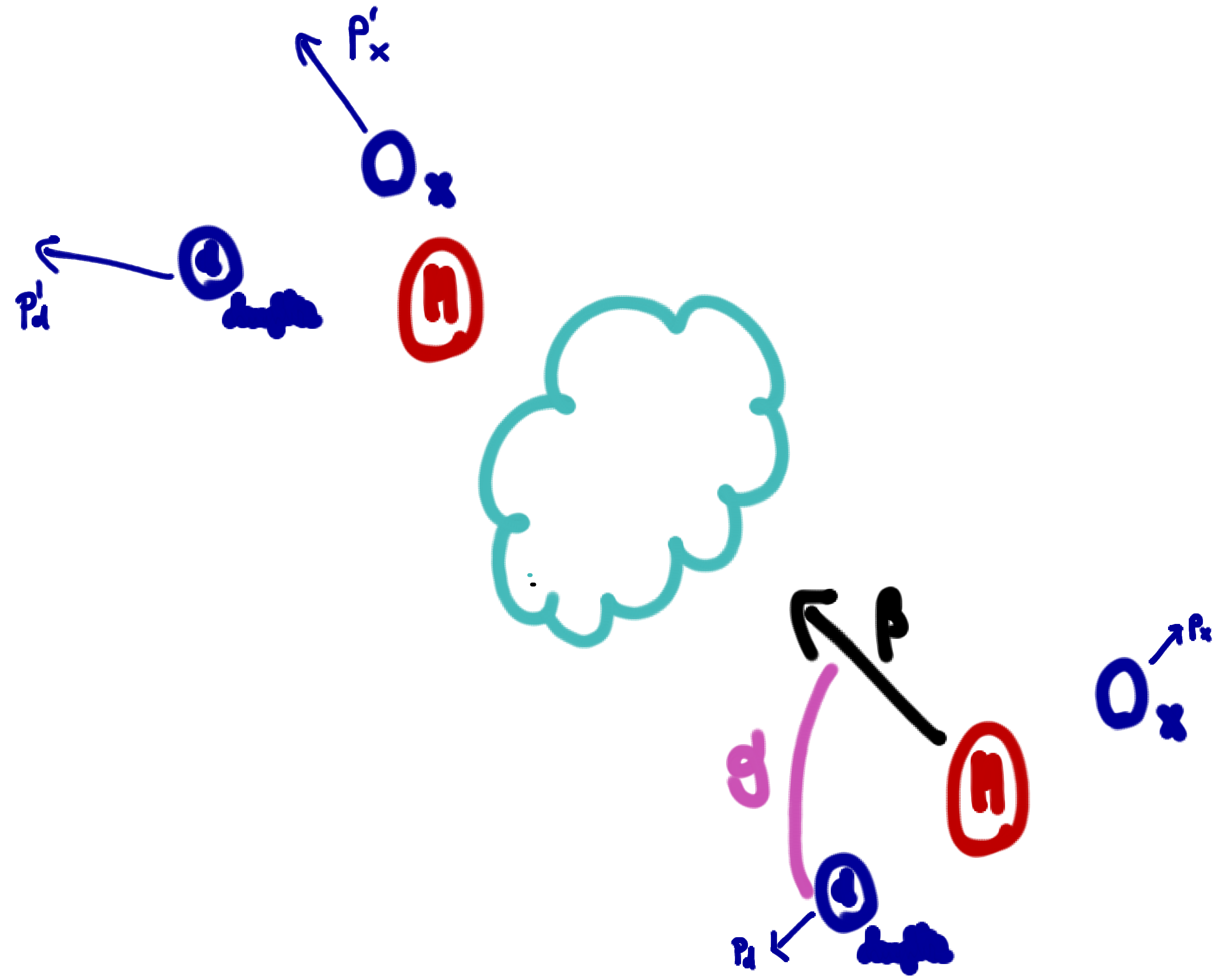
IN PHENOMENOLOGICALLY RELEVANT CASES (HIGH ENERGY COLLIDERS)

THE SPECTRUM OF ENERGY IN TWO BODY DECAYS ENCODES IN A SIMPLE WAY AN INVARIANT OF THE TWO BODY DECAY KINEMATICS



KINKS OR PLATEAUS ARE POSSIBLE AS WELL

Conclusions



THE PEAK OF THE ENERGY DISTRIBUTION IS ROBUST FOR MASSLESS AND MASSIVE DAUGHTERS

$$E_{peak} = \frac{m_M^2 - m_x^2}{2m_M}$$

$$E_{peak} \geq \frac{m_M^2 - m_x^2 + m_d^2}{2m_M}$$

LIMITING FACTORS:

