Primordial non-Gaussianity - a new frontier

Benjamin D. Wandelt
Amit P. Yadav
UIUC Physics/Astronomy
Why are we interested in cosmology?

The big questions are:

“What is the fundamental theory, valid at the highest energies?”

“What happened at $t = 0$?”
The Big Picture

DAWN OF TIME

tiny fraction of a second

inflation?

380,000 years

13.7 billion years

CMB

Credit: WMAP
How do we study what happens at the highest energy scales and at the shortest time scales?
Showdown

LHC at CERN

Hubble Ultra Deep Field

WMAP, Planck

\[ \text{HST - ACS} \]
Will accelerators work?

Planck energy (Quantum Gravity)

Unification of forces

CERN

Hard technological limit

Planck time $10^{-42}$s

... $10^{-26}$s

Everyday energies

Low energies

nanoseconds

seconds
The Physics of the Beginning

- Some process, e.g. inflation, seeds curvature perturbations in a huge, smooth Universe at.
- Due to gravity these density fluctuations start to grow. Eventually, overdensities become so large that they collapse to form galaxies and clusters.
Astronomical Probes of the Early Universe
Testing Inflationary Paradigm

• Probes of inflation:
  – Inflation generates primordial fluctuations in space-time
    • Fluctuations in radiation
      – CMB T
      – CMB E-polarization
    • Fluctuations in matter
      – Dark matter distribution (Gravitational lensing etc.)
      – Galaxy and gas distribution (Redshift surveys, Lyman-alpha clouds, cosmological 21-cm radiation, etc)
  • Fluctuations in space time itself
    – Primordial Gravitational Waves (eg. Primordial B-modes of CMB)
The CMB probes...

- Homogeneity and Isotropy
- Flatness
- Seed perturbations

John C. Mather
1978 Nobel Prize in Physics

George Smoot
CMB

COBE

Robert Wilson and Arno Penzias
2006 Nobel Prize in Physics

George Smoot
John C. Mather
Predictions of Standard Inflation

(i) Flat, homogeneous and isotropic

(ii) Seed perturbations: canonical models predict

- Nearly adiabatic:
  \[
  \frac{\delta \rho_i}{\dot{\rho}_i} = \frac{\delta \rho}{\dot{\rho}}
  \]

- Close to Gaussian

- Nearly Scale Invariant

\[
\langle \Phi(\vec{k}) \Phi(\vec{k}') \rangle = P_\Phi(k) \delta^3(\vec{k} - \vec{k}')
\]

\[
k^3 P_\Phi(k) = A k^{n_s - 1}
\]

Komatsu et al (WMAP5): 
+ e.g. HST: \(~2\%\)

Komatsu et al (WMAP5): 
\(~10\%\)

Komatsu et al (WMAP5): 
a few percent:
Primordial perturbations and Gaussianity

- Slow-roll -> shallow potential -> nearly free field; has Gaussian quantum perturbations (field modes in S.H.O. potential)
- Single-field: if multi-field, can have isocurvature perturbations that convert into non-Gaussian curvature perturbations outside horizon.
- Non-standard kinetic term: can inflate in spite of steep potential -> non-Gaussianity
- Standard choice of vacuum – can get flattened triangle contributions if not Bunch-Davies.
# Are primordial perturbations really Gaussian?

<table>
<thead>
<tr>
<th>Non-Gaussianity from the Early Universe</th>
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<tbody>
<tr>
<td>$f_{NL} \sim 0.05$ canonical inflation (single field, couple of derivatives)</td>
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<td>$f_{NL} \sim 0.1--100$ higher order derivatives</td>
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<tr>
<td>DBI inflation</td>
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<td>UV cutoff</td>
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<tr>
<td>$f_{NL} &gt; 10$ curvaton models</td>
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<tr>
<td>$f_{NL} \sim 100$ ghost inflation</td>
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<tr>
<td>$f_{NL} &gt; 20-100$ New Ekpyrotic models</td>
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<tr>
<td>$f_{NL} \sim -50-200$ Ekpyrotic models</td>
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</table>
Non-Gaussianity – a new frontier

• In addition to the information to be gained from 2-point correlations, non-Gaussianity opens a new window on the Physics of the Beginning.

