Calculating the NNLO corrections to Jet Observables

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Precision physics

Standard model

- well-established as theory of particle interactions
- testing now at per-mille (electroweak) to per-cent (QCD) level

LEP precision physics:

Electroweak processes

Tevatron/LHC precision physics:

QCD processes

QCD

Precision physics with QCD

- precise determination of
  - strong coupling constant
  - quark masses
  - electroweak parameters
  - parton distributions
  - LHC collider luminosity
- precise predictions for
  - new physics effects
  - and their backgrounds

Jet Observables

Observing ”free” quarks and gluons at colliders

QCD describes quarks and gluons; experiments observe hadrons

- describe parton $\longrightarrow$ hadron transition (fragmentation)
- define appropriate final states, independent of particle type in final state (jets)

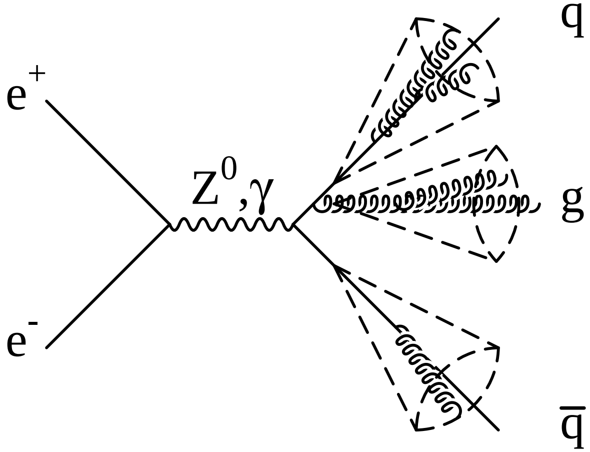
Jets

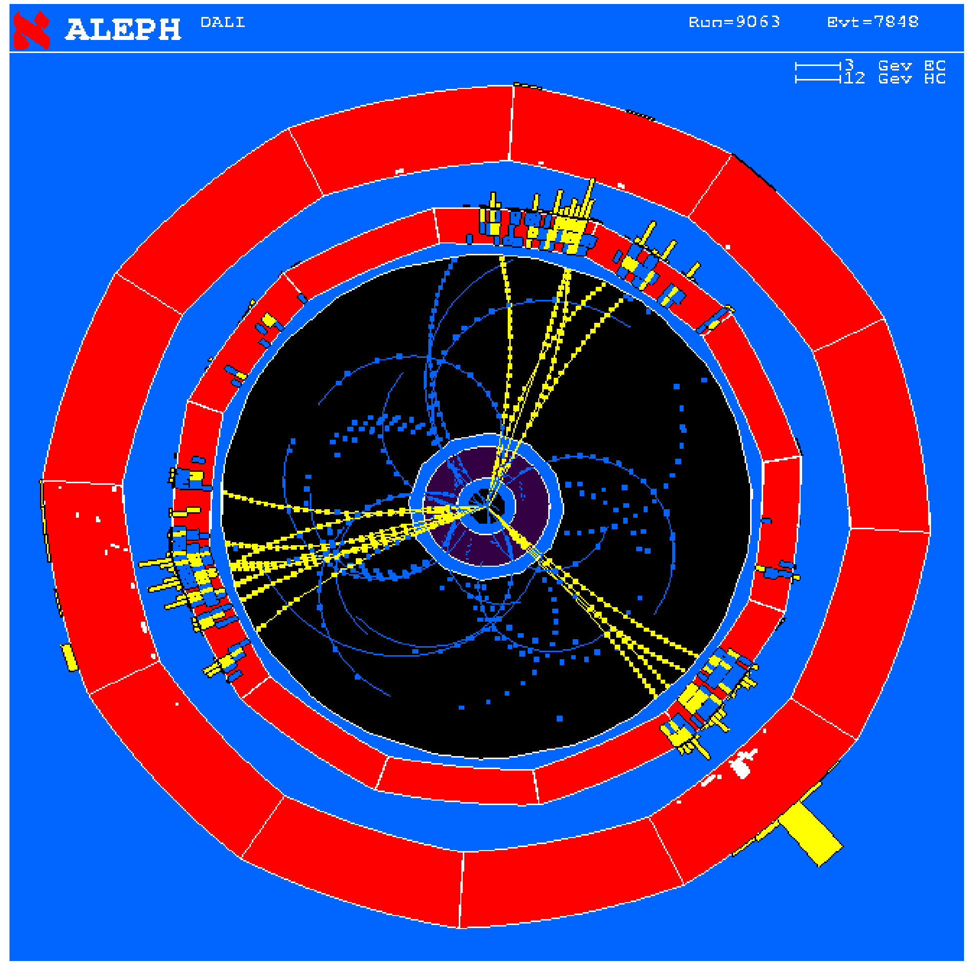
- experimentally: hadrons with common momentum direction
- theoretically: partons with common momentum direction

Jet Observables

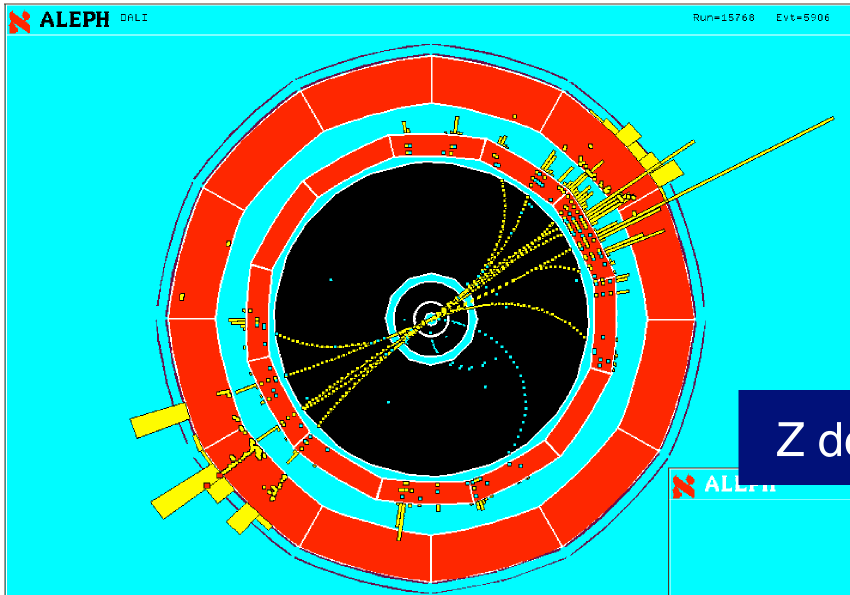
$e^+e^- \rightarrow 3$ jets

event at LEP





Event shape variables

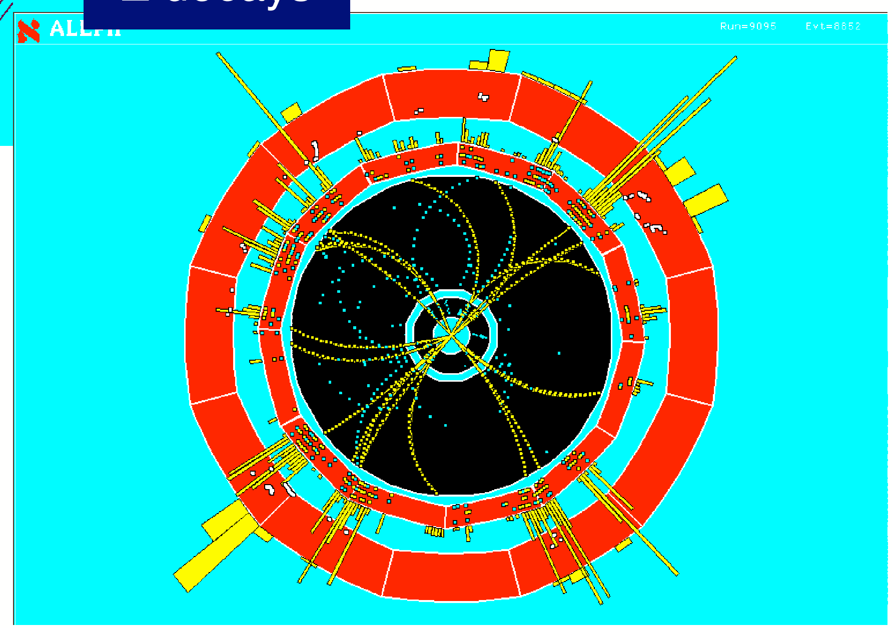


Thrust → 1

Thrust in e^+e^-

$$T = \max_{\vec{n}} \frac{\sum_{i=1}^{n} |\vec{p_i} \cdot \vec{n}|}{\sum_{i=1}^{n} |\vec{p_i}|}$$

Z decays



can be used for precision measurement of α_s

Thrust → 1/2

Jet Observables

- strong coupling constant from $e^+e^- \rightarrow 3j$ event shapes

e^+e^-

$$\alpha_s(M_Z) = 0.1202 \pm 0.0003(\text{stat}) \pm 0.0009(\text{sys}) \pm 0.0009(\text{had}) \pm 0.0047(\text{scale})$$

- study of non-perturbative power corrections from $e^+e^- \rightarrow 3j$

- strong coupling constant from $ep \rightarrow (2+1)j$

ep

$$\alpha_s^{\text{ZEUS}}(M_Z) = 0.1190 \pm 0.0017(\text{stat})^{+0.0049}_{-0.0023}(\text{sys}) \pm 0.0026(\text{th})$$

$$\alpha_s^{\text{H1}}(M_Z) = 0.1186 \pm 0.0030(\text{exp})^{+0.0039}_{-0.0045}(\text{scale}) \pm 0.0023(\text{pdf})$$

