Spinons and gauge degrees of freedom in spin liquids

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“Two-dimensional quantum antiferromagnets ”
Introduction

What is fractionalization?

Excitations with quantum numbers which are fraction of the elementary degrees of freedom.

Most famous example: $q = e/3$ in FQHE.

In magnetic systems:

An $s = \frac{1}{2}$ spinon (charge neutral) is a “fraction” of an electron.

(or a fraction of a $\Delta S^z = 1$ spin flip)

Very natural in 1D (domain wall or soliton)

But more complex in higher dimension…

This talk

Attempt to explain simply some (not so recent) theoretical approaches to confinement and deconfinement in magnets

Natural language: gauge theory.

Mention some recent spin (or related) models realizing deconfined phases
Spin-$\frac{1}{2}$ antiferromagnetic Heisenberg models

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + \cdots$$

Many possible phases characterized by different (spontaneously) broken symmetries

Square, triangular and hexagonal lattices:
- Néels states
- Spontaneously broken SU(2) symmetry
- Gapless spin waves ($\Delta S^z=1$)

S=0 plaquettes or dimer crystal.
- Spontaneous breakdown of some lattice symmetries
- Gapped magnons ($\Delta S=1$)

Even number of spins per unit cell:
- Possibility of no broken symmetry
- Gapped magnons ($\Delta S=1$)

So far no spinon…
Confinement/deconfinement in terms of valence-bonds

- **Valence-bond crystal**
  - Energy grows linearly with distance
  - \[ \approx \frac{1}{\sqrt{2}} (|\uparrow\rangle - |\downarrow\rangle) \]
  - One spinon is surrounded by a local reorganization of the (liquid-like) valence-bond background.
  - Confinement

- **Resonating valence-bond liquid**
  - (short-ranged)
  - [P. W. Anderson, 1973; 1987]
  - No broken symmetry
  - Short-ranged correlations only
  - One spinon is surrounded by a local reorganization of the (liquid-like) valence-bond background.
  - (possibility of) Deconfinement

Formalism? Examples?
Gauge theory descriptions of spin models

- Spin model: Local microscopic model
  - No explicit long-ranged interaction.
  - How to understand confinement / deconfinement?

- Gauge theories provide a natural framework...
  - Example: particles on a lattice
    \[ H = \sum_{rr'} t_{rr'} \left[ b_r^+ e^{-iA_{rr'}} b_{r'} + \text{H.c} \right] + \cdots \]
  - Local redundancy:
    \[
    \begin{align*}
    b_r &\rightarrow e^{i\theta(r)} b_r \\
    A_{rr'} &\rightarrow A_{rr'} + \theta(r) - \theta(r')
    \end{align*}
    \]
  - Physical quantum states must be invariant under such gauge transformations
  - Electric field, conjugate to \( A_{rr'} \):
    \[ \mathcal{E}_{rr'} = e^{-iA_{rr'}} \]
  - Local constraint (Gauss law)
  - (isolated) “charged” excitations have a finite energy = deconfinement
  - Only “neutral” excitations = confinement
  - Concrete example for a spin model?
Slave particles and U(1) Gauge symmetry - Schwinger bosons (I)

Example of a slave particle representation of the spins operators

- Schwinger boson representation of SU(2)
  & Heisenberg model with S=1/2

\[
\begin{align*}
S^z &= \frac{1}{2}(b^\dagger_\uparrow b_\uparrow - b^\dagger_\downarrow b_\downarrow) \\
S^+ &= b^\dagger_\uparrow b_\downarrow & S^- &= b^\dagger_\downarrow b_\uparrow \\
\vec{S}^2 &= \frac{1}{2}(\frac{1}{2} + 1) \Rightarrow b^\dagger_\uparrow b_\uparrow + b^\dagger_\downarrow b_\downarrow = 1 & (S = \frac{1}{2})
\end{align*}
\]

- \(b^\dagger_\uparrow\) (or \(b^\dagger_\downarrow\)) creates a spinon

\[
\chi_{\sigma r}^+ = \frac{1}{2}(b^\dagger_{\sigma_\uparrow} b_{\sigma_\downarrow}^+ - b_{\sigma_\downarrow}^+ b_{\sigma_\uparrow})
\]

- a spin singlet

\[
\vec{S}_r \cdot \vec{S}_r = \frac{1}{4} - 2\chi_{\sigma r}^+ \chi_{\sigma r}
\]

\[\theta(r)\) arbitrary angle at each site:

\[
\begin{cases}
 b_{\sigma \uparrow, r} \rightarrow e^{i\theta(r)} b_{\sigma r} \\
 \vec{S}_r \rightarrow \vec{S}_r
\end{cases}
\]

local U(1) redundancy

Review on slave particle approaches to the t-J model:
P. A. Lee, N. Nagaosa, X. G. Wen
cond-mat/0410445
Slave particles and U(1) Gauge symmetry - Schwinger bosons (II)