• What is the program?
  – Reliable theoretical prediction of non-Gaussianity from models of the early Universe
  – Characterization of non-Gaussian confusion effects
  – Development of efficient and practical statistical methods to draw inferences about non-Gaussianity from the data.
Our push at the frontier

- How to search for primordial non-Gaussianity
- How to search for $f_{NL}$
- What we find
- How to interpret our result
- Lessons for Planck
- Future prospects
How to search for (weak) primordial non-Gaussianity in 3 easy steps

• Reconstruct curvature perturbation from data
• Test for non-Gaussian features
• Compute error bars using Gaussian Monte Carlo realizations of the data
Reconstructed Primordial Perturbations

\[ \Phi_{lm} = \mathcal{O}_l a_{lm} \]

SW limit

\[ \frac{\delta \phi}{\phi} = \frac{-1}{3} \frac{\delta T}{T} \]

Reconstructed Primordial perturbations with T alone

Response function

\[ \mathcal{O}_l = \beta / C_l \]

\[ \beta^i_\ell (r) = \frac{2b^i_\ell}{\pi} \int k^2 dk P_\phi(k) g^i_\ell (k) j_\ell (kr) \]
The curvature perturbation leaves a unique signature in T & E

- Note negative response on large scales

T and E are out of phase

Yadav, and Wandelt, PRD (2005)
primordial map, using T alone

Yadav, and Wandelt, PRD (2005)
Yadav, and Wandelt, PRD (2005)
Reconstructed perturbations at different radii

Yadav, and Wandelt, PRD (2005)

Decoupling

Curvature fluctuations
Tomographic reconstruction of inflationary scalar curvature

We construct filters that invert linear radiative transport.
Generates a single scalar that contains all the information from T&E.
Anyone intending to test primordial non-Gaussianity (and anisotropy!) in T and/or E data should do so using curvature perturbations obtained with our filters.

Yadav and Wandelt 2006
\( f_{NL} \) – a specific parameterization of non-Gaussianity

\[
\Phi(x) = \Phi_G(x) + f_{NL} \Phi_G^2(x)
\]

Characterizes the amplitude of non-Gaussianity

- This non-Gaussianity creates a bispectrum signature (as well as higher order moments)

\[
\langle \Phi(k_1) \Phi(k_2) \Phi(k_3) \rangle = 2(2\pi)^3 f_{NL} \delta(k_1 + k_2 + k_3) P(k_1) P(k_2),
\]

where \((2\pi)^3 \delta(k_1 + k_2) P(k_1) = \langle \Phi(k_1) \Phi(k_2) \rangle\)

- This translates into a bispectrum signature in the CMB through

\[
\alpha_{lm} = 4\pi (-i)^l \int \frac{d^3k}{(2\pi)^3} \Phi(k) g_{Tl}(k) Y_{lm}^*(k)
\]

Salopek & Bond 1990
Komatsu & Spergel 2001
Statistics of local non-Gaussianity in the CMB

- Local non-Gaussianity smoothes hot spots and creates more structure in cold spots.
$f_{NL} = 0$

Temperature ($f_{NL} = 0$)

Liguori, Yadav, Hansen, Komatsu, Matarrese, Wandelt 2007
$f_{NL} = 10^1$

Temperature ($f_{NL} = 10$)

Liguori, Yadav, Hansen, Komatsu, Matarrese, Wandelt 2007
$f_{NL} = 10^2$

Temperature ($f_{NL} = 10^2$)

Liguori, Yadav, Hansen, Komatsu, Matarrese, Wandelt 2007
\[ f_{NL} = 10^3 \]

