Radiative $p_\perp$-broadening of high energy quarks and gluons in QCD matter

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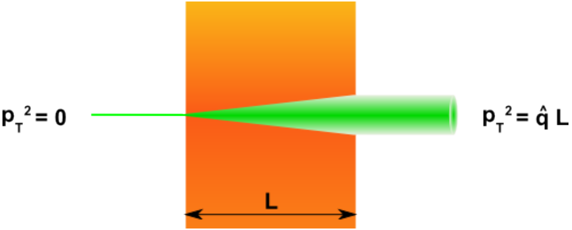
B. Wu, JHEP **1110***, 029 (2011) [arXiv:1102.0388]; T. Liou, A. H. Mueller and B. Wu, arXiv:1304.7677.*

Outline

1. **Motivation**
2. **Radiative $p_\perp$-broadening in single scattering**
3. **Radiative $p_\perp$-broadening in multiple scattering**
4. **Conclusions**

Motivation

- $p_\perp$-broadening of high energy partons
  - $p_\perp$ at leading order



Brownian motion of partons in the transverse plane: $\Rightarrow \hat{q}$.

  - **Radiative energy loss(the LPM effect)**

$$\Delta E \sim \alpha_s N_c \frac{\omega_c}{t_{form}} L \sim \alpha_s N_c \hat{q} L^2 = \alpha_s N_c \langle p_\perp^2 \rangle.$$

Also Edmond and Yacine's talks!

Motivation

- **Confrontation with RHIC's data**

- **Jet quenching**
  - **By fitting high-$p_\perp$ hadron spectra at RHIC**

    $$\hat{\mathbf{q}} \sim \mathbf{5} - \mathbf{15}\ \text{GeV}^2/\text{fm}$$

    Strongly interacting QGP?

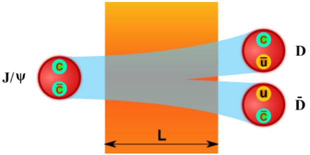
    See, *K. J. Eskola, H. Honkanen, C. A. Salgado and U. A. Wiedemann, Nucl. Phys. A* **747**, *511 (2005) [arXiv:hep-ph/0406319].* See also Jose Guilherme Milhano's talk!

  - **Paradoxical results obtained**

    $$\hat{\mathbf{q}} \sim \mathbf{2}\ \text{GeV}^2/\text{fm}$$

    See, *R. Baier and D. Schiff, JHEP* **0609**, *059 (2006) [arXiv:hep-ph/0605183].*

- **J/Ψ suppression**

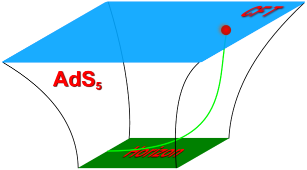


$$\hat{\mathbf{q}}(\tau_0) \lesssim \mathbf{1}\ \text{GeV}^2/\text{fm}$$

See, *B. Wu and B. Q. Ma, Nucl. Phys. A* **848**, *230 (2010) [arXiv:1003.1692 [hep-ph]].*

Motivation

- **What can we do?**
- **In AdS/CFT**

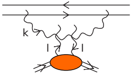


$p_\perp$-broadening is **radiation dominated**!

F. Dominguez, C. Marquet, A. H. Mueller, B. Wu and B. -W. Xiao, Nucl. Phys. A **811**, 197 (2008) [arXiv:0803.3234].

- **Q: $p_\perp$-broadening in pQCD?**
- **A: Double logarithmically enhanced by radiation!**

$\langle p_\perp^2 \rangle_{rad}$ =



$$= \frac{\alpha_s N_c}{8\pi} \hat{q} L \log^2 \left(\frac{L}{l_0}\right)^2$$

BDMPS Parametrically right: *B. Wu, JHEP* **1110**, *029 (2011) [arXiv:1102.0388].*

Zakharov Completely right:*T. Liou, A. H. Mueller and B. Wu, arXiv:1304.7677.*

Definition

- **Basic formula**

  - $p_\perp$**-broadening**

$$\langle p_\perp^2 \rangle \equiv \int d^2p_\perp p_\perp^2 \frac{dN}{d^2p_\perp} = -\left. \nabla^2 S(x_\perp)\right|_{x_\perp=0}$$