- RADIATIVE CORRECTIONS

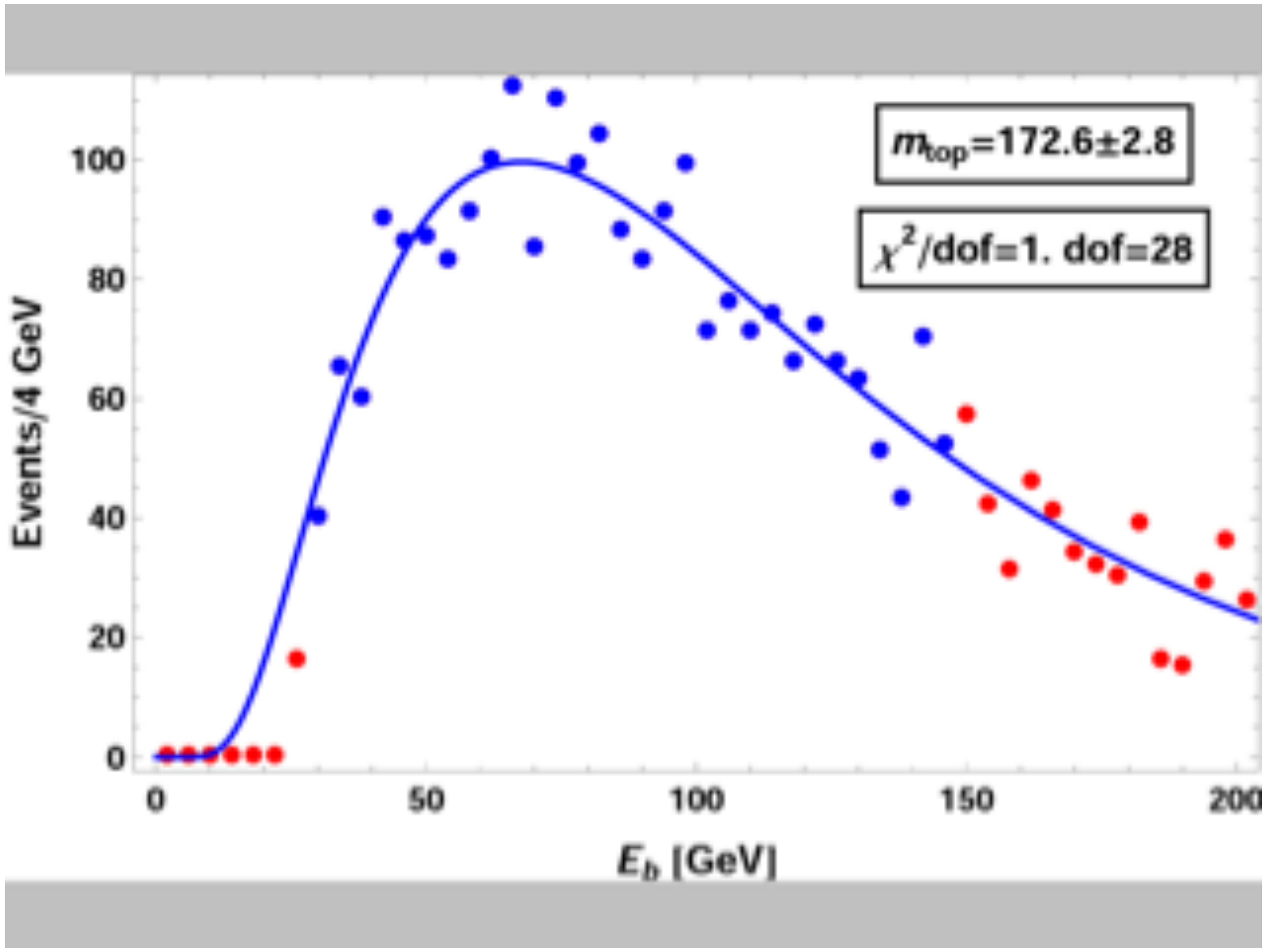
  EXTRA RADIATION MAKES THE DECAY 3-BODY

