Statistics of geodesics in large quadrangulations

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joint work with Emmanuel Guitter (arXiv:0712.2160)

~~Service~~ Institut de Physique Théorique, CEA Saclay

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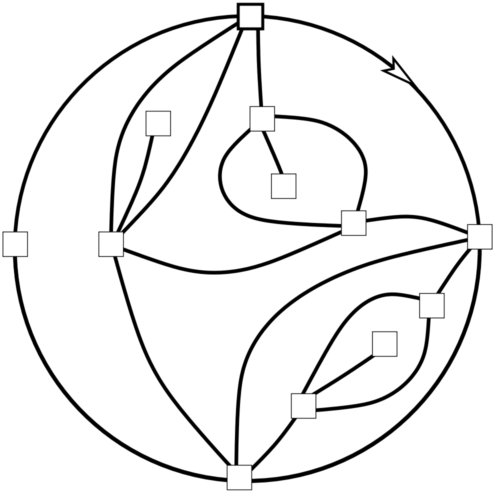
A (rooted) planar quadrangulation



Outline

1. Preliminaries
   - Marcus-Schaeffer bijection
   - Generating functions for well-labeled trees
2. Combinatorial results
   - From geodesics to spine trees
   - Confluent geodesics
   - Generating functions
3. Applications
   - Local limit
   - Continuum limit
4. Summary and conclusion

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There is a one-to-one correspondence between rooted planar quadrangulations with n faces and well-labeled trees with n edges.

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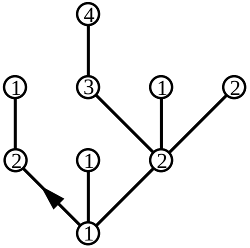
Well-labelled trees are rooted plane trees s.t.:

- each vertex has a positive integer label
- labels on adjacent vertices differ by at most 1
- the root has label 1.



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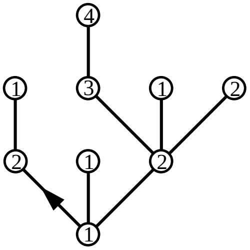

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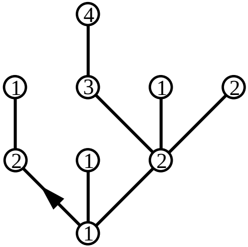
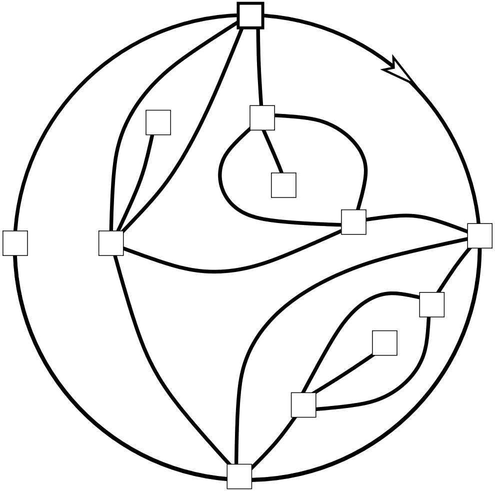


In the Marcus-Schaeffer construction, vertex labels correspond to distances from the origin in the quadrangulation.

MS construction : from quadrangulations to trees



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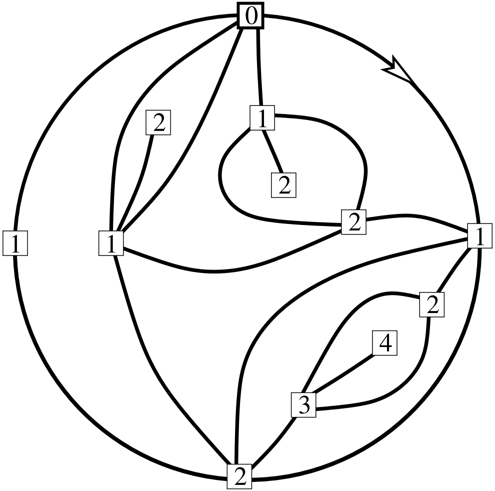
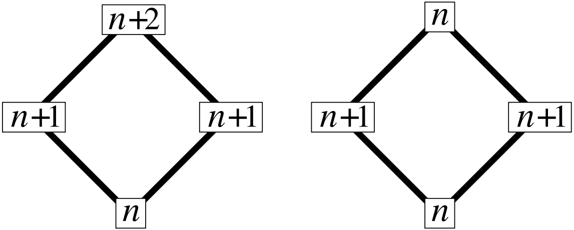