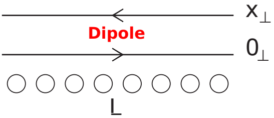
with

$$S(x_\perp) \equiv \int d^2p_\perp e^{ip_\perp \cdot x_\perp} \frac{dN}{d^2p_\perp}.$$

  - **Leading order: multiple scattering** $(L \gg \lambda \gg \frac{1}{m_D})$

$$\langle p_\perp \rangle = \hat{q}L$$

$S(x_\perp) =$

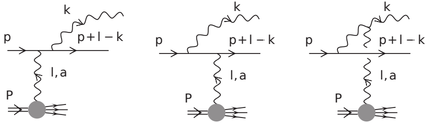


$= e^{-\frac{1}{4}\hat{q}Lx_\perp^2}$

  - **Corrections to $S(x_\perp)$ due to radiation?**

- **Real gluon emission**

$$\frac{dI_r}{d^2 p_{R\perp}} \equiv \left| \quad \right|^2$$



$$= \frac{L}{\lambda_R}\frac{\alpha_s N_c}{\pi^2}\int\frac{d\omega}{\omega}\int d^2k_\perp \frac{l_\perp^2}{k_\perp^2 p_{R\perp}^2}\frac{1}{\sigma_R}\frac{d\sigma_R}{d^2 l_\perp}\bigg|_{\vec{l}_\perp = \vec{k}_\perp + \vec{p}_{R\perp}}$$

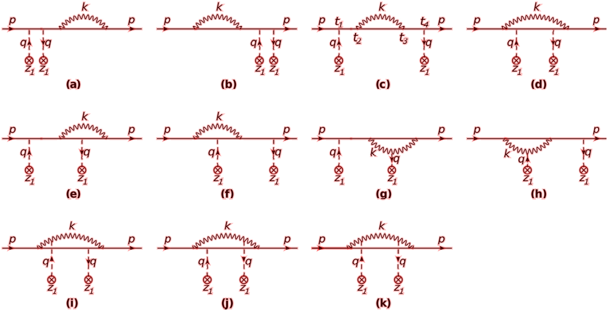
$$\langle p_{R\perp}^2\rangle_r = \underbrace{\frac{\alpha_s N_c L}{\pi}\int\frac{d\omega}{\omega}\int dk_\perp^2\frac{1}{k_\perp^2}}_{\textbf{Double-log}}\underbrace{\frac{1}{\lambda_R}\int d^2 l_\perp \frac{l_\perp^2}{\sigma_R}\frac{d\sigma_R}{d^2 l_\perp}}_{\hat{q}} = \langle k_\perp^2\rangle$$

where (Guy Moore's talk!)

$$\hat{q} \equiv \frac{1}{\lambda_R}\int d^2 q_\perp \frac{q_\perp^2}{\sigma_R}\frac{d\sigma_R}{d^2 q_\perp}$$

$$\propto g^2\int d^2q_\perp d^2 y_\perp dy^+ e^{-iq^- y^+ + iq_\perp\cdot y_\perp}\langle F_i^{a+}(y^+, y_\perp)F_i^{a+}(0)\rangle$$

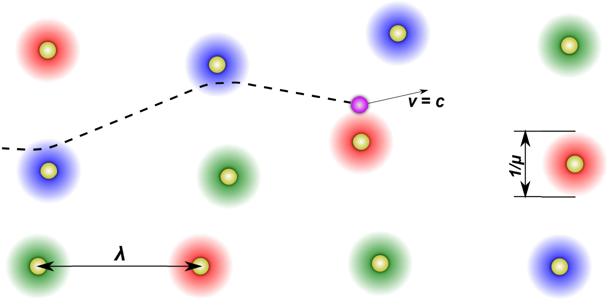
- **Including virtual gluon emission**



$$\langle p_\perp^2 \rangle_{rad} = \underbrace{\frac{\alpha_s N_c L}{\pi} \int \frac{d\omega}{\omega} \int dk_\perp^2 \frac{1}{k_\perp^2}}_{\text{Double-log}} \underbrace{\frac{1}{\lambda_R} \int dl_\perp^2 \theta(k_\perp^2 - l_\perp^2) \frac{l_\perp^2}{\sigma_R} \frac{d\sigma_R}{dl_\perp^2}}_{\hat{q}}$$

