Gravitational MHV Amplitudes in Twistor Space

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Based on work in progress with L. Mason

Wonders of Gauge Theory & Supergravity

23rd June 2008

MHV perturbation theory

- Can build SYM perturbation theory from MHV amplitudes, continued off-shell, together with $+-$ propagators

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- Can we find an action whose vertices gives these amplitudes?

  (Theisen, Ananth)

Gravity

Gravitational MHV amplitudes (Berends, Giele & Kuijf)

$$\mathcal{M} = [1n]^8 \left\{ \frac{\langle 12\rangle\langle n-2\; n-1\rangle}{[1\; n-1]} \frac{F}{N(n)} \prod_{i=1}^{n-3} \prod_{j=i+2}^{n-1} [ij] + \mathrm{P}_{(2,\ldots,n-2)} \right\}$$

where $N(n) := \prod_{i<j} [ij]$ and $F := \prod_{k=3}^{n-3} \langle k|p_{k+1} + p_{k+2} + \cdots + p_{n-1}|n]$

- Apology: with Penrose conventions for twistor space, natural amplitudes are mostly minus
- Can simplify a little...

Gravity

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Today, as a first step, we'll construct an on-shell generating function in twistor space, whose expansion gives the BGK amplitudes

(Nair)

Outline

- Brief review of Yang-Mills
  - Chalmers & Siegel action
  - Parke-Taylor amplitudes from Penrose transform of ${G^+}^2$

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  - Non-linear graviton
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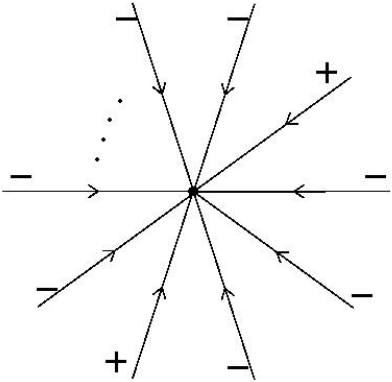
- Open questions & conclusions

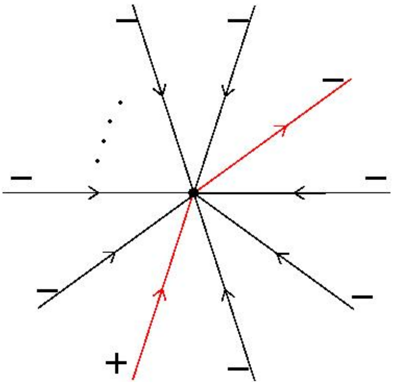
Brief Review of Yang-Mills

Scattering off an ASD background

MHV amplitudes involve 2 positive and $n-2$ negative helicity particles



A background made up entirely of negative helicity gluons/gravitons is anti self-dual



Scattering off an ASD background

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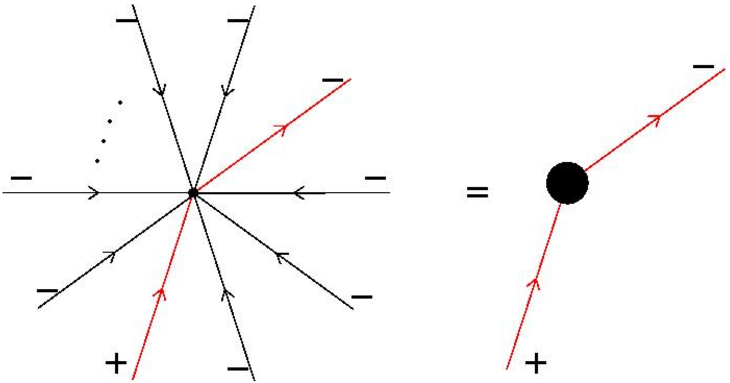

- Want an action that is well-adapted to describe this ASD background

Chalmers & Siegel Action

For Yang-Mills, appropriate action is

Yang-Mills (Chalmers & Siegel, 1996)

$$S = \int_M \mathrm{tr}\left(G^+ \wedge F_A - g^2 G^+ \wedge G^+\right)$$

where $F_A = dA + \frac{1}{2}[A, A]$ is usual YM curvature and G^+ is a Lie-algebra valued self-dual 2-form