- gluon distribution in proton from $ep \rightarrow (2+1)j$

- strong coupling constant from $p\bar{p} \rightarrow 1j + X$

$p\bar{p}$

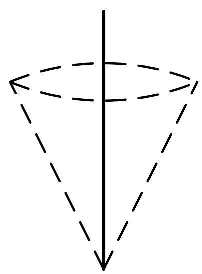
$$\alpha_s^{\text{CDF}}(M_Z) = 0.1178 \pm 0.0001(\text{stat})^{+0.0081}_{-0.0095}(\text{sys})\,^{+0.0071}_{-0.0047}(\text{scale}) \pm 0.0059(\text{pdf})$$

- gluon distribution in proton from $p\bar{p} \rightarrow 2j$
- multijet-signatures often background to new physics searches

Jets in Perturbation Theory

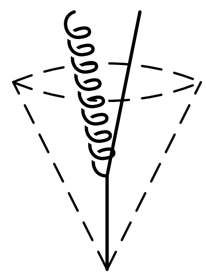
Jet Description

- Partons are combined into jets using the same jet algorithm as in experiment



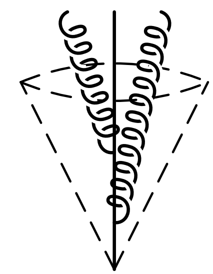
LO

each parton forms 1 jet on its own



NLO

2 partons in 1 jet, 1 parton experimentally unresolved



NNLO

3 partons in 1 jet, 2 partons experimentally unresolved

Current state-of-the-art: NLO plus resummation of all-order logarithms (NLLA)

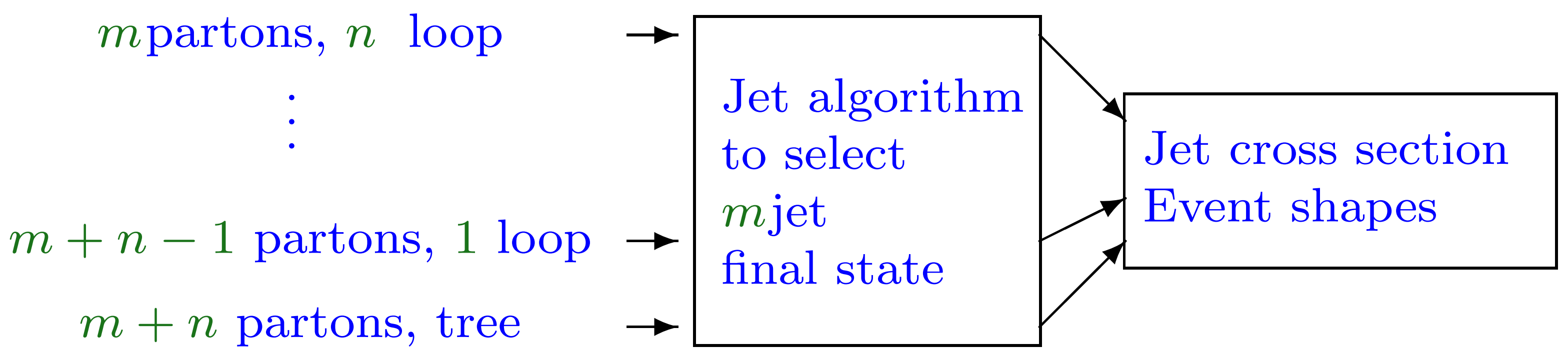
Need for higher orders:

- reduce error on α_s
- better matching of parton level and hadron level jet algorithm

Jets in Perturbation Theory

General structure:

m jets, n–th order in perturbation theory



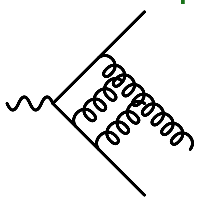
- Jet algorithm acts differently on different partonic final states
- Divergencies from soft and collinear real and virtual contributions must be extracted before application of jet algorithm

consider $e^+e^- \to 3$ jets

Ingredients to NNLO $e^+e^- \to$ 3-jet

- Two-loop matrix elements

$|\mathcal{M}|^2_{\text{2-loop},3\text{ partons}}$

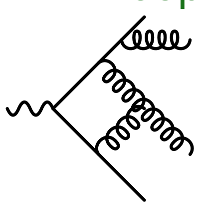


explicit infrared poles from loop integrals

L. Garland, N. Glover, A. Koukoutsakis, E. Remiddi, TG;
S. Moch, P. Uwer, S. Weinzierl

- One-loop matrix elements

$|\mathcal{M}|^2_{\text{1-loop},4\text{ partons}}$

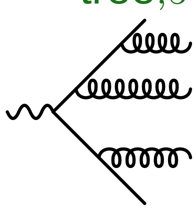


explicit infrared poles from loop integral and
implicit infrared poles due to single unresolved radiation

Z. Bern, L. Dixon, D. Kosower, S. Weinzierl;
J. Campbell, D.J. Miller, E.W.N. Glover

- Tree level matrix elements

$|\mathcal{M}|^2_{\text{tree},5\text{ partons}}$



implicit infrared poles due to double unresolved radiation

K. Hagiwara, D. Zeppenfeld;
F.A. Berends, W.T. Giele, H. Kuijf;
N. Falck, D. Graudenz, G. Kramer

Infrared Poles cancel in the sum

Virtual Corrections at NNLO

Virtual two-loop corrections feasible due to technical breakthroughs

- algorithms to reduce the ~ 10000's of integrals to a few $(10-30)$ master integrals
  - Integration-by-parts (IBP)
    K. Chetyrkin, F. Tkachov
  - Lorentz Invariance (LI)
    E. Remiddi, TG
  - and their implementation in computer algebra
    S. Laporta
- New methods to compute master integrals
  - Mellin-Barnes Transformation V. Smirnov, O. Veretin; B. Tausk;
    MB: M. Czakon; AMBRE: J. Gluza, K. Kajda, T. Riemann
  - Differential Equations E. Remiddi, TG
  - Sector Decomposition (numerically) T. Binoth, G. Heinrich
  - Nested Sums S. Moch, P. Uwer, S. Weinzierl

Virtual Corrections at NNLO

Reduction to master integrals

Identities:

- Integration-by-parts (IBP)
  K. Chetyrkin, F. Tkachov

$$\int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{\partial}{\partial a^\mu} \left[b^\mu f(k,l,p_i) \right] = 0$$

with: $a^\mu = k^\mu, l^\mu$ and $b^\mu = k^\mu, l^\mu, p_i^\mu$

- Lorentz Invariance (LI)
  E. Remiddi, TG

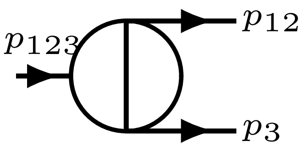
$$\int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\mathrm{d}^d l}{(2\pi)^d} \delta\varepsilon_\nu^\mu \left(\sum_i p_i^\nu \frac{\partial}{\partial p_i^\mu} \right) f(k,l,p_i) = 0$$

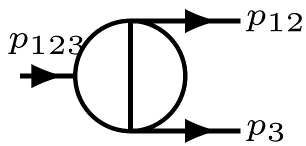
For each two-loop four-point integral, one has 10 IBP and 3 LI identities.

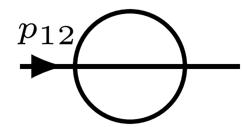
Virtual Corrections at NNLO

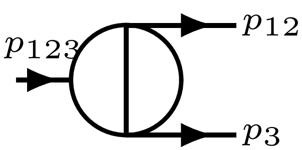
Master Integrals from differential equations

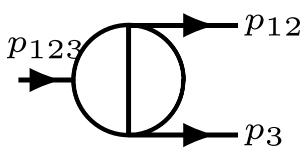
Example: two-loop off-shell vertex function

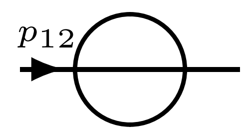
$$s_{123}\frac{\partial}{\partial s_{123}}$$


$$= +\frac{d-4}{2}\,\frac{2s_{123}-s_{12}}{s_{123}-s_{12}}$$


$$-\frac{3d-8}{2}\,\frac{1}{s_{123}-s_{12}}$$


$$s_{12}\frac{\partial}{\partial s_{12}}$$


$$= -\frac{d-4}{2}\,\frac{s_{12}}{s_{123}-s_{12}}$$


$$+\frac{3d-8}{2}\,\frac{1}{s_{123}-s_{12}}$$


- is a hypergeometric differential equation
- boundary conditions are two-point functions
- Laurent-series: expansion of hypergeometric functions in their parameters
  HypExp: T. Huber, D. Maître; XSummer: S. Moch, P. Uwer
- yields (generalized) harmonic polylogarithms
  E. Remiddi, J. Vermaseren; A. Goncharov; HPL: D. Maître

Virtual Corrections at NNLO

Virtual two-loop matrix elements have been computed for:

- Bhabha-Scattering: $e^+e^- \to e^+e^-$
  Z. Bern, L. Dixon, A. Ghinculov
- Hadron-Hadron 2-Jet production: $qq' \to qq'$, $q\bar{q} \to q\bar{q}$, $q\bar{q} \to gg$, $gg \to gg$
  C. Anastasiou, N. Glover, C. Oleari, M. Yeomans-Tejeda
  Z. Bern, A. De Freitas, L. Dixon [SUSY-YM]
- Photon pair production at LHC: $gg \to \gamma\gamma$, $q\bar{q} \to \gamma\gamma$
  Z. Bern, A. De Freitas, L. Dixon
  C. Anastasiou, N. Glover, M. Yeomans-Tejeda
- Three-jet production: $e^+e^- \to \gamma^* \to q\bar{q}g$
  L. Garland, N. Glover, A.Koukoutsakis, E. Remiddi, TG
  S. Moch, P. Uwer, S. Weinzierl
- DIS (2+1) jet production: $\gamma^* g \to q\bar{q}$, Hadronic (V+1) jet production: $qg \to Vq$
  E. Remiddi, TG
- Matrix elements with internal masses: $\gamma^* \to Q\bar{Q}$, $q\bar{q} \to Q\bar{Q}$, $gg \to Q\bar{Q}$
  M. Czakon, A. Mitov, S. Moch
  R. Bonciani, A. Ferroglia, D. Maître, C. Studerus, TG

