Learning about $\mathcal{N}=4$ $\Longrightarrow$ Learning about QCD?

John Joseph M. Carrasco

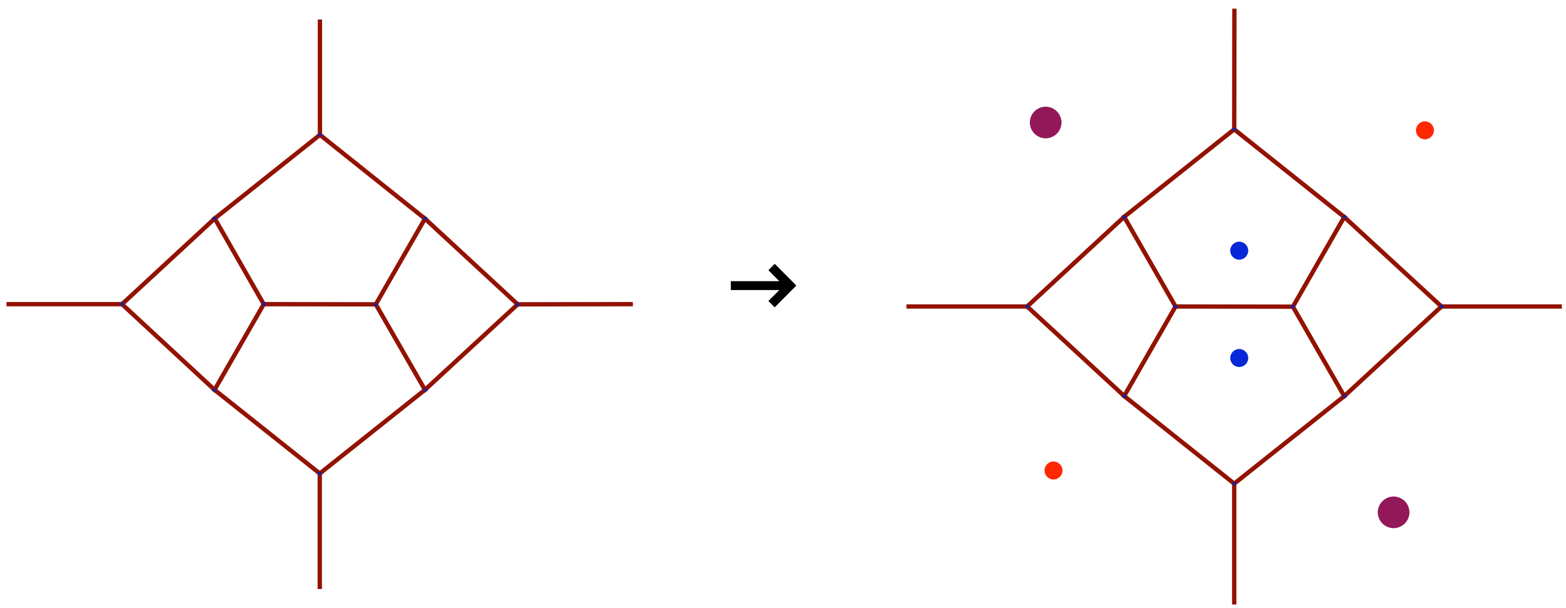
arXiv:0805.3993

Z.Bern, J.J.M.C, H. Johansson

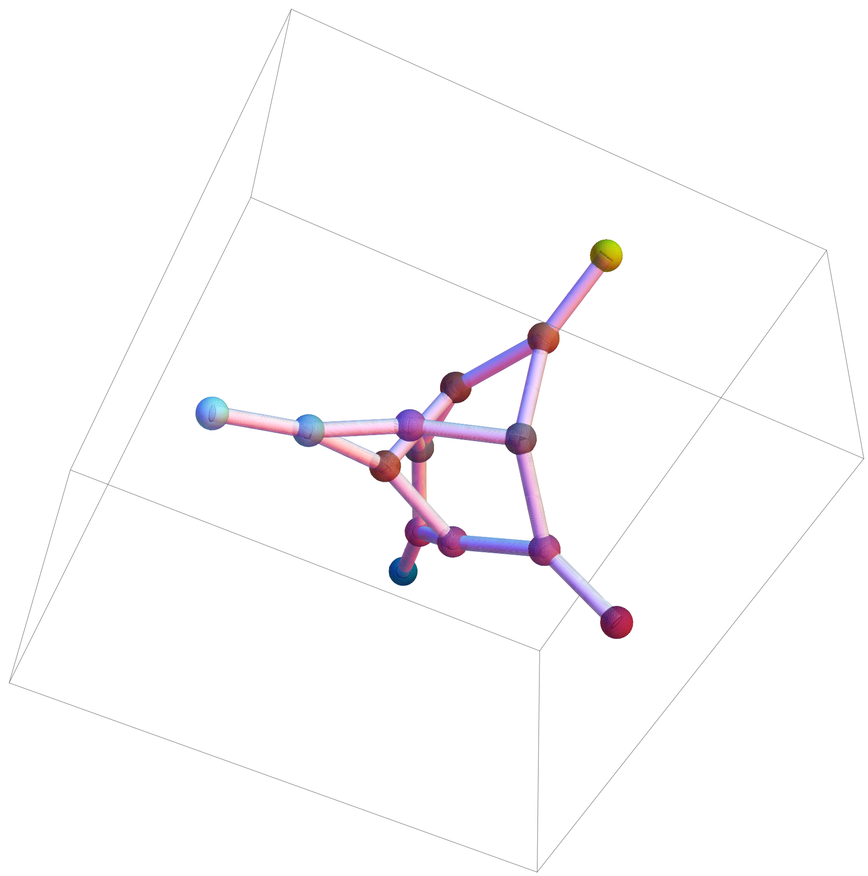