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Suggests MHV amplitudes are hiding in the ${G^+}^2$ term

The Penrose transform

According to the Penrose transform

$$\left\{\begin{array}{c}\text{elements of } H^1(\mathbb{PT}', \mathcal{O}(-2h-2)) \\ \text{on twistor space } \mathbb{PT}'\end{array}\right\} \simeq \left\{\begin{array}{c}\text{soln of wave eqn for massless} \\ \text{linearized field, helicity } h\end{array}\right\}$$

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So, for a gluon of $h = +1$ in Maxwell theory

$$G_{\dot\alpha\dot\beta}(x) = \int_{L_x} [\pi\, \mathrm{d}\pi]\ \pi_{\dot\alpha}\pi_{\dot\beta}\, \mathcal{G}|_{L_x}$$

where $G^+ = G_{\dot\alpha\dot\beta} dx^{\alpha(\dot\alpha} \wedge dx_\alpha{}^{\dot\beta)}$ and $\mathcal{G}$ is a (0,1)-form of weight -4,

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where $G^+ = G_{\dot\alpha\dot\beta} dx^{\alpha(\dot\alpha} \wedge dx_\alpha{}^{\dot\beta)}$ and $\mathcal{G}$ is a (0,1)-form of weight -4, valued in $\mathrm{End}(E)$

- $H(x, \pi)$ are holomorphic frames trivializing $E|_{L_x}$, ie $\mathcal{A}|_{L_x} = -\bar\partial H\, H^{-1}$
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- If $\mathcal{G} \in H^{0,1}(\mathbb{PT}', \mathcal{O}(-4) \otimes \mathrm{End}(E))$, then $d_A G^+ = 0$ by construction

The G^{+2} term in twistor space

Using this transform in Chalmers' & Siegel's action gives

$$\int \mathrm{tr}\, G^{+2} = \int \mathrm{d}^4x\, [\pi_1\, \mathrm{d}\pi_1][\pi_2\, \mathrm{d}\pi_2]\; [\pi_1\, \pi_2]^2\; \mathrm{tr}\left(H_2^{-1}\mathcal{G}_2\, H_2\;\; H_1^{-1}\mathcal{G}_1\, H_1\right)$$

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- Has susy extensions for $\mathcal{N} \leq 4$; $\mathcal{N} = 4$ version is (Nair; Boels, Mason, DS)

$$\int \mathrm{d}^{4|8}x \,\log\det(\bar\partial + \mathbf{A})$$

  where $\mathbf{A} = \mathcal{A} + \cdots + \psi^4\mathcal{G}$ and $\mathbf{A}|_{L_x} = \mathcal{A}|_{L_x} + \cdots + (\theta\cdot\pi)^4\mathcal{G}|_{L_x}$

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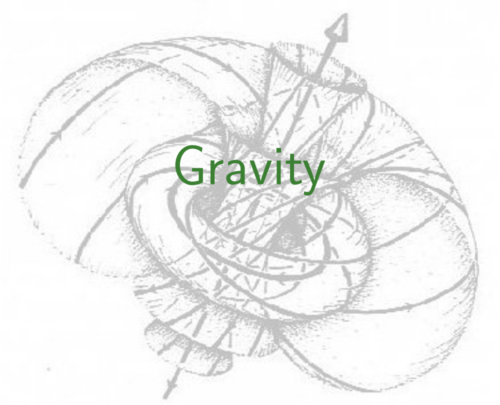
- Off-shell perturbation theory $\Rightarrow$ + holomorphic Chern-Simons theory

Gravity



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Plebanski action for gravity

$$S = \int_M \Sigma^{\dot\alpha\dot\beta} \wedge \left(d\Gamma + \kappa^2 \Gamma \wedge \Gamma\right)_{\dot\alpha\dot\beta}$$

where $\Sigma^{\dot\alpha\dot\beta} = e^{\alpha(\dot\alpha} \wedge e_\alpha{}^{\dot\beta)}$ in terms of vierbein 1-forms $e^{\alpha\dot\alpha} = \sigma_a^{\alpha\dot\alpha} e_\mu^a dx^\mu$

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where $\Sigma^{\dot\alpha\dot\beta} = e^{\alpha(\dot\alpha} \wedge e_\alpha{}^{\dot\beta)}$ in terms of vierbein 1-forms $e^{\alpha\dot\alpha} = \sigma_a^{\alpha\dot\alpha} e^a_\mu dx^\mu$

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- Analogous to Chalmers & Siegel (Abou-Zeid, Hull)
  Gravity MHV amplitudes from Penrose transform of Γ^2 term?