- TOO LARGE MASS OF THE OBSERVED DAUGHTER

- MAY BE SENSITIVE TO SELECTION CUTS

DESPITE THESE LIMITATIONS THE OBSERVATION CAN BE USED TO MEASURE PARTICLE MASSES WITH 10% ACCURACY OR BETTER

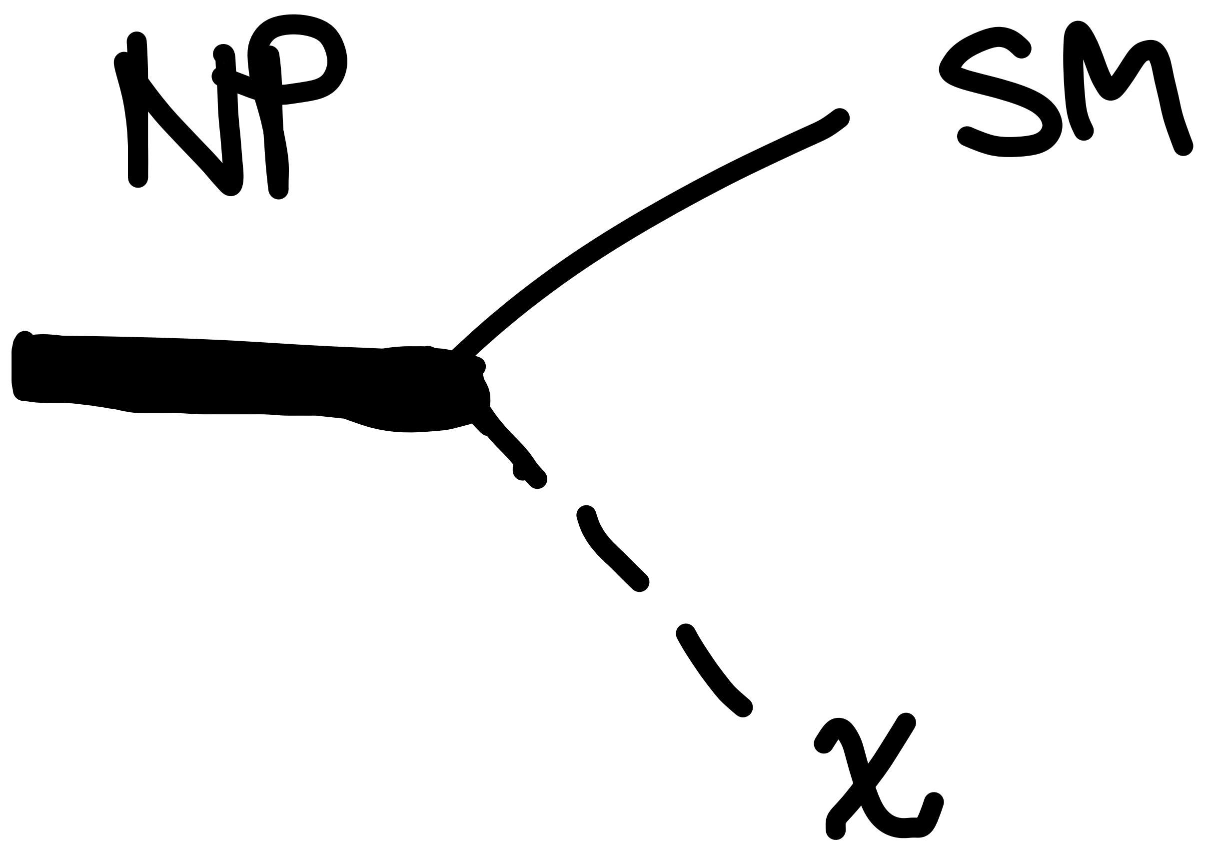
$t \to b \ell \nu$ in $pp \to t\bar{t}$ ⇨ m_{top} FROM $d\sigma/dE_b$