Formulation in terms of spinons interacting with bond fields $Q_{rr'}$

$$Z = \text{Tr}[\exp(-\beta H)]$$

$$S_{\text{eff}} = \int d\tau \sum_{rr'} \left\{ \frac{1}{2} \left| Q_{rr'} \right|^2 - Q_{rr'}^+ \left( b_{r\uparrow} b_{r'\downarrow} - b_{r\downarrow} b_{r'\uparrow} \right) + \text{h.c} \right\} + \cdots$$

Bipartite lattice:

$$(-1)^r = \begin{cases} +1 & r \in A \\ -1 & r \in B \end{cases}$$

$$b_{r\uparrow} \rightarrow e^{i\theta(r)} b_{r\uparrow} \Rightarrow 'electric' \text{ charge } +1$$

$$b_{r\downarrow} \rightarrow e^{-i\theta(r)} b_{r\downarrow} \Rightarrow 'electric' \text{ charge } -1$$

$$\arg(Q_{rr'}) = A_{rr'} \rightarrow A_{rr'} + \theta(r) - \theta(r') \Rightarrow \text{compact } U(1) \text{ gauge field}$$

Geometrical interpretation of the gauge field:

Solid angle defined by 2 spins and a reference $\vec{n}_0$

Gauge transformation $\Rightarrow$ change of reference direction

gauge flux $\sim$ non-collinearity of the spins

What is special with compact gauge fields?
Monopoles in U(1) gauge theory in D=2+1

- Analogy with vortices in the 2D classical O(2) model

\[ E = \sum_{\langle ij \rangle} \left[ 1 - \cos(\theta_i - \theta_j) \right] \]

\[ \theta_i \in [-\pi, \pi] \]

- Magnetic monopoles in a U(1) gauge theory

\[ S = \sum_{\langle ijkl \rangle} \left[ 1 - \cos(A_{ij} + A_{jk} + A_{kl} + A_{li}) \right] \]

2\pi flux tube

\[ \langle \Delta \theta \rangle \approx 2\pi \]

Energy \( \approx \log(L) \)

Space-time event = instanton (hedgehog for the spins)

[Polyakov 1975; 1977; 1987; Fradkin & Susskind 1979]

Finite action

\[ S \approx O(1) \]

\[ \sum_r B(r,t_2) = \varphi_0 + 2\pi \]

\[ \sum_r B(r,t_1) = \varphi_0 \]

Consequences?
Phases of U(1) Gauge theories in D=2+1 dimensions

What is known about such U(1) gauge theories?

- Monopoles proliferate [Polyakov 1975; 1977; 1987]
  - Confinement.
  - The spinons are glued in pairs by the strong gauge-field fluctuations and are not physical excitations.

For a S=½ system, monopoles have non-trivial Berry phases
[Haldane PRL 1988; Read & Sachdev PRL 1989]
- Analogy with the topological term which makes the difference between 2S odd and even in spin chains [Haldane 1983].
  - The confined phase is a valence-bond crystal

- Deconfinement possible in presence of gapless matter fields
  - So called U(1) spin liquid [Affleck & Marston 1988, …, Hermele et al. PRB 2004]
  - In presence of a charge-2 field the U(1) ‘symmetry’ can be broken down to Z₂, leading to deconfinement [Fradkin & Shenker, PRD 1979]

How can this happen in a spin system?
From a U(1) to $\mathbb{Z}_2$ gauge theory

$$S_{\text{eff}} = \int d\tau \sum_{rr'} \left\{ \frac{1}{2} \frac{|Q_{rr'}|^2}{J_{rr'}} - \left( \frac{Q_{rr'}^+ (b_{\uparrow r} b_{\downarrow r'} - b_{\downarrow r} b_{\uparrow r'}) + \text{h.c}}{\chi_{rr'}} \right) + \cdots \right\}$$

$Q_{ab} \mapsto e^{i(\theta(a) - \theta(b))} Q_{ab} \Rightarrow$ 'electric' charge 0

$Q_{aa'} \mapsto e^{i(\theta(a) + \theta(a'))} Q_{aa'} \Rightarrow$ 'electric' charge 2

Non-collinear spin-spin correlations

$\Rightarrow$ Some bond variables $Q_{aa'}$ connecting 2 sites on the same sublattice may acquire a finite expectation value.

