Dual superconformal symmetry of scattering amplitudes in $\mathcal{N} = 4$ super-Yang-Mills

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Based on work in collaboration with

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Plan of the talk

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1 Introduction

• Planar color-ordered n–particle scattering amplitudes in $\mathcal{N}=4$ SYM (gluons, gluinos, scalars) are functions of light-like momenta $p_i^2=0$ and helicities $h_i=\pm1,\pm1/2,0$ $(i=1\dots n)$, given by their perturbative expansion in $a=g^2N/8\pi^2$:

$$\mathcal{A}_n(p_i,h_i)=\mathcal{A}_{n;0}+a\sum_H \mathcal{A}^H_{n;1}\,M^H_{n;1}(p_i)+O(a^2)$$

$\mathcal{A}_{n;0}\longrightarrow$ tree-level amplitude depending on helicity distribution
$\mathcal{A}^H_{n;1}\longrightarrow$ one-loop coefficient function (helicity structure); the some goes over all independent helicity structures H
$M^H_{n;1}\longrightarrow$ one-loop scalar Feynman integrals
IR divergences $\Rightarrow$ dimensional regularization

• Simplest example: Maximally Helicity Violating (MHV) amplitudes of total helicity $n-4$, e.g. for gluons:
$(--+\dots+)$, $(-+-+\dots+)$, etc.

Unique helicity structure (tree):

$$\mathcal{A}_n^{\rm MHV}(p_1^-,p_2^-,p_3^+,\dots,p_n^+)=\mathcal{A}_{n;0}^{\rm MHV}\;M_n^{\rm MHV}(p_i)$$

$$M_n^{\rm MHV}=1+aM_n^{(1)}+O(a^2)$$

• $\mathcal{N}=4$ SYM is a (super)conformal theory $\Rightarrow$ conformal symmetry of $\mathcal{A}_n$? Two problems:

(i) Conformal boosts realized on momenta are 2nd-order differential operators (Witten)

(ii) IR divergences break conformal symmetry

• Dual conformal (super)symmetry – hidden symmetry of $\mathcal{A}_n$ of dynamical origin:

♣ Linear action on the particle momenta

♠ Exact symmetry of $\mathcal{A}_{n;k}$, $k = 0, 1, \dots$ (for the superamplitude)

♢ Anomalous symmetry of $M_{n;k}$ controlled by WI

♡ Exact symmetry of $\mathcal{N}_n$ where

$$\mathcal{A}_n = \mathcal{A}_n^{\mathrm{MHV}} \times \mathcal{N}_n$$

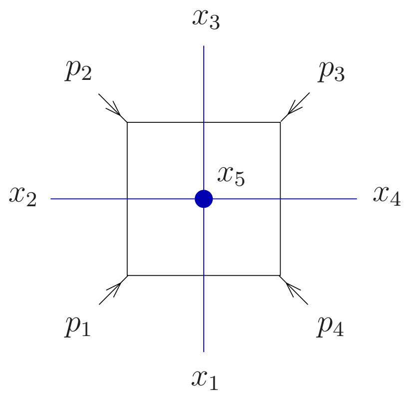
(conformal anomaly contained in MHV prefactor)

To show all this, we need a superamplitude – compact formulation of all amplitudes from the $\mathcal{N} = 4$ SYM supermultiplet $\Rightarrow$ superconformal symmetry

2 How it all started: pseudo-conformal momentum integrals

• Early evidence for dual conformal symmetry (Drummond, Henn, Smirnov, ES) from the study of loop momentum integrals appearing in M_4^{MHV} (Bern, Dixon, Dunbar, Kosower, Rozowski, Smirnov, ...)

• Simplest example: one-loop correction



Momentum integral in D dimensions

$$\int \frac{d^D k}{k^2(k-p_1)^2(k-p_1-p_2)^2(k+p_4)^2} \Rightarrow \int \frac{d^D x_0}{x_{10}^2 x_{20}^2 x_{30}^2 x_{40}^2}$$

rewritten in

Dual space:

$$p_i = x_i - x_{i+1} \equiv x_{i\ i+1} \quad \Leftrightarrow \quad \sum_i p_i = 0 \ \text{ if } \ x_{n+1} \equiv x_1$$

as well as $k = x_{10}$.