Real corrections at NNLO

Double real radiation

$$\mathrm{d}\sigma^{(m+2)} = |\mathcal{M}_{m+2}|^2 \mathrm{d}\Phi_{m+2} J_m^{(m+2)}(p_1, \ldots, p_{m+2}) \sim \frac{1}{\epsilon^4}$$

with $J_n^{(n+2)}$ jet definition for combining m+2 partons into m jets

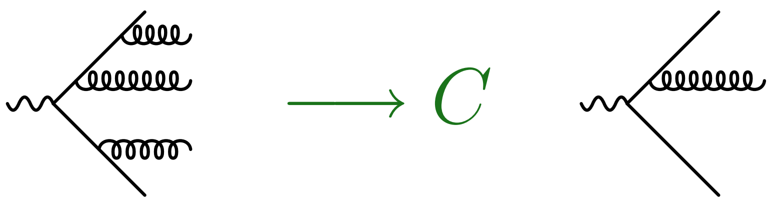
Two approaches:

- Direct evaluation
  C. Anastasiou, K. Melnikov, F. Petriello
  - expand $|\mathcal{M}_{m+2}|^2 \mathrm{d}\Phi_{m+2}$ in distributions
  - decompose $\mathrm{d}\Phi_{m+2}$ into sectors corresponding to different singular configurations (Iterated sector decomposition)
    T. Binoth, G. Heinrich
  - compute sector integrals numerically
    Results: $pp \to H + X$, $pp \to V + X$, $\mu \to e + \nu + \bar{\nu} + X$
- Evaluation with subtraction term

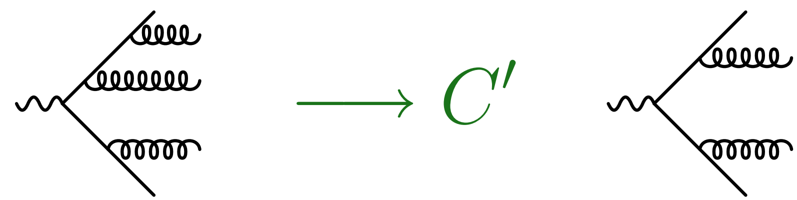
Real Corrections at NNLO

Infrared subtraction terms

$m+2$ partons $\to m$ jets:



$m+2 \to m+1$ pseudopartons $\to m$ jets:



- Double unresolved configurations:
  - triple collinear
  - double single collinear
  - soft/collinear
  - double soft
- Single unresolved configurations:
  - collinear
  - soft

J. Campbell, E.W.N. Glover; S. Catani, M. Grazzini

Issue: find subtraction functions which

- approximate full $m+2$ matrix element in all singular limits
- are sufficiently simple to be integrated analytically

NLO Subtraction

Structure of NLO m-jet cross section (subtraction formalism):

Z. Kunszt, D. Soper

$$\mathrm{d}\sigma_{NLO} = \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{R}_{NLO} - \mathrm{d}\sigma^{S}_{NLO}\right) + \left[\int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{S}_{NLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{V}_{NLO}\right]$$

- $\mathrm{d}\sigma^{S}_{NLO}$: local counter term for $\mathrm{d}\sigma^{R}_{NLO}$
- $\mathrm{d}\sigma^{R}_{NLO} - \mathrm{d}\sigma^{S}_{NLO}$: free of divergences, can be integrated numerically

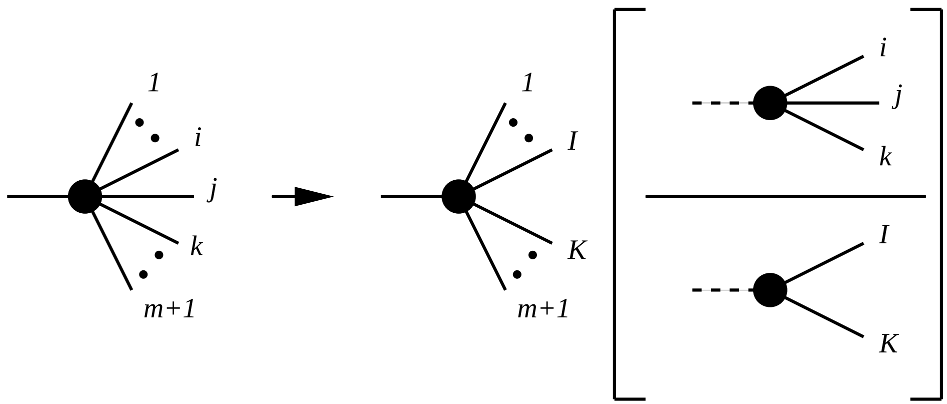
General methods at NLO

- Dipole subtraction
  S. Catani, M. Seymour; NNLO: S. Weinzierl
- $\mathcal{E}$-prescription
  S. Frixione, Z. Kunszt, A. Signer;
  NNLO: S. Frixione, M. Grazzini; V. Del Duca, G. Somogyi, Z. Trocsanyi
- Antenna subtraction
  D. Kosower; J. Campbell, M. Cullen, N. Glover; A. Daleo, D. Maître, TG
  NNLO: A. Gehrmann-De Ridder, E.W.N. Glover, TG

NLO Antenna Subtraction

Building block of $\mathrm{d}\sigma^S_{NLO}$: NLO-Antenna function X^0_{ijk}

Contains all singularities of parton j emitted between partons i and k



$$X^0_{ijk} = S_{ijk,IK} \frac{|M^0_{ijk}|^2}{|M^0_{IK}|^2}$$

$$\mathrm{d}\Phi_{X_{ijk}} = \frac{\mathrm{d}\Phi_3}{P_2}$$

Phase space factorisation

$$\mathrm{d}\Phi_{m+1}(p_1,\ldots,p_{m+1};q) = \mathrm{d}\Phi_m(p_1,\ldots,\tilde{p}_I,\tilde{p}_K,\ldots,p_{m+1};q)\cdot \mathrm{d}\Phi_{X_{ijk}}(p_i,p_j,p_k;\tilde{p}_I+\tilde{p}_K)$$

Integrated subtraction term (analytically)

$$|\mathcal{M}_m|^2\, J_m^{(m)}\, \mathrm{d}\Phi_m \int \mathrm{d}\Phi_{X_{ijk}}\, X^0_{ijk} \sim |\mathcal{M}_m|^2\, J_m^{(m)}\, \mathrm{d}\Phi_m \int \mathrm{d}\Phi_3 |M^0_{ijk}|^2$$

can be combined with $\mathrm{d}\sigma^V_{NLO}$

NNLO Infrared Subtraction

Structure of NNLO m-jet cross section:

$$
\begin{aligned}
\mathrm{d}\sigma_{NNLO} \quad = \quad & \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\sigma^{R}_{NNLO} - \mathrm{d}\sigma^{S}_{NNLO}\right) \\
& + \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO}\right) \\
& + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{V,2}_{NNLO} + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^{S}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{VS,1}_{NNLO} ,
\end{aligned}
$$

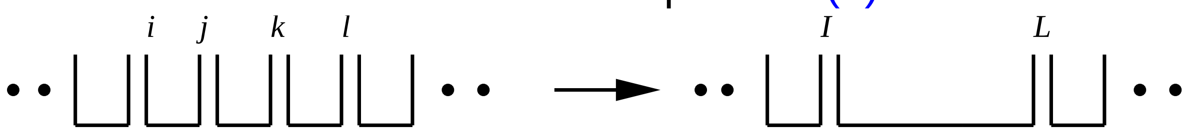
- $\mathrm{d}\sigma^{S}_{NNLO}$: real radiation subtraction term for $\mathrm{d}\sigma^{R}_{NNLO}$
- $\mathrm{d}\sigma^{VS,1}_{NNLO}$: one-loop virtual subtraction term for $\mathrm{d}\sigma^{V,1}_{NNLO}$
- $\mathrm{d}\sigma^{V,2}_{NNLO}$: two-loop virtual corrections

Each line above is finite numerically and free of infrared ϵ-poles $\longrightarrow$ numerical programme

Double Real Subtraction

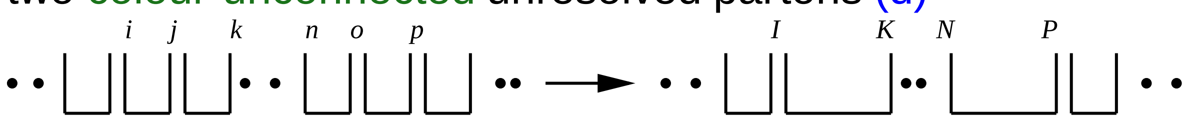
Distinct Configurations for $m+2$ partons $\rightarrow m$ jets

- one unresolved parton (a)
  - three parton antenna function X^0_{ijk} can be used (as at NLO)
  - this will not yield a finite contribution in all single unresolved limits
- two colour-connected unresolved partons (b)



  - four-parton antenna function X^0_{ijkl}
- two almost colour-unconnected unresolved partons (common radiator) (c)

