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From Supergravity Backgrounds to Pure Spinor Sigma Models

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Unversita' del Piemonte Orientale ad Alessandria
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Work in collaboration with Y. Oz, L. Mazzuccato, I. Adam, S. Yankelovich, P. Fré, M. Trigiante, R. D'Auria. J. Kluson

Outline

- Pure Spinor Strings on flat space (Brief Review)
- Pure Spinor Strings on curved spaces
- Rheonomic parametrization and pure spinors
- Anti-de Sitter backgrounds and non-maximally supersymmetric spaces
- Results and Future Projects

Brief Review of Pure Spinor String Theory

We start from the fields $x^m, \theta^\alpha, p_\alpha$ (θ^α are Majorana-Weyl spinors in $d = (9,1)$)

and the free field action

$$S = \int d^2z \Big(\partial x^m \bar{\partial} x_m + p_\alpha \bar{\partial}\theta^\alpha + \hat{p}_\alpha \partial \hat{\theta}^\alpha \Big)$$

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i) The total conformal charge is $c_T = (10)_x + (-32)_{p,\theta}$

ii) Inserting $p_\alpha = p^*_\alpha \equiv \frac{1}{2}\partial x_m \gamma^m_{\alpha\beta}\theta^\beta + \frac{1}{8}(\gamma^m_{\alpha\beta}\theta^\beta)(\theta\gamma_m\partial\theta)$ in S,

$$S|_{p=p^*} = S_{Green-Schwarz}$$

iii) So, $d_\alpha \equiv p_\alpha - p^*_\alpha \approx 0$ must be identified with the fundamental constraint.

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$$Q = \oint dz \lambda^\alpha d_\alpha$$

The nilpotency of the BRST charge implies that

$$\{Q,Q\} = \oint \lambda^\alpha(z) \oint \lambda^\beta(w) d_\alpha(z) d_\beta(w) = \oint \lambda^\alpha \gamma^m_{\alpha\beta} \lambda^\beta \Pi_m = 0$$

where $\Pi_m = \partial x_m + \frac{i}{2}\theta\gamma_m\partial\theta$

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$\Downarrow$

Pure Spinor Constraints

$$\boxed{\lambda^\alpha \gamma^m_{\alpha\beta} \lambda^\beta = 0}$$

These are first-class primary constraints and
they generate a gauge symmetry on the conjugate momenta w.

$$\delta_{\Lambda^m} w_\alpha = \Lambda_m (\gamma^m \lambda)_\alpha$$

Solving the pure spinor constraints one finds that there are 11 independent d.o.f.'s for the left movers and 11 d.o.f.'s for the right movers. Using the gauge symmetries, one finds 11 independent d.o.f. for left movers w's and 11 for the right movers.

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The action of the BRST charge on the fields is given by

$$\boxed{Q d_\alpha = \Pi^m (\gamma_m \lambda)_\alpha, \qquad Q w_\alpha = d_\alpha, \qquad Q^2 w_\alpha = \delta_{\Pi^m} w_\alpha}$$

Conformal Algebra

$$T = -\frac{1}{2}\Pi^m\Pi_m - d_\alpha\partial\theta^\alpha + \frac{1}{10} : N_{mn}N^{mn} : -\frac{1}{8} : JJ : +\partial J$$

$$J = w_\alpha\lambda^\alpha$$

$$j_{BRST} = \lambda^\alpha d_\alpha$$

where $N_{mn} = \frac{1}{2} w\gamma^{mn}\lambda$

$$TJ = -8\frac{1}{(z-w)^3} + \frac{J}{(z-w)^2} + \frac{\partial J}{(z-w)} \qquad JJ = -4\frac{1}{(z-w)^2}$$

$$TN^{mn} = \frac{N^{mn}}{(z-w)^2} + \frac{\partial N^{mn}}{(z-w)} \qquad N^{mn}N^{pq} = \frac{-3\eta^{m[p}\eta^{q]n}}{(z-w)^2} + \frac{\eta^{m[p}N^{q]n}}{(z-w)}$$

N=1 SYM D=10 OPEN STRING MASSLESS SPECTRUM

$$\mathcal{U}^{(1)} = \lambda^\alpha A_\alpha(x,\theta)$$

$$\{Q, \mathcal{U}^{(1)}\} = 0$$

$$\boxed{\gamma_{[5]}^{\alpha\beta} D_{(\alpha} A_{\beta)} = 0}$$

$$\gamma_{[5]}^{\alpha\beta} = (\gamma_{[m_1} \cdots \gamma_{m_5]})^{\alpha\beta}$$

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SUGRA IIA/B D =10 CLOSED STRING MASSLESS SPECTRUM

$$\mathcal{U}^{(1,1)} = \lambda_L^\alpha \lambda_R^{\hat\beta} \hat{A}_{\alpha\hat\beta}(x,\theta,\hat\theta) \qquad Q \to Q_L, \quad Q_R$$

$$\boxed{\gamma_{[5]}^{\alpha\beta} D_{(\alpha} A_{\beta)\hat\beta} = 0\,, \qquad \gamma_{[5]}^{\hat\alpha\hat\beta} \hat{D}_{(\hat\alpha} A_{\alpha\hat\beta)} = 0}$$