Drummond, Henn, Smirnov and Sokatchev, hep-th/0607160.

Bern, Czakon, Dixon, Kosower and Smirnov, hep-th/0610248.

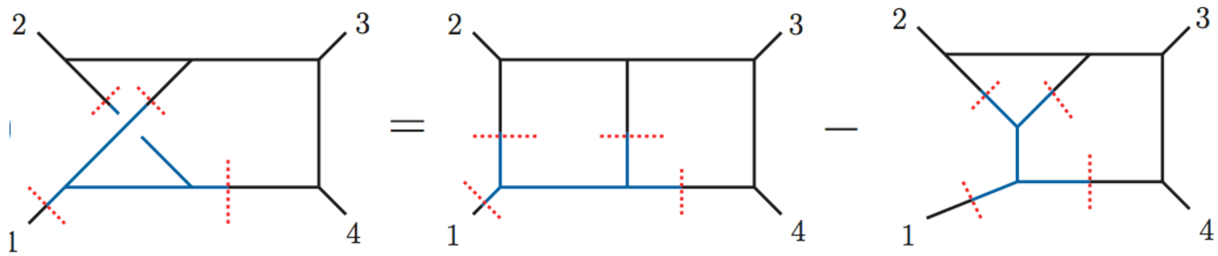
Feynman algorithm for planar $\mathcal{N}=4$



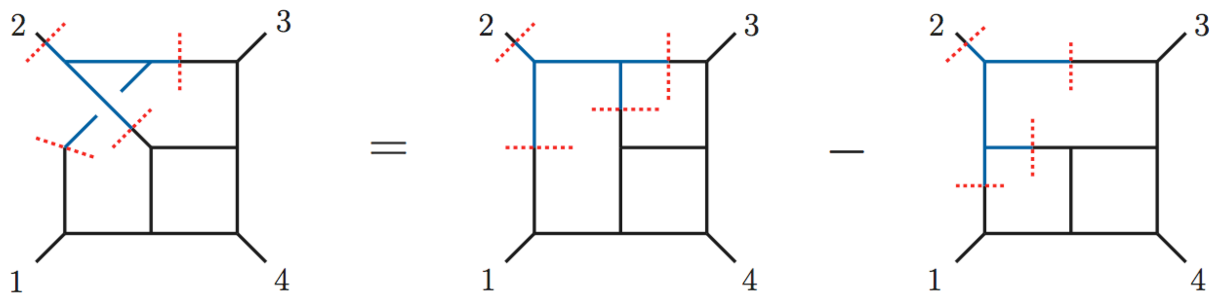
Nonplanar?