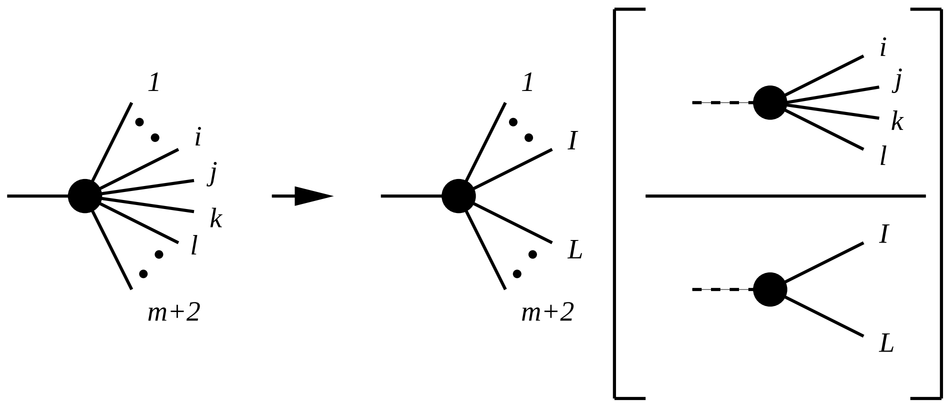
  - strongly ordered product of non-independent three-parton antenna functions
- two colour-unconnected unresolved partons (d)



  - product of independent three-parton antenna functions

Double Real Subtraction

Two colour-connected unresolved partons



$$X^0_{ijkl} = S_{ijkl,IL} \frac{|M^0_{ijkl}|^2}{|M^0_{IL}|^2}$$

$$\mathrm{d}\Phi_{X_{ijkl}} = \frac{\mathrm{d}\Phi_4}{P_2}$$

Phase space factorisation

$$\mathrm{d}\Phi_{m+2}(p_1,\ldots,p_{m+2};q) = \mathrm{d}\Phi_m(p_1,\ldots,\tilde{p}_I,\tilde{p}_L,\ldots,p_{m+2};q)\cdot\mathrm{d}\Phi_{X_{ijkl}}(p_i,p_j,p_k,p_l;\tilde{p}_I+\tilde{p}_L)$$

Integrated subtraction term (analytically)

$$|\mathcal{M}_m|^2\, J_m^{(m)}\,\mathrm{d}\Phi_m \int \mathrm{d}\Phi_{X_{ijkl}}\, X^0_{ijkl} \sim |\mathcal{M}_m|^2\, J_m^{(m)}\,\mathrm{d}\Phi_m \int \mathrm{d}\Phi_4 |M^0_{ijkl}|^2$$

Four-particle inclusive phase space integrals are known

A. Gehrmann-De Ridder, G. Heinrich, TG

Double Real Subtraction

Example: $1/N^2$ colour factor

Single unresolved parton subtraction

$$\mathrm{d}\sigma^{S,a}_{NNLO} = \frac{N_5}{N^2}\,\mathrm{d}\Phi_5(p_1,\ldots,p_5;q)\,\frac{1}{3!} \times \sum_{i,j,k\in P_C(3,4,5)} A_3^0(1_q,i_g,2_{\bar{q}})\,\tilde{A}_4^0(\widetilde{(1i)}_q,j_g,k_g,\widetilde{(2i)}_{\bar{q}})\;J_3^{(4)}(\widetilde{p_{1i}},p_j,p_k,\widetilde{p_{2i}})$$

Colour connected double unresolved subtraction

$$\mathrm{d}\sigma^{S,b}_{NNLO} = \frac{N_5}{N^2}\,\mathrm{d}\Phi_5(p_1,\ldots,p_5;q)\,\frac{1}{3!}\sum_{i,j,k\in P_C(3,4,5)}\Big(\tilde{A}_4^0(1_q,i_g,j_g,2_{\bar{q}}) - A_3^0(1_q,i_g,2_{\bar{q}})\,A_3^0(\widetilde{(1i)}_q,j_g,\widetilde{(2i)}_{\bar{q}}) - A_3^0(1_q,j_g,2_{\bar{q}})\,A_3^0(\widetilde{(1j)}_q,i_g,\widetilde{(2j)}_{\bar{q}})\Big) \times A_3^0(\widetilde{(1ij)}_q,k_g,\widetilde{(2ij)}_{\bar{q}})\,J_3^{(3)}(\widetilde{p_{1ij}},p_k,\widetilde{p_{2ij}})$$

$\mathrm{d}\sigma^{R}_{NNLO} - \mathrm{d}\sigma^{S,a}_{NNLO} - \mathrm{d}\sigma^{S,b}_{NNLO}$ is finite and can be integrated numerically over $\mathrm{d}\Phi_5$

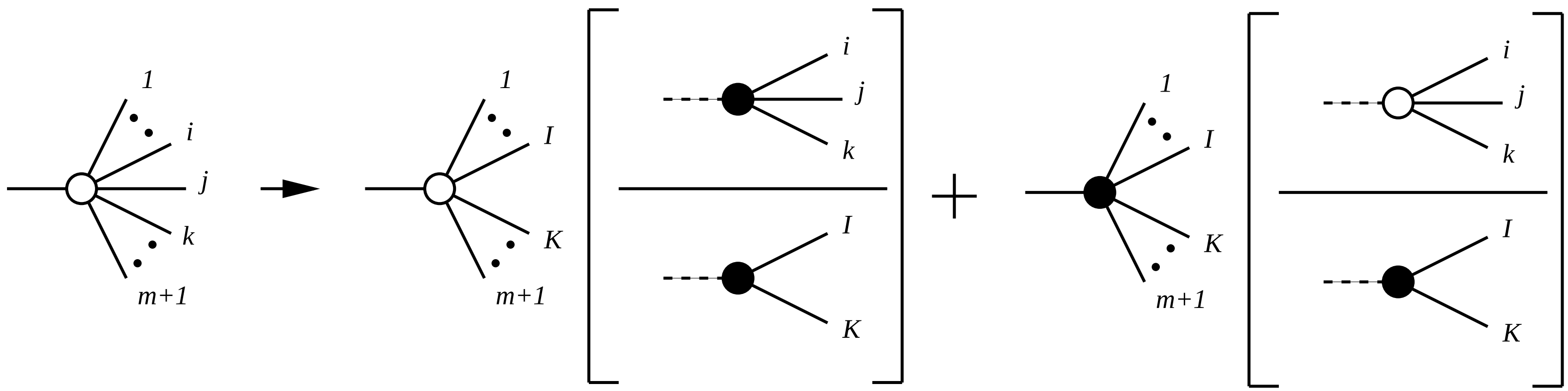
One-loop Real Subtraction

Single unresolved limit of one-loop amplitudes

$$Loop_{m+1} \stackrel{j\ unresolved}{\longrightarrow} Split_{tree} \times Loop_m + Split_{loop} \times Tree_m$$

Z. Bern, L.D. Dixon, D. Dunbar, D. Kosower; S. Catani, M. Grazzini; D. Kosower, P. Uwer
Z. Bern, V. Del Duca, W.B. Kilgore, C.R. Schmidt
Z. Bern, L.D. Dixon, D. Kosower; S. Badger, E.W.N. Glover

Accordingly: $Split_{tree} \rightarrow X^0_{ijk}$, $Split_{loop} \rightarrow X^1_{ijk}$



$$X^1_{ijk} = S_{ijk,IK} \frac{|\mathcal{M}^1_{ijk}|^2}{|\mathcal{M}^0_{IK}|^2} - X^0_{ijk} \frac{|\mathcal{M}^1_{IK}|^2}{|\mathcal{M}^0_{IK}|^2}$$