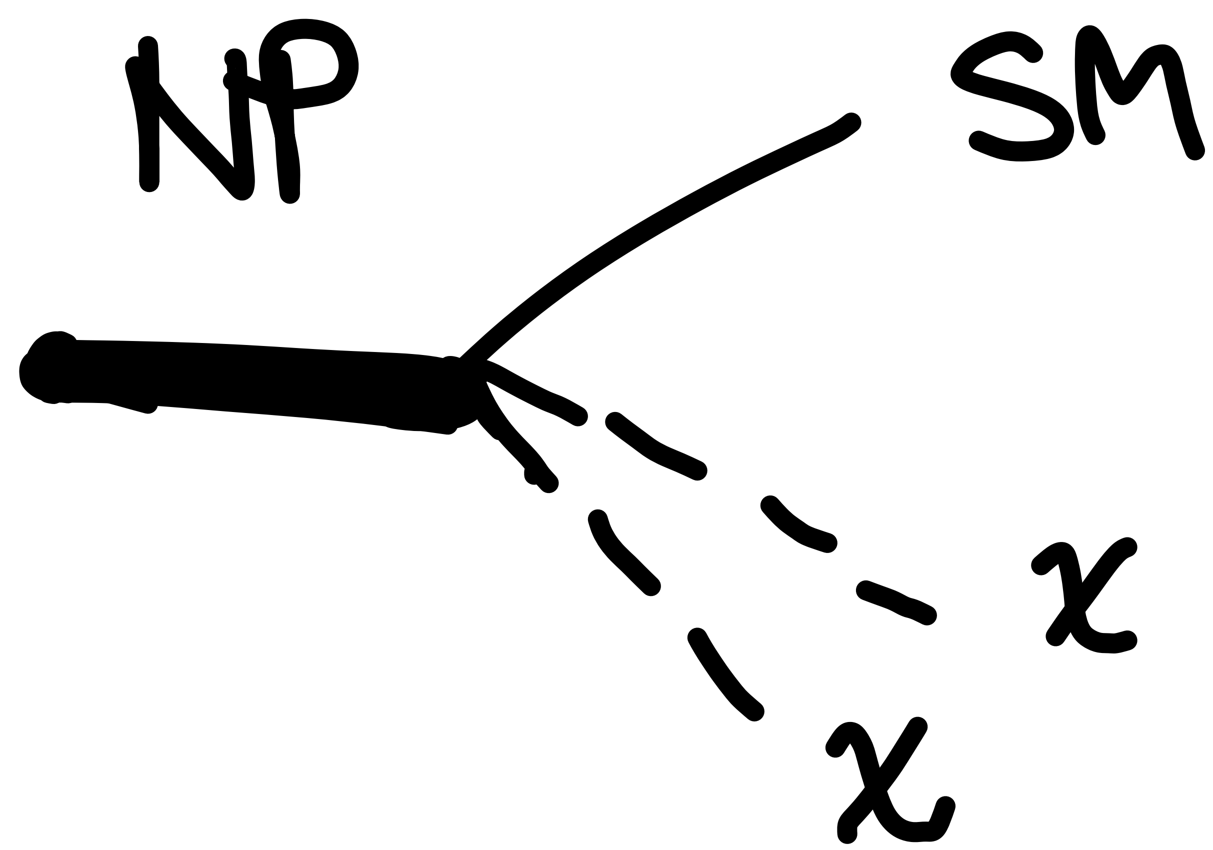
$g \to b\tilde{b} \to bb\chi$