This is a simple change of variables, not a Fourier transform!

• Consider the usual conformal group $SO(4,2)$ acting on the dual coordinates $\longrightarrow$ dual conformal group.

Conformal group = Poincaré + inversion:

$$x^\mu \longrightarrow \frac{x^\mu}{x^2} \ : \qquad x_{ij}^2 \longrightarrow \frac{x_{ij}^2}{x_i^2 x_j^2}\,, \qquad d^D x \longrightarrow \frac{d^D x}{(x^2)^D}$$

The integral is dual conformal covariant if $D = 4$.

But! Problem on shell:

$$p_i^2 = 0 \ \rightarrow \ x_{i\ i+1}^2 = 0 \ \rightarrow \ \text{IR divergence} \ \rightarrow \ D = 4 - 2\epsilon$$

Pseudo-conformal integral (not a true symmetry).

• Surprise: All loop integrals up to 4 (5?) loops appearing in Bern et al calculations of $M_4^{\rm MHV}$ are pseudo-conformal.

Why? What consequences for the amplitude? We do not know – yet!

Dual conformal symmetry $\neq$ pseudo-conformal integrals (may be related, though)

3 MHV amplitude = Wilson loop. Anomalous dual conformal symmetry

3.1 Strong coupling description of MHV amplitudes

New AdS/CFT duality by Alday and Maldacena:

• MHV amplitudes as minimal surfaces in AdS

• makes use of the same change of variables $p_i = x_i - x_{i+1}$ ("T-duality")

• mathematically equivalent to a Wilson loop on a contour defined by the particle momenta

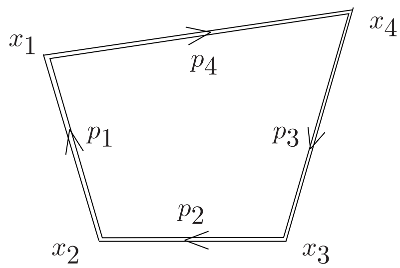
• efficient use of conformal invariance to find the classical solution for $n = 4$

• helicity blind – yet? Non-MHV amplitudes?

3.2 Weak coupling duality MHV amplitudes = Wilson loops

Generalization of the strong coupling duality to weak coupling (Drummond, Henn, Korchemsky, ES ; Brandhuber, Heslop, Travaglini)

$$W(C_4) = \frac{1}{N_c}\langle 0|\mathrm{Tr\,P}\exp\left(ig\oint_{C_4} dx^\mu A_\mu(x)\right)|0\rangle\,,$$

$C_4 =$ 

• UV divergences of WL $\Leftrightarrow$ IR divergences of MHV amplitude:

$$\ln W(C_n) = \ln Z_n + \ln F_n + O(\epsilon)$$

Divregent part: $\ln Z_n = \sum_{l\geq 1} a^l \sum_{i=1}^{n} \left(-x^2_{i-1,i+1}\mu^2\right)^{l\epsilon}\left(\frac{\Gamma^{(l)}_{\rm cusp}}{(l\epsilon)^2} + \frac{\Gamma^{(l)}}{l\epsilon}\right)$

• Conformal invariance of WL broken by UV in a controlled way $\Rightarrow$ all-order anomalous conformal Ward identity (Drummond, Henn, Korchemsky, ES):

$$K^\mu \ln F_n = \sum_{i=1}^{n}(2x_i^\nu x_i\cdot\partial_i - x_i^2\partial_i^\nu)\ln F_n = \frac{1}{2}\Gamma_{\rm cusp}(a)\sum_{i=1}^{n}\ln\frac{x^2_{i,i+2}}{x^2_{i-1,i+1}}x^\nu_{i,i+1}$$