Local face rules



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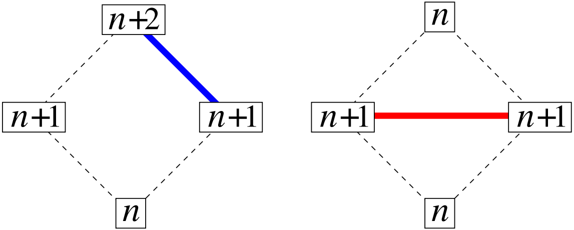

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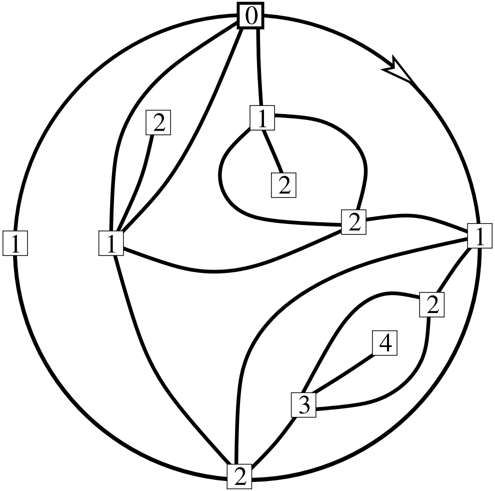

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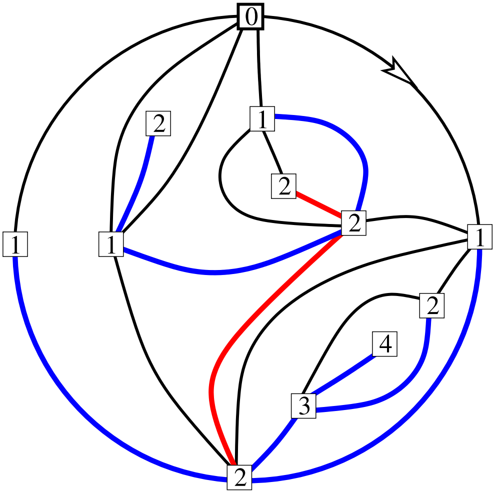

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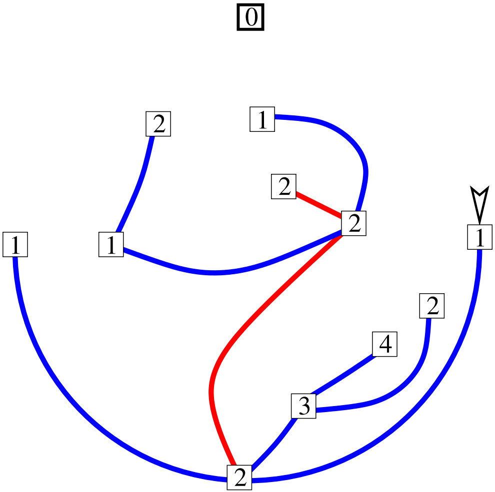

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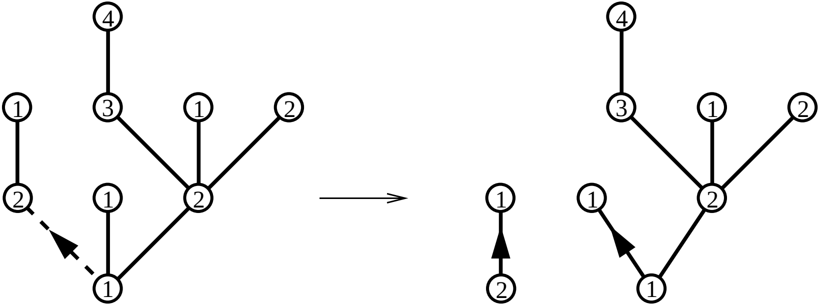
Introduce $R_i(g)$ the generating function for well-labeled trees with root label i, and weight g per edge. By recursive decomposition:

$$\forall i \geq 1, \quad R_i = 1 + g\, R_i \left(R_{i-1} + R_i + R_{i+1}\right) \qquad R_0 = 0$$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Without the $i > 0$ requirement we would have:

$$R_i \equiv R = 1 + 3gR^2 = \frac{1 - \sqrt{1 - 12g}}{6g} = \sum_{n=0}^{\infty} \frac{3^n}{n+1} \binom{2n}{n} g^n$$