Pure Spinor Strings on Curved Backgrounds

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Green-Schwarz action

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Action for the fermionic coordinates (gauge fixing of the Kappa symmetry)

$$\int d^2 z \Big[E_M^{\alpha} d_\alpha \bar{\partial} Z^M + E_M^{\hat{\alpha}} \hat{d}_{\hat{\alpha}} \partial Z^M + P^{\alpha\hat{\beta}} d_\alpha \hat{d}_{\hat{\beta}} \Big]$$

Quadratic Couplings with RR fields

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Quadratic Couplings with RR fields

Action for the ghost fields and couplings with the connection, the gravitino field strengths and the Riemann tensor

$$\int d^2z\Big[w_{z\alpha}(\bar{\partial}\lambda^\alpha+\Omega_M{}^{\alpha}{}_{\beta}\bar{\partial}Z^M\lambda^\beta)+\hat{w}_{\bar{z}\hat{\alpha}}(\partial\hat{\lambda}^{\hat{\alpha}}+\hat{\Omega}_M{}^{\hat{\alpha}}{}_{\hat{\beta}}\partial Z^M\hat{\lambda}^{\hat{\beta}})\Big]$$

$$\int d^2z\Big[C^{\hat{\gamma}\alpha}{}_{\beta}\hat{d}_{\hat{\gamma}}\,w_{z\alpha}\lambda^\beta+\hat{C}^{\gamma\hat{\alpha}}{}_{\hat{\beta}}d_\gamma\,\hat{w}_{\bar{z}\hat{\alpha}}\hat{\lambda}^{\hat{\beta}}+\boxed{S^{\alpha\hat{\gamma}}{}_{\beta\hat{\delta}}}w_{z\alpha}\lambda^\beta\,\hat{w}_{\bar{z}\hat{\gamma}}\hat{\lambda}^{\hat{\delta}}\Big]$$

Pure Spinor Strings on Curved Backgrounds

1. Consider a generic sigma model written in terms of the fundamental fields and their conjugate momenta

$$Z^M = (x^m, \theta^\alpha, \hat{\theta}^{\hat{\alpha}}), \quad p_\alpha, \hat{p}_{\hat{\alpha}}, \quad \lambda^\alpha, \hat{\lambda}^{\hat{\alpha}}, w_{z\alpha}, \hat{w}_{\bar{z}\hat{\alpha}}$$

2. Impose the gauge symmetries and require the pure spinor conditions. This selects some Lorentz structures such as

$$\Omega_{M\beta}^{\alpha} = \Omega_M^{(s)} \delta^\alpha_{\ \beta} + \Omega_M^{mn} (\gamma_{mn})^\alpha_{\ \beta}\,, \quad \hat{\Omega}_{M\hat{\beta}}^{\hat{\alpha}} = \hat{\Omega}_M^{(s)} \delta^{\hat{\alpha}}_{\ \hat{\beta}} + \hat{\Omega}_M^{mn} (\hat{\gamma}_{mn})^{\hat{\alpha}}_{\ \hat{\beta}}\,,$$

3. Compute the fermionic constraints by deriving the Lagrangian w.r.t. to the light-cone derivatives of the fermionic coordinates

$$D_\alpha = \frac{\partial \mathcal{L}}{\partial\, \partial\theta_\alpha}\,, \qquad \hat{D}_{\hat{\alpha}} = \frac{\partial \mathcal{L}}{\partial\, \bar{\partial}\hat{\theta}_{\hat{\alpha}}}$$

4. Then, compute the BRST charges

$$Q_L = \oint dz\, \lambda^{\alpha} d_{\alpha}\,, \qquad Q_R = \oint d\bar{z}\, \hat{\lambda}^{\hat{\alpha}} \hat{d}_{\hat{\alpha}}\,,$$

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Nilpotency of both BRST charges

Holomorphicity of BRST currents

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Nilpotency of both BRST charges

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Type IIA/B Supergravity Constraints !!!

Considerations about curved background sigma models

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- Some of these problems can be solved in the case of maximally supersymetric background
- What about less-supersymmetric backgrounds?

From supergravity to pure spinor

Super-Poincaré algebra (MC forms)

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Adding higher forms for NSNS and RR fields