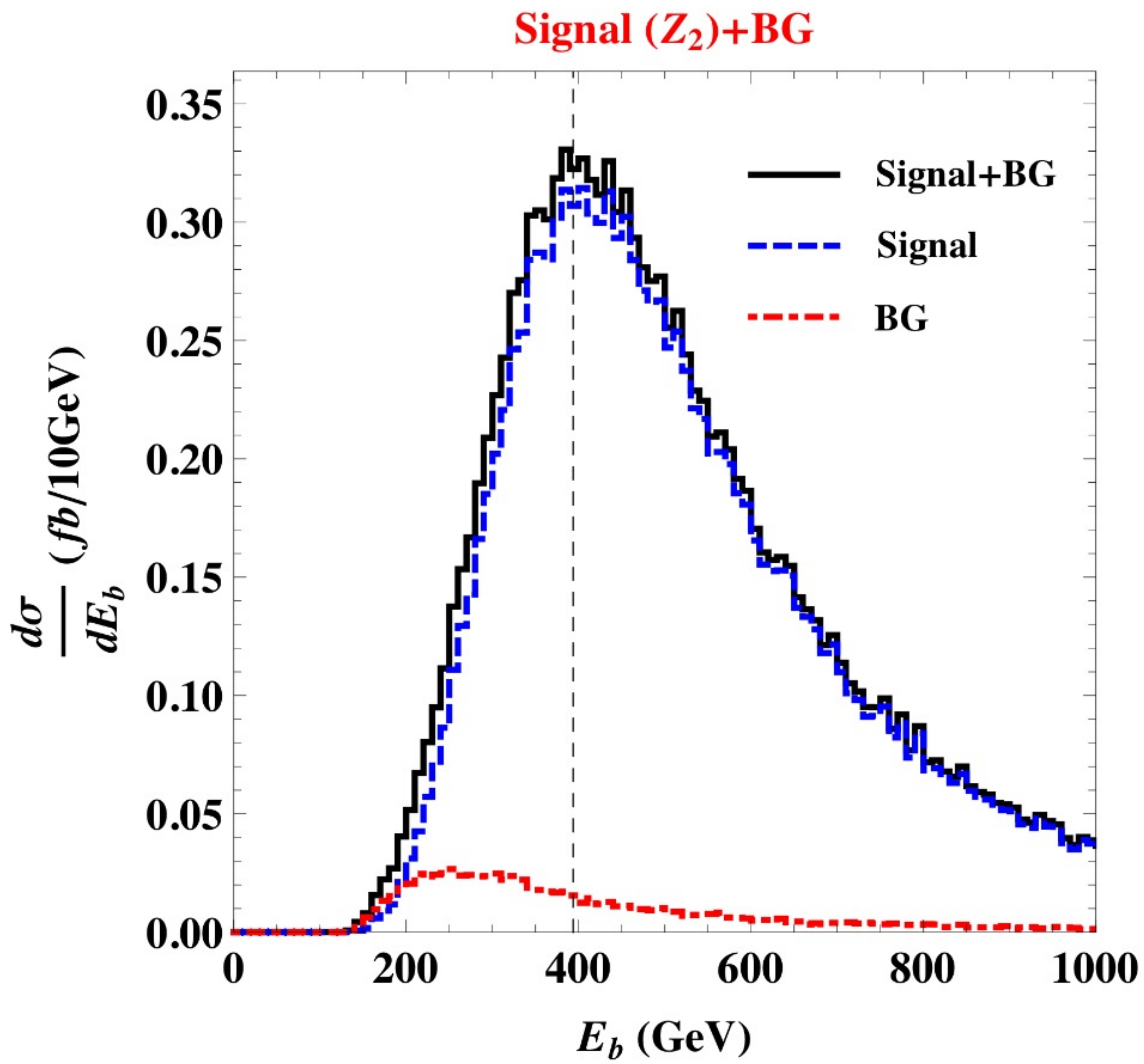
IN PROGRESS

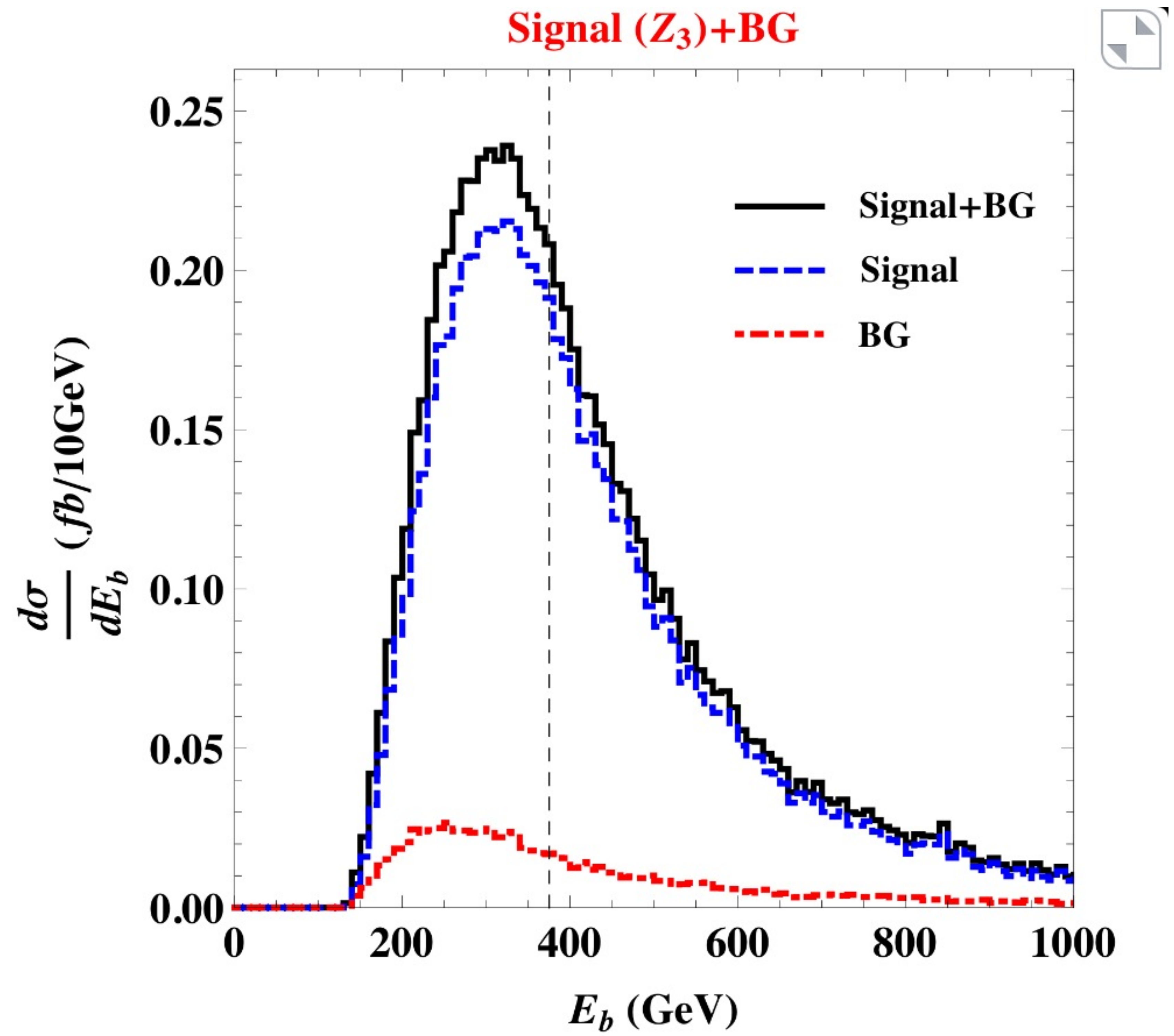
