2D quantum magnetism and spin liquids

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Outline

- What are the possible ground-states of 2D Heisenberg models, when magnetic long-range order has been destroyed by the zero-point fluctuations?
- Propose/discuss 3 three definitions of “spin liquids”:
  - A spin liquid is a state without mag. long range order
  - A spin liquid is a state without any spontaneously broken symmetry
  - A spin liquid is a state which sustains spin-½ excitations (spinon)
- Some basic ideas about spinon fractionalization
- Introduce a (fermionic) formalism to discuss some mean-field theories for spin liquids, and investigate fluctuations effects (gauge fields, confinement/deconfinement, etc).
What is specific to D=2?

- D=1: Mermin-Wagner $\Rightarrow$ no magnetic LRO. Powerful results and methods. LSM theorem: a spin chain is either i) gapped and ordered, or ii) critical. Bethe Anstaz. Bosonization. Tomonaga-Luttinger liquids. Conformal field theory. DMRG.
- D $\geq$ 3: Spin liquid are theoretically possible (and interesting!), but a priori more difficult to find in real systems, mostly because Néel ordered state are more stable in higher dimensions.
- D=2: Many phases are possible: different kinds of ordered states, different kinds of **spin liquids** (gapped & gapless). There is no *unique* method which is efficient to attack *all* problems/models.
Which spin models are we talking about?

- $T=0$
- Spin-$\frac{1}{2}$
- SU(2) symmetric models: Heisenberg, competing interactions ($J_1-J_2$), ring exchange and other multiple-spin interactions.

\[
H = J_1 \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + \ldots \\
H = J_1 \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\langle ijk \rangle} (P_{ijkl} + H.c) + \ldots
\]
“Moderate” quantum fluctuations $\Rightarrow$ Néel states

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

- The lattice breaks up in sub-lattices
- Spontaneously broken SU(2) symmetry
- Goldstone theorem
  $\Rightarrow$ Gapless spin waves ($\Delta S^z = 1$)
- The classical ground-state is "dressed" by zero-point fluctuations.
- But each sub-lattice keeps an extensive magnetization
- Possible description using a "1/S" expansion

What happens if quantum fluctuations are strong enough to destroy the magnetic order?

Anderson, PR 1953
Bernu et al., PRL 1992, PRB 1994
Lhuillier, cond-mat/0502464

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Mechanisms to destroy the mag. long range order

- Small spin $S$
- High density of low-energy classical modes:
  - Low space dimension
  - Low coordination
  - Frustration
  - Big (continuous) rotation symmetry group 
    $(SU(2), U(1), U(N), Sp(2N))$.

Spin wave theory for the $J_1$-$J_2$ model

$$H = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + \alpha \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

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Spin liquid – definition 1

**A spin liquid is a state without magnetic long-range order**

- More precisely, the structure factor $S(q)$ never diverges, whatever $q$.

$$S(q) = \frac{1}{N} \langle 0 | \sum_i \vec{S}_i \exp(i \mathbf{q} \cdot \mathbf{r}_i) | 0 \rangle$$

$$= \frac{1}{N} \sum_{ij} \langle 0 | \vec{S}_i \cdot \vec{S}_j | 0 \rangle \exp(i \mathbf{q} \cdot (\mathbf{r}_i - \mathbf{r}_j))$$

$$= \begin{cases} 
O(1) \forall q \Leftrightarrow \text{short- range mag. order} \\
\exists q_0 / S(q_0) \approx O(N) \Leftrightarrow \text{long- range mag. order}
\end{cases}$$

- Can be checked using neutron scattering. But also, $\mu$-SR, NMR, …
- Mermin-Wagner theorem $\Rightarrow$ *any* 2D Heisenberg model at $T>0$ is a S.L. according to this def. 😊
Valence-bond crystals

\[ \Delta > 0 \]

\[ \frac{1}{\sqrt{2}} (|\uparrow \downarrow\rangle - |\downarrow \uparrow\rangle) \]
Singlet, total spin S=0

\( J_1-J_2 \) Heisenberg model
(hexagonal lattice)
Fouet \textit{et al.} EPJB 2001

Properties:
- Short-ranged spin-spin correlations
- Spontaneous breakdown of some lattice symmetries
  \( \Rightarrow \) Ground-state degeneracy
- Gapped \( \Delta S=1 \) excitations ("magnons" or "triplons")
Valence-bond crystals (examples in 2D, from numerical studies)