Free Differential Algebra

From supergravity to pure spinor

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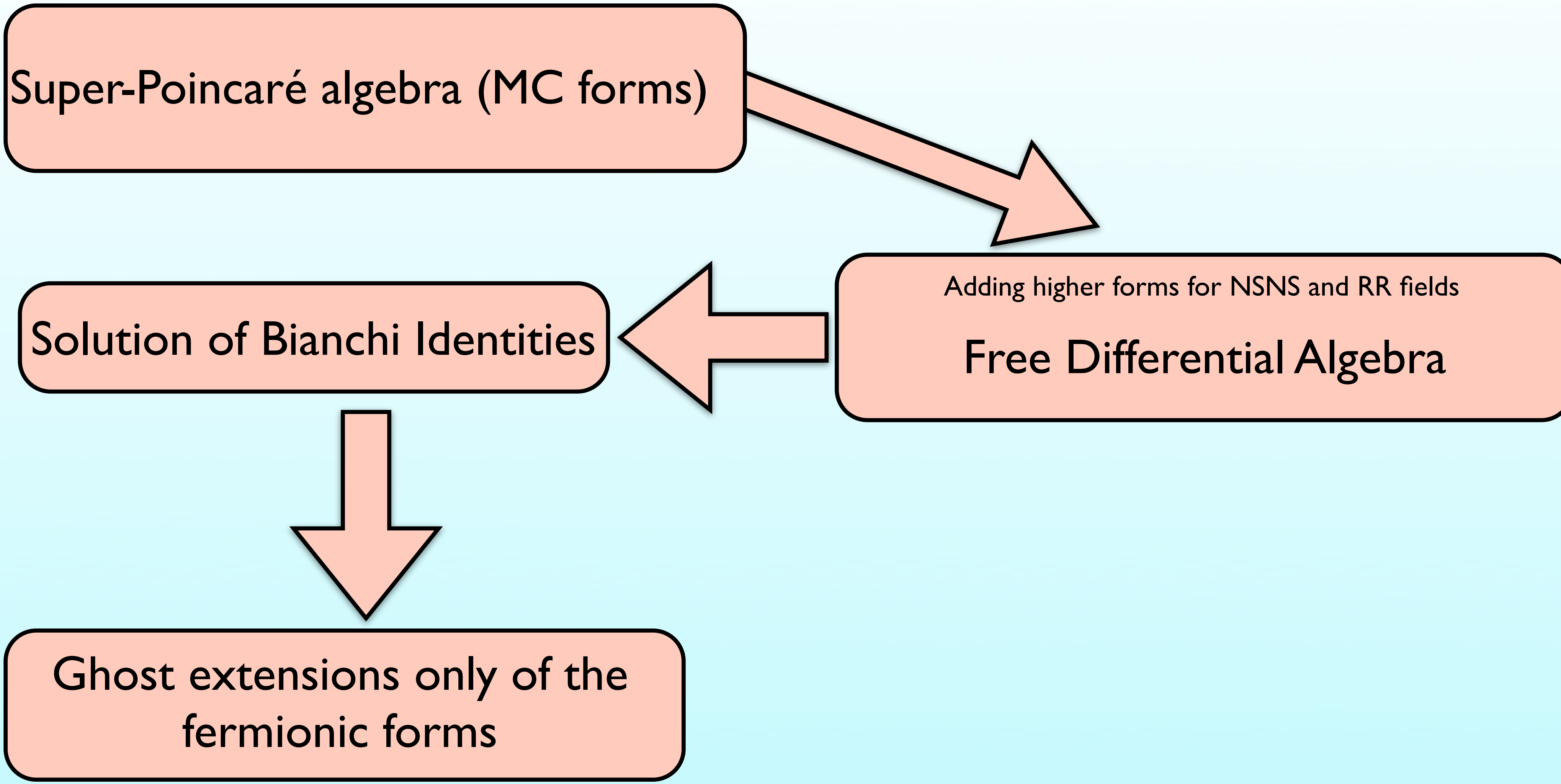
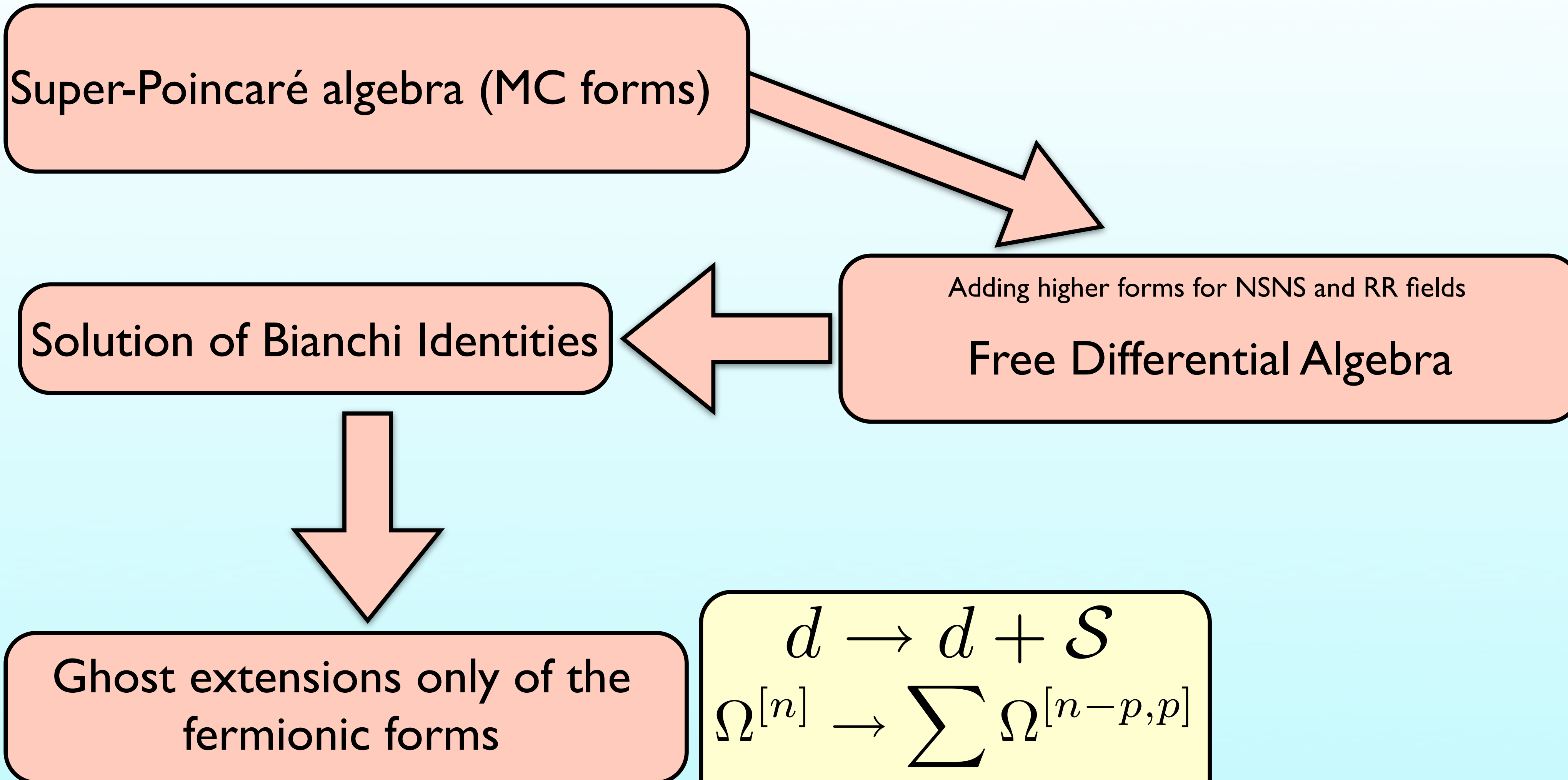
Solution of Bianchi Identities