2-BODIES



3-BODIES



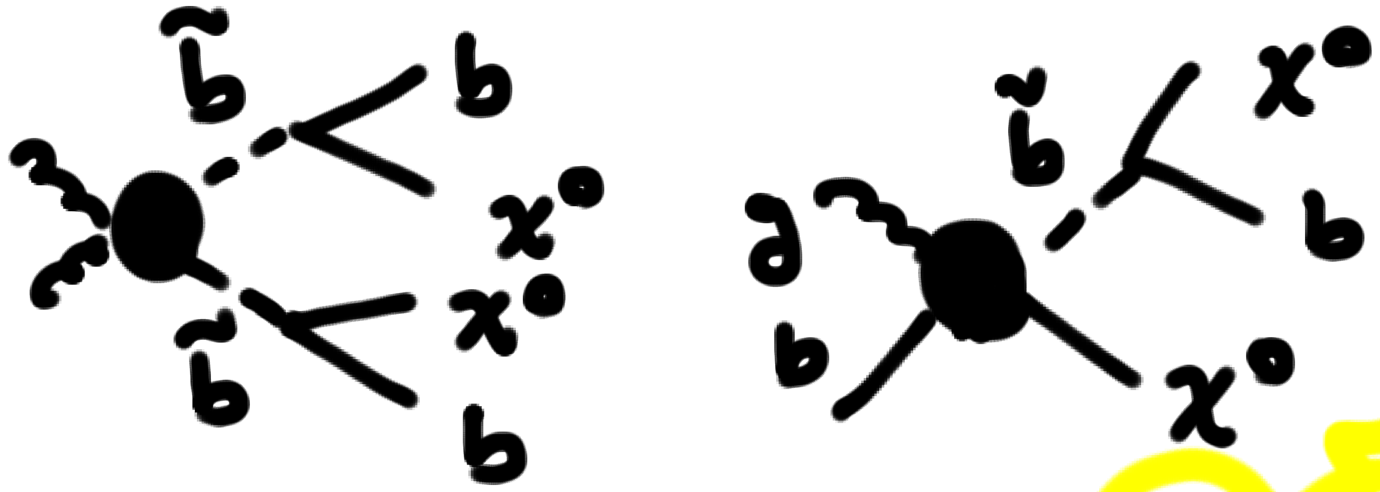




"LOCAL"