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[B.-Di Francesco-Guitter 2003] The actual solution is:

$$R_i = R\frac{(1 - x^i)(1 - x^{i+3})}{(1 - x^{i+1})(1 - x^{i+2})} \qquad x + \frac{1}{x} + 1 = \frac{1}{gR^2}$$

(there is a unique power series $x(g)$)

Interpretation for quadrangulations

The generating function for rooted quadrangulations is:

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Other quantities can be interpreted :

- $2R$: *pointed* rooted quadrangulations (extra marked vertex used as the origin for distances)
- R_i : pointed rooted quadrangulations, such that the root edge is of type $j-1 \to j$ with $j \le i$.
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Applications later...

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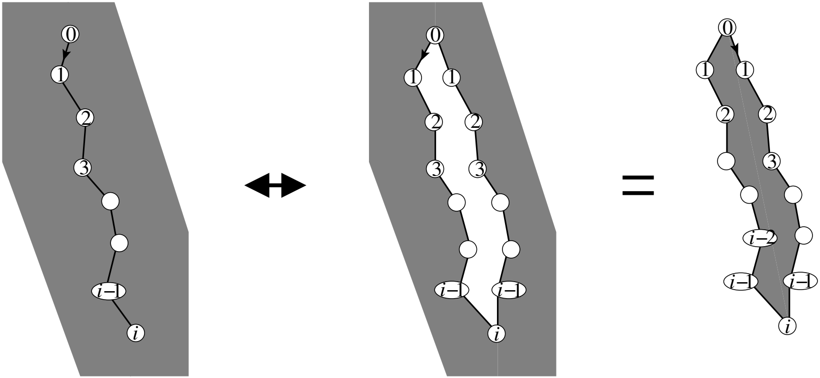
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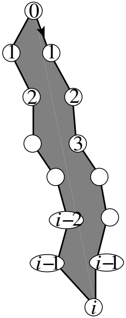
From quadrangulations to spine trees



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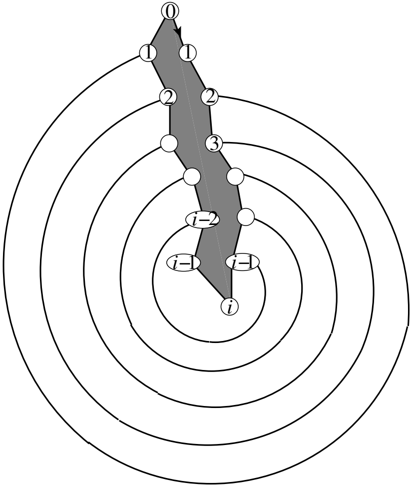

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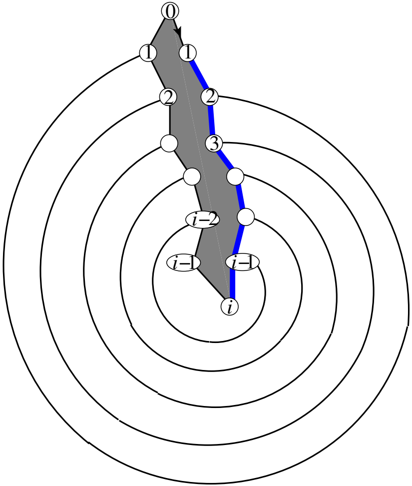

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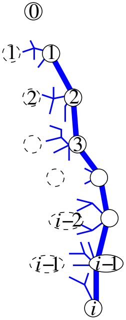

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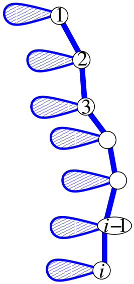