Temperature \((f_{NL} = 10^3)\)

Liguori, Yadav, Hansen, Komatsu, Matarrese, Wandelt 2007
\[ f_{NL} = 10^4 \]

Temperature \((f_{NL} = 10^4)\)

Liguori, Yadav, Hansen, Komatsu, Matarrese, Wandelt 2007
Why use the bispectrum?

\[
\begin{align*}
B_{\text{non-Gaussian}} &= 0 + f_{\text{NL}} b^2 \\
T_{\text{non-Gaussian}} &= T_{\text{Gaussian}} + f_{\text{NL}}^2 \delta T
\end{align*}
\]

For weak non-Gaussianity any even moment has a much larger contribution from Gaussian perturbations. This makes measuring the non-Gaussian component difficult.

Babich (2005): bispectrum contains nearly all the information about \( f_{\text{NL}} \). Kogo&Komatsu: Trispectrum contains complementary information.

Unfortunately evaluating all \( B_{111} \) is too expensive.
$f_{NL}$ phenomenology from the bispectrum

- Komatsu & Spergel 2001 – CMB bispectrum from $f_{NL}$
- Komatsu, Wandelt, Spergel, Banday, Gorski 2001 – $f_{NL}$ from COBE
- Komatsu Spergel & Wandelt 2003 – fast $f_{NL}$ estimator
- Komatsu et al (WMAP team) 2003 – WMAP1 analysis using KSW
- Babich and Zaldarriaga 2004 – temperature + polarization
- Creminelli, Nicolis, Senatore, Tegmark, Zaldarriaga 2006 – introduce linear term to improve KSW estimator
- Spergel et al (WMAP team) 2006 – WMAP3 analysis using KSW
- Creminelli, Senatore, Tegmark, Zaldarriaga 2006 – apply cubic + linear term to WMAP3 data
- Yadav & Wandelt 2005 – tomography of the curvature perturbations
- Yadav Komatsu & Wandelt 2007 – KSW generalized to T+P
- Liguori, Yadav, Hansen, Komatsu, Matarrese, Wandelt 2007 – calibrate YKW estimator against non-Gaussian simulations
- Yadav, Komatsu, Wandelt, Liguori, Hansen, Matarrese 2007 – Creminelli et al. corrected and generalized to T+P
- Yadav & Wandelt 2007 – application of YKWLH007 to WMAP3
- Komatsu et al 2008 – application of YKWLH007 to WMAP5
Fast, bispectrum based estimator of local $f_NL$

**Cubic Statistic:**

$$\hat{S}_{prim} = \frac{1}{f_{sky}} \int r^2 dr \int d^2 \hat{n} B(\hat{n}, r) B(\hat{n}, r) A(\hat{n}, r)$$

Komatsu, Spergel and Wandelt 2005

$$B(\hat{n}, r) \equiv \sum_{ip} \sum_{lm} (C^{-1})^{ip} a_{\ell m}^i \beta^p_\ell (r) Y_{\ell m}(\hat{n})$$

$B(r)$ is a map of reconstructed primordial perturbations

$$A(\hat{n}, r) \equiv \sum_{ip} \sum_{lm} (C^{-1})^{ip} a_{\ell m}^i \alpha^p_\ell (r) Y_{\ell m}(\hat{n})$$

$A(r)$ picks out relevant configurations of the bispectrum

Above statistics combine all configurations of bispectrum such that it most sensitive to “local” primordial non-Gaussianity i.e $f_NL$. 
Non-Gaussianity from Inflation

- \( f_{NL} \sim 0.05 \) canonical inflation (single field, couple of derivatives) (Maldacena 2003, Acquaviva et al. 2003)
- \( f_{NL} \sim 0.1\text{--}100 \) higher order derivatives
- \( f_{NL} > 10 \) curvaton models (Lyth, Ungarelli and Wands, 2003)
- \( f_{NL} \sim 100 \) ghost inflation (Arkani-Hamed et al., Cosmol, 2004)

We are far from \( \Delta f_{NL} \sim 1 \) but can already start putting constraints on some models like DBI inflation, ghost inflation etc.
Anisotropic sky coverage

- The KSW and YKWLHM estimators are optimal only for uniform sky coverage and noise distribution. Anisotropic noise distribution couples different $l$ and produces excess variance.

