The Standard model Higgs boson as the inflaton

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Outline

1. Inflation—virtues and problems
   - Cosmological requirements
   - “Standard” chaotic inflation
   - Problems with using the SM Higgs for inflation

2. SM with non-minimal coupling to gravity
   - The action
   - Conformal transformation
   - Inflation in the model
   - Radiative corrections—no danger

3. Predictions and expectations
   - CMB parameters—spectrum and tensor modes
   - Higgs mass

4. Conclusions
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Cosmological implications

Problems in cosmology

- Flatness problem (at $T \sim M_P$ density was tuned $|\Omega - 1| \lesssim 10^{-59}$)
- Entropy of the Universe $S \sim 10^{87}$
- Size of the Universe (at $T \sim M_P$ size was $10^{29} M_P^{-1}$)
- Horizon problem

Solution

Inflation!
# Cosmological implications

## Problems in cosmology

- **Flatness problem** (at $T \sim M_P$ density was tuned $|\Omega - 1| \lesssim 10^{-59}$)
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- **Horizon problem**

## Solution

Inflation!
**CMB**

Temperature fluctuations

Polarization

**CMB spectrum**

Angular Scale

\[ l(l+1)C_l/2\pi [\mu K^2] \]

- TT
- TE

Multipole moment \( l \)

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“Standard” chaotic inflation

Usually required for inflation

Scalar field
- quartic coupling constant $\lambda \sim 10^{-13}$
- mass $m \sim 10^{13}$ GeV

Present in the Standard Model

The Higgs boson
- $\lambda \sim 1$
- $m_H \sim 100$ GeV

Even if one writes a potential that flattens at large field values:
- Radiative corrections from $t, W$ generate $\delta V_{rad} \sim # h^4 \log h$

Solution: Non-minimal coupling to gravity
Scalar fields present in the SM

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![Potential](image)

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Possible operators in the SM (+gravity)

- Dimension $\leq 4$
- No new degrees of freedom (no higher derivatives)

\[
S = \int d^4x \sqrt{-g} \left[ \right.
\right.
\]

SM\{ 
\[
\text{Tr}(F_{\mu\nu}F^{\mu\nu}) + \frac{|D_\mu H|^2}{2} - V(H) + \bar{\psi} \slashed{D} \psi + YH \bar{\psi}_L \psi_R + m \bar{N}_c N
\]

- \[
- \frac{M_P^2}{2} R
\]

- \[
- \xi H^\dagger H R
\]

+ \[
R^2 + R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho} + \square R
\]

\]
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$$- \frac{M_P^2}{2} R$$

$$- \xi H^\dagger H R$$

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- \frac{M_P^2}{2} R \\
- \xi H^\dagger H R \\
+ R^2 + R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho} + \Box R
\end{array} \right. \]$$
Non-minimally coupled scalar field— inflation

Quite an old idea

Add $\phi^2 R$ term to/instead of the usual $M_P R$ term in the gravitational action

- A.Zee’78, L.Smolin’79, B.Spokoiny’84
- D.Salopek J.Bond J.Bardeen’89

\[ S_J = \int d^4x \sqrt{-g} \left\{ -\frac{M^2 + \xi h^2}{2} R + g_{\mu\nu} \frac{\partial^\mu h \partial^\nu h}{2} - \frac{\lambda}{4} (h^2 - v^2)^2 \right\} \]

- $h$ is the Higgs field
- $M \gg v \sqrt{\xi}$ so $M \simeq M_P = \frac{1}{\sqrt{8\pi G_N}} = 2.4 \times 10^{18}$ GeV
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Conformal transformation

It is possible to get rid of the non-minimal coupling by the conformal transformation (field redefinition)

\[ \hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2} \]

and also redefinition of the Higgs field to make canonical kinetic term

\[ \frac{d'\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2 / M_P^2}{\Omega^4}} \quad \Rightarrow \quad \begin{cases} h \approx \chi & \text{for } h < M_P / \xi \\ \Omega^2 \approx \exp \left( \frac{2\chi}{\sqrt{6}M_P} \right) & \text{for } h > M_P / \xi \end{cases} \]

Resulting action (Einstein frame action)

\[ S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2} \hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} - \frac{1}{\Omega(\chi)^4} \frac{\lambda}{4} \left( h(\chi)^2 - v^2 \right)^2 \right\} \]
Inflationary potential

\[ U(\chi) \approx \frac{\lambda M_P^4}{4\xi^2} \left( 1 - \exp \left( -\frac{2\chi}{\sqrt{6}M_P} \right) \right)^2 \]

For \( \chi \gtrsim M_P \):
Slow roll stage

\[ \varepsilon = \frac{M_P^2}{2} \left( \frac{dU/d\chi}{U} \right)^2 \approx \frac{4}{3} \exp \left( -\frac{4\chi}{\sqrt{6}M_P} \right) \]

\[ \eta = M_P^2 \frac{d^2U/d\chi^2}{U} \approx -\frac{4}{3} \exp \left( -\frac{2\chi}{\sqrt{6}M_P} \right) \]

Slow roll ends at \( \chi_{\text{end}} \approx M_P \)

Number of e-folds of inflation at the moment \( h_N \) is \( N \approx \frac{6}{8} \frac{h_N^2 - h_{\text{end}}^2}{M_P^2/\xi} \)

\( \chi_{60} \approx 5M_P \)

COBE normalization \( U/\varepsilon = (0.027M_P)^4 \) gives

\[ \xi \approx \sqrt{\frac{\lambda}{3}} \frac{N_{\text{COBE}}}{0.027^2} \approx 49000\sqrt{\lambda} = 49000 \frac{m_H}{\sqrt{2}v} \]