$$Q_{aa'} \mapsto Q_{aa'} e^{2i\theta} \langle Q_{aa'} \rangle \neq 0 \Rightarrow U(1) \text{ redundancy broken to } \mathbb{Z}_2 : \theta \in \{0, \pi\}$$

(Condensation of a charged particle $\Rightarrow$ Anderson-Higgs mechanism, Meissner effect)

$\mathbb{Z}_2$ gauge theories $[A_{rr} = 0 \text{ or } \pi]$ do have a deconfined phase 😊

A concrete example?
Solvable dimer (toy) model realizing a $\mathbb{Z}_2$ liquid (I)

- Arrow representation of dimer coverings on the kagome lattice

Constraint
Number of incoming arrows must be even on every triangle

- Hamiltonian
$\tau^z(i) = \text{Flips the arrow } i$

$$H = -\sum_h \prod_{i=1}^{6} \tau_i^z$$

Where is the $\mathbb{Z}_2$ gauge theory?
Solvable dimer (toy) model realizing a $\mathbb{Z}_2$ liquid (II)

- Arrow = $\mathbb{Z}_2$ Gauge field

Gauge field

$$\tau^z(i) = \text{Flips the arrow } i = e^{iA_{rr}},$$

Electric field

$$\tau^x(i) = \begin{cases} +1 & \text{If the arrow } i \text{ is the same as in some reference configuration} \\ -1 & \text{Otherwise} \end{cases}$$

- Hamiltonian = magnetic energy

$$H = -\sum_h \prod_{i=1}^{6} \tau^z_i - \Gamma \sum_i \tau^x_i$$

- Constrain= Gauss law

$$\prod_{i=1}^{3} \tau^x_i = 1 \iff \text{div } \vec{E} = 0$$

- Ground-state: uniform dimer liquid

$$\forall h \ B(h)\langle 0 \rangle = 0$$

Not that trivial in terms of the original dimers…
What is a deconfined $Z_2$ state?

- No broken symmetry
  - No long-ranged correlations
  - No local order parameter
  - Short-ranged RVB state: dimer~spin singlet

- Gapped excitations
  = Elementary flux (vortex) of a $Z_2$ gauge theory = visons

- Deconfined fractional excitations (spinons)

- Topological degeneracy – topological order
  [G. X. Wen PRB 1991]

Ground-states are *locally* indistinguishable.
Degeneracy *robust* to any local *perturbation*.

[Disc, Cylinder, Torus]

[Furukawa, GM, Oshikawa (unpublished); GM, Pasquier, Lhuillier & Mila PRB 2005]

Other examples?
Examples of deconfined $Z_2$ Liquids (I)

- **Heisenberg-like models** (SU(2) or larger continuous symmetry)
  - Large-$N$ frustrated antiferromagnets with Sp(N) symmetry [Read & Sachdev PRL 1991]
  - Multiple-spin exchange model on the triangular lattice?
    - Exact diagonalization study [GM, Lhuillier, Bernu, Waldtmann PRB 1999]
  - $J_1$-$J_2$ model on the honeycomb lattice? [Fouet et al. PRB 2003]
  - J_1-J_2 model on the square lattice? [Capriotti et al. PRL 2001]
  - Perturbed Klein models [S. Fujimoto PRB 2005; Raman, Moessner & Sondhi PRB 2005]
  - CsCuCl_3: Spinon-like continuum observed with inelastic neutron scattering.
    - [Coldea, Tennant et al. PRL 2001; PRB 2003]

- **Ising-like models**
  - Ising like model with multiple-spin interactions.
    - [Kitaev, cond-mat 1997], [Nayak & Shtengel PRB 2001] [X. G. Wen PRL 2003]
  - $J_1$-$J_2$-$J_3$ Heisenberg model on the kagome lattice with easy axis (Ising) anisotropy:
    - [Balents, Fisher & Girvin PRB 2002; D. N. Sheng and Balents 2004]
Examples of deconfined $Z_2$ Liquids (II)

- **Quantum dimer models**
  - Effective description of singlet valence-bond dynamics
    - Triangular lattice
      \[ H = -J \sum \left| \begin{array}{l} \text{\ding{42}} \text{\ding{43}} \text{\ding{43}} \end{array} \right| + \left| \begin{array}{l} \text{\ding{43}} \text{\ding{42}} \text{\ding{42}} \end{array} \right| + V \sum \left| \begin{array}{l} \text{\ding{43}} \text{\ding{43}} \text{\ding{42}} \end{array} \right| + \left| \begin{array}{l} \text{\ding{43}} \text{\ding{42}} \text{\ding{42}} \end{array} \right| \]
    - [Moessner & Sondhi PRL (2001)]
    - Kagome lattice
      - Completely solvable! [GM, Serban & Pasquier PRL (2002)]
    - 3D non-bipartite lattices (fcc, …)
      - [Moessner & Sondhi PRB (2003)]

- **Other**
  - Josephson junction arrays
    - [Ioffe et al.; Douçot, Fegeil’man & Ioffe PRL (2003)]
  - Bose-Hubbard models
    - [Senthil & Motrunich PRB (2002); PRL (2002)]
  - Classification with Projective Symmetry Groups [X. G. Wen PRB 2002]
Deconfined phase of a U(1) Gauge theory in D=3+1 dimensions

- « Coulomb phase »
  - A phase without isolated magnetic monopoles
    Analogy with the 2D XY model
    at low temperature where vortices are bound in pairs.