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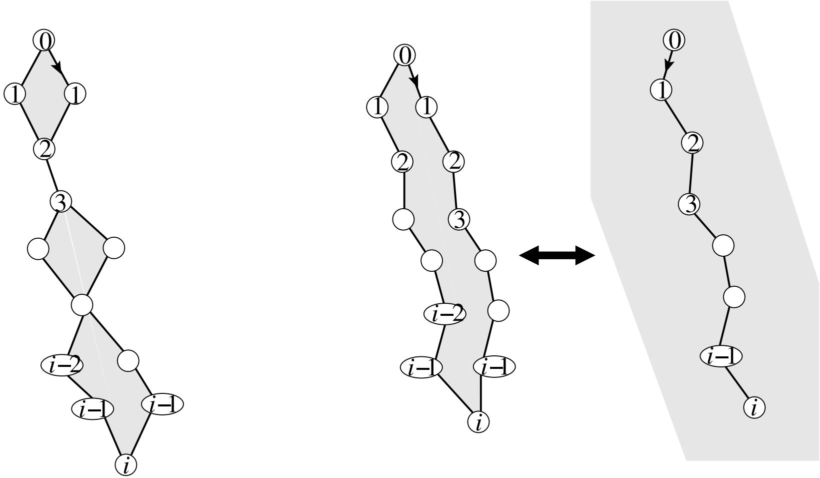
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- To have a bijection we must actually consider general geodesic boundaries with "pinch points". Quadrangulations with a marked geodesic correspond to an irreducible boundary.
- We can easily enumerate quadrangulations with an arbitrary geodesic boundary, then quadrangulations with an irreducible boundary.

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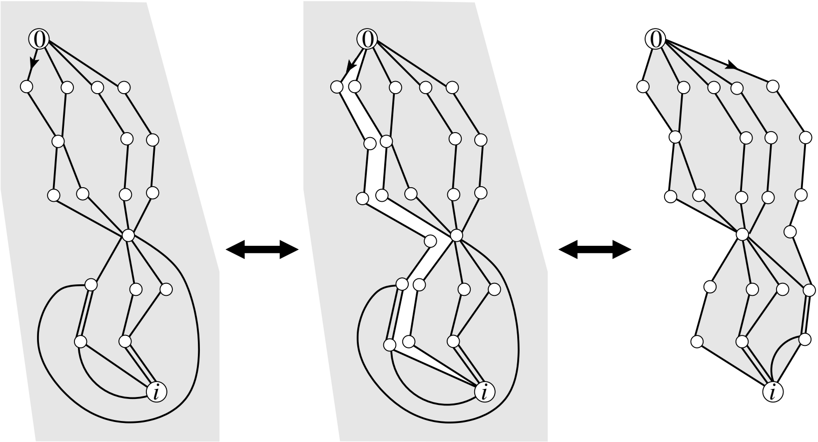
As a simple extension we can study quadrangulations with k confluent and non-crossing geodesics. These correspond to k quadrangulations with geodesic boundaries placed “side by side”.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Arbitrary (crossing) geodesics seem to be harder to deal with.

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Knowing R_i we can easily compute the generating function for spine trees:

$$Z_i = \prod_{j=1}^{i} R_j = R^i \frac{(1-x)(1-x^{i+3})}{(1-x^3)(1-x^{i+1})}$$

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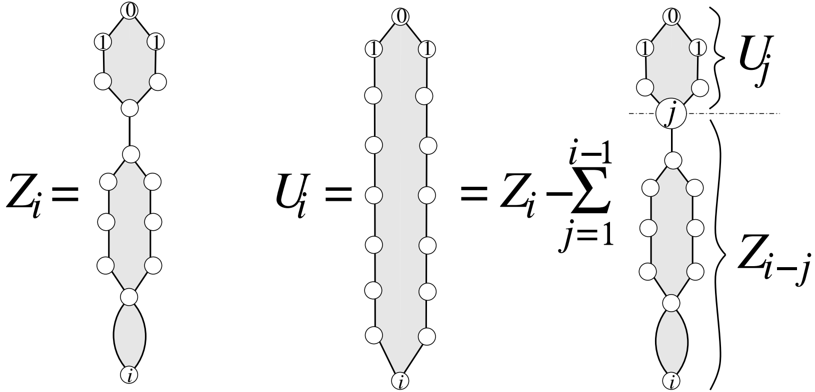
Irreducible quadrangulations (corresponding to quadrangulations with a marked geodesic) are obtained through:

$$U_i = Z_i - \sum_{j=1}^{i-1} U_j Z_{i-j} \qquad \text{i.e.} \qquad \hat{U}(t) = \frac{\hat{Z}(t)}{1+\hat{Z}(t)}$$