Colour-ordered antenna functions

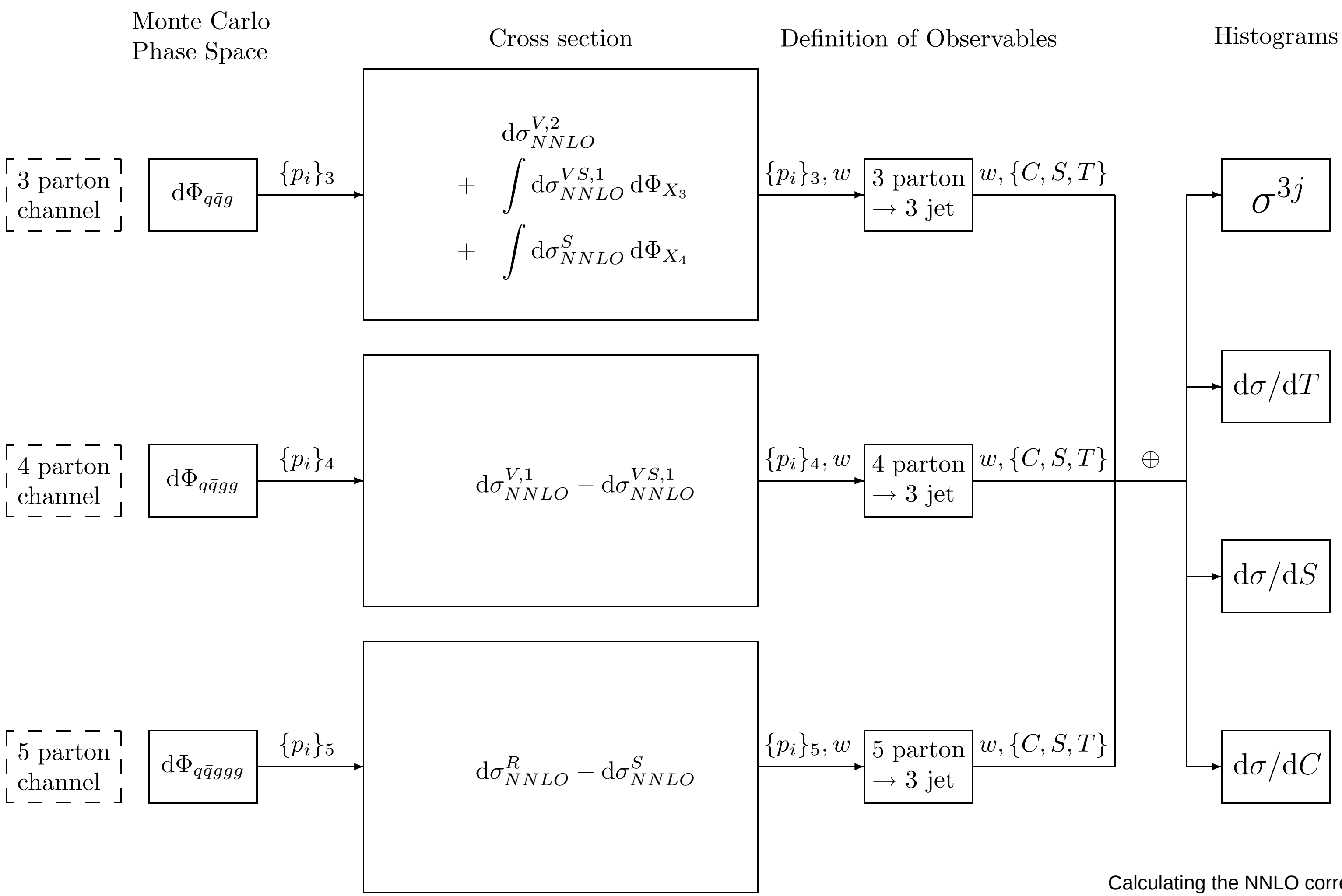
Antenna Functions

- colour-ordered pair of hard partons (radiators) with radiation in between
  - hard quark-antiquark pair
  - hard quark-gluon pair
  - hard gluon-gluon pair
- three-parton antenna $\longrightarrow$ one unresolved parton
- four-parton antenna $\longrightarrow$ two unresolved partons
- can be at tree level or at one loop
- all three-parton and four-parton antenna functions can be derived from physical matrix elements, normalised to two-parton matrix elements
  - $q\bar{q}$ from $\gamma^* \to q\bar{q} + X$
  - qg from $\tilde{\chi} \to \tilde{g}g + X$
  - gg from $H \to gg + X$

$e^+e^- \to 3$ jets at NNLO

Structure of $e^+e^- \to 3$ jets program:

EERAD3: A. Gehrmann-De Ridder, E.W.N. Glover, G. Heinrich, TG



Three-jet cross section at NNLO

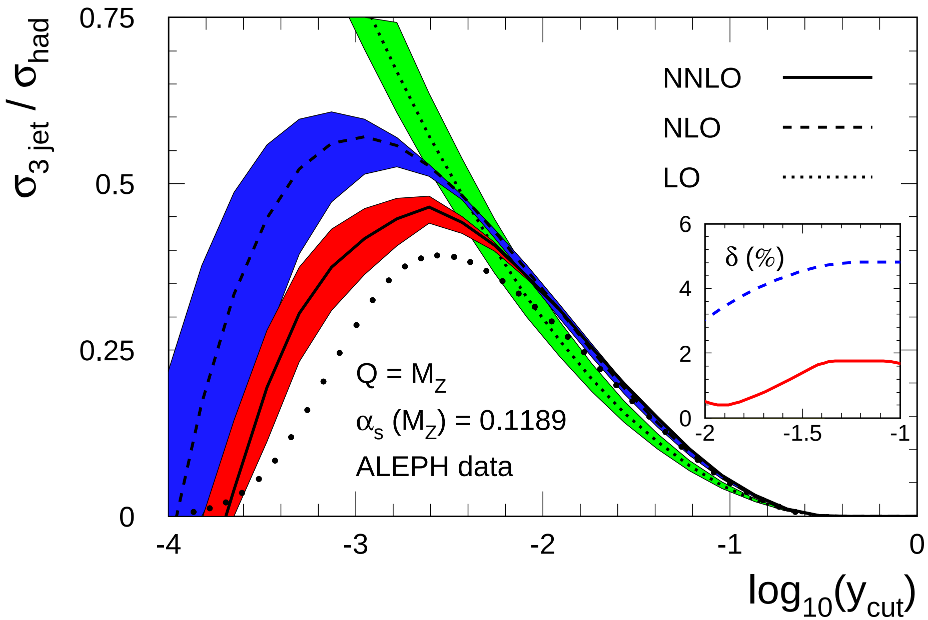
NNLO corrections: jet rates

Three-jet fraction in Durham jet algorithm

$$y_{i,j,D} = \frac{2\min(E_i^2, E_j^2)\,(1-\cos\theta_{ij})}{E_{vis}^2}$$

- vary $\mu = [M_Z/2\,;\,2\,M_Z]$
- determine minimal and maximal values

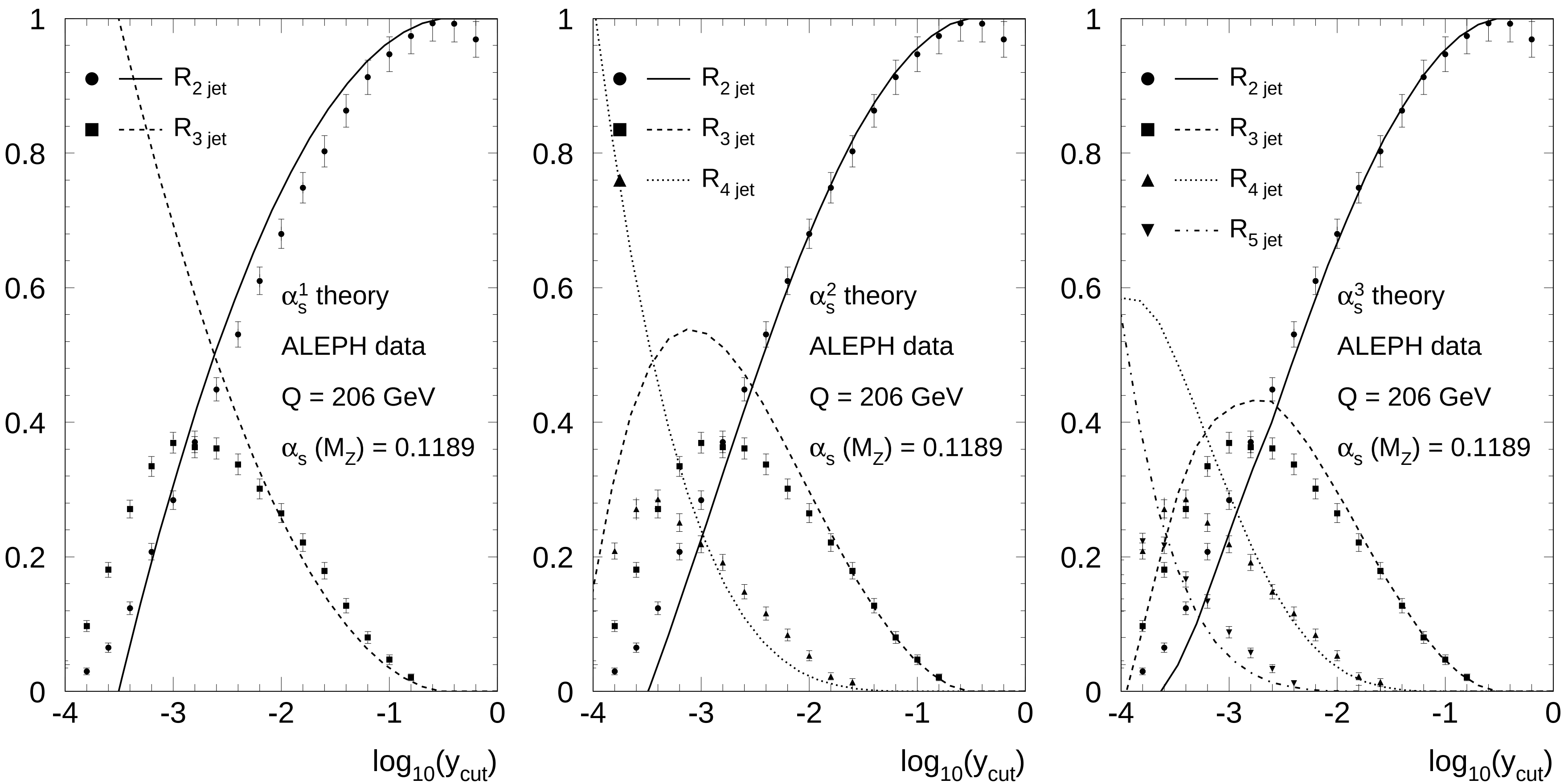
$$\delta = \frac{\max(\sigma) - \min(\sigma)}{2\sigma(\mu = M_Z)}$$



- NNLO corrections small
- substantial reduction of scale dependence
- better description towards lower jet resolution

Three-jet cross section at NNLO

NNLO corrections: jet rates



- substantial improvement towards lower $y_{\rm cut}$
- two-jet rate now NNNLO

$e^+e^- \to 3$ jets and event shapes

Standard Set of LEP

- Thrust (E. Farhi)

$$T = \max_{\vec{n}} \left(\sum_{i=1}^{n} |\vec{p_i} \cdot \vec{n}| \right) / \left(\sum_{i=1}^{n} |\vec{p_i}| \right)$$

- Heavy jet mass (L. Clavelli, D. Wyler)

$$\rho = M_i^2/s = \frac{1}{E_{\rm vis}^2} \left(\sum_{k \in H_i} |\vec{p_k}| \right)^2$$

- C-parameter: eigenvalues of the tensor (G. Parisi)