Fixes the form of $\ln F_n$ for $n = 4, 5$ but not for $n \geq 6$

• New all-order duality

$$\ln F_n^{\rm WL} = \ln F_n^{\rm MHV}$$

Checked at one loop for all n and at two loops for $n = 4, 5, 6$, also at strong coupling for $n = 4$

• Duality WL = MHV $\Rightarrow$ dual conformal invariance of MHV $\rightarrow$ BDS conjecture (see talk by Gregory Korchemsky)

4 Dual conformal symmetry of gluon amplitudes

Question: Can we generalize dual conformal symmetry to non-MHV amplitudes?

$$\mathcal{A}_n(p_i, h_i) = \mathcal{A}_{n;0} + a \sum_H \mathcal{A}^H_{n;1} \; M^H_{n;1}(p_i) + O(a^2)$$

Need to examine both the helicity structures $\mathcal{A}_{n;0}$, $\mathcal{A}^H_{n;1}$ and loop corrections $M^H_{n;1}$.

Start with the simplest case of MHV amplitudes $\rightarrow$ unique helicity structure. Is it dual conformal?

4.1 MHV tree level

- Spinor helicity formalism: introduce commuting spinors λ^α, $\tilde\lambda^{\dot\alpha}$

$$p_i^2 = 0 \;\Leftrightarrow\; p_i^{\alpha\dot\alpha} \equiv p_i^\mu (\sigma_\mu)^{\alpha\dot\alpha} = \lambda_i^\alpha \, \tilde\lambda_i^{\dot\alpha}$$

$$\mathcal{A}^{\rm MHV}_{n;0}(\ldots(-i)\ldots(-j)\ldots) = \delta^{(4)}(\sum_{k=1}^n p_k) \, \frac{\langle i\, j\rangle^4}{\langle 1\, 2\rangle\langle 2\, 3\rangle \ldots \langle n\, 1\rangle}$$

Lorentz invariant spinor contractions

$$\langle i\, j\rangle = -\langle j\, i\rangle = \epsilon^{\alpha\beta}\lambda_{i\,\alpha}\, \lambda_{j\,\beta}$$

carrying helicities $-1/2$ at points i and j

- Is this amplitude dual conformal?

4.2 Dual conformal transformations of spinors

• Dual coordinates $\rightarrow$ spinor variables:

$$p_i^{\alpha\dot{\alpha}} = (x_i - x_{i+1})^{\alpha\dot{\alpha}} = \lambda_i^\alpha \, \tilde{\lambda}_i^{\dot{\alpha}} \qquad \Rightarrow \; \lambda_i^\alpha \, (x_i - x_{i+1})_{\alpha\dot{\alpha}} = 0$$

• Conformal inversion in dual space:

$$I[x_{ij}] = x_i^{-1}(x_i - x_j)x_j^{-1} \;\Rightarrow\; I[x_i - x_{i+1}] = x_i^{-1}(\lambda_i \, \tilde{\lambda}_i)x_{i+1}^{-1} \;\Rightarrow$$

$$\begin{aligned} I[\lambda_i^\alpha] \;&=\; \frac{\lambda_i^\alpha (x_i)_{\alpha\dot{\alpha}}}{x_i^2} \equiv \lambda_i \, x_i^{-1} \\ &=\; \lambda_i^\alpha \frac{(x_{i+1})_{\alpha\dot{\alpha}}}{x_i^2} \end{aligned}$$

• Conformal properties of $\langle i \, j \rangle$:

$$I[\langle i \; i+1 \rangle] = \langle i | \frac{x_{i+1}}{x_i^2} x_{i+1}^{-1} | i+1 \rangle = \frac{\langle i \; i+1 \rangle}{x_i^2}$$

So, $\langle i \; i+1 \rangle$ is dual conformal, but $\langle i \, j \rangle$ for $j \neq i+1$ is not !

• The rational factor in $\mathcal{A}_{n;0}^{\mathrm{MHV}}$ is dual covariant only if the negative-helicity gluons are adjacent ('split-helicity' amplitudes).

