Revenge of the S-Matrix

or

What is the simplest QFT?

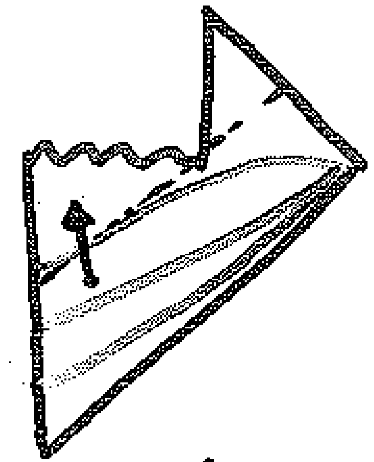
N.A-H.
with Jared Kaplan
+ in progress with
Freddy Cachazo

Biggest Crises / Oppurtunity in fundamental physis:

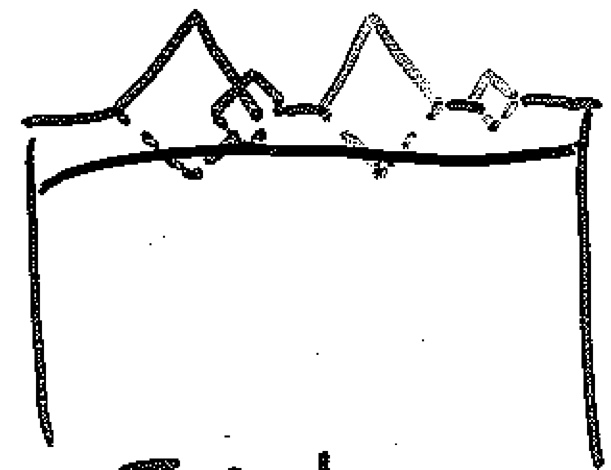
- The Landscape
- Vacuum Selection
- Fundamental issues of
  - QM
  - Gravity
  - Cosmology

MUST BE DEALT WITH.

Something goes wrong with locality + gravity, not just @ l_{pl}.



Information Paradox



Infinities in eternal inflation.

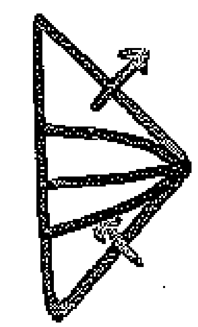
Can we talk about local things?

e.g. infalling observer into BH.

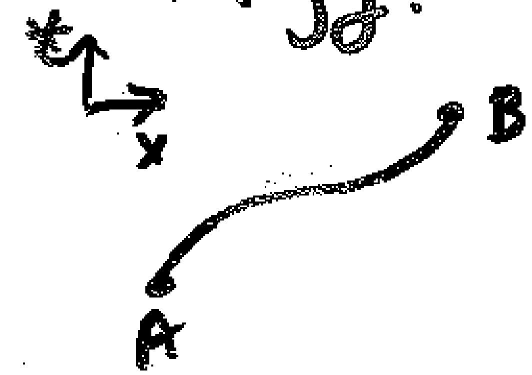
In cosmology, we are like these guys!

Local physics $\longrightarrow$ Flat Space

Can we talk about ordinary QFT in a different way, not manif. local?



Analogy:



$m\ddot{x} = -V'(x)$ manif. deterministic

$S[A,B]$ extremized not manif. det.

$\longrightarrow$ better jumping off point to QM

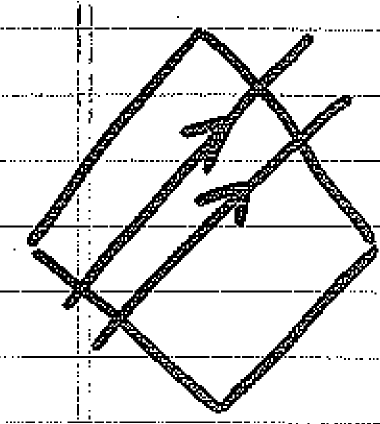
Pragmatic evidence that such a theory exists:

locality $\longrightarrow$ huge amount of redundancy (field redef; gauge + diffeo)

$\longrightarrow$ Enormous # of diagrams for n-pt amp. But amps much simpler!

Beautiful subject developed in '80s + '90s [Parke–Taylor ... Unitarity methods + Bern, Dixon, Kosower, ..] from "bottom up".

2000's "top down" attempt: Witten's twistor formulation of SYM



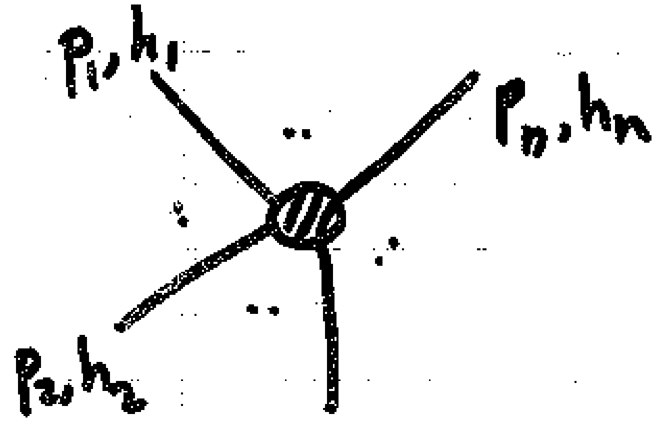
$\sim$ boundary theory for flat space.

CSW recursion relations

Very special to 4D, MHV amp. play special role, related to $F = \pm i\tilde{F}$ solutions of YM.

$\hookrightarrow$ led to BCF + W recursion relations, which can be described from "bottom up", and are much more general.

BCFW Redux



$$\mathcal{M}(p_i, h_i)$$

Complexify 2 momenta $p_{j,k}$, keeping them on shell:

$$p_j^\mu \longrightarrow p_j^\mu + q^\mu z \equiv p_j^\mu(z)$$

$$p_k^\mu \longrightarrow p_k^\mu - q^\mu z \equiv p_k^\mu(z)$$

$$0 = p_{j,k}^2(z) \Longrightarrow \boxed{q \cdot p_{j,k} = 0 \,,\ q^2 = 0}$$

$$q^2 = 0 \; , \; q \cdot p_{j,k} = 0$$

$$p_{j,k} = (1, \pm 1, 0, 0; 0, \dots 0) \; , \; q = \frac{1}{\sqrt{2}}(0,0,1,i;0,\dots,0)$$

[Or keep momenta real, $(D-2,2)$ sig.].