From supergravity to pure spinor



From supergravity to pure spinor



$$d \to d + \mathcal{S}$$

$$\Omega^{[n]} \to \sum_p \Omega^{[n-p,p]}$$

From supergravity to pure spinor

Super-Poincaré algebra (MC forms)

Adding higher forms for NSNS and RR fields

Free Differential Algebra

Solution of Bianchi Identities

Ghost extensions only of the fermionic forms

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Projection on the worldsheet
and requiring the nilpotency of the BRST charge

Super-Poincaré algebra (MC forms)

We construct the rheonomic parametrization for type IIA directly in the string frame (this is the first time that the solution of the Bianchi ids' is computed directly in the string frame).

$$
\begin{aligned}
R^{\underline{ab}} &\equiv d\omega^{\underline{ab}} - \omega^{\underline{ac}} \wedge \omega^{\underline{cb}} \\
T^{\underline{a}} &\equiv \mathcal{D} V^{\underline{a}} - \mathrm{i}\, \tfrac{1}{2} \left(\overline{\psi}_L \wedge \Gamma^{\underline{a}} \psi_L + \overline{\psi}_R \wedge \Gamma^{\underline{a}} \psi_R \right) \\
\rho_{L,R} &\equiv \mathcal{D}\psi_{L,R} \equiv d\psi_{L,R} - \tfrac{1}{4}\omega^{\underline{ab}} \wedge \Gamma_{\underline{ab}} \psi_{L,R}
\end{aligned}
$$

The super-Poincaré algebra is common to type IIA and to type IIB, the only difference is the chirality of the spinors.

Adding higher forms for NSNS and RR fields

Free Differential Algebra

In order to describe sugra type IIA/B in 10 dimensions we need to add some form fields to complete the spectrum with NSNS and RR fields

$$
\begin{aligned}
R^{\underline{ab}} &\equiv d\omega^{\underline{ab}} - \omega^{\underline{ac}} \wedge \omega^{\underline{cb}} \\
T^{\underline{a}} &\equiv \mathcal{D} V^{\underline{a}} - \mathrm{i}\, \tfrac{1}{2} \left(\overline{\psi}_L \wedge \Gamma^{\underline{a}} \psi_L + \overline{\psi}_R \wedge \Gamma^{\underline{a}} \psi_R \right) \\
\rho_{L,R} &\equiv \mathcal{D}\psi_{L,R} \equiv d\psi_{L,R} - \tfrac{1}{4}\omega^{\underline{ab}} \wedge \Gamma_{\underline{ab}} \psi_{L,R} \\
\mathbf{G}^{[2]} &\equiv d\mathbf{C}^{[1]} + \exp\left[-\varphi\right] \overline{\psi}_R \wedge \psi_L \\
\mathbf{f}^{[1]} &\equiv d\varphi \\
\nabla \chi_{L/R} &\equiv d\chi_{L,R} - \tfrac{1}{4}\omega^{\underline{ab}} \wedge \Gamma_{\underline{ab}} \chi_{L,R}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{H}^{[3]} &= d\mathbf{B}^{[2]} + \mathrm{i} \left(\overline{\psi}_L \wedge \Gamma_{\underline{a}} \psi_L - \overline{\psi}_R \wedge \Gamma_{\underline{a}} \psi_R \right) \wedge V^{\underline{a}} \\
\mathbf{G}^{[4]} &= d\mathbf{C}^{[3]} + \mathbf{B}^{[2]} \wedge d\mathbf{C}^{[1]} \\
&\quad - \tfrac{1}{2} \exp\left[-\varphi\right] \left(\overline{\psi}_L \wedge \Gamma_{\underline{ab}} \psi_R + \overline{\psi}_R \wedge \Gamma_{\underline{ab}} \psi_L \right) \wedge V^{\underline{a}} \wedge V^{\underline{b}}
\end{aligned}
$$