- $J_1$-$J_2$-$J_3$ model
  Fouet et al. EPJB 2001

- Shastry-Sutherland lattice
  Koga & Kawakami, PRL 2000
  Läuchli, Wessel & Sigrist PRB 2002

- Heisenberg model & 4-spin “ring” exchange
  Läuchli et al. PRL 2005

- $J_1$-$J_2$-$J_3$ model
  Mambrini et al., cond-mat/0606776

- Gellé et al. arxiv:0704.2352
  (⇒tutorial)

+ others…
A simple (tensor) product of singlet is usually not an exact eigenstate for realistic Hamiltonians. The true VBC ground-state is a regular singlet arrangement « dressed » by fluctuations:

Remark: Comparing a typical v.-bond configuration with the appropriate “parent” columnar state, one gets a collection of small loops (length of order one).
A spin liquid is a state without any spontaneously broken symmetry

- This def. excludes Néel ordered states, which break the SU(2) sym. (also spin nematics)
- This def. excludes valence-bond crystals, which break some lattice sym.
Quantum paramagnets

- Some magnetic insulators without any broken sym.

\[ S = \frac{1}{\sqrt{2}} (|\uparrow \downarrow \rangle - |\downarrow \uparrow \rangle) \]  
S=0 spin singlet, or dimer

- Even number of spin-½ in the crystal unit cell
- No broken symmetry
- Adiabatically connected to the (trivial) limit of decoupled blocks
- No phase transition between \( T=0 \) and \( T=\infty \)

\[ \Delta \approx 100 \text{ K} \] - 1st 2D spin-gap system

**Other examples**: coupled dimer systems: \( \text{TICuCl}_3 \), etc.

\[ \Delta > 0 \]

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Quantum paramagnets – perturbation theory

Perturbation theory about a decoupled limit

$$\Delta E = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Exercise:
compute $\Delta E(k)$ to first order in $\lambda$ for a two-leg ladder. What about a three-leg ladder?
Magnetic excitations in the quantum spin system TlCuCl$_3$

N. Cavadini,$^1$ G. Heigold,$^1$ W. Henggeler,$^1$ A. Furrer,$^1$ H.-U. Güdel,$^2$ K. Krämer,$^2$ and H. Mutka$^3$

FIG. 2. Observed energy dispersion of the magnetic excitation modes in TlCuCl$_3$ at $T = 1.5$ K. Full circles from the relevant directions of reciprocal space are arranged in a reduced scheme representation, with $A = E = (1/2,0,0)$, $B = (0,1,0)$, $D(0,0,0)$ [r.l.u.]. Zone centering corresponds to $C = (0,0,1)$ for $\Delta q = (h,0,l)$, $C = (0,0,0)$ for $\Delta q = (0,k,0)$. Lines are fits to the model expectations explained in the text with the parameters reported in Table I.
Quantum paramagnets (and VBC) are *not* fractionalized

Remark: the same picture applies to valence-bond crystals.

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In a quantum paramagnet, the unit cell contains an even number of spin-$\frac{1}{2}$.

In a VBC, the unit cell is spontaneously enlarged to enclose a even number of spin-$\frac{1}{2}$.

Are there other types of wave-functions with short-range spin-spin correlations? (with just one spin $\frac{1}{2}$ per unit cell in particular?)
Lieb-Schultz-Mattis-Hastings theorem

  [See also: Affleck 1988; Bonesteel 1989; Oshikawa 2000; GM et al. 2002]

“A system with a half-odd-integer spin in the unit cell
(+ periodic boundary conditions, + dimensions $L_1 \times L_2 \times \ldots \times L_D$ with $L_2 \times \ldots \times L_d=odd$)
cannot have a gap and a unique ground-state
(in the thermodynamic limit).”

