Generalized geometric compactifications

Mariana Graña
CEA / Saclay
France

12 Marcel Grossmann meeting - Paris - July 2009

Type II sugra on $\mathcal{M}_{10} = \mathcal{M}_4 \times \mathcal{M}_6$

Susy : $\mathcal{M}_6$ is CY

→ moduli problem

Turn on fluxes : $\mathcal{M}_6$ ~~CY~~ Fluxes ⟷ Geometry

$$\delta_\epsilon \Psi_M = \underbrace{\nabla_M}_{\substack{\text{connection} \\ \text{der. of metric}}} \epsilon + \underbrace{H_M \epsilon + F_M}_{\substack{\text{fluxes} \\ \text{der. of gauge fields}}} \epsilon$$

Can we combine metric and gauge fields into some "geometry" ?

Generalized geometry → metric and gauge fields combined into algebraic structures on larger spaces

Conventional 6D geometry: TM

SUSY: $\delta_\epsilon \Psi_M = \nabla_M \epsilon + H_M \epsilon + F_M \epsilon$ $\qquad \epsilon = \theta_+ \otimes \eta_+ + c.c.$

Off-shell SUSY: ∃ nowhere vanishing η

Stabilizer group G = {g ∈ Spin(6): g · η = η }

- G = SU(3)⊂SU(4) ≃ Spin(6), η ∈ **4** $\longrightarrow$ **3 + 1**

SO(6) $\longrightarrow$ SU(3)

2-form 15 $\longrightarrow$ $8 + 3 + \bar{3} + 1 \leftarrow J$ $\qquad$ J : Sp(6,R) structure

3-form 20 $\longrightarrow$ $6 + \bar{6} + 3 + \bar{3} + 1 + 1 \leftarrow \Omega$ $\qquad$ Ω:SL(3,C) structure

- Equivalent to nowhere vanishing Ω and J: $J \wedge \Omega = 0$

$\cap$

SU(3)

structure group of TM

- g=g(J, Ω)

Generalized complex geometry : TM⊕T*M , 12D

Hitchin 02
Gualtieri 04

Structure on T⊕T*: defined by O(6,6) pure spinors

$$\Phi^o_\pm = \eta^1_+ \otimes \eta^{2\,\dagger}_\pm$$ sum of forms $\longleftrightarrow$ 1-1 correpondance with GACS

$$(\Phi^o_\pm)_{m1\ldots mp} = Tr(\Phi^o_\pm \gamma_{m1\ldots mp}) = \eta^{2\,\dagger}_\pm \gamma_{m1\ldots mp} \eta^1_+$$

$\mathcal{J}$: T⊕T* → T⊕T*
$\mathcal{J}^2 = -1_{12}$

Ex:

- $\eta^1 = \eta^2$
  SU(3) ⊂ O(6)
  - $\Phi^o_- = \Omega_3 \longrightarrow$ complex structure
  - $\Phi^o_+ = e^{iJ} \longrightarrow$ symplectic structure
- $\eta^2 = (v + iv')_m \gamma^m \eta^1$
  SU(2) ⊂ O(6)
  - $\Phi^o_- = (v + iv')e^{ij} \longrightarrow$ 1d complex, 2d symplectic
  - $\Phi^o_+ = \Omega_2 \wedge e^{iv\wedge v'} \longrightarrow$ 2d complex, 1d symplectic

define metric

generalized almost complex structures SU(3,3) ⊂ O(6,6)

One application

- Recast on shell SUSY conditions into differential properties of the pure spinors

Generalized complex geometry : TM⊕T*M , 12D

Hitchin 02
Gualtieri 04

Structure on T⊕T*: defined by O(6,6) pure spinors

$$\Phi_\pm = e^B \eta^1_+ \otimes \eta^{2\,\dagger}_\pm = e^B \Phi^0_\pm \in S^\pm(E) \longleftrightarrow$$

1-1 correpondance with GACS

$$(\Phi^o_\pm)_{m1\dots mp} = Tr(\Phi^o_\pm \gamma_{m1\dots mp})$$
$$= \eta^{2\,\dagger}_\pm \gamma_{m1\dots mp} \eta^1_+$$

$\mathcal{J}$: T⊕T* → T⊕T*

$\mathcal{J}^2 = -1_{12}$

Ex:

- $\eta^1 = \eta^2$

SU(3) ⊂ O(6)

$\Phi^o_- = \Omega_3$ → complex structure

$\Phi^o_+ = e^{iJ}$ → symplectic structure

- $\eta^2 = (v + iv')_m \gamma^m \eta^1$

SU(2) ⊂ O(6)

$\Phi^o_- = (v + iv')e^{ij}$ → 1d complex, 2d symplectic

$\Phi^o_+ = \Omega_2 \wedge e^{iv\wedge v'}$ → 2d complex, 1d symplectic

define metric

generalized almost complex structures SU(3,3) ⊂ O(6,6)

$$0 \to T^*M \to E \xrightarrow{\Pi} TM \to 0$$

$$B_\alpha = B_\beta + dA_{\alpha\beta}$$

encodes the twisting by H_3

$$x_\alpha + \xi_\alpha = x_\beta + \left(\xi_\beta + i_{x_\beta} dA_{\alpha\beta}\right)$$

One application

- Recast on shell SUSY conditions into differential properties of the pure spinors