Solution of Bianchi Identities

As is well-known there are consistency condition (a.k.a. Bianchi identities) which can be solved if we choose a given parametrization (rheonomic) of the supergravity fields.

$$
\begin{aligned}
T^{\underline{a}} &= 0 \\
R^{\underline{ab}} &= R^{\underline{ab}}{}_{\underline{mn}} V^{\underline{m}} \wedge V^{\underline{n}} + \overline{\psi}_R \Theta^{\underline{ab}}_{\underline{m}|L} \wedge V^{\underline{m}} + \overline{\psi}_L \Theta^{\underline{ab}}_{\underline{m}|R} \wedge V^{\underline{m}} \\
&\quad + \mathrm{i}\, \tfrac{3}{4} \left(\overline{\psi}_L \wedge \Gamma_{\underline{c}} \psi_L - \overline{\psi}_R \wedge \Gamma_{\underline{c}} \psi_R \right) \mathcal{H}^{\underline{abc}} \\
&\quad + \overline{\psi}_L \wedge \Gamma^{[\underline{a}} \mathcal{Z} \Gamma^{\underline{b}]} \psi_R \\
\mathbf{H}^{[3]} &= \mathcal{H}_{\underline{abc}} V^{\underline{a}} \wedge V^{\underline{b}} \wedge V^{\underline{c}} \\
\mathbf{G}^{[2]} &= \mathcal{G}_{\underline{ab}} V^{\underline{a}} \wedge V^{\underline{b}} + \mathrm{i}\, \tfrac{3}{2} \exp[-\varphi] \left(\overline{\chi}_L \Gamma_{\underline{a}} \psi_L + \overline{\chi}_R \Gamma_{\underline{a}} \psi_R \right) \wedge V^{\underline{a}} \\
\mathbf{f}^{[1]} &= f_{\underline{a}} V^{\underline{a}} + \tfrac{3}{2} \left(\overline{\chi}_R \psi_L - \overline{\chi}_L \psi_R \right) \\
\mathbf{G}^{[4]} &= \mathcal{G}_{\underline{abcd}} V^{\underline{a}} \wedge V^{\underline{b}} \wedge V^{\underline{c}} \wedge V^{\underline{d}} \\
&\quad - \mathrm{i}\, \tfrac{1}{2} \exp[-\varphi] \left(\overline{\chi}_L \Gamma_{\underline{abc}} \psi_L - \overline{\chi}_R \Gamma_{\underline{abc}} \psi_R \right) \wedge V^{\underline{a}} \wedge V^{\underline{b}} \wedge V^{\underline{c}}
\end{aligned}
$$

Fermionic curvatures

$$
\begin{aligned}
\rho_{L/R} &= \rho^{L/R}_{\underline{ab}} V^{\underline{a}} \wedge V^{\underline{b}} + \mathcal{L}^{(even)}_{\underline{a}\,\pm} \psi_{L/R} + \mathcal{L}^{(odd)}_{\underline{a}\,\mp} \psi_{R/L} + \rho^{(0,2)}_{L/R} \\
\nabla \chi_{L/R} &= \mathcal{D}_{\underline{a}} \chi_{L/R} V^{\underline{a}} + \mathcal{N}^{(even)}_{\pm} \psi_{L/R} + \mathcal{N}^{(odd)}_{\mp} \psi_{R/L}
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Ghost extensions only of the fermionic forms

We extend of the fermionic forms and of the differential

$$d \longrightarrow d + \mathcal{S}$$

$$\boxed{\psi_{L/R} \mapsto \psi_{L/R} + \lambda_{L/R}}$$

$$\mathcal{S} = \mathcal{S}_L + \mathcal{S}_R$$

$$\Omega^{[n]} \longrightarrow \sum_p \Omega^{[n-p,p]}$$

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$$\Omega^{[n]} \rightarrow \sum_p \Omega^{[n-p,p]}$$