Twistor basics

The twistor space of flat spacetime is called $\mathbb{PT}'$

- $M \simeq \mathbb{C}^4$ (complexified) spacetime with coordinates $x^{\alpha\dot{\alpha}} = \sigma_a^{\alpha\dot{\alpha}} x^a$
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- Plugging $\omega^\alpha = \mathrm{i}x^{\alpha\dot{\alpha}}\pi_{\dot{\alpha}}$ into fields on twistor space really pulls back to projectivized spin bundle $P(\mathbb{S}^+)$; coordinates $(x^{\alpha\dot{\alpha}}, [\pi_{\dot{\beta}}])$

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- $\epsilon_{\alpha\beta}\partial_\alpha B_\beta = \tilde{h}_{-6}$ ensures $d\Gamma^{\dot\alpha}{}_{\dot\beta} = (\delta R)^{\dot\alpha}{}_{\dot\beta\dot\gamma\dot\delta}\, dx^{\gamma\dot\gamma} \wedge dx_\gamma{}^{\dot\delta}$

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- Unknown what $\tilde{h}_{-6}$ deforms $\Rightarrow$ only get ASD spacetime

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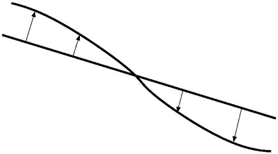

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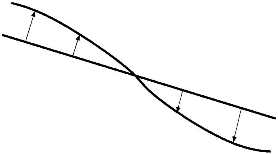

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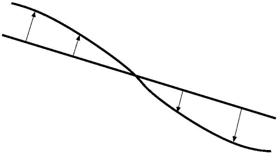

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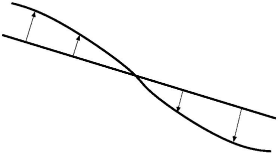

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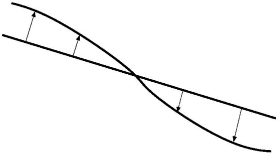

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Generalizing from flat space, can show $d\omega^\alpha \wedge d\omega_\alpha|_{L_x} = e^{\alpha\dot\alpha} \wedge e_\alpha{}^{\dot\beta}\pi_{\dot\alpha}\pi_{\dot\beta}$

- $d\omega^\alpha|_{L_x} = e^{\beta\dot\beta}\Lambda_\beta{}^\alpha\pi_{\dot\beta}\ \Rightarrow$ spacetime vierbein ($\Lambda \in SL(2,\mathbb{C})$ a frame)

The Γ^2 term

Penrose transform of spin connection

$$\Gamma^{\dot\alpha}{}_{\dot\beta}(x) = e^{\gamma\dot\gamma}\Gamma_{\gamma\dot\gamma}{}^{\dot\alpha}{}_{\dot\beta} = \int_{L_x}[\pi\,\mathrm{d}\pi]\,\pi^{\dot\alpha}\pi_{\dot\beta}B|_{L_x} = e^{\gamma\dot\gamma}\int_{L_x}[\pi\,\mathrm{d}\pi]\,\pi^{\dot\alpha}\pi_{\dot\beta}\pi_{\dot\gamma}\Lambda_{\gamma}{}^{\delta}B_{\delta}|_{L_x}$$