Connection of the parameter \( \xi \) and the Higgs mass!
After inflation—reheating

\[ \frac{\lambda M^4}{\xi^2/4} \]

Reheating

\[ \frac{\lambda M^4}{\xi^2/16} \]

\[ \frac{M_P}{\xi} < \chi < M_P : \quad U(\chi) \simeq \frac{\lambda M_P^4}{4\xi^2} \left(1 - \exp\left(-\frac{2\chi}{\sqrt{6}M_P}\right)\right)^2 \]

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After inflation—reheating

\[ \frac{\lambda M^4}{\xi^2/4} \]

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\[ U(\chi) \]

\[ \chi_{\text{end}} \]

\[ \chi_{\text{COBE}} \]

\[ \lambda M^4/\xi^2/4 \]

\[ \lambda M^4/\xi^2/16 \]

\[ \frac{M_p}{\xi} < \chi < M_p : \quad U \approx \frac{\lambda M_p^2}{6\xi^2} \chi^2, \quad \Omega \approx 1, \quad \chi \approx \sqrt{\frac{3}{2}} \frac{\xi h^2}{M_p}, \quad T_{\text{reh}} \gtrsim 10^{13} \text{GeV} \]
After inflation—back to the SM

For $\chi \lesssim M_P/\xi$: the Standard Model
Radiative corrections

Could be a problem

In the ordinary situation effective potential is generated

\[ \Delta U(h) \sim \frac{m^4(h)}{64\pi^2} \log \frac{m^2(h)}{\mu^2} + A\Lambda^2 + B\Lambda^4 \]

We suppose that quadratic divergences are dealt with (eg. in dimensional regularization)
Radiative corrections

Could be a problem

In the ordinary situation effective potential is generated

\[ \Delta U(h) \sim \frac{m^4(h)}{64\pi^2} \log \frac{m^2(h)}{\mu^2} \]

standard Yukawa interaction \( m = y \cdot h \)

\[ \Delta U \propto -y^4 h^4 \log \frac{h^2}{\mu^2} \]

Spoils flatness of the potential (for top quark \( y \sim 1 \) !)
Radiative corrections

This is also cured by non-minimal coupling!

Effective potential is still generated

\[
\Delta U(\chi) \sim \frac{m^4(\chi)}{64\pi^2} \log \frac{m^2(\chi)}{\mu^2}
\]

Conformal transformation: fermions

\[
S_J = \int d^4x \sqrt{-g} \left\{ \bar{\psi} \partial \psi + yh \bar{\psi} \psi \right\}
\]

\[
\hat{\psi} = \Omega^{-3/2} \psi
\]

\[
S_E = \int d^4x \sqrt{-\hat{g}} \left\{ \bar{\hat{\psi}} \partial \hat{\psi} + y \frac{h(\chi)}{\Omega(\chi)} \bar{\hat{\psi}} \hat{\psi} \right\}
\]
Radiative corrections

This is also cured by non-minimal coupling!

Effective potential is still generated

\[ \Delta U(\chi) \sim \frac{m^4(\chi)}{64\pi^2} \log \frac{m^2(\chi)}{\mu^2} \]

The interactions are suppressed now!

\[ m(\chi) = y \frac{h(\chi)}{\Omega(\chi)} \xrightarrow{\chi \to \infty} \text{const} \]

(where \( \Omega(\chi) \propto h(\chi) \) for large \( \chi \))

\[ \implies \Delta U(\chi) \to y^4 \frac{M_P^4}{\xi^2} \left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2 \log \left(\frac{m^2(\chi)}{\mu^2}\right) \to \text{const} \]
Radiative corrections

This is also cured by non-minimal coupling!

Effective potential is still generated

\[ \Delta U(\chi) \sim \frac{m^4(\chi)}{64\pi^2} \log \frac{m^2(\chi)}{\mu^2} \]

The same for self interactions

\[ m^2(\chi) = U''(\chi) = \frac{\lambda M_P^2}{3\xi^2} \left(2e^{-\frac{2\chi}{\sqrt{6}M_P}} - 1\right)e^{-\frac{2\chi}{\sqrt{6}M_P}} \xrightarrow{\chi \to \infty} 0 \]

\[ \implies \Delta U(\chi) \to 0 \]
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\[ n = 1 - 6\varepsilon + 2\eta \approx 1 - \frac{8(4N + 9)}{(4N + 3)^2} \approx 0.97 \]

\[ r = 16\varepsilon \approx \frac{192}{(4N + 3)^2} \approx 0.0033 \]
Expected window for the Higgs mass

Standard Model should remain applicable up to

\[ M_P / \xi \simeq 10^{14} \text{ GeV} \]

We expect the Higgs mass

\[ 130 \text{ GeV} < M_H < 190 \text{ GeV} \]

Discovery of the Higgs with different mass will close the model!
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- Adding non-minimal coupling $\xi H^\dagger HR$ of the Higgs field to the gravity makes inflation possible without introduction of new fields
  - The new parameter of the model, non-minimal coupling $\xi$, relates the normalization of CMB fluctuations and the Higgs mass $\xi \simeq 49000 m_H / \sqrt{2} v$
- Predicted for CMB
  - $n_s \simeq 0.97$
  - $r \simeq 0.0033$
- Expected for LHC
  - Higgs mass $130 \text{ GeV} < M_H < 190 \text{ GeV}$
Appendix Outline
Note about fourth family

Disallowed by vacuum metastability or strong coupling before $M_P$.

Pirogov, Zenin, 98