Interesting Observation

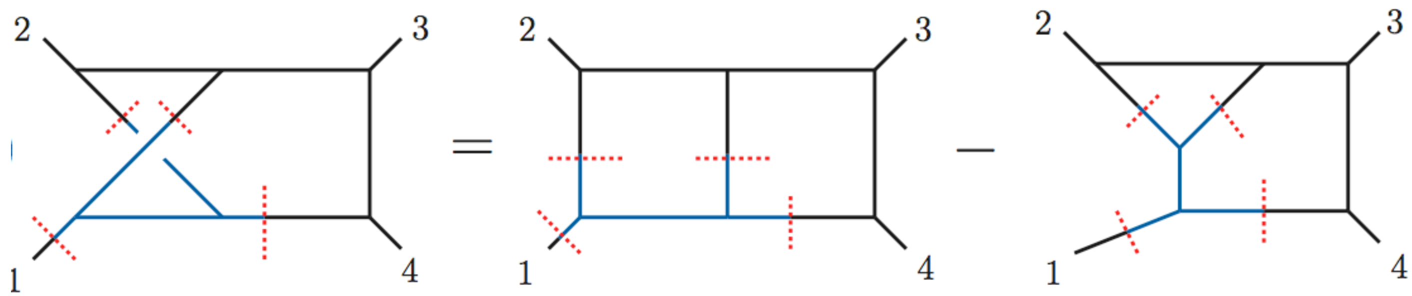


Relation between numerator factors of diagrams on the cut for all massless gauge theories. All edges cut except for blue propagator.

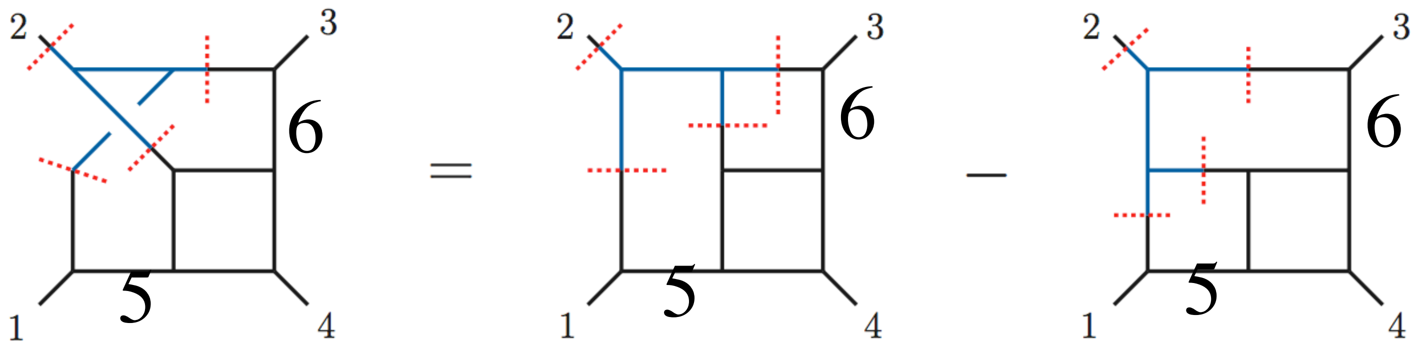


Easiest to see in $\mathcal{N}$ =4(this is where it was discovered)