4.3 Properties of the delta function

$\delta^{(4)}(\sum_{i=1}^n p_i)$ imposes momentum conservation:

$$\sum_{i=1}^n p_i = 0 \;\Leftrightarrow\; \sum_{i=1}^n (x_i - x_{i+1}) = 0 \;\text{ iff }\; x_{n+1} \equiv x_1$$

$\rightarrow$ cyclic symmetry

• Break the cycle by relaxing $x_1 = x_{n+1}$ and then impose it by

$$\delta^{(4)}(x_1 - x_{n+1}) \;\rightarrow\; \text{manifestly dual conformal}$$

4.4 Split-helicity non-MHV tree amplitudes

• Split-helicity MHV tree amplitudes are dual conformal, e.g.

$$\mathcal{A}_n^{\rm MHV}(- - + \ldots +) = \delta^{(4)}(x_1 - x_{n+1}) \, \frac{\langle 1\, 2\rangle^4}{\langle 1\, 2\rangle\langle 2\, 3\rangle \ldots \langle n\, 1\rangle}$$

• All split-helicity non-MHV tree amplitudes are dual conformal. Checked directly using the recursion relations of Britto, Cachazo, Feng, Roiban, Spradlin, Volovich, Witten

• Non-split-helicity tree amplitudes are not dual conformal

• Accidental property of split-helicity amplitudes?

No, general property! To see it, we need dual supersymmetry.

5 Superamplitudes in on-shell superspace

• Superamplitudes: a compact way of writing all $\mathcal{N} = 4$ SYM amplitudes (gluons, gluinos and scalars) in dual superspace.

• In this form dual (super)conformal symmetry is manifest

5.1 Nair's formulation of MHV amplitudes

• Nair proposed to describe tree MHV amplitudes in superspace

$$\mathcal{A}_n^{\mathrm{MHV}} = \frac{\delta^{(4)}(\sum_{i=1}^n p_i)\, \delta^{(8)}(\sum_{j=1}^n \lambda_{j\,\alpha}\, \eta_j^A)}{\langle 1\,2\rangle\langle 2\,3\rangle \ldots \langle n\,1\rangle}$$

Here η_i^A ($A = 1 \ldots 4$ is an $SU(4)$ R symmetry index), with helicity 1/2, are Grassmann variables of on-shell superspace

• $\mathcal{N} = 4$ gluon supermultiplet $\rightarrow$ PCT self-conjugate $\rightarrow$ holomorphic (chiral) description

$$\begin{aligned}\Phi(p,\eta) \ &= \ G^+(p) + \eta^A \Gamma_A(p) + \eta^A \eta^B S_{AB}(p) + \eta^A \eta^B \eta^C \epsilon_{ABCD} \bar{\Gamma}^D(p) \\ &\quad + \eta^A \eta^B \eta^C \eta^D \epsilon_{ABCD} G^-(p)\end{aligned}$$

Particle wave functions: $G^\pm$ – gluons (helicity ± 1); Γ_A, $\bar{\Gamma}^A$ – gluinos (helicity $\pm 1/2$); S_{AB} – scalars (helicity 0)

• Extracting, e.g., the gluon component $(- - + \ldots +)$: collect η^4 terms at negative-helicity sites

$$\delta^{(8)}(\sum_{i=1}^n \lambda_{i\,\alpha}\, \eta_i^A) \ \rightarrow \ \langle 12\rangle^4 \ \eta_1^4 \eta_2^4 \eta_3^0 \ldots \eta_n^0$$

5.2 On-shell $\mathcal{N} = 4$ supersymmetry

• Clifford algebra of creators and annihilators for massless Poincaré states:

$$q^A = \eta^A\,, \qquad \bar{q}_A = \frac{\partial}{\partial \eta^A}\,, \qquad \{q^A, \bar{q}_B\} = \delta^A_B$$

• Usually considered on the light cone (breaking Lorentz)
• Covariant description with the help of λ_α:

$$q^A_\alpha = \lambda_\alpha \eta^A\,, \qquad \bar{q}_{A\,\dot\alpha} = \tilde{\lambda}_{\dot\alpha}\frac{\partial}{\partial \eta^A}$$