1) Ground-state degeneracy
   a- “Conventional” broken symmetry
      (valence bond crystal for instance)
   b- Resonating valence bond liquid ($Z_2$) or more complex topological structure

2) Gapless spectrum
   a- Continuous broken sym. (Néel order)
   b- Critical state

Gapped paramagnet = forbidden at $T=0$

Spin liquids with fractional excitations
Short-range RVB picture

- P. W. Anderson’s idea (1973): (short-ranged) resonating valence-bond (RVB)
  Linear superposition of many (exponential) low-energy short-range valence-bond configurations

- Spin-$\frac{1}{2}$ excitations?
  VBC $\Rightarrow$ linear potential between spinons
  no dimer order $\Rightarrow$ we may expect deconfined spinons

- Topological degeneracy & spinon fractionalization

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Definition 3:
A spin liquid is a state which sustains fractional (spin-$\frac{1}{2}$) excitations

What is fractionalization?

- Existence of (finite energy) excitations with quantum number(s) which are fraction of the elementary degrees of freedom.
  Most famous example: charges $q=e/3$ in the fractional quantum hall effect.
- In magnetic systems:
  A spinon is a neutral spin-$\frac{1}{2}$ excitation, “one half” of a $\Delta S^z=1$ spin flip.
  (it has the same spin as an electron, but is has no charge)
- Spinons can only be created by pairs in finite systems (combining $S^+$ and $S^-$ operators can only change $S^z$ by some integer) The question is to understand if they then can propagate at large distances from each other, as two elementary particles.
What is a fractional excitation? (very) simple example in 1D

- Majumdar-Gosh chain

\[ H = \sum_i \vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{2} \sum_j \vec{S}_j \cdot \vec{S}_{j+2} \]

The initial $S^z=+1$ excitation can *decay* into two spatially separated spin-$\frac{1}{2}$ excitations (spinons)

Apply $S_i^+$ here

Finite-energy state with an *isolated* spinon (the other is far apart)
A few examples of fractionalized systems in \( D>1 \)

- **Easy-axis kagome model**
  

\[
H = J_z \sum_{h \text{ hexagon}} (S_{h1}^z + \cdots + S_{h6}^z)^2 \\
+ J_\perp \sum_{h \text{ hexagon}} \left( (S_{h1}^x + \cdots + S_{h6}^x)^2 + (S_{h1}^y + \cdots + S_{h6}^y)^2 \right)
\]

\( J_\perp \ll J_z \)

- **Kitaev’s “toric code” model** ⇒ tutorial
- **SU(2) symmetric spin models**
  
  GM *et al.*, 1999


- **Experiments? Some candidates:**
  
  - **Cs\textsubscript{2}CuCl\textsubscript{4}** [Anisotropic S=1/2 triangular lattice, Coldea *et al.* 2003]
  - **\( \kappa \)-(BEDT-TTF)\textsubscript{2}Cu\textsubscript{2}(CN)\textsubscript{3}** [Shimizu *et al.* 2003]
  - **NiGa\textsubscript{2}S\textsubscript{4}** [Spin-1 on a triangular lattice, Nakatsuji *et al.*, 2005]
  - **ZnCu\textsubscript{3}(OH)\textsubscript{6}Cl\textsubscript{2}** [Helton *et al.* 2007, Mendels *et al.* 2007, Ofer *et al.* 2007, Imai *et al.* 2007]
  - **Na\textsubscript{4}Ir\textsubscript{3}O\textsubscript{8}** [3D lattice of corner sharing triangles, “hyper kagome”, Okamoto *et al.* 2007]
  - **He\textsuperscript{3} films** [Nuclear magnetism on a triangular lattice, Masutomi *et al.* 2004]
How to detect deconfined spinons?

- Neutron scattering
- Non magnetic impurities $\Rightarrow$ tutorial
Inelastic neutron scattering – spinon continuum

Inelastic neutron scattering: probe for the dynamical structure factor $S(q, \omega)$.

\[
S(q, \omega) = \int dt \langle 0 | S^-_{-q}(t) S^+_{q}(0) | 0 \rangle e^{-i \omega t}
\]

- If the elementary excitations are spin-1 magnons: $S(q, \omega)$ has single-particle pole at $\omega = \omega(q)$.

- If the spin flip decays into two spin-$\frac{1}{2}$ excitations, $S(q, \omega)$ exhibits a two-particle continuum.

$q_1, \omega(q_1), S=\frac{1}{2}$

$q = q_1 + q_2$

$\omega = \omega(q_1) + \omega(q_2)$

$S=0$ or $1$

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Inelastic neutron scattering – spinon continuum

Neutron scattering on Cs$_2$CuCl$_4$
R. Coldea et al. (2000)
Formalisms and methods to investigate fractionalized spin liquids?