- For non-uniform noise the addition of a linear term reduces the variance of the estimator (Creminelli et al. 2005).

- We (Yadav, Komatsu, Wandelt, et al. arxiv:0711.4933) generalized this estimator to include polarization; and discovered and corrected an error in the linear term.

\[
\hat{S}_{\text{prim}} = \frac{-3}{f_{\text{sky}}} \int r^2 dr \int d^2 \hat{n} \left\{ B(\hat{n}, r) S_{AB}(\hat{n}, r) + S_{BB}(\hat{n}, r) A(\hat{n}, r) \right\}
\]

\[
A(\hat{n}, r) = \sum_{ip} \sum_{lm} (C^{-1})^{ip} a_{ip}^i \alpha_{lp}^p(r) Y_{lp}(\hat{n})
\]

\[
B(\hat{n}, r) = \sum_{ip} \sum_{lm} (C^{-1})^{ip} a_{ip}^i \beta_{lp}^p(r) Y_{lp}(\hat{n})
\]
What is new in Yadav & Wandelt 2008?

Can only analyze to $l_{\text{max}} \approx 375$
Our result:

$27 < f_{NL} < 147$ at 95% Confidence
Questions you might ask

Might his result be due to...

- Instrument systematics?
- Foregrounds?
- Secondary anisotropies?
- Just rediscovery of other non-Gaussian signals?
- Noise fluctuation?
Instrument systematics?

1) Beam asymmetries

- If the CMB is Gaussian, no asymmetry of the main beam can produce non-vanishing bispectrum.

- If there are large side-lobes that spread foreground around the sky they will produce large scale features – unlikely to affect the high $l$ regime. Further, we do not see evidence for frequency dependence.
Instrument systematics? II: WMAP Noise

- **Noise correlations (striping)**
  - As long as noise is Gaussian, no noise correlations will produce a bispectrum.

- **Non-Gaussian noise?**
  Analyzed differences of WMAP yearly maps
  - year1-year2 \( f_{NL} = 1.1 \) (\(+/- \sim 60\) at 95% C.L.)
  - year2-year3 \( f_{NL} = 1.8 \)
  - year1-year3 \( f_{NL} = -3.4 \)

- **So to explain our results an instrumental systematic has to be 1) non-Gaussian, 2) the same in individual years and 3) mimic the specific bispectrum signature of \( f_{NL} \).**
We test the impact of foregrounds as a function of frequency and as a function of mask. V and W channels are the least foreground contaminated. Choice of V+W is driven by foreground considerations. Analysis on raw maps to avoid FG oversubtraction.

<table>
<thead>
<tr>
<th>$\ell_{\text{max}}$</th>
<th>$f_{\text{sky}} = 94.2%$</th>
<th>$f_{\text{sky}} = 84.7%$</th>
<th>$f_{\text{sky}} = 76.8%$</th>
<th>$f_{\text{sky}} = 64.3%$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kp12</td>
<td>Kp2</td>
<td>Kp0</td>
<td>Kp0++</td>
</tr>
<tr>
<td>350</td>
<td>-3145.22</td>
<td>-26.68</td>
<td>34.62</td>
<td>19.24</td>
</tr>
<tr>
<td>450</td>
<td>-1425.06</td>
<td>-15.63</td>
<td>67.94</td>
<td>64.69</td>
</tr>
<tr>
<td>550</td>
<td>-1509.92</td>
<td>-13.09</td>
<td>79.18</td>
<td>81.29</td>
</tr>
<tr>
<td>650</td>
<td>-1559.91</td>
<td>-22.43</td>
<td>86.81</td>
<td>86.52</td>
</tr>
<tr>
<td>750</td>
<td>-1575.11</td>
<td>-22.81</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

stable beyond kp0

no frequency dependence
Foregrounds? (II)