Pol vectors:

$z=0$: $\epsilon_j^- = \epsilon_k^+ = q \; , \; \epsilon_j^+ = \epsilon_k^- = \bar{q} \; , \; \epsilon_T = (0,0,0,0; \dots 1 \dots)$

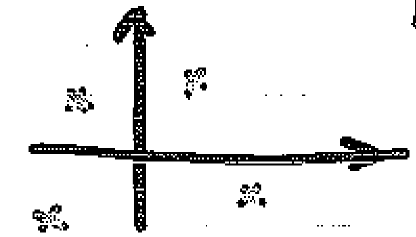
$\epsilon_j^-(z) = \epsilon_k^+(z) = q \; , \; \epsilon_T(z) = (0,0,0,0; \dots 1 \dots)$

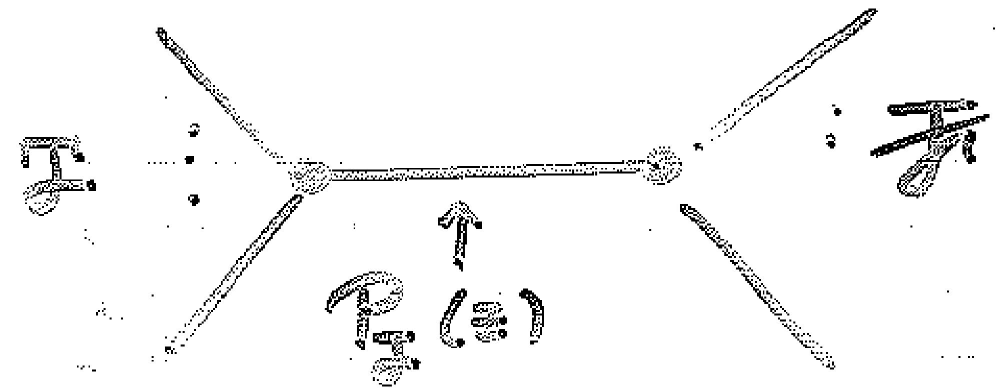
$\epsilon_j^+(z) = \bar{q} - z p_k \; , \; \epsilon_k^- = \bar{q} + z p_j$

[$p_{j,k\,\mu}\, \epsilon_{j,k}^{\mu}(z) = 0, \;\; \epsilon_j^- \epsilon_j^+ = \epsilon_k^- \epsilon_k^+ = 1, \dots$].

$$\mathcal{M}(p_i, h_i) \longrightarrow \mathcal{M}^{h_j h_k}(z)$$

$\mathcal{M}(z)$: only simple poles



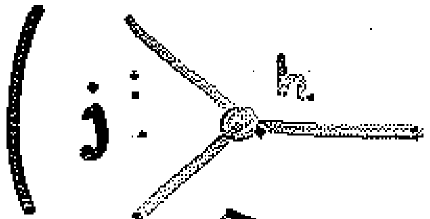
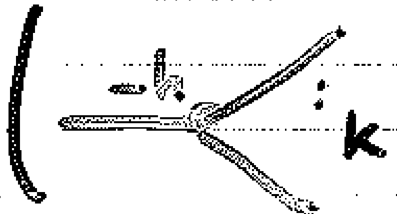


$$\mathcal{P}_J(z) = \begin{cases} P_J & j,k \in J \\ P_J + zq & j \in J \\ P_J - zq & k \in J \end{cases}$$

$$\mathcal{P}_J^2(z) = P_J^2 + 2P_J\cdot q\, z\,,$$ poles at

$$z \to z_j = -\mathcal{P}_J^2/2P_J\cdot q$$

res $\mathcal{M}(z\to z_j)$

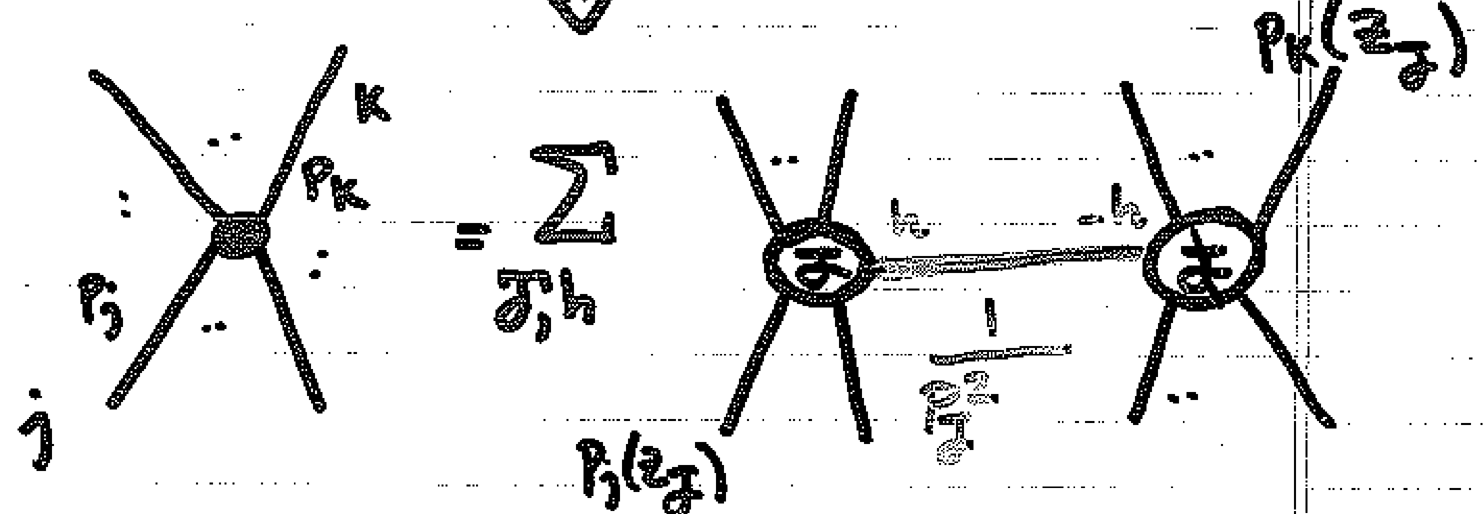
$= \sum_h \Big($  $\Big) \times \Big($  $\Big)$

Lower point, on shell (at complex z_J!) amp.

If $M(z\to\infty)\to 0$,

$$0 = \frac{1}{2\pi i}\oint \frac{dz}{z} M(z) = M(0) + \text{other res.}$$

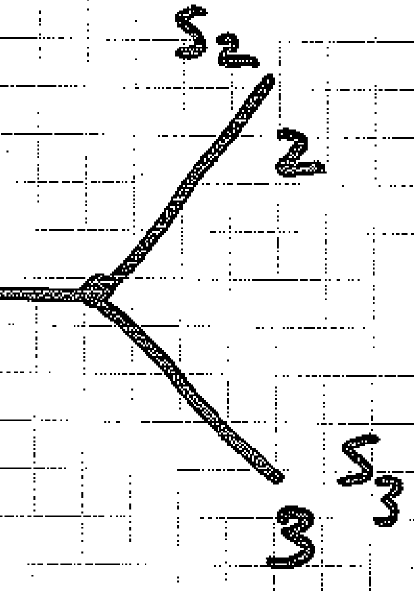
$$\Downarrow$$



On-shell BCFW recursion relations.