- Numerics (exact diag., quantum Monte-Carlo, …)
- Effective models (dimer models)
- Large-N/slave particle approaches:
  - Bosonic mean-field (Schwinger bosons) + fluctuation effects
  - Fermionic mean-field (Abrikosov fermions) + fluctuation effects

See Lee, Nagaosa & Wen, Rev. Mod. Phys. 78, 17 (2006)
Fermionic representation of a spin-1/2

\[ S^z = \frac{1}{2} \left( c_{\uparrow}^+ c_{\uparrow} - c_{\downarrow}^+ c_{\downarrow} \right) \]
\[ S^+ = c_{\uparrow}^+ c_{\downarrow} \quad S^- = c_{\downarrow}^+ c_{\uparrow} \]
\[ c_{\uparrow}^+ c_{\uparrow} + c_{\downarrow}^+ c_{\downarrow} = 1 \]
\[ S^a = c_{\mu}^+ \sigma_{\mu \nu}^a c_{\nu} \quad a = x, y, z \quad \mu, \nu = \uparrow, \downarrow \]

- Compact notations using a 2x2 matrix
  \[ \psi_i = \begin{bmatrix} c_{i \uparrow} & c_{i \downarrow} \\ c_{i \uparrow}^+ & -c_{i \downarrow}^+ \end{bmatrix} \]
  \[ S^a = \frac{1}{2} \text{Tr} \left[ \psi_i^+ \psi_i (\sigma^a)^T \right] \]
  \[ \bar{S}_i \cdot \bar{S}_j = \frac{1}{4} \sum_A \text{Tr} \left[ \psi_i^+ \psi_i (\sigma^a)^T \right] \text{Tr} \left[ \psi_j^+ \psi_j (\sigma^a)^T \right] \]
  \[ = \frac{1}{8} \text{Tr} \left[ \psi_i \psi_j^+ \psi_j \psi_i^+ \right] \]
Mean-field decoupling

\[ \tilde{S}_i \cdot \tilde{S}_j = \frac{1}{8} \text{Tr}[\psi_i \psi_j^+ \psi_j \psi_i^+] \]

\[ \psi_i \psi_j^+ \psi_j \psi_i^+ \to \langle \psi_i \psi_j^+ \rangle \psi_j \psi_i^+ + \psi_i \psi_j^+ \langle \psi_j \psi_i^+ \rangle - \langle \psi_i \psi_j^+ \rangle \langle \psi_j \psi_i^+ \rangle \]

Mean-field approx.

\[ J_{ij} \tilde{S}_i \cdot \tilde{S}_j \to \text{Tr} \left[ U_{ij}^0 \psi_j \psi_i^+ + \psi_i \psi_j^+ (U_{ij}^0)^\dagger - U_{ij}^0 U_{ij}^{0+} \right] \]

\[ U_{ij}^0 = \frac{J_{ij}}{8} \langle \psi_i \psi_j^+ \rangle = \frac{J_{ij}}{8} \begin{bmatrix} \langle c_{i \uparrow} c_{j \uparrow}^+ + c_{i \downarrow} c_{j \downarrow}^+ \rangle & \langle c_{i \uparrow} c_{j \downarrow} - c_{i \downarrow} c_{j \uparrow} \rangle \\ \langle c_{i \downarrow} c_{j \uparrow}^+ - c_{i \uparrow} c_{j \downarrow}^+ \rangle & \langle c_{i \downarrow} c_{j \downarrow}^+ + c_{i \uparrow} c_{j \uparrow}^+ \rangle \end{bmatrix} = \begin{bmatrix} -\chi_{ij}^+ & \eta_{ij} \\ \eta_{ij}^+ & \chi_{ij} \end{bmatrix} \]

Mean-field parameters

\[ H_{MF} = \sum_{\langle ij \rangle} \chi_{ij} \left( c_{i \downarrow}^+ c_{j \downarrow} + c_{i \uparrow}^+ c_{j \uparrow} \right) + \eta_{ij} \left( c_{i \uparrow} c_{j \downarrow} - c_{i \downarrow} c_{j \uparrow} \right) + H.c \]

Spinon “hopping”