On-shell $\mathcal{N} = 4$ supersymmetry ($p^2 = 0$):

$$\{q^A_\alpha, \bar{q}_{B\,\dot\alpha}\} = \delta^A_B\,\lambda_\alpha\tilde{\lambda}_{\dot\alpha} = \delta^A_B\,p_{\alpha\dot\alpha}$$

• λ^α serves as a bridge between Lorentz and Poincaré $\rightarrow$ Lorentz harmonics (twistor variables)

5.3 General superamplitudes

• Origin of delta function:

$$\text{Translation invariance } p = \sum_{i=1}^{n} p_i \;\Rightarrow\; \delta^{(4)}(\sum_{i=1}^{n} p_i) = \delta^{(4)}(\sum_{i=1}^{n} \lambda_i\,\tilde{\lambda}_i)$$

$$\text{On-shell } q\text{–supersymmetry } \; q^A_\alpha = \sum_{i=1}^{n} (q_i)^A_\alpha \;\Rightarrow\; \delta^{(8)}(\sum_{i=1}^{n} \lambda_{i\,\alpha}\,\eta^A_i)$$

• General superamplitude

$$\mathcal{A}_n(\lambda,\tilde\lambda,\eta) = \delta^{(4)}(\sum_{i=1}^{n} \lambda_i\,\tilde\lambda_i)\,\delta^{(8)}(\sum_{j=1}^{n} \lambda_j\,\eta_j)\left[\mathcal{A}_n^{(0)}+\mathcal{A}_n^{(4)}+\ldots+\mathcal{A}_n^{(4n-16)}\right]$$

$\mathcal{A}_n^{(4k)}(\eta)$ – homogeneous polynomials of degree $4k$:

$$\begin{aligned} &k=0 \;\rightarrow\; \text{MHV} \\ &k=1 \;\rightarrow\; \text{Next-to-MHV} \\ &\ldots \\ &k=n-4 \;\rightarrow\; \overline{\text{MHV}} \end{aligned}$$

• Simplest case – MHV:

$$\mathcal{A}_n^{(0)} = \frac{1}{\langle 1\,2\rangle\langle 2\,3\rangle\ldots\langle n\,1\rangle}\,M_n(p)$$

Perturbative corrections in $M_n(p) = 1 + aM_n^{(1)} + a^2M_n^{(2)} + \ldots$ $\rightarrow$ IR divergences

Complete all-order MHV superamplitude:

$$\mathcal{A}_n^{\text{MHV}}(\lambda,\tilde\lambda,\eta) = \frac{\delta^{(4)}(\sum_{i=1}^{n} p_i)\;\delta^{(8)}(\sum_{j=1}^{n}\;\lambda_{j\,\alpha}\,\eta_j^A)}{\langle 1\,2\rangle\langle 2\,3\rangle\ldots\langle n\,1\rangle}\,M_n(p)$$

• Rewrite the general superamplitude by pulling out MHV:

$$\mathcal{A}_n = \mathcal{A}_n^{\text{MHV}}\left[1+\mathcal{N}_n^{(4)}+\ldots+\mathcal{N}_n^{(4n-16)}\right]$$

$\mathcal{N}_n^{(4k)}$ are finite and nilpotent. They contain helicity structures and loop corrections for all non-MHV superamplitudes.

• Conjecture: all $\mathcal{N}_n^{(4k)}$ are exactly dual superconformal. The dual conformal anomaly is confined to the IR divergent MHV prefactor.