$$\begin{aligned}
\mathcal{S}_{L/R}\,\mathbf{B}^{[2]} &= \mp 2\,\mathrm{i}\,\overline{\psi}_{L/R}\,\Gamma_{\underline{a}}\,\lambda_{L/R}\,V^{\underline{a}} \\
\mathcal{S}_{L/R}\,\mathbf{C}^{[1]} &= \mp \exp[-\varphi]\,\overline{\psi}_{R/L}\,\lambda_{L/R} + \tfrac{3}{2}\,\mathrm{i}\,\exp[-\varphi]\,\overline{\chi}_{L/R}\,\Gamma_{\underline{a}}\,\lambda_{L/R}\,V^{\underline{a}} \\
\mathcal{S}_{L/R}\,\mathbf{C}^{[3]} &= \overline{\psi}_{R/L}\,\Gamma_{\underline{ab}}\,\lambda_{L/R}\,V^{\underline{a}} \wedge V^{\underline{b}} - B^{[2]} \wedge \mathcal{S}_{L/R}C^{[1]} \\
&\quad \mp \mathrm{i}\,\tfrac{1}{2}\,\exp[-\varphi]\,\overline{\chi}_{L/R}\,\Gamma_{\underline{abc}}\,\lambda_{L/R}\,V^{\underline{a}} \wedge V^{\underline{b}} \wedge V^{\underline{c}} \\
\mathcal{S}_{L/R}\,V^{\underline{a}} &= \mathrm{i}\,\overline{\psi}_{L/R}\,\Gamma^{\underline{a}}\,\lambda_{L/R} \\
\mathcal{S}_{L/R}\psi_{L/R} &= -\mathcal{D}\,\lambda_{L/R} \mp \tfrac{3}{8}\,\Gamma^{\underline{a_1 a_2}}\,\lambda_{L/R}\,V^{\underline{a_3}}\,\mathcal{H}_{\underline{a_1 a_2 a_3}} \pm \tfrac{21}{16}\,\Gamma_{\underline{a}}\chi_{R/L}\,(\overline{\psi}_{L/R}\,\Gamma^{\underline{a}}\lambda_{L/R}) \\
&\quad \mp \tfrac{1}{1280}\,\Gamma_{\underline{a_1}\dots\underline{a_5}}\chi_{R/L}\,(\overline{\psi}_{L/R}\,\Gamma^{\underline{a_1}\dots\underline{a_5}}\lambda_{L/R}) \\
\mathcal{S}_{R/L}\psi_{L/R} &= \mathcal{M}_{\pm}\,\Gamma_{\underline{b}}\lambda_{R/L}\,V^{\underline{b}} \\
\mathcal{S}_{L/R}\lambda_{L/R} &= \pm \tfrac{21}{16}\,\Gamma_{\underline{a}}\chi_{R/L}\,(\overline{\lambda}_{L/R}\,\Gamma^{\underline{a}}\lambda_{L/R}) \\
&\quad \mp \tfrac{1}{1280}\,\Gamma_{\underline{a_1}\dots\underline{a_5}}\chi_{R/L}\,(\overline{\lambda}_{L/R}\,\Gamma^{\underline{a_1}\dots\underline{a_5}}\lambda_{L/R}) \\
\mathcal{S}_{R/L}\lambda_{L/R} &= 0
\end{aligned}$$

Projection on the worldsheet and requiring the nilpotency of the BRST charge

As they stand, by requiring the nilpotency, one obtaines strong constraints on the spinors λ, then we need to project onto 2d surface

$$\lambda_{L/R}(x,\theta,\bar{\theta}) \to \lambda_{L/R}(z,\bar{z}) \qquad\qquad V^{\underline{a}} = \Pi^{\underline{a}}_{+}\, e^{+} + \Pi^{\underline{a}}_{-}\, e^{-}$$

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Pure Spinor Constraints

$$\begin{aligned}
0 &= \bar\lambda_L\,\Gamma_{\underline{a}}\,\lambda_L + \bar\lambda_R\,\Gamma_{\underline{a}}\,\lambda_R \\
0 &= \left(\bar\lambda_L\,\Gamma_{\underline{a}}\,\lambda_L - \bar\lambda_R\,\Gamma_{\underline{a}}\,\lambda_R\right) \wedge V^{\underline{a}} \\
0 &= \exp[-\varphi]\,\left(\bar\lambda_R\,\lambda_L\right) \\
0 &= \exp[-\varphi]\,\bar\lambda_R\,\Gamma_{\underline{ab}}\,\lambda_L\, V^{\underline{a}} \wedge V^{\underline{b}}
\end{aligned}$$

Solution of the pure spinor constraints

- Using an adapted basis, we find that there are 22 independent degrees of freedom for λ
- We find the map between our solution and the pure spinors used by Berkovits et al.
- Analogously, we find that the constraints for type IIB. It is easy to show the T-duality of the constraints and of the formulation.

Pure Spinor Sigma model

$$
\begin{aligned}
\mathcal{A} &= \mathcal{A}_{GS} + \mathcal{A}_{gf}^{\mathrm{IIA}} \\
\mathcal{A}_{GS} &= \int \left(\Pi_{+}^{\underline{a}} V^{\underline{b}} \eta_{\underline{ab}} \wedge e^{+} - \Pi_{-}^{\underline{a}} V^{\underline{b}} \eta_{\underline{ab}} \wedge e^{-} + \tfrac{1}{2} \Pi_{i}^{\underline{a}} \Pi_{j}^{\underline{b}} \eta^{ij} \eta_{\underline{ab}} e^{+} \wedge e^{-} + \tfrac{1}{2} \mathbf{B}^{[2]} \right) \\
\mathcal{A}_{gf}^{\mathrm{IIA}} &= \int \Big(\overline{\mathbf{d}}_{+} \psi_R \wedge e^{+} + \overline{\mathbf{d}}_{-} \psi_L \wedge e^{-} + \tfrac{\mathrm{i}}{2} \overline{\mathbf{d}}_{+} \mathcal{M}_{-} \mathbf{d}_{-} \\
&- \overline{w}_{+} (\mathcal{S}_R \psi_R) \wedge e^{+} - \overline{w}_{-} (\mathcal{S}_L \psi_L) \wedge e^{-} \\
&- \tfrac{\mathrm{i}}{2} \overline{w}_{+} (\mathcal{S}_R \mathcal{M}_{-}) \mathbf{d}_{-} + \tfrac{\mathrm{i}}{2} \overline{\mathbf{d}}_{+} (\mathcal{S}_L \mathcal{M}_{-}) w_{-} - \tfrac{\mathrm{i}}{2} \overline{w}_{+} \left(\mathcal{S}_R \mathcal{S}_L \mathcal{M}_{-} \right) w_{-} \Big) \, .
\end{aligned}
$$