Spinon “pairing”
Spin rotation symmetry

\[ V \in SU(2), \text{ global spin rotation} \]

\[
\psi_i = \begin{bmatrix}
    c_{i\uparrow} & c_{i\downarrow} \\
    c_{i\uparrow} & -c_{i\downarrow}
\end{bmatrix} \rightarrow \psi_i V
\]

\[
\tilde{S}_i \cdot \tilde{S}_j = \frac{1}{8} \text{Tr} [\psi_i \psi_j^+ \psi_j \psi_i^+] \rightarrow \tilde{S}_i \cdot \tilde{S}_j
\]

\[
U_{ij}^0 = \frac{J_{ij}}{8} \langle \psi_i \psi_j^+ \rangle \rightarrow U_{ij}^0
\]

\[ H_{MF} \rightarrow H_{MF} \]

⇒ Mean-field Hamiltonian and its ground-state are rotation invariant (can describe a “spin liquid”)

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Beyond mean-field

- **Is the mean field state stable under the inclusions of fluctuations?**
- Spinons are deconfined (free fermions!) at the mean-field level, but is this a robust property? Will the inclusion of fluctuations confine the spinons?
- This is usually a difficult question…
  - If yes: fluctuations are strong, they induce long-range interactions between spinons and this mean-field is not a very useful starting point.
  - If no (fluctuations do not confine the spinon), the mean-field approximation is a good starting point to describe the spin liquid.

- One way to address these questions: numerical Gutzwiller projection
  Example of recent study on the kagome lattice: Ran, et al., PRL (2007).

- Other point of view: analyze the qualitative structure of the (potentially important) low-energy mode/fluctuations about the mean-field state. Some important modes for the long-distance physics are *gauge* modes.
Redundancy – gauge transformations

$W_i$, arbitrary $SU(2)$ matrix at each site $i$:

$$\psi_i \rightarrow W_i \psi_i$$

$$\bar{S}_r \rightarrow \bar{S}_r \quad \text{local redundancy}$$

$$U_{ij}^0 = \frac{J_{ij}}{8} \langle \psi_i \psi_j^+ \rangle \rightarrow W_i U_{ij}^0 W_j^+$$

$$\psi_i = \begin{bmatrix} c_{i \uparrow} & c_{i \downarrow} \\ c_{i \downarrow}^+ & -c_{i \uparrow}^+ \end{bmatrix}$$

$$S^a = \frac{1}{2} \text{Tr} \left[ \psi_i^+ \psi_i (\sigma^a)^T \right]$$

$$\bar{S}_i \cdot \bar{S}_j = \frac{1}{8} \text{Tr} \left[ \psi_i \psi_j^+ \psi_j \psi_i^+ \right]$$

- The same physical mean-field state can be represented by many different sets of parameters $U^0$ (differing from each other by some gauge-transformation).
- The bond parameters $U$ are a redundant way of labeling physical states. This redundancy is unavoidable if we want to use a formalism which include spinon operators.
- This redundancy has important consequences on the structure of the fluctuation modes around a given mean-field state $\Rightarrow$ gauge modes.

Remark: can be extended to time-dependent gauge transformations – see the Appendix and Affleck, Zou, Hsu & Anderson, Phys. Rev. B 38, 745 (1988)
Projective symmetry group (PSG)

- Two sets of $U_{ij}$ which differ by a gauge transformation describe the same physical spin wave-function.
- Definition of PSG

\[ T : \text{lattice symmetry} \]
\[ W : \text{gauge transformation} \]
\[ U_{ij}^0 \xrightarrow{\text{Lattice sym.}} U_{T(i)T(j)}^0 \xrightarrow{\text{gauge transf.}} W_i U_{T(i)T(j)}^0 W_j^+ = U_{ij}^0 \]

\((T, W) \in \text{PSG}\)
\[ \iff W_i U_{T(i)T(j)}^0 W_j^+ = U_{ij}^0 \]

- We have defined the PSG of the mean-field state, but the PSG is in fact a universal property of the whole phase. Including fluctuations should not affect the PSG (unless an instability or phase transition occurs).
- Even in the absence of any spontaneously broken symmetry, the PSG is generally non-trivial. It characterizes ‘how’ the lattice symmetries are realized in the wave-functions. Distinct spin-liquid phase can have the same lattice symmetries (they can be completely symmetric for instance), but different PSG. The PSG of a fractionalized phase plays a role analog to that of the symmetry group for usual ordered phases.
The invariant gauge group of a mean-field state is defined as the set of all gauge transformations which leaves the mean-field parameter $U^0$ invariant.