- **WMAP raw maps vs WMAP cleaned maps**
  - Foreground subtracted maps do not show negative $f_{NL}$ behavior
  - Same level of $f_{NL}$, uniformly higher for FG subtracted maps
  - *We quote the result from raw maps to be conservative and because the cleaned maps could contain oversubtracted foregrounds giving a positive bias.*
Foregrounds (III)

- **Simulations of Gaussian CMB + Foregrounds + WMAP Noise**
  - negative for smaller masks
  - goes to zero by the time you reach Kp0 mask
  - is consistent with zero for masks greater than k0

<table>
<thead>
<tr>
<th>$\ell_{\text{max}}$</th>
<th>VW</th>
<th>Q</th>
<th>QVW</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kp12</td>
<td>Kp2</td>
<td>Kp0</td>
</tr>
<tr>
<td>350</td>
<td>-1290</td>
<td>-27</td>
<td>35</td>
</tr>
<tr>
<td>450</td>
<td>-1425</td>
<td>-16</td>
<td>68</td>
</tr>
<tr>
<td>550</td>
<td>-1510</td>
<td>-13</td>
<td>80</td>
</tr>
<tr>
<td>650</td>
<td>-1560</td>
<td>-22</td>
<td>79</td>
</tr>
<tr>
<td>750</td>
<td>-1575</td>
<td>-23</td>
<td>87</td>
</tr>
<tr>
<td>750*</td>
<td>-1105</td>
<td>-42</td>
<td>-6</td>
</tr>
<tr>
<td></td>
<td>$\pm \frac{19}{19}$</td>
<td>$\pm \frac{5}{5}$</td>
<td>$\pm \frac{4}{4}$</td>
</tr>
</tbody>
</table>
Secondary Anisotropies?

- Point sources, including SZ
  - Orthogonal overlap with primordial bispectrum. Bias of $|f_{NL}| < 1$. SZ and point sources have opposite signs.

- Serra and Cooray (arxiv:0801.3276)
  - Dominant secondary confusion level to WMAP bispectrum arises from
    - ISW-lensing bispectrum (positive bias)
    - SZ-lensing bispectrum (negative bias)
  - If $f_{NL} = 20$ effective bias around 10%. **Negligible** for $f_{NL} > 20$, because effects add in quadrature.
Re-discovery of another non-Gaussian signal?

- Larson/Wandelt (hot and cold spots not hot or cold enough):
  - at smaller angular scales
  - symmetric -> no odd correlation. Probably noise model.

- The Cold Spot (Vielva et al. 2004) is localized in the map and covers a particular range in scale.
  Preliminary result: \( f_{NL} = 94 \pm 60 \) (95% C.L.)

- Large Scale anomaly? Can check by removing large scale signal. Preliminary result:
  Removing \( l<21 \), \( f_{NL} = 135 \pm 96 \) (95% C.L.)
Sensitivity to assumed cosmology

- The filters depend weakly on assumed cosmology. We used $n=1$.
- Choosing $n=0.95$ reduces the error bars by 10%, and reduces the central values between 5% and 15%.
- At $l_{\text{max}}=750$, significance increases to just over 3 sigma; at lower $l_{\text{max}}$, significance decreases slightly.
Noise fluctuation?