Multiple scattering

- **Decorrelated multiple scattering:** $\lambda \gg \frac{1}{\mu}$



Multiple scattering

- **A detour: the Path integral approach for high energy partons**
  - **2-d Schrödinger equation**

$$i\frac{\partial}{\partial z}\psi_i(x_\perp, z) = H_i\psi_i(x_\perp, z) = \left(\frac{-\nabla^2 + m_i}{E_i} + gA^+(x_\perp, z)\right)\psi_i(x_\perp, z)$$

    for species i with energy E_i.
  - **Integrate out the medium particle**
    - **Uncorrelated multiple scattering**

$$\langle A^+(x,z)A^+(y,z')\rangle \propto \delta(z-z') \Rightarrow \text{Infrared cutoff } l_0$$

      The same assumption as that in the Langevin equation (Tuomas Lappi's talk!)
    - **Gaussian distribution of scatterers**

$$\langle A^+A^+A^+A^+\cdots\rangle = \langle A^+A^+\rangle\langle A^+A^+\rangle\cdots$$

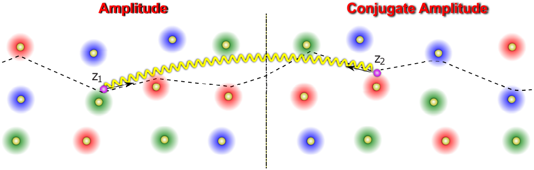
  - **A dipole-like picture:**

    Partons in conjugate amplitude $\Leftrightarrow$ their anti-particles in amplitude.

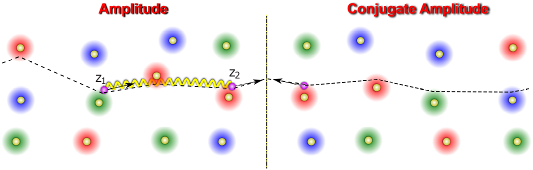
R. Baier, D. Schiff and B. G. Zakharov, Ann. Rev. Nucl. Part. Sci. **50***, 37 (2000) [hep-ph/0002198].*

Multiple scattering

- **Medium-induced gluon emission**

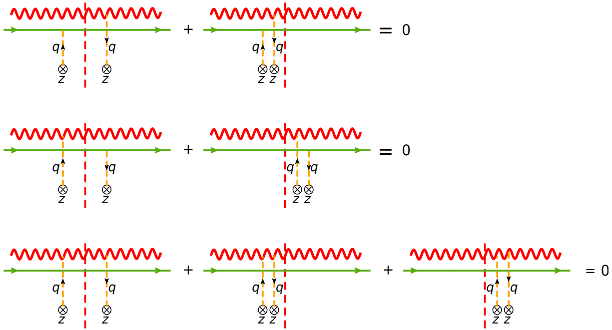


- **Medium-induced "self-energy"**



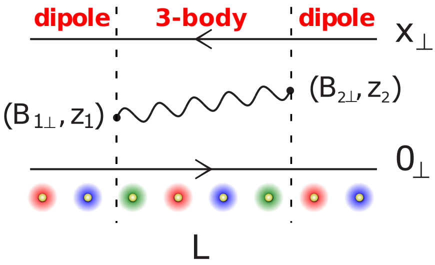
Multiple scattering

- **Cancellation of diagrams after z_2**



Multiple scattering

- **In Zakharov's path integral formalism**



- **One needs to solve for 3-body propagator**

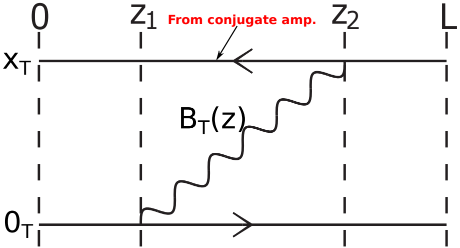
$$\left(i\frac{\partial}{\partial z} - H\right) G\left(B_\perp, z; B_{1\perp}, z_1\right) = i\delta(z - z_1)\delta(B_\perp - B_{1\perp})$$

with

$$H \simeq -\frac{\nabla^2_{cb}}{2\omega} - \frac{i}{4}\hat{q}_R\left\{x_\perp^2 + \frac{C_A}{2C_R}\left[B_\perp^2 + (B_\perp - x_\perp)^2 - x_\perp^2\right]\right\}$$

