Holographic thermalization at intermediate coupling

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R. Baier, S. Stricker, O. Taanila, AV, 1205.2998 (JHEP), 1207.1116 (PRD)
D. Steineder, S. Stricker, AV, 1209.0291 (PRL), 1304.3404 (JHEP)

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Why holography?

Two surprising lessons (and nontrivial theory challenges) from comparison of RHIC data with hydro simulations:

1. Very small, yet nonzero shear viscosity $\eta/s = \mathcal{O}(0.1)$
2. Early onset of hydrodynamic behavior: $\tau_{\text{hydro}} \sim 0.5$ fm/c, close to causal limit

Both results in glaring contradiction with perturbative estimates ⇒ Standard conclusion: QGP (@ RHIC) a strongly coupled liquid, out of the realm of weak coupling methods

QGP a multi scale system — optimal approach probably an efficient combination of different methods. Clearly missing: Tool to access strongly coupled dynamics

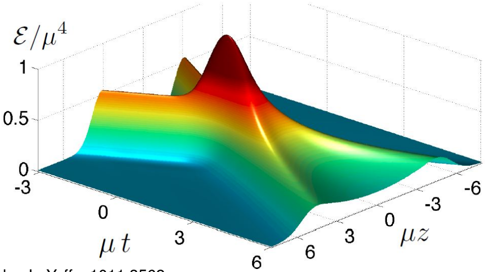
All approaches to (thermal) QCD some types of *systematically improvable* approximations: pQCD, lattice QCD, effective theories, ...

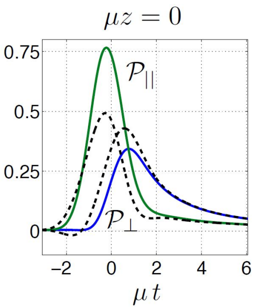
Why not consider a different expansion point: SU(N_c) gauge theory w/

- N_c taken to infinity
- Large 't Hooft coupling $\lambda = g^2 N_c$
- Conformal invariance
- Additional adjoint fermions and scalars to make theory $\mathcal{N} = 4$ supersymmetric

AdS/CFT conjecture (Maldacena, 1997):

- String theory in AdS_5 exactly dual to $\mathcal{N} = 4$ Super Yang-Mills theory on 4d boundary
- Strongly coupled, $N_c \to \infty$ SYM $\leftrightarrow$ Classical supergravity in AdS_5





P. Chesler, L. Yaffe, 1011.3562

Theory very different from QCD at $T = 0$. However:

- Most of the above limits systematically improvable
- At finite T, systems much more similar
  - Conformality and SUSY broken
  - Deconfinement, Debye screening, finite static correlation length, ...
- *Very* nontrivial field theory problems mapped to solvable classical gravity calculations

By now several results extending our qualitative understanding of strongly coupled non-Abelian plasmas:

- $\lambda \to \infty$ behavior of many bulk thermodynamic quantities and transport constants
- Insights into thermalization process at strong coupling — cf. the 'early thermalization puzzle'
- $\cdots$

However, direct quantitative application of holographic results to heavy ion physics problematic because of (even qualitative) issues related to $N_c \to \infty$ and $\lambda \to \infty$ limits

- Example: At infinite coupling, thermalization pattern always top-down, i.e. most energetic field modes reach equilibrium first independent of initial conditions

Clearly, trying to bring solvable gravity setup closer to the real life heavy ion system is a worthwhile task

Rest of the talk: Attempt to relax $\lambda = \infty$ limit in the context of holographic thermalization

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AdS/CFT duality: $T = 0$

- Original conjecture relates SU(N_c) $\mathcal{N} = 4$ SYM theory living in $\mathbb{R}^{1,3}$ to type IIB String Theory in $AdS_5 \times S_5$