• Possible. Noise couples to any bispectrum form.
• It's a 2.5-2.8 sigma result. \( P \leq 0.01 \)

2.5 sigma when after conservative increase of error bar to model uncertainty in residual systematics

[The most aggressive interpretation of the WMAP3 data would be a 3.3 sigma effect (correcting for negative foreground bias and using best fit WMAP parameters)]
Summary of Yadav & Wandelt 2008

- $\Delta f_{NL} \sim 30$ for all of WMAP 3 using YKWLHLM07 and WMAP best fit parameters (statistical)
- First bispectrum-based analysis of the full WMAP3 data
- First significant departure of $f_{NL}$ from 0 at $>99\%$ C.L.
- Estimators tested against Gaussian and non-Gaussian simulations with and without inhomogeneous noise
- If any bias, it is likely to be negative.
- 2.5-2.8 sigma, depending on choices and assumptions
WMAP 5-year analysis

- Komatsu et al. 2008
- Somewhat more conservative analysis:
  - mask shape that enhances the statistical error compared to the 3-yr mask;
  - stop at $l_{\text{max}} = 500$
  - subtract very generous estimate of point source bias.

- Quoted result: $f_{\text{NL \ local}} = 51 +/- 60 \ (95\%)$
- Significance: 1.7 sigma
- 2.3 sigma quoted for analysis closer to ours
- Differences understood $\Rightarrow$ Consistent with our analysis
WMAP 5 year constraint on $f_{\text{equil}}$

$$-151 < f_{\text{NL}}^{\text{equil}} < 253; \quad \Delta f_{\text{NL}}^{\text{equil}} = 201$$

- Of interest for DBI inflation, ghost condensation

(for reference: Planck should get $\Delta f_{\text{NL}}^{\text{equil}} \sim 35$)
A very preliminary result by K. Smith et al., obtained at the Perimeter Workshop a few weeks ago:

\[ f_{NL}^{\text{local}} = 21 \pm 44 \text{ (95\%)} \]

Note that this uses the exact same data as the WMAP 5, so the difference is entirely due to different weighting in the estimator.
- Smaller error bar due to optimal weighting
- This remains to be checked and the differences remain to be understood.
Next generation $f_{\text{NL}}$ statistics: Fully Bayesian non-Gaussianity analysis

- Instead of going via the bispectrum, build full statistical model of the data, including non-Gaussianity, and a detailed model of the observations
- Returns the full $P(f_{\text{NL}} | \text{data})$

Toy model

Elsner, Wandelt, Schneider 2008
Non-Gaussianity post WMAP

For an Ideal CMB experiment and using both temperature and polarization we can get down to \( \Delta f_{NL} \sim 1 \) (maybe less using lensing).

For Planck the Cramer Rao limit is \( \Delta f_{NL} \sim 3 \)

$f_{\text{NL}}$ from large scale structure

- Halo mass function: Verde, Matarrese, Jimenez (2000); LoVerde, Miller, Shandera & Verde (2007)
- Afshordi (2008): $f_{\text{NL}} = 240 \pm 120$ from CMB/LSS $X$-correlations Ho et al 2008
- Ultimately, $\Delta f_{\text{NL}} \sim 5$! 
Outlook

- Analysis of WMAP 5 year data continues
- New data to come soon! Forecasts:
  - WMAP 8 year: $\Delta f_{NL} \sim 21$
- $\Delta f_{NL} \sim 5$ from Planck T and E polarization (in <5 yrs!)
- Cross-checks using temperature and polarization
- Implications of large non-Gaussianity:
We wrote:

“If our result holds up under the statistical weight of future data, it will have profound implications for our understanding of the physics of the early Universe. As it stands, the data disfavors canonical single field slow-roll inflation.”
Final thoughts

- Non-Gaussianity is a powerful probe of the physics of the beginning
  - currently the *highest precision* test of inflation
    (flatness in second place $\sim 1.5\%$, non-Gaussianity at $\sim 0.1\%$)
  - a way to distinguish between classes of models that give similar predictions for the two-point correlations
  - complementary to tensor modes
- A new, exciting and fast-moving frontier
- Great news for cosmology!