where we have set

$$
\mathcal{M}_{\pm} = \mathrm{i} \left(\mp \mathcal{M}_{\underline{ab}} \Gamma^{\underline{ab}} + \mathcal{M}_{\underline{abcd}} \Gamma^{\underline{abcd}} \right)
$$

$$
\begin{aligned}
\mathcal{M}_{\underline{ab}} &= \left(\frac{1}{8} \exp[\varphi] \, \mathcal{G}_{\underline{ab}} + \frac{9}{64} \overline{\chi}_R \Gamma_{\underline{ab}} \chi_L \right) \\
\mathcal{M}_{\underline{abcd}} &= -\frac{1}{16} \exp[\varphi] \, \mathcal{G}_{\underline{abcd}} - \frac{3i}{256} \overline{\chi}_L \Gamma_{\underline{abcd}} \chi_R
\end{aligned}
$$

Maximally Supersymmetric Anti-de Sitter Background

The most famous solution is the $AdS_5 \times S^5$ of type IIB supergravity

This background can be completely characterized by the supercoset

$$\boxed{\frac{PSU(2,2|4)}{SO(1,4)\times SO(5)}}$$

Using the Maurer-Cartan equations, one reconstructs the entire supergravity solution in terms of the vielbein, the spin connection, the gravitinos and the RR fields

$$E^m, \quad \Omega^{mn}, \quad \psi^\alpha, \quad \hat{\psi}^{\hat{\alpha}}, \quad F_5$$

Using these fields and inserting them in the sigma model, one can obtain a simplified sigma model. Indeed, the dependence on $d_\alpha, \ \hat{d}_{\hat{\alpha}}$ is only quadratic and therefore they can be eliminated.

Solution of Bianchi identities in the case of AdS x S background

$$T^a = dV^a + \omega^a{}_b \wedge V^b + i\,\overline{\Psi}\gamma^a\Psi = 0\,,$$

$$T^{a'} = dV^{a'} + \omega^{a'}{}_{b'} \wedge V^{b'} + i\,\overline{\Psi}\gamma^{a'}\Psi = 0\,,$$

$$\nabla_{AdS_5}\Psi = 0\,, \qquad \nabla_{S^5}\Psi = 0$$

$$F^{[5]}_{AdS_5} = \epsilon_{abcde} V^a \wedge \ldots \wedge V^b\,, \qquad F^{[5]}_{S^5} = \epsilon_{a'b'c'd'e'} V^{a'} \wedge \ldots \wedge V^{b'}\,.$$

where the F-curvature is parametrized as follows

$$F^{[5]}_{AdS_5} = dC^{[4]}_{AdS_5} + \epsilon_{abcde} i\overline{\Psi}\gamma^{de}\Psi \wedge V^a \wedge \ldots \wedge V^c\,,$$

$$F^{[5]}_{S^5} = dC^{[4]}_{S^5} + \epsilon_{a'b'c'd'e'} i\overline{\Psi}\gamma^{d'e'}\Psi \wedge V^{a'} \wedge \ldots \wedge V^{c'}$$

where the induced NSNS B field is written in terms of the MC forms

$$\mathcal{B}^{[2]} = \overline{\Psi} \wedge \Psi = (\overline{\chi}_A \otimes \overline{\chi}_{A'} \chi_B \otimes \chi_{B'})\overline{L}^{A'B} \wedge L^{AB'}$$

$$= \overline{L}^{A'B} \wedge L^{AB'} C_{AB} C_{A'B'}\,.$$

It is convenient to use the decomposition of the original supercoset

$$\frac{SU(2,2)}{SO(1,4)} \times \frac{PSU(2,2|4)}{SU(2,2)\times SU(4)} \times \frac{SU(4)}{SO(5)}$$

The 5-beins, the spin connections and the gravitinos are given in terms of the MC of the supercoset

$$V^a = B^a + \overline{\chi}_A \gamma^a \chi_B L^{AB}\,, \qquad V^{a'} = B^{a'} + \overline{\chi}_{A'} \gamma^{a'} \chi_{B'} L^{A'B'}\,,$$

$$\omega^{ab} = B^{ab} + \overline{\chi}_A \gamma^{ab} \chi_B L^{AB}\,, \qquad \omega^{a'b'} = B^{a'b'} + \overline{\chi}_{A'} \gamma^{a'b'} \chi_{B'} L^{A'B'}\,,$$

$$\Psi = \chi_A \otimes \chi_{A'} L^{AA'}\,, \qquad \overline{\Psi} = \overline{\chi}_A \otimes \overline{\chi}_{A'} \overline{L}^{AA'}$$

A, A',B, B'=1,..,4. a, a' = 1,..,5

The pure spinor constraints are identical to those of Berkovits and the map between the two set of pure spinor in this case is trivial.

