Spontaneous symmetry breaking and finite-size spectra of quantum frustrated antiferromagnets

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M. Oshikawa, V. Pasquier, K. Penc, L. Pierre & N. Shannon
Outline

- Exact diagonalizations and quantum numbers
- Spontaneous symmetry breaking
- Discrete SSB: some examples of valence-bond crystals
  - Kagome lattice, expanded kagome lattice, others…
- Continuous SSB: Néel and nematic phases
Exact diagonalizations and quantum numbers

- Quantum lattice model $\Rightarrow$ symmetry group $G$

- $G \Rightarrow$ Irreducible representations (irreps)

Possible “automatic” implementation. Using GAP/GRAPE for example (www.gap-system.org)

- Irrep. $\Rightarrow$ block diagonalization

In most practical cases, irreps are induced by 1d representations of some subgroup.

First fix momentum $\mathbf{k}$, then look for irrep. of the “little” (point) group of $\mathbf{k}$

- Example. Irreps of the first levels of a 36-site kagome Heisenberg model

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[Waldtmann, EPJB 1998 & Sindzingre, unpublished]
Spontaneous symmetry breaking and quantum numbers

- Broken symmetry states ≠ eigenstates

\[ |A\rangle, |B\rangle, |C\rangle \]

- SSB ⇒ quasi-degenerate ground-state in finite-size system, several irreps.

- Group theory ⇒ Predict the irreps associated to a given SSB phase

\[ n_\gamma = \frac{1}{|G|} \sum_{g \in G} \chi_\gamma(g^{-1}) \sum_{i=1}^{d} \langle i | \hat{g} | i \rangle \]

- Basis of the ground-state subspace

- A simple (no fluctuations) trial ground-state can be used to compute \( n_\gamma \)

- Simple for discrete SSB, slightly more involved for continuous ones
Examples of valence-bond crystals (from ED studies)

- \(J_1-J_2-J_3\) model
  Fouet et al. EPJB 2001

- Shastry-Sutherland lattice
  Läuchli, Wessel & Sigrist PRB 2002

- Kagome, 1/3 magnetization plateau
  Cabra et al., PRB 2005

- Heisenberg model & 4-spin “ring” exchange
  Läuchli et al. PRL 2005

- \(J_1-J_2-J_3\) model
  Mambrini et al., cond-mat/0606776

+ others…
Valence bond crystals on kagome?

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Marston & Zeng 1991; Nikolic & Senthil 2003
Degeneracy=12

VBC-1

VBC-2

Syromyatnikov & Maleyev 2004
Degeneracy=4

VBC-3

Budnik & Auerbach 2004
Degeneracy=24
Expanded kagome (star) lattice $J_e - J_t$ model

Similarities with kagome:
- same classical AF degeneracy
- same # of dimer coverings

Isolated dimers - trivial limit
Single singlet ground-state
Large gap to all excitations
“quantum paramagnet”
At $J_t = J_e$, the model is in this phase
[Richter et al. PRB 2004]

Extensive degeneracy
Degenerate perturbation theory
Effective spin-chirality Hamiltonian [Subrahmanyam PRB 1995]

Mean-field approximation
Degenerate solutions = super coverings
[same as in Mila PRL 1998 !]

How is this degeneracy lifted beyond mean-field ?
⇒ We tried ED in the 1st neighbor valence-bond subspace
Expanded kagome lattice $J_e$-$J_t$ model: a VBC when $J_t \gg J_e$?

ED in the 1st neighbor valence-bond subspace
[Zeng & Elser PRB 1995; Mambrini & Mila EPJB 2000]

# of super coverings (mean-field) states

G.S. →

72 sites

108 sites
Néel phases – Anderson tower

Anderson, PR 1953
Bernu et al., PRL 1992, PRB 1994
Lhuillier, cond-mat/0502464

Example with 12 sublattices!
\( J_1 - J_2 \) model on the kagome lattice
J.-C. Domenge, P. Sindzingre, C. Lhuillier and L. Pierre
PRB 2005

\[ E \approx \frac{1}{N} \]
\[ E \approx \frac{1}{N} S(S+1) \]
Spontaneous breakdown of SU(2) symmetry but short-ranged spin-spin correlations. The order parameter involves 2 spins operators

\[ \langle \hat{S}_r \rangle = 0 \]

Spontaneous selection of a (oriented or non oriented) plane

\[ N = 32 \quad \theta / \pi \approx 0.31 \quad (K/J = 1.5) \]

p-nematic
Spin-½ model with 4-spin ring exchange (square lattice)
Läuchli et al., PRL 2005

n-nematic
Shannon, Momoi & Sindzingre, PRL 2006

[See Also: Tsunetsugu & Arikawa JPSJ 2006, Läuchli, Penc & Mila, cond-mat/0605234]
Conclusions

- Exact diagonalizations, when coupled to a full symmetry analysis is a powerful tool specially for D>1 quantum frustrated magnets.
- The spectrum itself is a rich source of information concerning broken symmetries.
- Very useful if only very small systems are numerically available (spin > ½, spin-orbital models, doped magnets, …)