- Similar to conventional QED:
  - Deconfined spinons (“electric” charges) with 1/r interaction
  - Gapped “magnetic” monopoles (also with 1/r interaction)
  - Gapless transverse excitation,
    = “photon” with linear dispersion relation \( \omega(k) = c|k| \)
    BUT NO SPONTANEOUSLY BROKEN SYMMETRY!
  - Algebraic correlations.

- Some microscopic models
  - Bose-Hubbard models
    [Motrunich & Senthil PRL (2002); PRB (2005); Wen PRB (2003)]
  - Quantum dimer model on the cubic lattice
    [Huse et al. PRL 2003; Moessner & Sondhi PRB (2003)]
  - S=1/2 model on the pyrochlore lattice
    with strong Ising anisotropy
    [Hermele, Fisher & Balents PRB (2004)]
  - SU(2) spin models [Raman, Moessner & Sondhi PRB 2005]
Conclusions

- Fractionalization in frustrated magnets exists!
  - Several microscopic models are now available
  - No clear experimental evidence in D>1 so far…

- U(1) and Z_2 gauge theories provide a natural language to describe these “exotic” phases.
  - Relation to topological order [see also Oshikawa & Senthil cond-mat/0506008]
  - Gauge excitations (and even fermionic excitations)

- Recent developments:
  - “String-net condensation” picture [X. G. Wen 2002-2005]
    - to explain the origin of gauge degrees of freedom
  - “Deconfined critical points” [Senthil et al. Science (2004)]
    - New kind of continuous quantum phase transitions between phases with different broken symmetries. Fractional excitations at the critical point.
    - Role of gauge theory description, topological defects.
  - “Application” to topological quantum bits?
What about the kagome spin-$\frac{1}{2}$ AF Heisenberg model?

- Spin liquid with gapless singlet excitations?
  - No magnetic long-ranged order (almost surely)
  - Triplet probably gapped

- Maybe deconfined
  - [Dommange et al. PRB (2003); Läuchli & Poilblanc PRL (2004)]
  - see also [GM, Serban & Pasquier PRB (2003), J. Phys Cond. Mat. (2004)]

- Should be a $\mathbb{Z}_2$ liquid according to Sp(N) approach…
  - [Sachdev PRB 1992]

- Might also be a valence-bond crystal…
Lattice gauge theory

(Abelian)

**Continuum**

\[ H = \frac{1}{2m} \left( \vec{p} - q \vec{A} \right)^2 + \cdots \]

\[ \psi(r) \rightarrow e^{iq\theta(r)}\psi(r) \]

\[ \vec{A} \rightarrow \vec{A} + \vec{\nabla}\theta \]

\[ \vec{B} = \vec{\nabla} \times \vec{A} \]

\[ \bar{E} = -\vec{\nabla}A_0 - \partial_t \vec{A} \]

Gauss law \quad \text{div} \, \bar{E} = \rho

**Lattice**

\[ H = \sum_{rr'} t_{rr'} \left[ b^+_r e^{-iA_{rr'}} b_{rr'}^* + \text{H.c} \right] + \cdots \]

\[ b_r \rightarrow e^{i\theta(r)} b_r \]

\[ A_{rr'} \rightarrow A_{rr'} + \theta(r) - \theta(r') \]

\[ \bar{E}_{rr'} : \text{Electric field, conjugate to } A_{rr'} \]

\[ \left( b^+_r b_r - \sum_{rr'} E_{rr'} \right) \psi = 0 \quad \text{Local constraint} \]

\[ A_{rr'} \in \begin{cases} [-\pi, \pi] \Rightarrow U(1) \text{gauge theory} \\ \{0, \pi\} \Rightarrow Z_2 \text{gauge theory} \end{cases} \]
Quantum dimer models in 3D & Coulomb phase

[Bi-partite lattice: sublattices A & B
Dimer $A \rightarrow B = 5$ units of electric field
No dimer $= -1$ unit of electric field
One dimer per site $\Rightarrow \text{div } \vec{E} = 0$]