SIMPLE

- NO NEED TO KNOW ANYTHING ABOUT THE REST OF THE EVENT



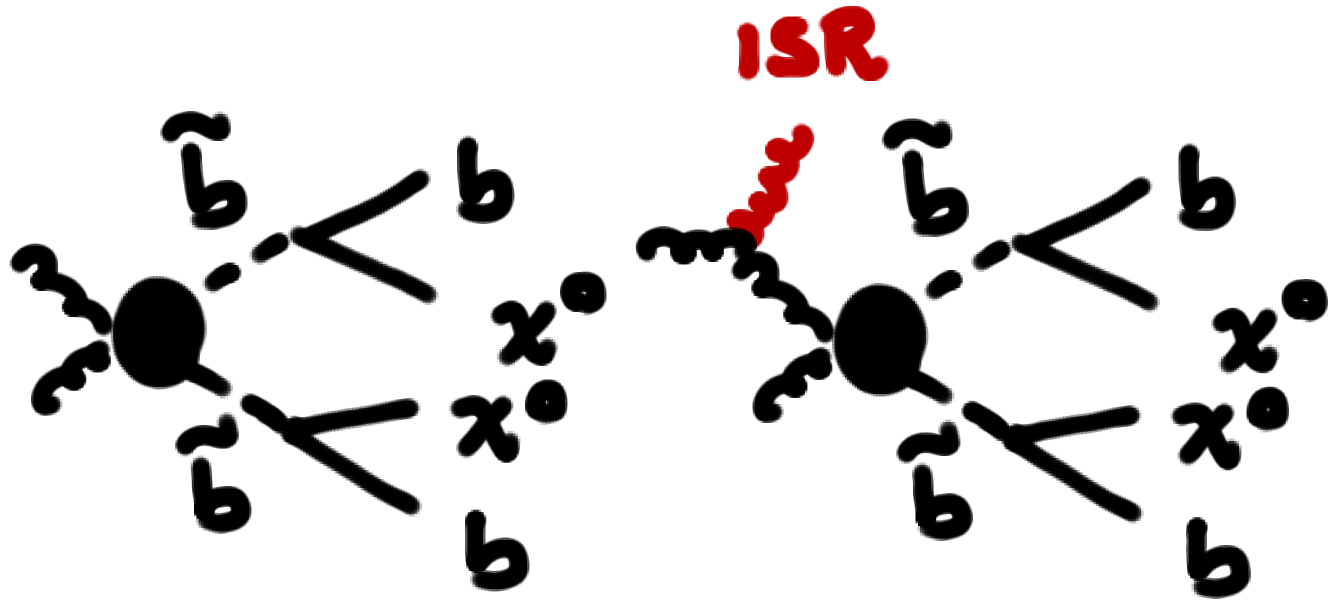
- NO NEED TO MEASURE THE OTHER DECAY PRODUCT

$\tilde{b} \to b\chi^0$

$W \to \ell\nu$

$t \to bW \to b\ell\nu$

- NO NEED TO KNOW ANYTHING ABOUT THE REST OF THE EVENT



ROBUST

Also, since

$$\log E_\gamma = \tfrac{1}{2}(\log E_{\gamma,\,\mathrm{min}} + \log E_{\gamma,\,\mathrm{max}}) = \log \mu \qquad (1\text{–}225)$$

it follows that, on logarithmic plots of the energy spectra of these γ-rays, the rest-system energy μ will lie halfway between the extremum energies.

We are particularly concerned with decays that are isotropic in the rest system of the decaying particle, such as the π^0 and Σ^0 decays, which we have previously considered. For these decays, we have already shown that the resultant γ-ray energy distribution function is only a function of the momentum of the primary; indeed this function is a constant which is inversely proportional to this momentum for a given primary, within a range proportional to the momentum of the primary, and vanishes outside this range. Thus, for decays of parent particles with a wide range of primary energies, γ-ray spectra are generated which are made up of a superposition of rectangular spectra, as shown in figure 1–11. Higher energy primaries produce the γ-rays at the extremes of the spectrum. We therefore deduce a second important kinematic property, which holds for two-body decays that produce γ-rays isotropically in the rest system of the decaying primary; viz,

> The energy spectra of γ-rays produced isotropically in the rest system of the decaying primary will be symmetric on a logarithmic plot with respect to $E_\gamma = \mu$ and will peak at $E_\gamma = \mu$.