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Γ^2 term in Twistor Space

$$\int \mathrm{d}^4x\, e\,\Gamma_{\alpha\dot\alpha\dot\beta\dot\gamma}\Gamma^{\alpha\dot\alpha\dot\beta\dot\gamma} = \int \mathrm{d}^4x\, e\,[\pi_2\,\mathrm{d}\pi_2][\pi_1\,\mathrm{d}\pi_1]\,[\pi_2\,\pi_1]^3\,\Lambda_{2\alpha}^{\ \beta}B_{2\beta}\,\Lambda_1^{\alpha\gamma}B_{1\gamma}$$

$$= \int \mathrm{d}^4x\, e\,[\pi_2\,\mathrm{d}\pi_2]\,\Lambda_{2\alpha}^{\ \beta}B_{2\beta}\,\frac{1}{\bar\partial_{21}}\left(\Lambda_1^{\alpha\gamma}B_{1\gamma}[\pi_2\,\pi_1]^4\right)$$

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- To obtain the BGK amplitudes, must expand deformed twistor lines around $L \subset \mathbb{PT}'$

A spin-bundle diffeomorphism

The integral $\int \mathrm{d}^4 x\, e\, [\pi\, \mathrm{d}\pi]$ is really over the projectivized spin bundle $P(\mathbb{S}^+)$ with coordinates $(x^{\alpha\dot{\alpha}}, [\pi_{\dot{\beta}}])$

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$$\phi : P(\mathbb{S}^+) \to P(\mathbb{S}^+) \qquad \phi : (x^{\alpha\dot{\alpha}}, \pi_{\dot{\beta}}) \mapsto (y^{\alpha\dot{\alpha}}(x,\pi), \pi_{\dot{\beta}})$$

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- Deformation of $\mathbb{C}$-str $\mathbb{PT}' \to \mathcal{PT}$ is not a diffeo, but pullback to $P(\mathbb{S}^+)$ is
  - *cf* $\mathcal{A} \neq -\overline{\partial}H\,H^{-1}$ but $\mathcal{A}|_{L_x} = -\overline{\partial}H\,H^{-1}$

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Γ^2 term in Twistor Space

$$\int_{P(\mathbb{S}^+)} \mathrm{d}^4x\, e\, [\pi_2\, \mathrm{d}\pi_2]\ \pi_{2\dot{\alpha}}\pi_{2\dot{\beta}}\pi_{2\dot{\gamma}}\pi_{2\dot{\delta}}\Lambda_{2\alpha}^{\ \ \beta} B_{2\beta}\, \frac{1}{\bar{\partial}_{21}} \left(\pi_1^{\dot{\alpha}}\pi_1^{\dot{\beta}}\pi_1^{\dot{\gamma}}\pi_1^{\dot{\delta}}\Lambda_1^{\alpha\gamma} B_{1\gamma}\right)$$

A spin-bundle diffeomorphism

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- Just as $H\frac{1}{\bar\partial}H^{-1} = \frac{1}{\bar\partial + \mathcal{A}}$ in Yang-Mills

Berends-Giele-Kuijf amplitudes in twistor space

Expanding in powers of V gives (after some simplification)

Gravity MHV generating function

$$\sum_{n=3}^{\infty} \int \mathrm{d}^4 y \, [\pi_n \, \mathrm{d}\pi_n] \, B_{n\alpha} \left(\frac{1}{\bar{\partial}} \frac{V_{n-1}^{\alpha} \beta^{\dot{\alpha}}}{[\pi_{n-1} \, \beta]} \frac{1}{\bar{\partial}} V_{n-2} \frac{1}{\bar{\partial}} V_{n-3} \cdots \frac{1}{\bar{\partial}} V_2 \frac{1}{\bar{\partial}} \tilde{h}_1 [\pi_1 \, \pi_n]^4 \beta_{\dot{\alpha}} \right)$$

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- Derivative of δ-fn support $\to$ perturbative description of support on deformed twistor lines

Open Questions & Conclusions

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$$S_{\rm asd} = \int \Omega \wedge \tilde{h} \left(\overline{\partial} h + \frac{1}{2}\{h, h\} \right) = \int \Omega \wedge \epsilon^{\alpha\beta} B_\alpha \partial_\beta \left(\overline{\partial} h + \frac{1}{2}\{h, h\} \right)$$

Analogue of hol Chern-Simons for YM (Closely related: Karnas, Ketov; Sokatchev)

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- Does this approach give successful MHV perturbation theory?

  cf (Bianchi, Elvang, Freedman)

Conclusions

Covered in this talk:

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- Review of non-linear graviton
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Thanks for listening!