$Z_i =$
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$= Z_i - \sum_{j=1}^{i-1}$
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U_j
Z_{i-j}

Similarly:

- k weakly avoiding geodesics:

$$U_i^{(k)} = (Z_i)^k - \sum_{j=1}^{i-1} U_j^{(k)} (Z_{i-j})^k$$

- k strongly avoing geodesics:

$$\tilde{U}_i^{(k)} = (U_i)^k$$

That's it.

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All our g.f. have radius of convergence $1/12$, with expansion:

$$X(g) = A - C\epsilon + \frac{2}{3}D\epsilon^{3/2} + O\left(\epsilon^2\right) \qquad g = \frac{1}{12}(1-\epsilon)$$

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$q_n^\bullet = (12^n)/(2\sqrt{\pi} n^{5/2})$ is the asymptotic number of pointed quadrangulations with n faces, hence D can be understood as the expectation value for the quantity enumerated by X, in the *ensemble* of large pointed quadrangulations.

Doubly-pointed quadrangulations with marked vertices at distance i:

$$\lim_{n\to\infty} \frac{\log(R_i/R_{i-1})|_{g^n}}{q_n^\bullet} = \frac{3}{35}(5i^3 + 15i^2 + 12i + 2) \sim \frac{3}{7} i^3 \quad (i \gg 1)$$

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Quadrangulations with a geodesic boundary of length i:

$$Z_i = A_i - C_i\epsilon + \frac{2}{3}D_i\epsilon^{3/2} + O\left(\epsilon^2\right)$$

$$A_i = \frac{2^i(i+3)}{3(i+1)} \quad D_i = \frac{2^i i(i+2)(i+3)(i+4)(3i^2+12i+13)}{420(i+1)} \sim \frac{2^i i^5}{140}$$

(not immediately related to the ensemble of pointed quadrangulations)

Quadrangulations with a marked geodesic of length i:

$$U_i = \alpha_i - \gamma_i \epsilon + \frac{2}{3}\delta_i \epsilon^{3/2} + O\left(\epsilon^2\right)$$

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Coefficients are determined through $U_i = Z_i - \sum_{j=1}^{i-1} U_j Z_{i-j}$:

$$\alpha_i = A_i - \sum_{j=1}^{i-1} \alpha_j A_{i-j} \qquad \delta_i = D_i - \sum_{j=1}^{i-1} (\alpha_j D_{i-j} + \delta_j A_{i-j})$$

$$\hat{\alpha}(t) = \frac{\hat{A}(t)}{1+\hat{A}(t)} \qquad \hat{\delta}(t) = \frac{\hat{D}(t)}{\left(1+\hat{A}(t)\right)^2}$$

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In the ensemble of large doubly-pointed quadrangulations with marked vertices at distance $i \gg 1$, we find that the average number of geodesics joining the marked vertices is 3×2^i.

Quadrangulations with k weakly avoiding confluent geodesics:

$$\delta_i^{(k)} \sim k \cdot \left(3 \cdot 2^i\right)^k \cdot \frac{3}{7} i^3$$

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Other computations ($k = 2$):

- two weakly avoiding geodesics of length $i \gg 1$ have in average $i/3$ common vertices
- they delimit two regions with respective areas n vs $O(i^3)$

Quadrangulations with k strongly avoiding confluent geodesics:

$$\tilde{\delta}_i^{(k)} = k(\alpha_i)^{k-1}\delta_i \sim k \cdot (3 \cdot 2^i)^k \cdot \frac{3 \cdot 4^{k-1}}{7} i^{6-3k}$$

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Interpretation:

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- a small number of points are reachable by two distinct geodesics from the origin, none by three.

Two strongly avoiding geodesics delimit two regions with respective areas n vs $O(i^4)$. These will be of the same order in the continuum ($n \propto i^4$) limit.