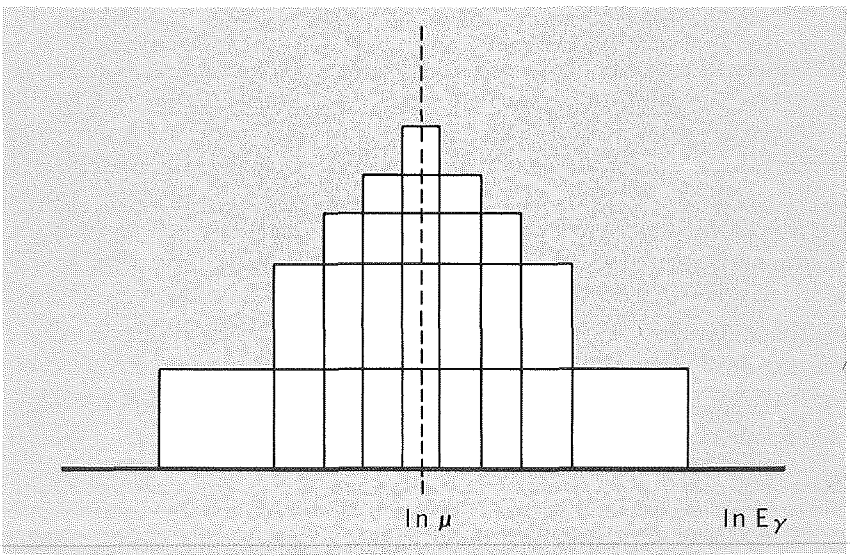
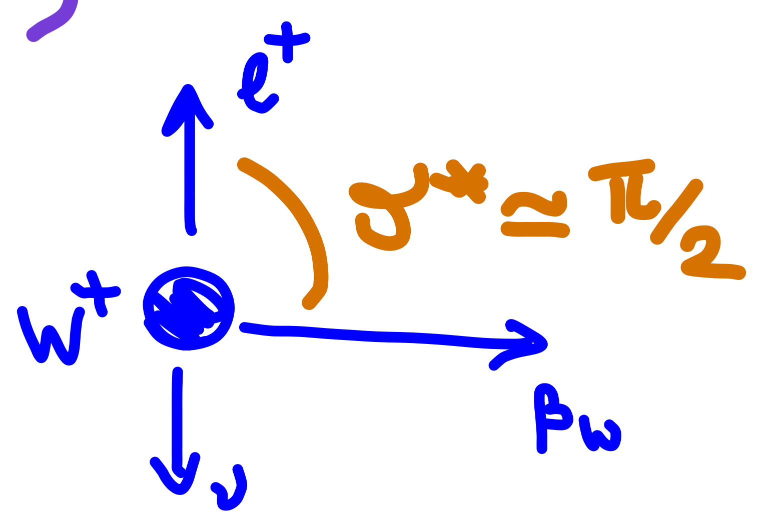


FIGURE 1–11.—Ideal superposition of γ-ray energy spectra from π^0 of Σ^0 particles having discrete values of energy.

$\frac{d\sigma}{dp_T}$ JACOBIAN PEAK

- p_T ($E > p_T \Rightarrow$ NO PEAK IN E FROM p_T)
- USUALLY APPLIED FOR PARENT PARTICLES MOVING ALONG $\hat{z}$
- POLARIZATION DOES NOT MATTER
- END-POINT (UP TO RADIATION EFF.)
-



$$\boxed{\cos\vartheta^* \simeq 0}$$

ENERGY PEAK

- ENERGY
- VALID FOR PARENT PARTICLES MOVING ALONG ANY DIRECTION
- UNPOLARIZED PARENT PARTICLE
- MAXIMUM
-

$$E = E^* \gamma (1 + \cos\vartheta^* \beta)$$

$E = E^*$ at the peak implies

$$\boxed{\cos\vartheta^* = -\sqrt{\frac{\gamma - 1}{\gamma + 1}}}$$

FOR SOME BOOSTS THE EVENTS AT THE PEAK HAVE C.O.M. ANGLE $\neq \pi/2$