- **Real gluon emission**

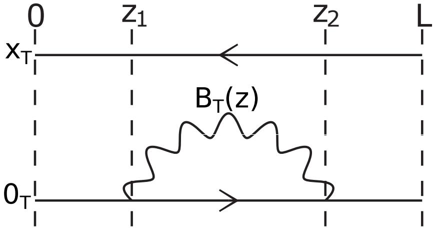
$S_r(x_\perp) \equiv$



$$
\begin{aligned}
&= 2\alpha_s C_F \mathrm{Re} \int \frac{d\omega}{\omega} \int_0^L dz(L-z) \\
&\times \nabla_{B_{1\perp}} \cdot \nabla_{B_{2\perp}} \left[e^{-\frac{1}{4}\hat{q}x_\perp^2(L-z)} G(B_{2\perp}, z_2; B_{1\perp}, z_1) \right. \\
&- \left. G_0(B_{2\perp}, z_2; B_{1\perp}, z_1) \right] \Big|_{B_{1\perp}=0, B_{2\perp}=x_\perp}
\end{aligned}
$$

- **Medium-induced "self-energy"**

$$S_v(x_\perp) \equiv$$



$$= -2\alpha_s C_F \mathrm{Re} \int \frac{d\omega}{\omega} \int_0^L dz(L-z)$$
$$\times \nabla_{B_{1\perp}} \cdot \nabla_{B_{2\perp}} \Big[e^{-\frac{1}{4}\hat{q}x_\perp^2 (L-z)} G(B_{2\perp}, z_2; B_{1\perp}, z_1)$$
$$- G_0(B_{2\perp}, z_2; B_{1\perp}, z_1) \Big] \Big|_{B_{1\perp}=0=B_{2\perp}}$$

$S_r(0) + S_v(0) = 0 \Leftarrow$ Probability conservation!

Multiple scattering

- **Correction from radiation (real + virtual gluons)**

$$\langle p_\perp^2 \rangle_{rad} = -\nabla^2 \left[S_r(x_\perp) + S_v(x_\perp)\right]\Big|_{x_\perp=0}$$
$$= \mathrm{Re}\frac{i\alpha_s N_c}{\pi}\int d\omega \int_{t_0}^{L} dz\, \frac{L-z}{z^3}\left\{\left(\frac{\omega_0 z}{\sin\omega_0 z}\right)^3\left[4-\sin^2\omega_0 z\right]-4\right\}$$

where

$$\omega_0 = \frac{1+i}{\sqrt{2}}\sqrt{\frac{\hat{q}}{\omega}}$$

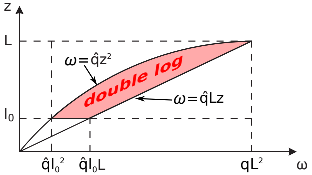
- **Recover the double log:** $z \lesssim \frac{1}{|\omega_0|} \simeq t_{form} = \sqrt{\frac{\omega}{\hat{q}}}$

$$\langle p_\perp^2 \rangle_{rad} \simeq \frac{\alpha_s N_c}{\pi}\hat{q}L\int\frac{d\omega}{\omega}\int\frac{dz}{z}$$

$z \simeq \frac{\omega}{k_\perp^2} \Rightarrow$ **result from single scattering!**

- **Setting limits of integration: Multiple scattering**

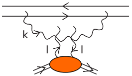
- **Double log region**



- "Single scattering": $z \leq \sqrt{\frac{\omega}{\hat{q}}}$
- Length of medium: $L \geq \frac{\omega}{\hat{q}z}$
- Infrared cutoff: $z > l_0$