6 Dual superconformal symmetry: MHV superamplitudes

6.1 Dual superspace

- Introduce dual superspace coordinates:

$$\sum_{i=1}^{n} p_i = 0 \;\rightarrow\; \text{even coordinates} \quad p_i = x_i - x_{i+1}\,, \quad x_{n+1} = x_1$$

$$\sum_{i=1}^{n} \lambda_i \eta_i = 0 \;\rightarrow\; \text{odd coordinates} \quad \lambda_{i\,\alpha}\,\eta_i^A = (\theta_i - \theta_{i+1})^A_\alpha\,, \quad \theta_{n+1} = \theta_1$$

- Dual chiral superspace

$$(x_{\alpha\dot\alpha}\,,\, \theta^A_\alpha\,,\, \lambda_\alpha)$$

Defining constraints:

$$\begin{aligned} \lambda^\alpha_i\,(x_i - x_{i+1})_{\alpha\dot\alpha} = 0 \;&\rightarrow\; \text{derive } \tilde\lambda^{\dot\alpha}_i \\ \lambda^\alpha_i\,(\theta^A_i - \theta^A_{i+1})_\alpha = 0 \;&\rightarrow\; \text{derive } \eta^A_i \end{aligned}$$

6.2 Dual $\mathcal{N} = 4$ superconformal symmetry

- $\mathcal{N} = 4$ super-Poincaré algebra realized in dual superspace

$$Q_{A\,\alpha} = \sum_{i=1}^{n} \frac{\partial}{\partial \theta_i^{A\,\alpha}}\,, \quad \bar Q^A_{\dot\alpha} = \sum_{i=1}^{n} \theta_i^{A\,\alpha} \frac{\partial}{\partial x_i^{\dot\alpha\alpha}}\,, \quad P_{\alpha\dot\alpha} = \sum_{i=1}^{n} \frac{\partial}{\partial x_i^{\dot\alpha\alpha}}$$

$$\{Q_{A\,\alpha}, \bar Q^B_{\dot\alpha}\} = \delta^B_A\; P_{\alpha\dot\alpha}$$

• Conformal inversion rules for dual superspace coordinates

$$I[x_i] = x_i^{-1}\,, \quad I[\theta_i] = \theta_i x_i^{-1}\,, \quad I[\lambda_i] = \lambda_i x_i^{-1}$$

$\rightarrow$ compatible with the defining constraints

• From Poincaré to conformal supersymmetry:

$\rightarrow$ Conformal boosts: $K = IPI$

$\rightarrow$ Special conformal supersymmetry : $(S, \bar{S}) = I(Q, \bar{Q})I$

6.3 Dual superconformal symmetry of MHV superamplitudes

• Properties of the delta functions:

Relax the cyclic conditions $x_{n+1} = x_1$, $\theta_{n+1} = \theta_1$ and impose them through delta function. Then, only in $\mathcal{N} = 4$,

$$\begin{aligned} I[\delta^{(4)}(x_1 - x_{n+1})] &\rightarrow x_1^8\, \delta^{(4)}(x_1 - x_{n+1}) \\ I[\delta^{(8)}(\theta_1 - \theta_{n+1})] &\rightarrow x_1^{-8}\, \delta^{(8)}(\theta_1 - \theta_{n+1}) \end{aligned}$$

• MHV superamplitude in dual superspace

$$\mathcal{A}_n^{\mathrm{MHV}}(x, \theta, \lambda) = \frac{\delta^{(4)}(x_1 - x_{n+1})\ \delta^{(8)}(\theta_1 - \theta_{n+1})}{\langle 1\, 2\rangle \langle 2\, 3\rangle \ldots \langle n\, 1\rangle}\ M_n(x_{i\ i+1})$$

Tree – manifestly dual (super)conformal covariant.

Loops – IR divergent factor $M_n(x_{i\ i+1})$ satisfies anomalous dual conformal Ward identity

• Part of the superconformal algebra $(Q, \bar{S}, P, K)$ is a symmetry of the whole amplitude, and $(\bar{Q}, S)$ only of the tree amplitude

7 Dual superconformal symmetry: non-MHV superamplitudes

7.1 Conjecture

Recall the general structure of the superamplitude

$$\mathcal{A}_n = \mathcal{A}_n^{\rm MHV} \left[1 + \mathcal{N}_n^{(4)} + \ldots + \mathcal{N}_n^{(4n-16)}\right]$$

- $\mathcal{A}_n^{\rm MHV}$ is the full MHV amplitude, containing the IR divergences and satisfying an anomalous dual CWI $\Leftrightarrow$ Wilson loop
- All $\mathcal{N}_n^{(4)}$ are dual superconformal nilpotent invariants