[Sufficient $\mathcal{M}^{-,\text{any}}(z\to\infty)\to 0$ Gauge

$\mathcal{M}^{-,\text{any}}(z\to\infty)\to 0$ Grav]



Remarkable object.
Completely det. by Lorentz Inv.
[Cachazo, Benincasa]

$p_i \cdot p_j = 0$

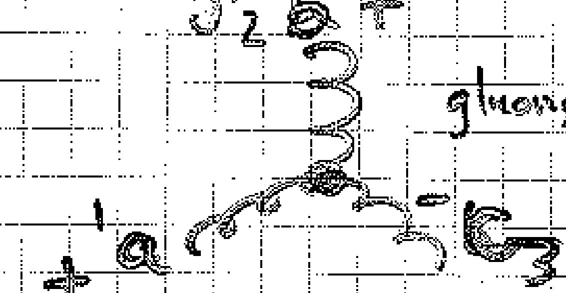
$$p_i^{\alpha\dot\alpha} = \lambda_i^{\alpha}\tilde\lambda_i^{\dot\alpha}$$

$$\langle ij\rangle[ij] = 0 \overset{\text{mom. cons.}}{\Longrightarrow} \langle ij\rangle = 0 \ \text{ or } \ [ij] = 0$$

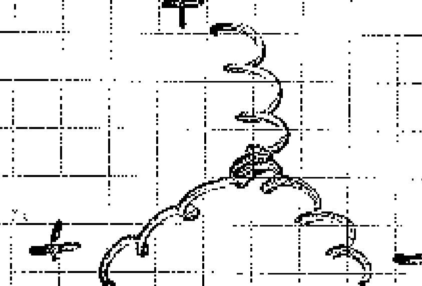
$$M^{123\,(\text{mostly}\,+)} = \langle 12\rangle^{\frac{s_1+s_2-s_3}{2}}\,\langle 23\rangle^{\cdot\cdot}\,\langle 31\rangle^{\cdot\cdot}$$

$$M^{123\,(\text{mostly}\,-)} = [12]^{-(\quad)}\,[23]^{-\cdot\cdot}\,[31]^{-\cdot\cdot}$$

e.g.



gluons

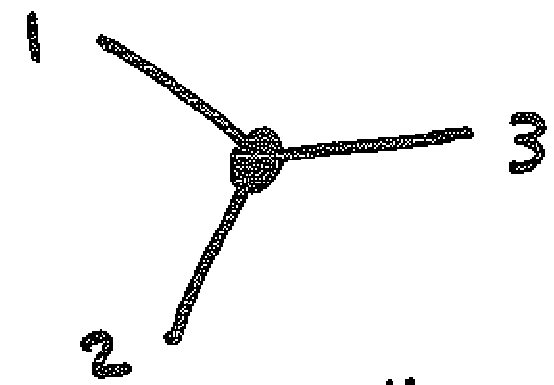


gravitons

$$f^{abc}\,\frac{\langle 12\rangle^3}{\langle 13\rangle\langle 23\rangle} \qquad \left[\frac{\langle 12\rangle^3}{\langle 13\rangle\langle 23\rangle}\right]^2$$

Grav $\sim$ Gauge^2

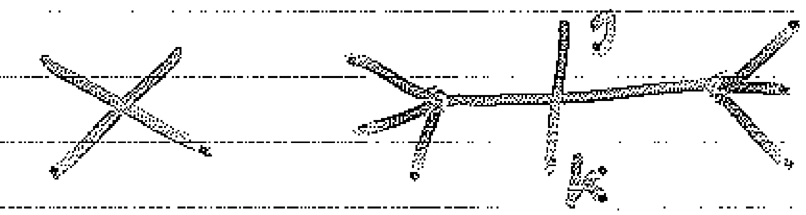
Can recursively reduce all amplitudes to



which normally can't be on-shell, but can be for complex momenta (or in $(D-2,2)$ signature).

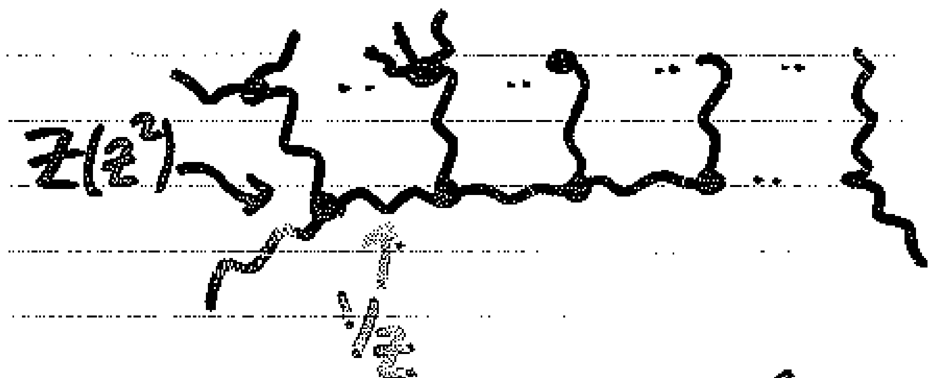
Behavior of $\mathcal{M}(z\to\infty)$ is Surprising

Naively, $\mathcal{M}(z\to\infty)\to 0$ is never true! e.g. ϕ^4 theory



$\mathcal{M}^{\phi^4}(z) \to z^0$

Gauge / Gravity is worse!



then fold in ϵ's, $\to z^0$ or z^1 ...

Actual

Easy $\rightarrow \frac{1}{z}$

BCFW

Needed CSW rules $\rightarrow \frac{1}{z}$

z^3

Cachazo, Boels et al.