Generalized complex geometry : TM⊕T*M , 12D

Hitchin 02
Gualtieri 04

Structure on T⊕T*: defined by O(6,6) pure spinors

$$\Phi_\pm = e^B \eta^1_+ \otimes \eta^{2\,\dagger}_\pm = e^B \Phi^0_\pm \in S^\pm(E) \longleftrightarrow$$ 1-1 correpondance with GACS

$$(\Phi^o_\pm)_{m1\ldots mp} = Tr(\Phi^o_\pm \gamma_{m1\ldots mp}) = \eta^{2\,\dagger}_\pm \gamma_{m1\ldots mp} \eta^1_+$$

$\mathcal{J}$: T⊕T* → T⊕T*
$\mathcal{J}^2 = -1_{12}$

Ex:

- $\eta^1 = \eta^2$

SU(3) ⊂ O(6)

$\Phi_- = e^B \Omega_3 \longrightarrow$ complex structure

$\Phi_+ = e^B e^{iJ} \longrightarrow$ symplectic structure

- $\eta^2 = (v + iv')_m \gamma^m \eta^1$

SU(2) ⊂ O(6)

$\Phi_- = e^B (v + iv') e^{ij} \longrightarrow$ 1d complex, 2d symplectic

$\Phi_+ = e^B \Omega_2 \wedge e^{iv\wedge v'} \longrightarrow$ 2d complex, 1d symplectic

generalized almost complex structures SU(3,3) ⊂ O(6,6)

define metric and B-field

$$0 \to T^*M \to E \xrightarrow{\Pi} TM \to 0$$

$$B_\alpha = B_\beta + dA_{\alpha\beta}$$ encodes the twisting by H_3

$$x_\alpha + \xi_\alpha = x_\beta + \left(\xi_\beta + i_{x_\beta} dA_{\alpha\beta}\right)$$

One application

- Recast on shell SUSY conditions into differential properties of the pure spinors

N=1 SUSY

M.G, Minasian, Petrini, Tomasiello 05

IIA

$$d\Phi_+ = 0$$

$$d\Phi_- = dA \wedge \bar{\Phi}_- + i * F_A$$

$\downarrow$

Φ_+ is closed

IIB

$$d\Phi_- = 0$$

$$d\Phi_+ = dA \wedge \bar{\Phi}_+ + i * F_B$$

$\downarrow$

Φ_- is closed

$\xleftrightarrow[\text{mirror symmetry}]{\text{generalized}}$

CY: $d\Phi_+ = 0$ and $d\Phi_- = 0$

GCY: $d\Phi_+ = 0$ or $d\Phi_- = 0$

Susy vacua are all generalized Calabi-Yau's !

$$\Phi_\pm = e^B \overbrace{e^{2A-\phi} \eta_+^1 \otimes \eta_\pm^{2\,\dagger}}^{\Phi^0}$$

$$\underbrace{(d - H\wedge)}_{d_H}\Phi_\pm = e^B d\Phi^0_\pm$$

$$F_A = e^{3A}(F_0 + F_2 + F_4 + F_6)$$

$$F_B = e^{3A}(F_1 + F_3 + F_5)$$

Metric and B-field are geometrized

What about RR fields? → Extend the bundle

IIA

NS5-brane charge

$TM \xrightarrow{\text{GCG}} TM \oplus T^*M \xrightarrow{\text{EGG}} TM \oplus T^*M \oplus S^+ \oplus \wedge^5 TM \oplus \wedge^5 T^*M$

p_m w^m

D-brane charges

KKmonop. charges

Hull 07
Pires Pacheco , Waldram 08

O(6) → O(6,6) → E_7

Exceptional Generalized geometry : E , 56D

56 = (**2**,**12**) + (**1**,**32**)

$$J \rightarrow \Phi_+ = e^B e^{iJ} \rightarrow \lambda = e^{C^-} e^B e^{B_6} (0, \Phi_+)$$

Sp(6,R) ⊂O(6)

SU(3,3) ⊂O(6,6)

E_6 ⊂ E_7

MG, Louis, Slm, Waldram 09

133 = (**3**,**1**) + (**1**,**66**) + (**2**,**32**)

$$\Omega \rightarrow \Phi_- = e^B \Omega \rightarrow K = e^{C^-} e^B e^{B_6} (0, 0, u^i \Phi_-)$$

axion-dilaton

SL(3,C) ⊂O(6)

SU(3,3) ⊂O(6,6)

SO(12) ⊂ E_7

E_7 → SL(2,R) x O(6,6)

Conclusions

- GCG, EGG are natural language for description of flux backgrounds

O(6) → O(6,6) → E_7

Geometrized B-field — Geometrized B and C-fields

- O(6,6) : - Relevant structures containing metric and B-field dof: ϕ_+, ϕ_-
  - Kähler potential : quartic invariant of O(6,6) (Hitchin functional)
  - N=1 SUSY: $d\phi_+=0$, $d\phi_-=i*F$ (ϕ_+ is integrable)

- E_7 : - Relevant structures containing all dof: λ, K
  - Special Kähler and quaternionic moduli spaces: orbits of λ, K
  - Kähler and hyperK potentials : invariants of E_7 specialized to orbits λ, K
  - Gravitino mass matrix (or Killing prepotentials) $P_a = S(\lambda, D \cdot K_a)$
  - N=1 vacua equations : λ, K'_3 integrable structures