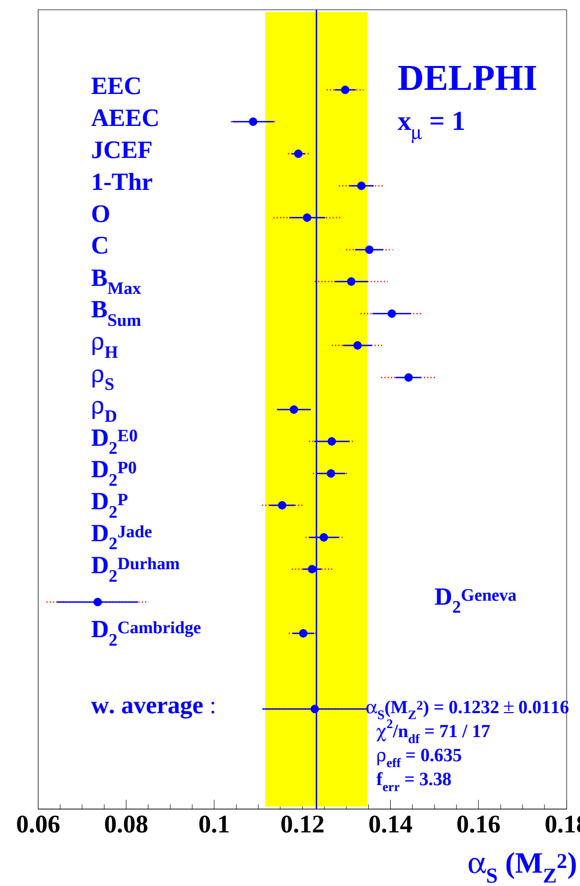
$$\Theta^{\alpha\beta} = \frac{1}{\sum_k |\vec{p_k}|} \frac{\sum_k p_k^\alpha p_k^\beta}{\sum_k |\vec{p_k}|}$$

- Jet broadenings (S. Catani, G. Turnock, B. Webber)

$$B_i = \left(\sum_{k \in H_i} |\vec{p_k} \times \vec{n}_T| \right) / \left(2 \sum_k |\vec{p_k}| \right)$$

$$B_W = \max(B_1, B_2) \qquad B_T = B_1 + B_2$$

- $3j \to 2j$ transition parameter in Durham algorithm y_{23}^D
  S.Catani, Y.L.Dokshitzer, M.Olsson, G.Turnock, B.Webber

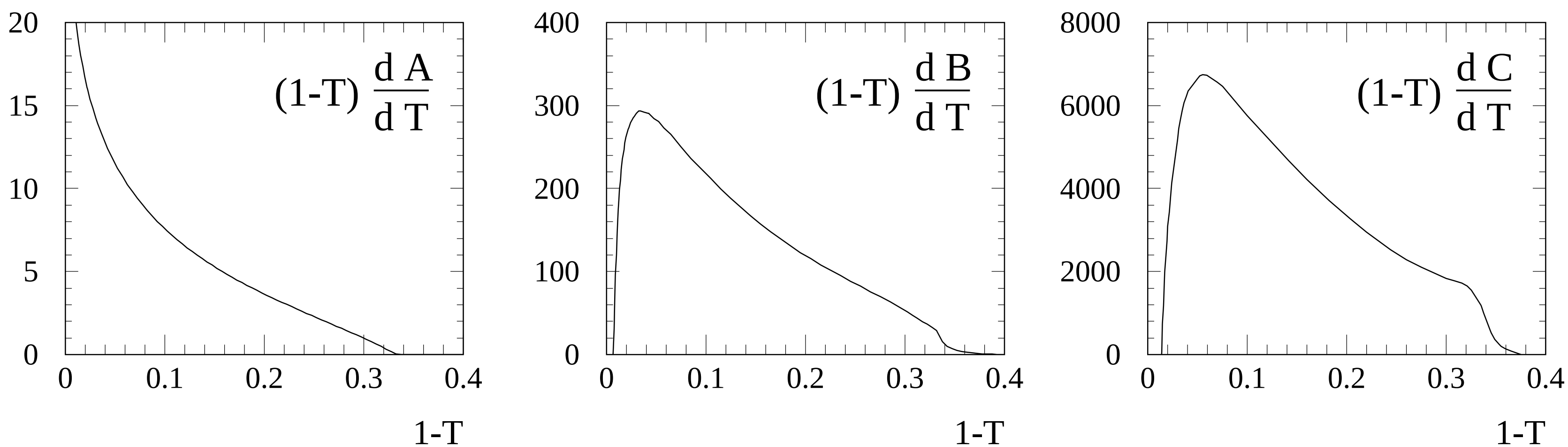


Event shapes at NNLO

NNLO expression for Thrust

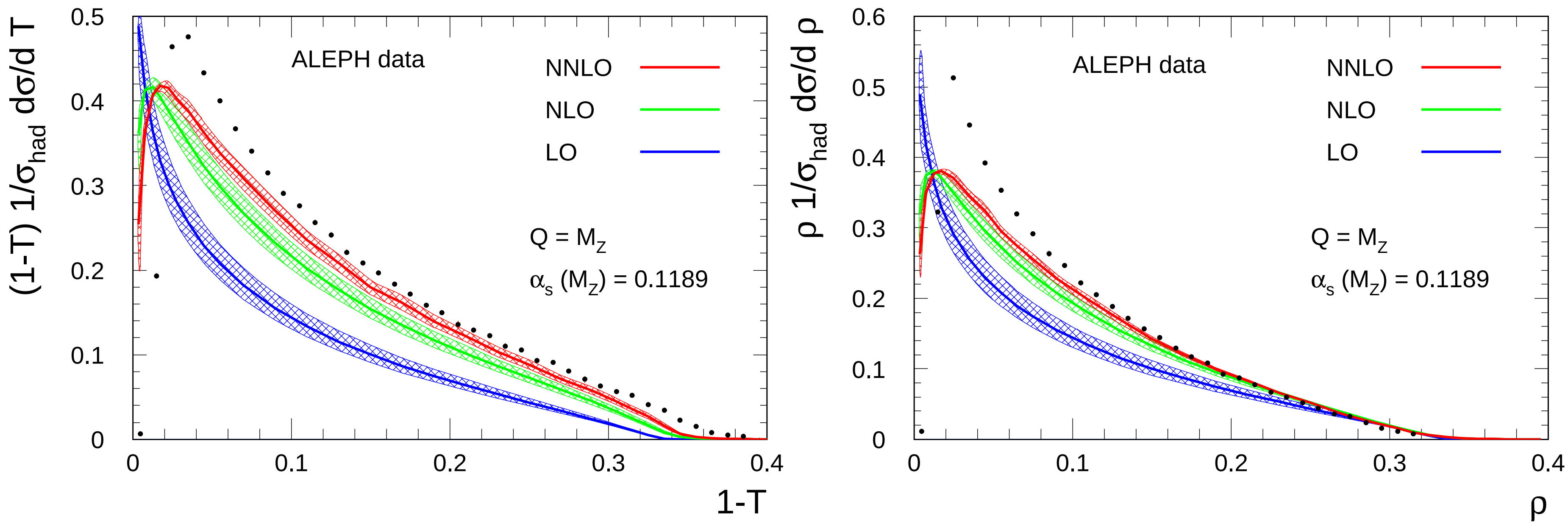
$$(1-T)\,\frac{1}{\sigma_{\rm had}}\,\frac{\mathrm{d}\sigma}{\mathrm{d}T} = \left(\frac{\alpha_s}{2\pi}\right) A(T) + \left(\frac{\alpha_s}{2\pi}\right)^2 (B(T) - 2A(T)) + \left(\frac{\alpha_s}{2\pi}\right)^3 (C(T) - 2\,B(T) - 1.64\,A(T))$$

with LO contribution $A(T)$, NLO contribution $B(T)$, NNLO contribution $C(T)$



Event shapes at NNLO

NNLO thrust and heavy mass distributions

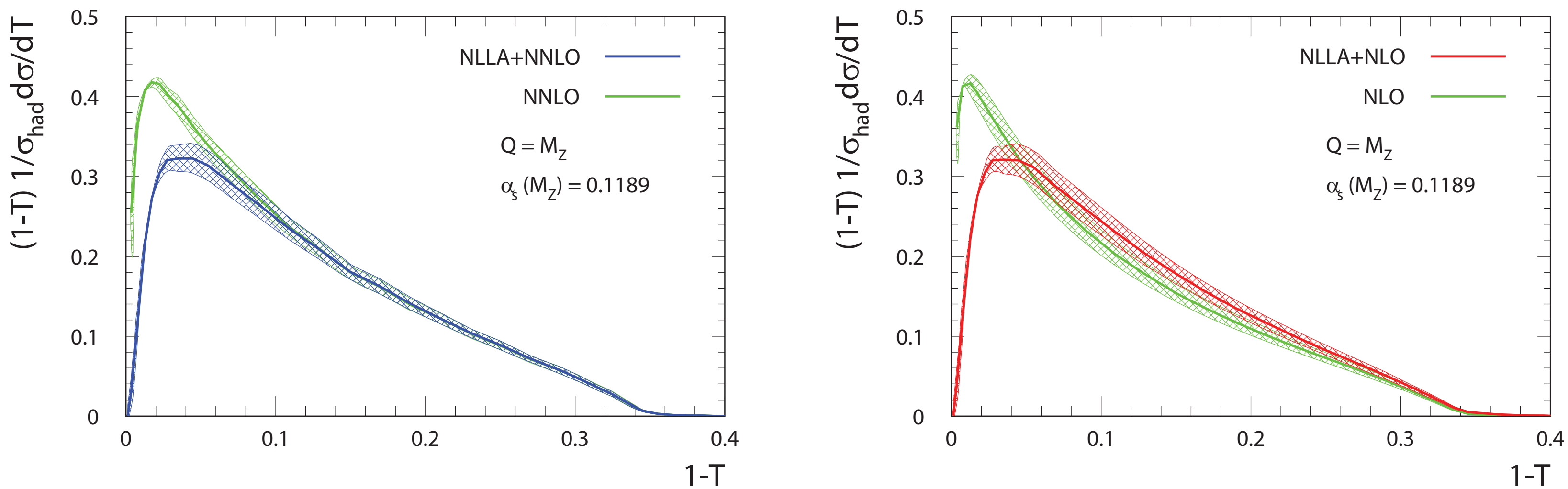


- NNLO corrections sizable: 15-20% in T, 10% in ρ
- theory uncertainty reduced by about 50 %
- large $1 - T, \rho > 0.33$: kinematically forbidden at LO
- small $1 - T, \rho$: two-jet region, need matching onto NLL resummation
- NNLO corrections for B_W smaller than for B_T
- observe: small corrections for Y_3; large corrections for C