Grav $\sim$ Gauge^2 $\left\{ \begin{array}{l} \frac{1}{z^2} \\ \vdots \\ z^6 \end{array} \right.$

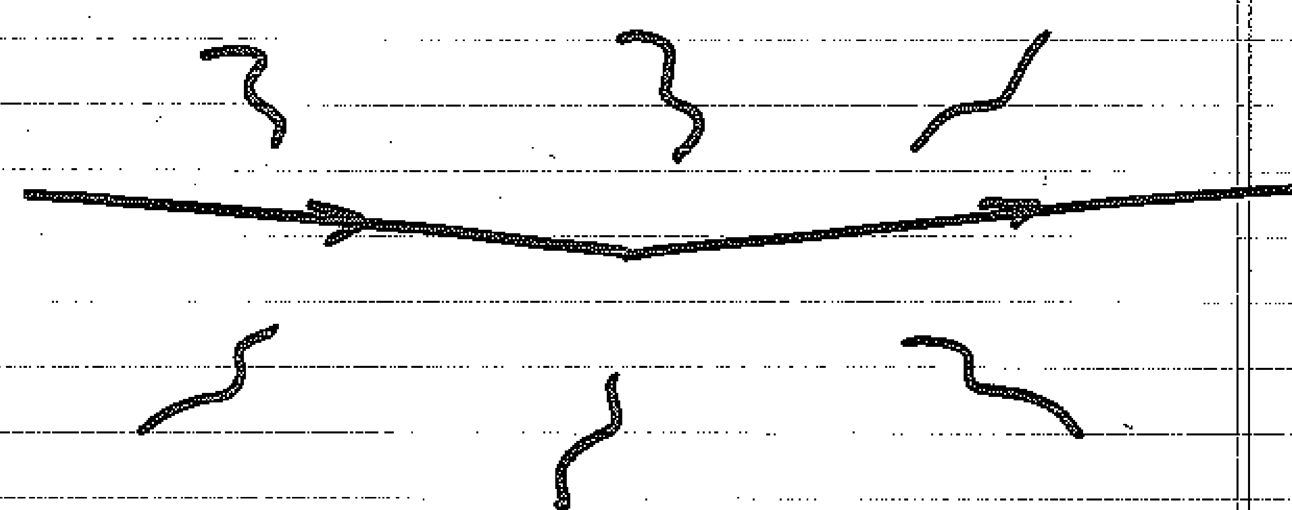
Unexpectedly good behavior of $\mathcal{M}(z \to \infty)$ encapsulates heavy cancellations in explicit diagram calculations.

Gauge	Naive
$-+$	z
$--/++$	z^2
$+-$	z^3

Grav.	
$--,++$	z^{n-1}
$\vdots$	
$++,--$	z^{n+3}

Understanding $\mathcal{M}(z \to \infty)$

$$P_{\substack{j\\k}}(z) = P_{\substack{j\\k}} \pm z\, q$$



$z \to \infty$: hard (complex) light-like particle blasting through soft background. Familiar for real momenta (eikonal). "Not much" scattering, "helicity conserved". We'll formalize + extend to apply to complex mom. of interest.

Scalar QED

$\mathcal{M}^{\text{naive}} \longrightarrow z$

$\mathcal{L} = D_\mu \phi^* D^\mu \phi$, general back. A_μ.

dangerous vertex:

$$\phi^* \partial^\mu \phi A_\mu \to z \phi^* \phi q^\mu A_\mu .$$

Eliminated in q-lightcone gauge

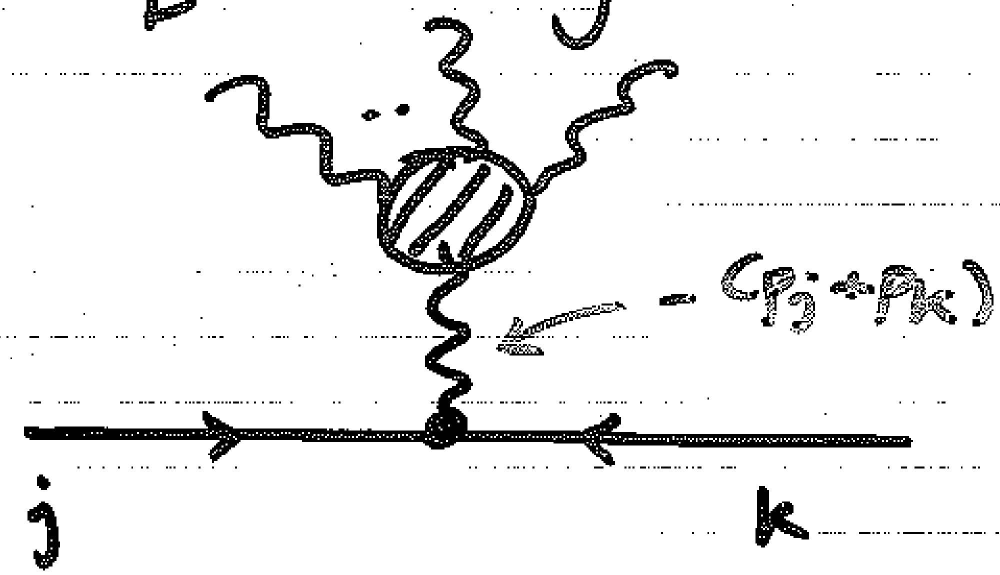
$$q^\mu A_\mu \equiv A_- = 0 .$$

$\to z^0$, $\mathcal{M}^{n\geq 2} \to \frac{1}{z}$!

Subtlety: if $(p \cdot q) = 0$, can't go to L.C. gauge for $\tilde{A}_\mu(p)$.

[Familiar $1/p_+$ poles in LC].

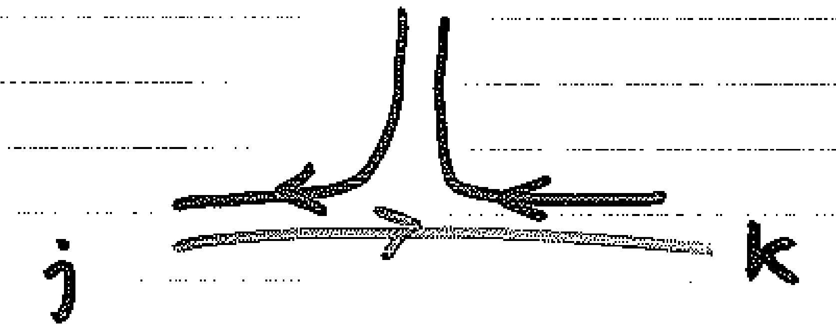
Unique diagram:



Dominates $z \to \infty$, naive z-scaling.