\[
W : i \mapsto W_i \in SU(2) \\
W \in IGG \\
\iff W_i U^0_{ij} W_j^+ = U^0_{ij} \quad \forall ij
\]

Why is this useful? The IGG of a mean-field state is the gauge group associated to the fluctuations around this mean-field state.
Invariant gauge group & and gauge fluctuations

- Consider some mean-field state defined by the bond parameters $U_{ij}^0$.
- Assume, for simplicity, that the associated IGG is $\sim U(1)$ and that its elements can be parameterized in the following way (but the final result is in fact general):

$$W^\theta \in \text{IGG} : W_j^\theta = \exp\left(i\theta \bar{\sigma} \cdot \vec{n}\right), \theta \in [\pi, \pi], \vec{n}^2 = 1$$

$$W_i^\theta U_{ij}^0 W_j^{\theta^+} = U_{ij}^0 \quad \forall ij, \forall \theta \quad \text{(by definition of the IGG)}$$

- We will now show that some fluctuations about this mean field state are described by a \textbf{U(1) gauge field}.
- Consider the following fluctuations:

$$\psi_i \psi_j^+ = U_{ij}^0 \exp(i \bar{\sigma} \cdot \vec{n}) \psi_i \psi_j^+$$

Remark: $A_{ij}$ is rotation invariant $\Rightarrow$ describes $S=0$ modes

- Now we perform the following U(1) gauge transformation:

$$\psi_i \rightarrow \exp(i \bar{\sigma} \cdot \vec{n}) \psi_i$$

and see how the field $A_{ij}$ transforms:

$$\psi_i \psi_j^+ \rightarrow \exp(i \theta_i \bar{\sigma} \cdot \vec{n}) \psi_i \psi_j^+ \exp(-i \theta_j \bar{\sigma} \cdot \vec{n})$$

$$= \exp(i \theta_i \bar{\sigma} \cdot \vec{n}) U_{ij}^0 \exp(i A_{ij} \bar{\sigma} \cdot \vec{n}) \exp(-i \theta_j \bar{\sigma} \cdot \vec{n})$$

$$= U_{ij}^0 \exp[i(A_{ij} + \theta_i - \theta_j)\bar{\sigma} \cdot \vec{n}]$$

$$A_{ij} \rightarrow A_{ij} + \theta_i - \theta_j \quad \Rightarrow A \text{ is a gauge field}$$

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Fluctuations (about mean-field) and gauge fields

- The IGG gives the gauge structure of the fluctuations about a given mean-field state.
- The associated gauge field(s) may or may not provide gapless excitations, may or may not confine the spinon.
- Non-trivial (non-perturbative) results about gauge theories coupled to matter may be ‘imported’ to discuss the stability/instability of a given mean-field state.
- For instance (D=2):
  - $\mathbb{Z}_2$ gauge field + gapped spinons may be in a stable deconfined phase.
    =short-range RVB physics, Read & Sachdev PRL 1991
  - $U(1)$ gauge field + gapped spinon: instability
    usually toward confinement and VBC, Read & Sachdev PRL 1989
  - $U(1)$ gauge field + Dirac gapless spinons: may be stable
    (so-called “algebraic spin liquids”) Hermele, Senthil, Fisher, Lee, Nagaosa, Wen, PRB 2004
- Remark: gauge modes are not the only source of instability: interactions between fermions should also be investigated. The projective symmetry group (PSG) constrains the possible interaction terms.
Summary

- There are several possible definitions for “spin liquids”
  - A spin liquid is a state without magnet long range order
  - A spin liquid is a state without any spontaneously broken symmetry
  - A spin liquid is a state which sustains spin-$\frac{1}{2}$ excitations (spinon)
- From the theoretical point of view, the richest structures are found in “fractionalized” spin liquids.
- Gauge theories are the natural language to describe these fractionalized phases.
- There are many kinds of fractionalized spin liquids, they are characterized by different gauge groups, and different PSG.
- In the recent years, many spin models are be shown to be fractionalized.
- Several compounds have emerged as ‘candidates’ but there is not yet any definitive experimental evidence for a fractionalized spin liquid in D>1.