Event shapes at NLLA+NNLO

Matching onto resummation

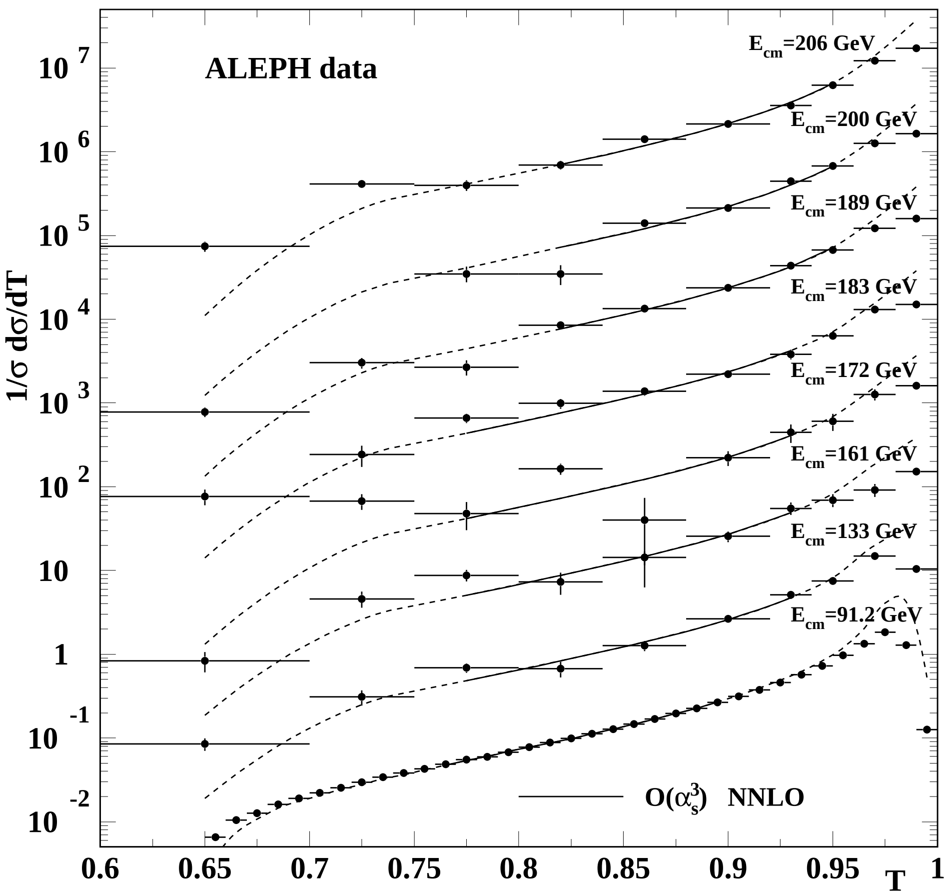
G. Luisoni, H. Stenzel, TG



- resummation to NLLA (S. Catani, L. Trentadue, G. Turnock, B. Webber; Y.L. Dokshitzer, A. Lucenti, G. Marchesini, G.P. Salam; A. Banfi, G. Zanderighi)
- normalisation in three-jet region was modified between NLO and NLLA+NLO
- normalisation in three-jet region stable between NNLO and NLLA+NNLO
- improved scale-dependence in three-jet region
- scale-dependence of NLLA dominant $\longrightarrow$ need higher orders in resummation
  T. Becher, M. Schwartz: thrust beyond NLLA

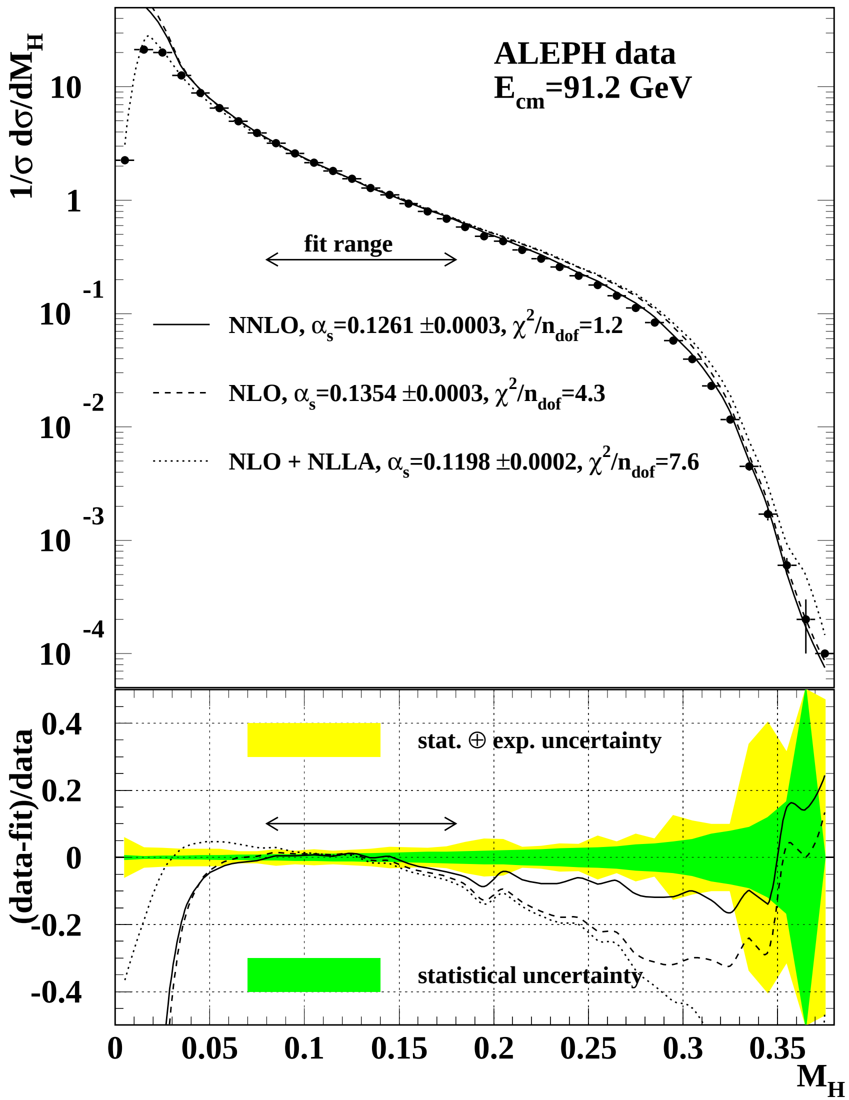
Comparison with data

High precision data from all LEP experiments, compare here to ALEPH



- include quark mass effects to NLO
  P. Nason, C. Oleari
  W. Bernreuther, A. Brandenburg, P. Uwer
  G. Rodrigo, A. Santamaria
- include hadronization corrections
  HERWIG: B. Webber et al.
  ARIADNE: T. Sjostrand et al.
- try new fit of α_s, based on ALEPH analysis
  G. Dissertori, A. Gehrmann-De Ridder,
  G. Heinrich, H. Stenzel, TG