[Of course irrelevant in QED]

In non-Ab YM with adjoint scalar,



Unique diagram only exists for j,k adj. colors. So

$$\mathcal{M}_{jk\,\text{adj. col.}} \longrightarrow Z \quad \text{[Unique diag]}$$

$$\mathcal{M}_{jk\,\substack{\text{non}\\ \text{adj. col}}} \longrightarrow Z^0$$

$$\boxed{\mathcal{M}^{ab} = (cz + \dots)\eta^{ab} + A^{ab} + \frac{1}{z}B^{ab} + \dots}$$

$$\mathcal{M}^{-+} = \epsilon^{-}_{ja}\mathcal{M}^{ab}\epsilon^{+}_{kb} = q_a \mathcal{M}^{ab} q_b$$

$$\longrightarrow \frac{1}{z}$$

$$\mathcal{M}^{--} = q_a \mathcal{M}^{ab}(\bar{q}_b + z p_{j\,b})$$

WI

$$= \frac{1}{z} p_{ja} \mathcal{M}^{ab}(\bar{q}_b + z p_{jb})$$

$$\longrightarrow \frac{1}{z}$$

$$\mathcal{M}^{+-} = \epsilon^{+}_{ja}\mathcal{M}^{ab}\epsilon^{-}_{b}$$

$$= (\bar{q}_a - z p_{ka})\mathcal{M}^{ab}(\bar{q}_b + z p_{ja})$$

$$\longrightarrow z^3$$

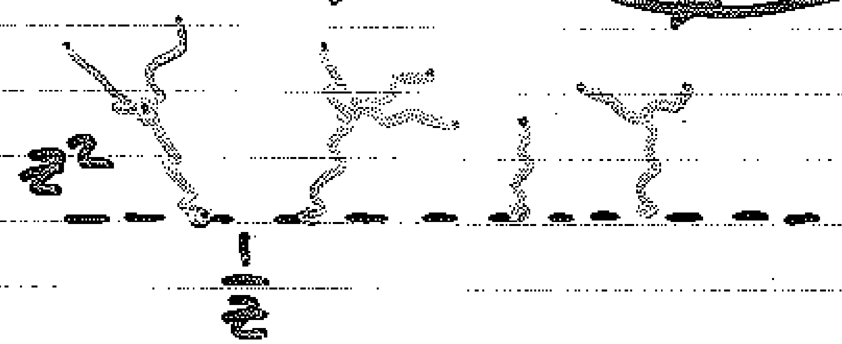
ϵ_1 / ϵ_2	$-$	$+$	T
$-$	$\frac{1}{z}$	$\frac{1}{z}$	$\frac{1}{z}$
$+$	z^3	$\frac{1}{z}$	z
$T1$	z	$\frac{1}{z}$	z
$T2$	z	$\frac{1}{z}$	1

– any vanishing

$\Rightarrow$ $\boxed{\text{BCFW applies to YM in any } D \geq 4.}$

[Note: for non adj colors, extra $1/z$].

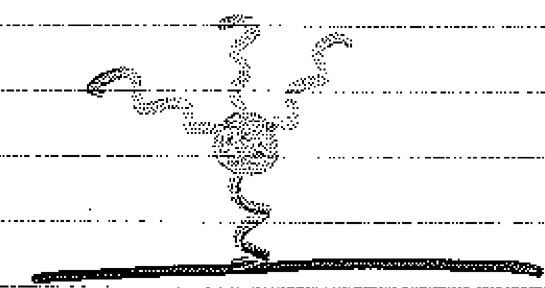
Scalar Gravity



$\mathcal{M}^{naive} \to z^{n-1}$

$\mathcal{L} = \sqrt{-g}\, g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$

Fix diffs by choosing $g^{++} = g^{+i} = 0$, $g^{+-} = 1$ (again q-lightcone gauge) $\Rightarrow$ eliminates z^2 vertices.



Unique diagram $\propto z^2$

$\mathcal{M}(z) \to z^2$

Photon-Graviton

$$\mathcal{L} = -\frac{1}{4}\sqrt{-g}\, g^{\mu\alpha} g^{\nu\beta}\, \nabla_{[\mu} A_{\nu]} \nabla_{[\alpha} A_{\beta]}$$

$$\xrightarrow{GF} -\frac{1}{2}\sqrt{-g}\, g^{\mu\alpha} g^{\nu\beta}\, \nabla_\mu A_\nu \nabla_\alpha A_\beta$$

to see spin L.I., introduce more redundancy $A_\mu = e^a_\mu A_a$

$$\mathcal{L} = -\sqrt{g}\, g^{\mu\nu} \eta^{ab} (\partial_\mu A_a + \omega_{\mu a}{}^c A_c)(\partial_\nu A_b + \omega_{\nu b}{}^d A_d)$$

Choose L.C. gauge for diffs + local Lorentz:

$$g^{++} = g^{+i} = 0,\ g^{+-} = 1\ ;\ \omega^+_{ab} = 0.$$

$$\boxed{\mathcal{M}^{ab} = c\, z^2 \eta^{ab} + z A^{ab} + B^{ab} + \dots}$$

Unique diag.

because ω_{ab} is antisymm.

ϵ_1 / ϵ_2	$-$	$+$	T
$-$	1	1	1
$+$	z^4	1	z^2
T1	z^2	1	z^2
T2	z^2	1	z

Pure Gravity

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{4} g^{\mu\nu} g^{\alpha\rho} g^{\beta\sigma} \nabla_\mu h_{\alpha\beta} \nabla_\nu h_{\rho\sigma} - \frac{1}{2} h_{\alpha\beta} h_{\mu\nu} R^{\beta\mu\alpha\nu}\right]$$

(Bern-Grant trick used).

L, R $h_{\alpha\beta}$ indices separately contracted.

$\Rightarrow \underline{\underline{2}}$ copies of spin L.I.

$$h_{\mu\nu} = e^{a}_{\mu} \tilde{e}^{\tilde{a}}_{\nu} h_{a\tilde{a}}$$

Gauge $\omega^{+}_{ab} = \tilde{\omega}^{+}_{\tilde{a}\tilde{b}} = 0$

$$g^{++} = g^{+i} = 0, \; g^{+-} = 1.$$

$$\mathcal{M}^{a\tilde{a}b\tilde{b}} =$$

$$c\, z^2 \eta^{ab} \tilde{\eta}^{\tilde{a}\tilde{b}}$$ ← Unique diag.

$$+\ z\left(\eta^{ab} \tilde{A}^{\tilde{a}\tilde{b}} + \tilde{\eta}^{\tilde{a}\tilde{b}} A^{ab}\right)$$ ← single $\omega, \tilde{\omega}$ ins.

$$+\ \mathcal{A}^{ab\tilde{a}\tilde{b}}$$ ← $\omega \times \tilde{\omega}$, R

$$+\ \eta^{ab} \tilde{B}^{\tilde{a}\tilde{b}} + \tilde{\eta}^{\tilde{a}\tilde{b}} B^{ab}$$ $\vdots$

$$+\ \frac{1}{z} C^{ab\tilde{a}\tilde{b}}$$

$$+ \ldots$$

Precisely z+ tensor structure of $\mathcal{M}^{ab}_{Gauge} \times \mathcal{M}^{\tilde{a}\tilde{b}}_{Gauge}$!