Inserting these data into the action discussed above, one obtains (Berkovits) Pure Spinor AdSxS action.

$AdS_5 \times S^5$ pure spinor sigma-model

pure spinor sigma-model

$$S = \int d^2z \Big[\frac{1}{2}\Big(\eta_{ab} J^a \bar{J}^b + \eta_{a'b'} J^{a'} \bar{J}^{b'}\Big) + \delta_{\alpha\hat{\beta}}\Big(3 J^{\hat{\beta}} \bar{J}^{\alpha} - J^{\alpha} \bar{J}^{\hat{\beta}}\Big) +$$

$$\Big(w_\alpha \bar{\partial}\lambda^\alpha + N_{ab} \bar{J}^{[ab]} + N_{a'b'} \bar{J}^{[a'b']}\Big) + \Big(\hat{w}_{\hat{\alpha}} \partial \hat{\lambda}^{\hat{\alpha}} + \hat{N}_{ab} J^{[ab]} + \hat{N}_{a'b'} J^{[a'b']}\Big) +$$

$$+ \frac{1}{2}(N_{ab}\hat{N}_{ab} - N_{a'b'}\hat{N}^{a'b'}) \Big]$$

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$$\Big(w_\alpha \bar{\partial} \lambda^\alpha + N_{ab} \bar{J}^{[ab]} + N_{a'b'} \bar{J}^{[a'b']} \Big) + \Big(\hat{w}_{\hat{\alpha}} \partial \hat{\lambda}^{\hat{\alpha}} + \hat{N}_{ab} J^{[ab]} + \hat{N}_{a'b'} J^{[a'b']} \Big) +$$
$$+ \frac{1}{2} (N_{ab} \hat{N}_{ab} - N_{a'b'} \hat{N}^{a'b'}) \Big]$$

where $$J^A = (g^{-1} \partial g)^A \, , \qquad \bar{J}^A = (g^{-1} \bar{\partial} g)^A$$

are the left-invariant currents constructed from an element of the supergroups PSU(2,2|4)

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$$+ \frac{1}{2}(N_{ab}\hat{N}_{ab} - N_{a'b'}\hat{N}^{a'b'})\Big]$$

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The action has the SO(5) x SO(1,4) isometry
(a,b refer to SO(1,4), a',b' refer to SO(5), and α,β =1,...,16)

Some results for $AdS_5 \times S^5$ pure spinor sigma-model

- Conformal invariance to all orders
- Infinite number of conserved non-local currents (at the classical and quantum level)
- Beta-deformed backgrounds
- One-, two-loop computations of the monodromy matrix
- New limits for the gauge/gravity correspondence (see N. Berkovits talk)

Pure Spinor String Theory and Non-Critical Strings

- The supergravity approximation is no longer valid
- AdS + RR backgrounds for non-critical strings
- We need to keep supersymmetry and Poincare' symmetry manifest at all stages of the computation
- We need a gauge-fixed action to get a conformal field theory model
- Study of physical states spectrum
- Computation of amplitudes in manifestly supersymmetric.

AdS Backgrounds for Non-Critical Models

As an example, we construct the sigma model for AdS in D=4 with N=2 supersymmetry.

The basic supergroup is $$\boxed{\frac{OSp(2|4)}{SO(1,3)\times SO(2)}}$$

$$g^{-1}dg = L_\mu P^\mu + L_{\mu\nu}J^{\mu\nu} + L_{IJ}J^{IJ} + L^I_\alpha Q^\alpha_I$$

The action is decomposed into $$S = S_{GS} + S_d + S_{ghost}$$

$$S_{GS} = \int_\Sigma d^2z\eta_{\mu\nu}L^\mu\bar{L}^\nu + \int_{\mathcal{M}} d^3yL^\mu L^\alpha_I(\gamma_5\gamma_\mu)_{\alpha\beta}L^\beta_J\epsilon^{IJ}$$

$$S_d = \int_\Sigma d^2z(\delta^{ij} + i\epsilon^{ij})d_{\alpha i}\bar{L}^\alpha_j + (\delta^{ij} - i\epsilon^{ij})\bar{d}_{\alpha i}L^\alpha_j \boxed{+ q_{RR}d_{\alpha i}\gamma^{5\alpha\beta}d_{\beta j}\delta^{ij}}$$

Coupling with RR field strengths

Non-maximally symmetric backgrounds

- Several interesting models of the type AdS x M are not maximally supersymmetric backgrounds (they provide very useful checks on AdS/CFT correspondence with lower supersymmetry)
- One of the problem is: Pure Spinor string theory seems to be very suitable for maximally supersymmetric background since the superspace contains all needed θ's, but what happen for less susy backgrounds, how they enter the model?
- Two examples: $AdS_5 \times T^{(1,1)}\,, \qquad AdS_4 \times \mathbf{CP}^3\,,$
- Of course there are several questions (from worldsheet point of view) such as conformal invariance, conserved currents, radiative corrections