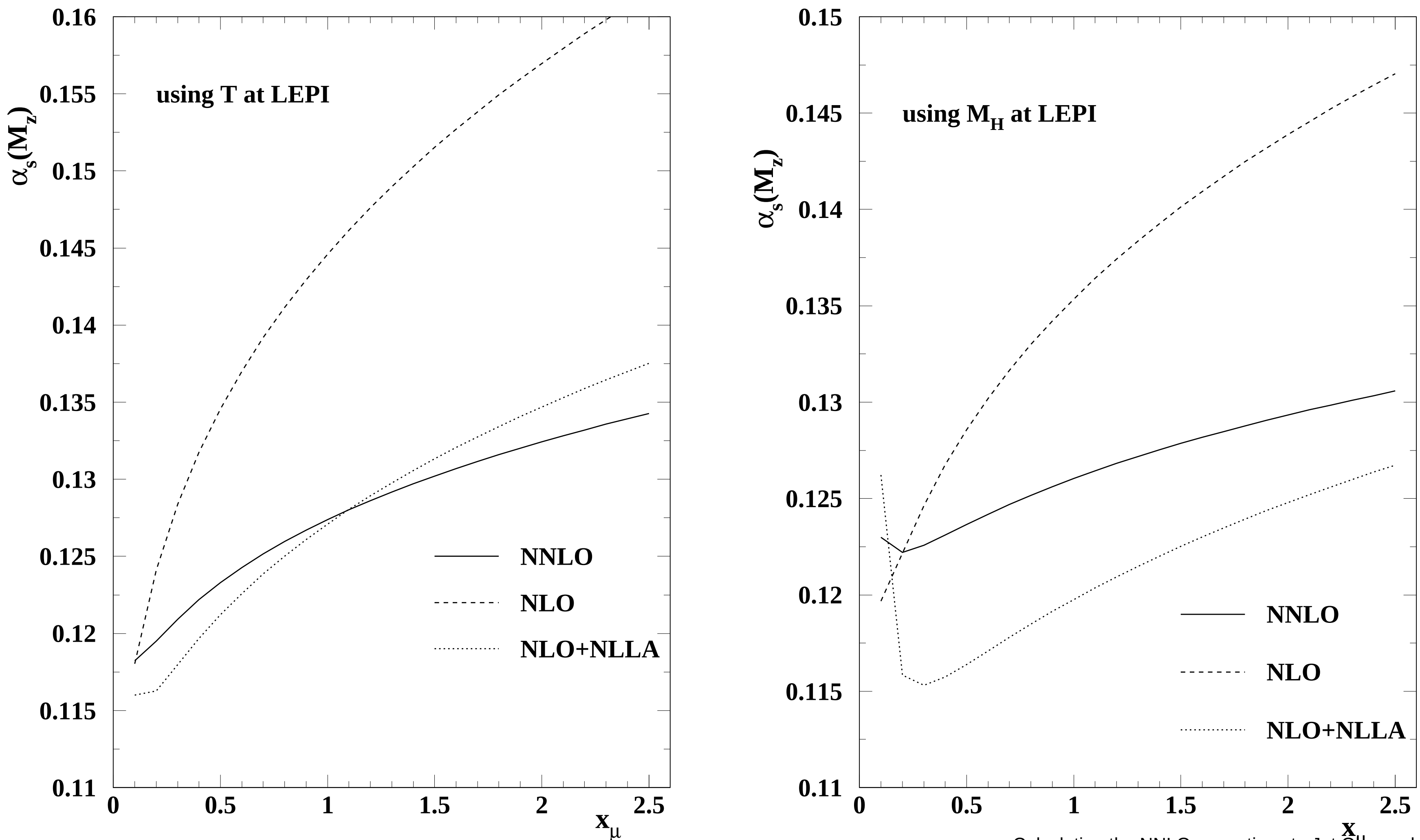
Extraction of α_s



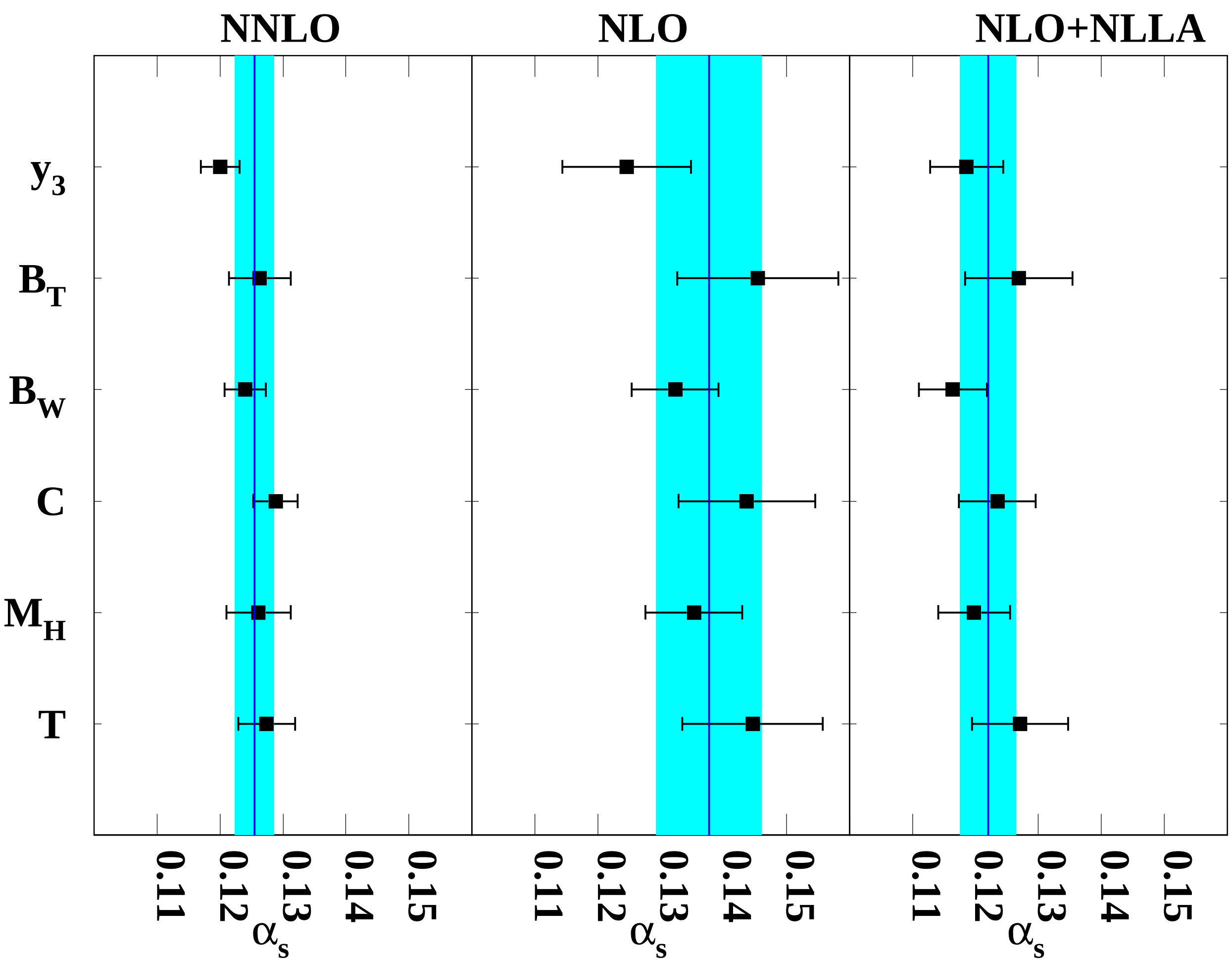
- clear improvement of NNLO over NLO
- good fit quality
- extended range of good description in 3-jet region
- matched NLO+NNLA still yields a better prediction in 2-jet region
- value of α_s lower than at NLO, but still rather high

Extraction of α_s

Uncertainty from renormalisation scale



Extraction of α_s



- scale uncertainty reduced by factor 2 compared to NLO; factor 1.3 compared to NLLA+NLO
- scatter among values from different observables reduced very substantially at NNLO $\longrightarrow$ genuine NNLO effect

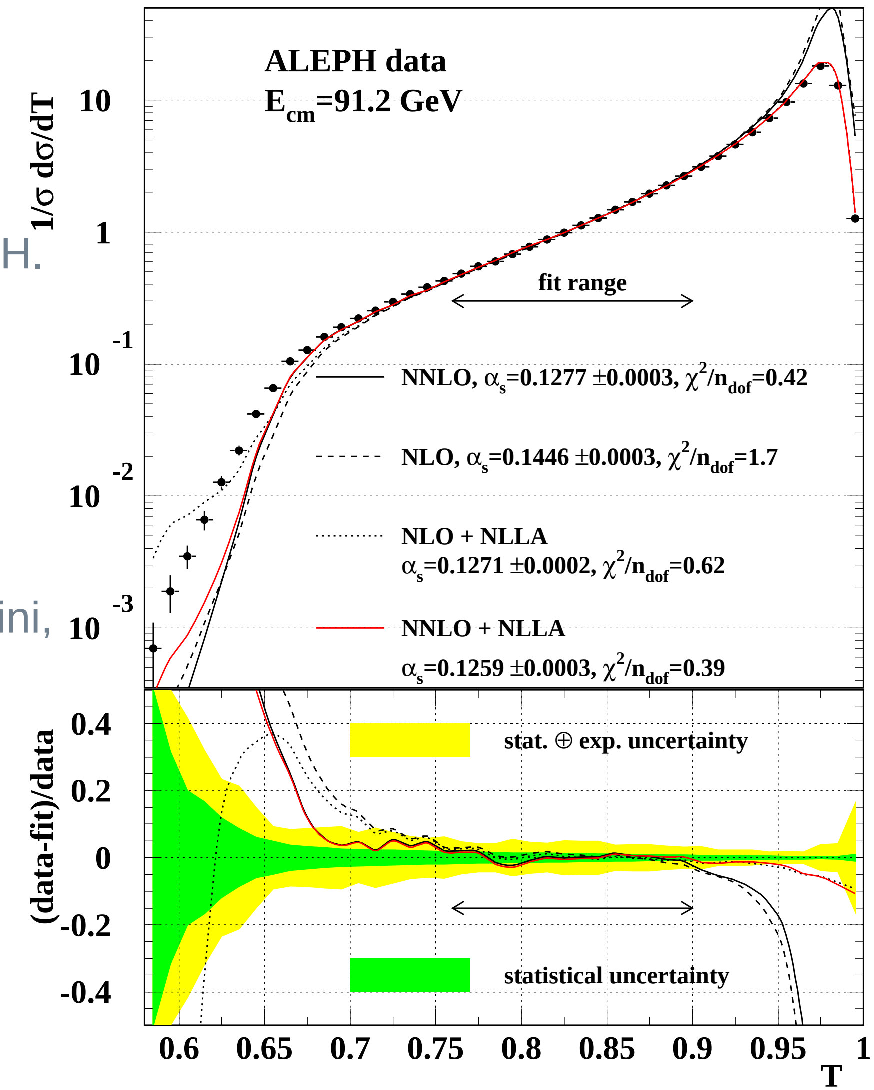
Result for all ALEPH event shapes of LEP1/LEP2

$$\alpha_s(M_Z) = 0.1240 \pm 0.0008(stat) \pm 0.0010(exp) \pm 0.0011(had) \pm 0.0029(theo)$$

Outlook

Next steps:

- α_s from NLLA+NNLO
  G. Dissertori, A. Gehrmann-De Ridder, E.W.N. Glover, G. Heinrich, G. Luisoni, H. Stenzel, TG
- study jet rates in different algorithms
- study moments of event shapes
- revisit analytic power corrections
  Y.L. Dokshitzer, A. Lucenti, G. Marchesini, G.P. Salam
- include electroweak corrections
  C. Carloni-Calame, S. Moretti, F. Piccinini, D. Ross
- resummation and matching at NNLLA



Summary and Conclusions

- High precision data on jet observables demand theoretical accuracy beyond NLO
- Principal ingredients to NNLO jet calculations
  - two-loop virtual corrections
  - generic algorithm for singular real emission
- Presented results for event shapes in e^+e^- annihilation
  - improved theoretical uncertainty
  - considerably better consistency between observables
  - new NNLO extraction of α_s, more phenomenology to come
- Precision calculations for jet observables at LHC in progress