Yang Mills

$\mathcal{A}_\mu = A_\mu + a_\mu$. Usual a_μ G Fixing

$\mathcal{L} = -\frac{1}{4} \operatorname{tr} D_\mu a_a D^\mu a_b \eta^{ab}$

$+ \frac{i}{2} \operatorname{tr} [a_a, a_b] F^{ab}$

$z \to \infty$: "spin Lorentz invariance".

$$\boxed{M^{ab} = (c z + \cdots) \eta^{ab} + A^{ab} + \frac{1}{z} B^{ab} + \cdots}$$

also Ward id. : $P_j(z)_a M^{ab} \epsilon_b = 0$

$\Rightarrow q_a M^{ab} \epsilon_b = -\frac{1}{z} P_{ja} M^{ab} \epsilon_b$

Interesting that in order to expose $M_{Grav} \sim M_{Gauge} \times M_{Gauge}$, we had to add a lot of ~~extra~~ redundancy + Gauge fix it judiciously. c.f. fascinating recent Bern et. al. paper

$$M_{Gauge} \sim \sum \frac{c_i n_i}{D_i}$$

$$M_{Grav} \sim \sum \frac{\tilde{n}_i n_i}{D_i}$$

But redundant

ϵ_1/ϵ_2	$--$	$-+$	$++$	$-T$	$+T$	TT
$--$	$\frac{1}{z^2}$	$\frac{1}{z^2}$	$\frac{1}{z^2}$	$\frac{1}{z^2}$	$\frac{1}{z^2}$	$\frac{1}{z^2}$
$-+$	z^2	z^2	$\frac{1}{z^2}$	z^2	1	1
$++$	z^6	z^2	$\frac{1}{z^2}$	z^4	1	z^2
$-T$	1	1	$\frac{1}{z^2}$	1	1 or $\frac{1}{z}$	1 or $\frac{1}{z}$
$+T$	z^4	z^2	1	z^4 or z^3	1	z^2 or z
TT	z^2	1	$1/z^2$	z^2 or z	1 or $1/z$	z^2, z or 1

BCFW applies to Gravity Ampl. in any D [redacted].

More generally [Cheung, Kaplan]

For Gauge + anything

$$\mathcal{M}^{-\ \text{anything}} \longrightarrow \frac{1}{z}$$

For Gravity + anything

$$\mathcal{M}^{--,\ \text{anything}} \longrightarrow \frac{1}{z^2}.$$

Can BCF any YM, Grav. line.

Many remarkable
properties in
pert. theory
+ beyond...

Why? We've seen that amps of $s \geq 1$ particles are much nicer than scalars. But: fundamentally <u>discrete</u> objects $M^{++--+++\cdots-}$

	YM	Grav
$+2$		1
$+\frac{3}{2}$		
$+1$	1	
$+\frac{1}{2}$		
0		
$-\frac{1}{2}$		
-1	1	
$-\frac{3}{2}$		
-2		1

Why? We've seen that amps of $s \geq 1$ particles are much nicer than scalars. But: fundamentally <u>discrete</u> objects $M^{++--+++\dots-}$

	YM	$\mathcal{N}=4$		Grav	$\mathcal{N}=8$	
$+2$				1	1	
$+\frac{3}{2}$					8	
$+1$	1	1			28	All related by $\mathcal{N}=8$
$+\frac{1}{2}$		4			$\vdots$	
0		6	All related by $\mathcal{N}=4$			
$-\frac{1}{2}$		4				
-1	1	1			28	
$-\frac{3}{2}$					8	
-2				1	1	

Label states of mom. $(\lambda, \tilde{\lambda})$ [Nair]

$$|\eta\rangle = e^{(\tilde{\omega}\tilde{Q}^I)\eta_I} |-\rangle \quad ([\omega\lambda]=1)$$

or

$$|\tilde{\eta}\rangle = e^{(\omega Q_I)\tilde{\eta}^I} |+\rangle \quad (\langle\omega\lambda\rangle=1)$$

$$Q_{\alpha I}|\eta\rangle = \lambda_\alpha \eta_I |\eta\rangle ; \quad \tilde{Q}^{\dot{\alpha} I}|\tilde{\eta}\rangle = \tilde{\lambda}^{\dot{\alpha}} \tilde{\eta}^I |\tilde{\eta}\rangle$$

$$|\eta\rangle \longleftrightarrow |\tilde{\eta}\rangle$$

$$\text{MHV} \longleftrightarrow \overline{\text{MHV}}$$ Complementary

$$|\tilde{\eta}\rangle = \int d^8\eta \, e^{\tilde{\eta}\eta} |\eta\rangle$$

What is simplest QFT?

$$\begin{array}{ccccc}\phi^4 & \longrightarrow & \text{YM} & \longrightarrow & \text{Gravity}\\ & & \downarrow & & \downarrow \\ & & \mathcal{N}=4\text{ SYM} & & \mathcal{N}=8\text{ SUGRA}\end{array}$$

What is simplest QFT?

$\phi^4 \longrightarrow \text{YM} \longrightarrow \text{Gravity}$

$\mathcal{N}=4$ SYM

$\mathcal{N}=8$ SUGRA

The simplest QFT??

Largest "spin Lorentz" invariance + ...

Finite?? (Bern, Dixon, Kosower, ..)

* SUSY:

$$e^{\tilde{Q}\tilde{\xi}} |\eta\rangle = |\eta + [\tilde{\lambda}\tilde{\xi}]\rangle$$

$$e^{Q\xi} |\eta\rangle = e^{\langle\lambda\xi\rangle\eta} |\eta\rangle$$

* Amplitudes

$$\mathcal{M}[\lambda_1, \tilde{\lambda}_1, \eta_1; \ldots; \lambda_n, \tilde{\lambda}_n, \eta_n]$$

completely smooth !

* SUSY

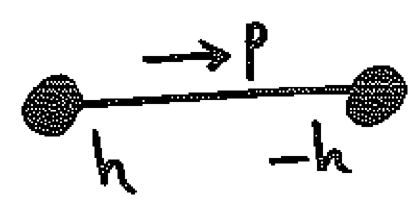
$$M(\eta_1, \ldots, \eta_n) = e^{\sum_i \langle\lambda_i\xi\rangle\eta_i} M(\eta_i + [\tilde{\lambda}_i\xi])$$

Note: $\overline{Q}^{I}_{\dot\alpha}$ SUSY can be used to translate 2 η_I's to the origin:

Choose $$\bar\xi_{\dot\alpha I} = \frac{\bar\lambda^1_{\dot\alpha}\eta^2_I - \bar\lambda^2_{\dot\alpha}\eta^1_I}{[12]}$$

sends $\eta_{1,2} \longrightarrow 0$.