Outline

Reminder: the continuum limit is obtained for $n \to \infty$, $i = r \cdot n^{1/4}$ with r fixed:

$$\frac{\log(R_i)|_{g^n}}{\log(R)|_{g^n}} \to \frac{4}{\pi} \int_0^\infty d\xi \, \xi^2 e^{-\xi^2} \left(1 - 6 \frac{1 - \cosh(r\sqrt{3\xi}) \cos(r\sqrt{3\xi})}{(\cosh(r\sqrt{3\xi}) - \cos(r\sqrt{3\xi}))^2} \right)$$

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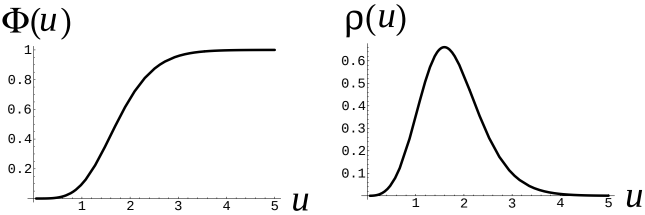
$$\frac{\log(R_i)|_{g^n}}{\log(R)|_{g^n}} \to \Phi(r)$$

$\Phi(r)$ is the probability that two vertices in a large random quadrangulation are at rescaled distance $< r$, $\rho(r) = \Phi'(r)$ is the associated density.

Reminder: the continuum limit is obtained for $n \to \infty$, $i = r \cdot n^{1/4}$ with r fixed:

$$\frac{\log(R_i)|_{g^n}}{\log(R)|_{g^n}} \to \Phi(r)$$

$\Phi(r)$ is the probability that two vertices in a large random quadrangulation are at rescaled distance $< r$, $\rho(r) = \Phi'(r)$ is the associated density.



Quadrangulations with a geodesic boundary:

$$\left.\frac{Z_i}{2^i}\right|_{g^n} \sim \frac{12^n}{\pi n^{5/4}} \int_0^\infty d\xi\, \xi e^{-\xi^2} \left(\frac{2r\xi}{3} - 2\sqrt{\frac{\xi}{3}} \frac{\sinh(r\sqrt{3\xi}) - \sin(r\sqrt{3\xi})}{\cosh(r\sqrt{3\xi}) - \cos(r\sqrt{3\xi})} \right)$$

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By a careful analysis we deduce:

$$U_i|_{g^n} \sim \frac{12^n}{2\sqrt{\pi} n^{7/4}} \left(3 \cdot 2^{r \cdot n^{1/4}}\right) \rho(r)$$

We find no new scaling function but the average number of geodesics is 3×2^i in the whole scaling range (not only the previous case $1 \ll i \ll n^{1/4}$).

Similarly for weakly avoiding geodesics:

$$U_i^{(k)}\Big|_{g^n} \sim \frac{12^n}{2\sqrt{\pi}n^{7/4}} k \left(3 \cdot 2^{r \cdot n^{1/4}}\right)^k \rho(r)$$

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With strongly avoiding geodesics we find new scaling functions:

$$U_i^{(k)}\Big|_{g^n} \sim \frac{12^n}{2\sqrt{\pi}n^{3k/4+1}} k \left(3 \cdot 2^{r \cdot n^{1/4}}\right)^k \sigma^{(k)}(r)$$

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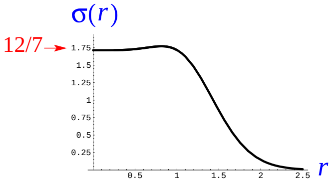

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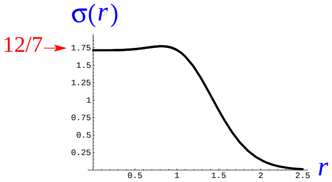

Another computation shows that two strongly avoiding geodesics delimit two regions both of area $\propto n$.

Outline

- We have extended the Marcus-Schaeffer bijection to the case of marked geodesics, and derived exact expressions for the related generating functions.

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- In large planar quadrangulations, there are in average 3×2^i geodesics between two points at distance $i \gg 1$, and such geodesics tend to "stick" to each other (extensive number of contacts, negligible area inbetween).
- Only a few exceptional pairs of points can be connected by $k \geq 2$ strongly avoiding geodesics. The number of such pairs is of order: $n^{(11-3k)/4}$. This seems in agreement with recent probabilistic approaches [Miermont, Le Gall].