7.2 Evidence: one-loop NMHV superamplitudes

The complete one-loop NMHV superamplitude of Bern, Dixon, Kosower is described by the dual superconformal invariant

$$\mathcal{N}_n^{(4)} = \sum_{p,q,r=1}^{n} \frac{\delta^{(4)}(\Xi_{pqr})}{D_{pqr}}\, M_{pqr}(x_{i\ i+1})$$

- $$\begin{aligned}\Xi_{pqr} &= \langle p|\left[x_{pq}x_{qr}(|\theta_r\rangle - |\theta_p\rangle) + x_{pr}x_{rq}(|\theta_q\rangle - |\theta_p\rangle)\right] \\ &= -\langle p|\left(x_{pq}x_{qr}\sum_{i=p}^{r-1}|i\rangle\eta_i + x_{pr}x_{rq}\sum_{i=p}^{q-1}|i\rangle\eta_i\right)\end{aligned}$$

are 3-point dual superspace covariants of degree 4

- $$D_{pqr} = \frac{x_{qr}^2\langle p|x_{pr}x_{r\,q-1}|q-1\rangle\langle p|x_{pr}x_{r\,q}|q\rangle\langle p|x_{pq}x_{q\,r-1}|r-1\rangle\langle p|x_{pq}x_{q\,r}|r\rangle}{\langle q-1\,q\rangle\langle r-1\,r\rangle}$$

is a dual conformal covariant

- $$M_{pqr}(x_{i\ i+1}) = 1 + aM_{pqr}^{(\rm one-loop)} + \ ?\ O(a^2)$$

are dual conformal invariant functions, made of finite combinations of one-loop scalar box integrals

7.3 Comments

• The superstructure

$$\frac{\delta^{(8)}(\sum_{i=1}^{n}\ \lambda_{i\,\alpha}\,\eta_i^A)\ \delta^{(4)}(\Xi_{pqr})}{D_{pqr}}$$

encodes all helicity structures for gluons, gluinos, scalars. In particular

$$C_{m_1 m_2 m_3}\ \eta_{m_1}^4\ \eta_{m_2}^4\ \eta_{m_2}^4$$

describes gluon NMHV amplitudes with negative-helicity gluons at sites m_1, m_2, m_3.

$$C_{m_1 m_2 m_3}\ \Leftrightarrow\ \text{3-mass-box coefficients of Bern, Dixon, Kosower}$$

• Expanding in η_i breaks manifest dual conformal symmetry, except for split-helicity terms. The non-split-helicity ones transform into each other

• As a byproduct, we get a new, manifestly Lorentz covariant form of the NMHV tree superamplitude

$$\mathcal{A}_{n;0}^{\mathrm{NMHV}} = \delta^{(8)}(\sum_{i=1}^{n}\lambda_{i\,\alpha}\,\eta_i^A)\ \sum_{p,q,r}\ \frac{\delta^{(4)}(\Xi_{pqr})}{D_{pqr}}$$

No need for reference spinor!

Compare to Georgio, Glover, Khoze and to Bianchi, Freedman, Elvang (talk from yesterday)

• An early result for $n = 6$ NMHV in a paper by Huang.

8 Conclusions and outlook

• Dual (super)conformal symmetry is a universal feature of $\mathcal{N} = 4$ scattering amplitudes

• Its origin is unknown (dynamical)

• What fixes the form of the superstructures

$$\frac{\delta^{(4)}(\Xi_{pqr})}{D_{pqr}} \ ?$$

Dual superconformal symmetry allows considerable freedom of choice. Need further constraints (dynamical symmetries)

• Probably the "tip of an iceberg" of an (infinite?) set of symmetries $\rightarrow$ integrability?

• non-MHV amplitudes provide us with finite exactly dual conformal functions. Can we find differential equations for them? $\rightarrow$ integrability?

• How to adapt the Wilson loop and the string description to see helicity? Does the duality NMHV = (N)WL hold?