Also supermultiplet sums:



$$\sum_{\substack{\text{multip}\\ h}} F(\lambda,\tilde\lambda,h)\,G(-\lambda,\tilde\lambda,-h)$$

$$= \int d^8\eta\, F(\lambda,\tilde\lambda,\eta)\,G(-\lambda,\tilde\lambda,\eta)$$

SMOOTH: $\Sigma \to \int$

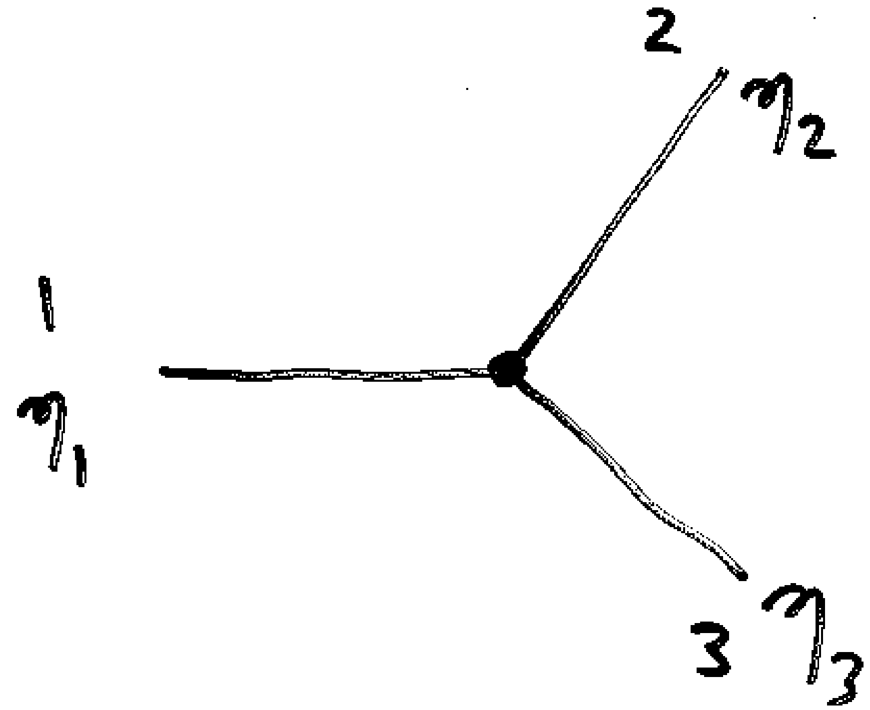
Reproduce Classic SUSY facts:

$$* \; M^{++\cdots+} = \int d^8\eta_1 \cdots d^8\eta_n \, M(\eta_1, \ldots, \eta_n)$$

$$= \int d^8\eta_1 \, d^8\eta_2 \cdots d^8\eta_n \, M(0, \ldots, \eta_n) = 0$$

$$* \; M^{++\cdots+-} = \int d^8\eta_1 \cdots d^8\eta_{n-1} \, d^8\bar{\eta}_n \, M(\eta_1, \ldots, \eta_n)$$

$$= \int d^8\eta_1 \, d^8\eta_2 \cdots d^8\bar{\eta}_n \, e^{(A\eta_1 + B\eta_2)\bar{\eta}_n} M(0, 0, \ldots) = 0$$



$$M(\eta_1,\eta_2,\eta_3) = \frac{\delta^8\left(\sum_i \lambda_i \eta_i\right)}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}$$

$$+ \int d^8\bar{\eta}\, e^{\eta\bar{\eta}}\, \frac{\delta^8\left(\sum_i \tilde{\lambda}_i \bar{\eta}_i\right)}{[12][23][31]}$$

BCFW (slightly extended)

for any amplitudes:

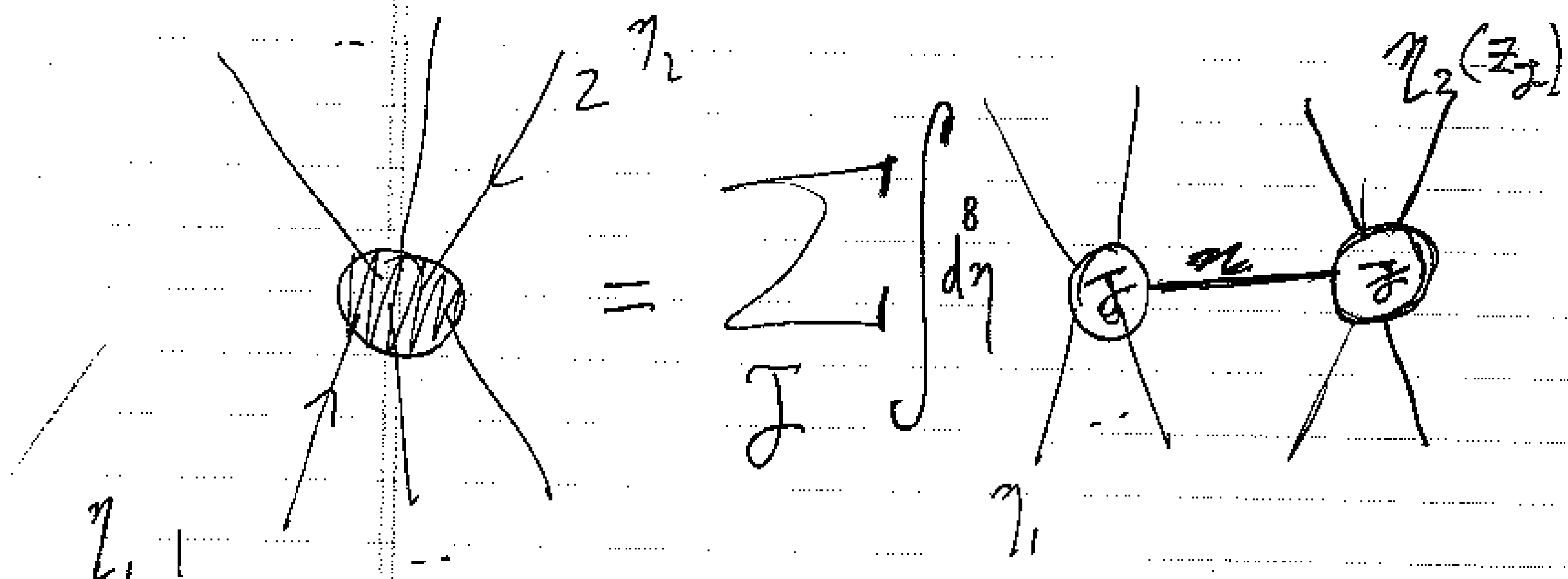
$$M(\eta_1, \eta_2, \eta_i)$$

$\downarrow$ $\bar{Q}$ translate

$$M(0, 0, \eta_i')$$

$\longleftarrow$ Can be BCF'd since $(- - \text{anything}) \to 0$ as $z \to \infty$.