$$s_{a,b} \equiv (k_a + k_b)^2$$



$$s_{1,2} = s_{1,2} - 0$$



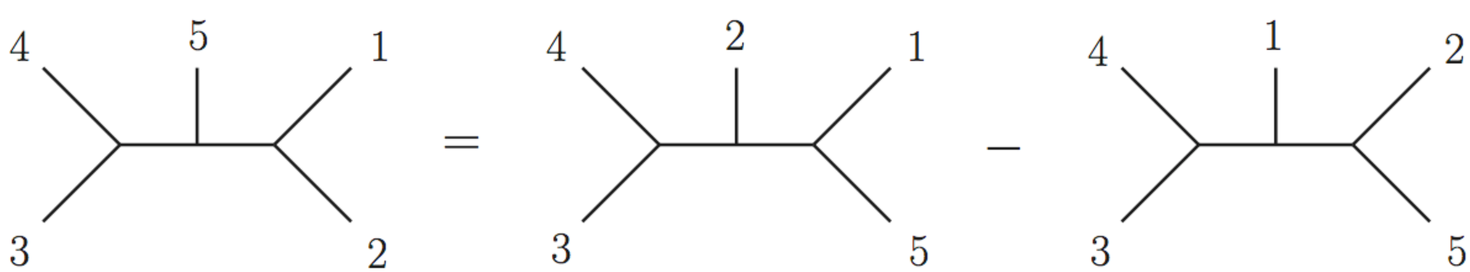
$$(s_{1,2}\, s_{4,5} - s_{1,4}\, s_{4,6}) = (s_{1,2}\, s_{4,5}) - (s_{1,4}\, s_{4,6})$$

Tree-level Gauge Theory

$$\mathcal{A}_n^{\text{tree}}(1, 2, 3, \ldots, n) = g^{n-2} \sum_i \left(c_i\, n_i \times \text{Tree Diag}_i\right)$$

$$= g^{n-2} \sum_i \frac{c_i\, n_i}{\left[\Pi_j (p_j)^2\right]_i}; \quad c_i \equiv f^{\text{abc}} \text{ dressed vertices of Tree Diag}_i$$

Jacobi Identities obeyed : $c_i = c_j - c_k \Longrightarrow n_i = n_j - n_k$



c_3 c_5 c_8

Tree-level Gauge Theory

Only (n-3)! independent color-ordered tree partial-amplitudes for n point interaction. (c.f. (n-2)! from Kleiss-Kuijf)

5 point $\Longrightarrow$ 2 amplitudes:

$$A_5^{\text{tree}}(12345),\ \ A_t^{\text{tree}}(12354)$$

$$A_t^{\text{tree}}(12435)\, s_{24} = -A_5^{\text{tree}}(12345)\,(s_{14}+s_{15}) - A_5^{\text{tree}}(12354)\, s_{14}$$

$$A_t^{\text{tree}}(12453)\, s_{24}\, s_{245} = -A_5^{\text{tree}}(12345)\, s_{34}\, s_{15} - A_5^{\text{tree}}(12354)\, s_{14}(s_{245}+s_{35})$$

Gravity via KLT

$$\mathcal{M}_n^{\text{tree}}(1, 2, \ldots, n) = i\left(\frac{\kappa}{2}\right)^{n+2} (-1)^{n+1} \times$$

$$\Big[A_n^{\text{tree}}(1, 2, \ldots, n) \sum_{\text{perms}} \tilde{A}_n^{\text{tree}}(i_1, \ldots, i_j, 1, n-1, l_1, \ldots, l_{j'}, n) \times$$

$$f(i_1, \ldots, i_j)\, \overline{f}(l_1, \ldots, l_{j'})\Big] + \mathcal{P}(2, \ldots, n-2) \quad (\textit{obscure}!)$$

express each YM $A, \tilde{A}$:

$$\mathcal{A}_n^{\text{tree}}(1, \ldots, n) = g^{n-2} \sum_i \frac{c_i\, n_i}{\left(\prod_j (p_j)^2\right)_i} \;,$$

$$s.t.\; A_n^{\text{tree}}(\mathcal{P}_k(1, \ldots, n)) = \sum_{i \in d(\mathcal{P}_k)} \frac{n_i}{\left(\prod_j (p_j)^2\right)_i}$$

then Gravity:

$$\mathcal{M}_n^{\text{tree}}(1, 2, 3, \ldots, n) = i\left(\frac{\kappa}{2}\right)^{n-2} \sum_i \frac{\tilde{n}_i\, n_i}{\left(\prod_j (p_j)^2\right)_i}$$