- **Double logarithmical result:** $B_\perp^2 \gtrsim 1/\hat{q}L$

$$\langle p_\perp^2 \rangle_{rad} =$$



$$= \frac{\alpha_s N_c}{\pi} \hat{q}L \int_{l_0}^{L} \frac{dz}{z} \int_{\hat{q}z^2}^{\hat{q}Lz} \frac{d\omega}{\omega}$$

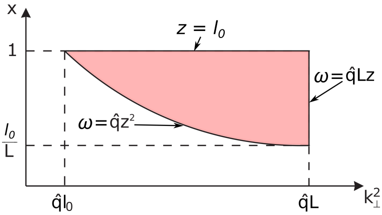
$$= \frac{\alpha_s N_c}{8\pi} \hat{q}L \log^2\left(\frac{L}{l_0}\right)^2$$

- **Resummation of Double Logs**

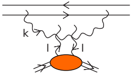
- **Transformation**

$$\begin{cases} x = \frac{l_0}{z} \\ k_\perp^2 = \frac{\omega}{z} \end{cases}$$

- **Double log region**



- **Double logarithmical result: again**

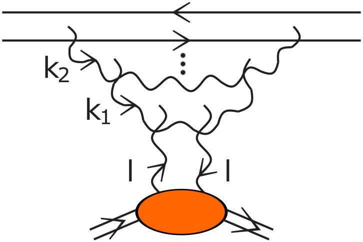
$\langle p_\perp^2 \rangle_{rad} =$ 

$$= \frac{\alpha_s N_c}{\pi} \hat{q} L \int_{\hat{q} l_0}^{\hat{q} L} \frac{dk_\perp^2}{k_\perp^2} \int_{\frac{\hat{q} l_0}{k_\perp^2}}^{1} \frac{dx}{x}$$

$$= \frac{\alpha_s N_c}{4\pi} \hat{q} L \frac{1}{1!2!} \log^2 \left(\frac{L}{l_0} \right)^2$$

Multiple scattering

- **Resummation of Double Logs**
  - **Many gluon emission**



  - **Leading double log region:**
    - **Formation time:** $x_1 \geq x_2 \geq x_3 \geq \cdots$
    - **Transverse momentum:** $k_{1\perp} \leq k_{2\perp} \leq k_{3\perp} \leq \cdots$
    - **"Single scatter":** $x_1 \geq \frac{\hat{q} l_0}{{k_{1\perp}}^2}, x_2 \geq \frac{\hat{q} l_0}{{k_{2\perp}}^2}, x_3 \geq \frac{\hat{q} l_0}{{k_{3\perp}}^2} \cdots$

Multiple scattering

- **Resummation of Double Logs**

  For n gluon emission

$$\langle p_\perp^2 \rangle_{ng} = \hat{q} L \frac{1}{n!(n+1)!} \left[\frac{\alpha N_c}{4\pi} \log^2 \frac{L^2}{l_0^2} \right]^n$$

  As a result

$$\langle p_\perp^2 \rangle = \hat{q} L \sqrt{\frac{4\pi}{\alpha_s N_c}} \frac{1}{\ln \frac{L^2}{l_0^2}} I_1 \left[\sqrt{\frac{\alpha_s N_c}{\pi}} \ln \frac{L^2}{l_0^2} \right]$$

Conclusions

1. **Radiative $p_\perp$-broadening is double logarithmically enhanced**

$$\langle p_\perp^2 \rangle_{rad} = \frac{\alpha_s N_c}{8\pi} \hat{q} L \log^2 \left(\frac{L}{l_0} \right)^2$$

2. **Leading double log can be resummed!**

$$\boxed{\langle p_\perp^2 \rangle = \hat{q} L \sqrt{\frac{4\pi}{\alpha_s N_c}} \frac{1}{\ln \frac{L^2}{l_0^2}} I_1 \left[\sqrt{\frac{\alpha_s N_c}{\pi}} \ln \frac{L^2}{l_0^2} \right]}$$

3. **Imply a parametrical large radiative energy loss at NLO?**

   Possibly! **Yacine's talk!**