$\downarrow$ $\bar{Q}$ translate back



$$\boxed{\eta_2(z) = \eta_2 - z\eta_1}$$

$$M\left(\{\eta_1, \lambda_1(z), \tilde{\lambda}_1\}, \{\eta_2, \lambda_2, \tilde{\lambda}_2(z)\}, \eta_i\right)$$

$$\not\longrightarrow 0 \quad \text{as } z \to \infty$$

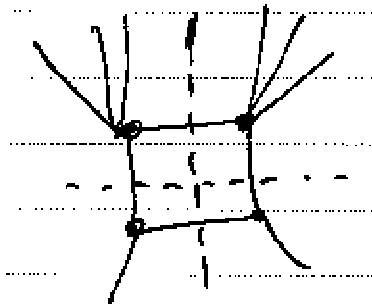
but

$$M\left(\{\eta_1, \lambda_1(z), \tilde{\lambda}_1(z)\}, \{\eta_2(z), \lambda_2, \tilde{\lambda}_2(z)\}, \eta_i\right)$$

$$\longrightarrow \frac{1}{z^2} \quad \text{as } z \longrightarrow \infty.$$

Recursion from IR equation:

. Initial motivation of BCF:

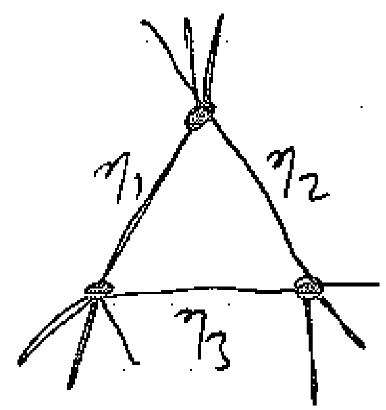
M^{tree} =  ← Some linear comb. of

$$M^{IR}_{loop} = \left(\sum_i -\frac{s_{i,i+1}^{\epsilon}}{\epsilon^2}\right) M_{tree}$$

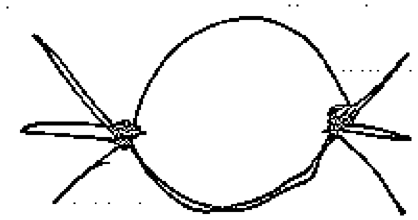
$||$

Our (extended) BCFW recursion relation.

This formalism also makes it very easy to prove the absence of triangles + bubbles in $\mathcal{N}=8$ and $\mathcal{N}=4$ [Very different recent pf. by Bohr + Vanhove].

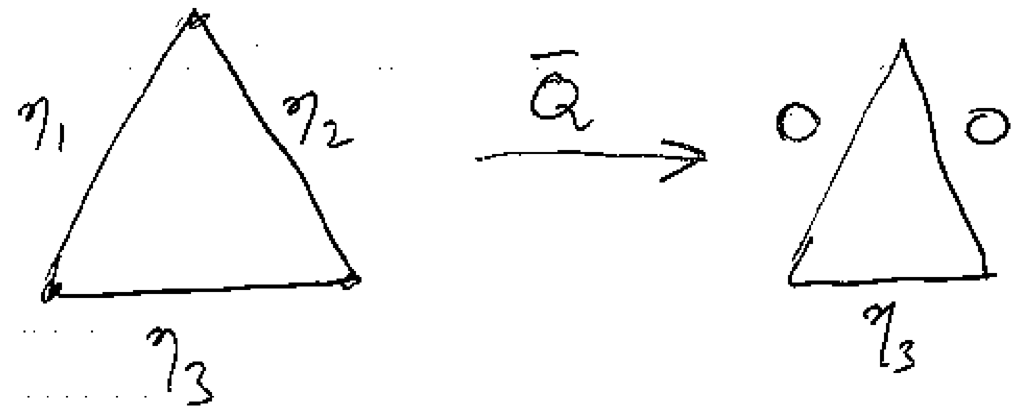


Triangle $\propto \int d^8\eta_1\, d^8\eta_2\, d^8\eta_3$
$M_1(\eta_1,\eta_2)\, M_2(\eta_2,\eta_3)$
$M_3(\eta_3,\eta_1)$ [∞ momentum]



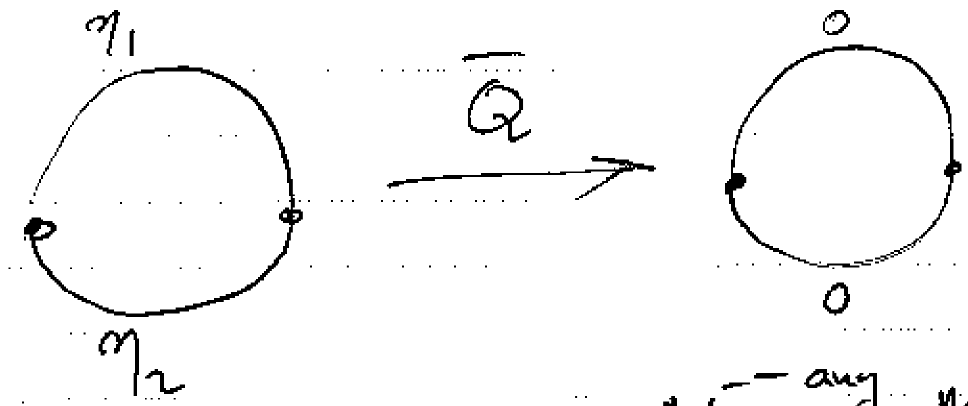
Bubble $\propto \int d^8\eta_1\, d^8\eta_2\, M_1(\eta_1,\eta_2)$
$M_2(\eta_1,\eta_2)$
[∞ momentum]

[Forde, also ms in progress].



$$\longrightarrow M^{--\text{any}}\, M^{-\text{any}'}\, M^{-\text{any}''}$$

$$\longrightarrow \frac{1}{z^6}$$ @ infinite momentum.



$$\longrightarrow M^{--\text{any}}\, M^{--\text{any}'}$$

$$\longrightarrow \frac{1}{z^4}$$ @ infinite momentum.

[Relations among box coeff. in $\mathcal{N}=8$ absent in $\mathcal{N}=4$; better UV behavior]

Fundamental Q:

* What is the boundary holographic theory underlying this rich structure? Perhaps $D=4$ shouldn't play a special role, given BCFW holds for any D
* The twistor string made CSW rules obvious, what makes BCFW "obvious"?

The excellent behavior of tree amplitudes very naturally has an imprint @ 1-loop; explains + proves in general the "no triangle hypothesis" for 1-loop $\mathcal{N}=8$ amplitudes [Bern et. al.]