"center" of AdS — $r = 0$

boundary — $r = \infty$

- Pure AdS metric corresponds to vacuum state of the CFT

$$ds^2 = L^2\Big(- r^2 dt^2 + \frac{dr^2}{r^2} + r^2 d\mathbf{x}^2 \Big)$$

- Dictionary: CFT operators $\leftrightarrow$ bulk fields, with identification

$$(L/l_s)^4 = \lambda, \quad g_s = \lambda/(4\pi N_c)$$

$\Rightarrow$ Strongly coupled, large-N_c QFT $\leftrightarrow$ Classical sugra

AdS/CFT duality: $T \neq 0$

- Strongly coupled large-N_c SYM plasma in thermal equilibrium $\leftrightarrow$ Classical gravity in AdS black hole background

center	horizon	boundary
$r = 0$	$r = r_h$	$r = \infty$

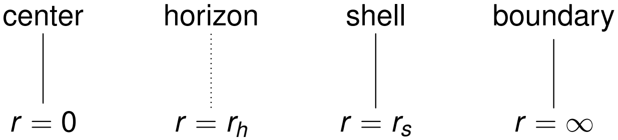
- Metric now features event horizon at $r = r_h$ ($L \equiv 1$ from now on)

$$ds^2 = -r^2(1 - r_h^4/r^4)dt^2 + \frac{dr^2}{r^2(1 - r_h^4/r^4)} + r^2 d\mathbf{x}^2$$

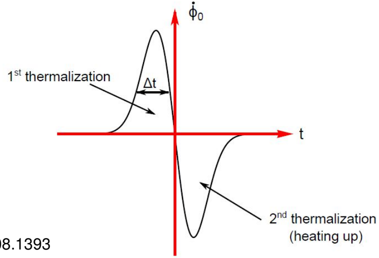
- Identification of field theory temperature with Hawking temperature of black hole fixes $T = r_h/\pi$

AdS/CFT duality: Thermalizing system

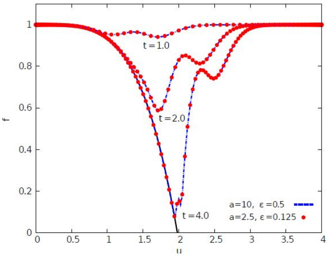
- Simplest way to take system out of equilibrium: Begin with a thin massive shell at $r = r_s > r_h$ and let it collapse towards $r_s = r_h$



- Shell fills entire three-space $\Rightarrow$ System translationally and rotationally invariant
- Corresponds to rapid, spatially homogenous injection of energy to the field theory system
- Model proposed by Danielsson, Keski-Vakkuri, Kruczenski (1999); analyzed in more detail by e.g. Lin, Shuryak (2008) and Erdmenger, Lin (2011, 2012)



Bin Wu, 1208.1393



- Can be realized by turning on a spatially homogenous scalar source in the 5d bulk, coupled to
  - A marginal composite operator on the boundary
  - Metric components through Einstein equations

$$ds^2 = \frac{1}{u^2}\Big(-f(u,t)\, e^{-2\delta(u,t)} dt^2 + 1/f(u,t)\, du^2 + d\mathbf{x}^2 \Big), \quad u = r_h^2/r^2$$

- Special feature: In the limit of a slowly moving thin shell, off-equilibrium Green's functions straightforwardly available

Holographic Green's functions

Off-equilibrium correlators offer a useful window to thermalization:

- Probe how fast different length (energy) scales equilibrate
- Related to measurable quantities, e.g. particle production rates

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Example: Photon production rate in quasistatic limit (valid for large ω) obtainable from spectral function of an external U(1) gauge field, i.e. imaginary part of a retarded correlator

$$k^0 \frac{d\Gamma_\gamma}{d^3k} = \frac{\alpha_{\rm EM}}{4\pi^2}\eta^{\mu\nu}\Pi^{<}_{\mu\nu}(k_0 \equiv \omega, k) = \frac{\alpha_{\rm EM}}{4\pi^2} n_B(\omega)\chi(\omega, k)$$

Evaluating χ, we can both study photon production out of equilibrium and investigate *relative deviation* of spectral function from thermal limit

$$R(\omega, k) \equiv \frac{\chi(\omega, k) - \chi_{\rm therm}(\omega, k)}{\chi_{\rm therm}(\omega, k)}$$

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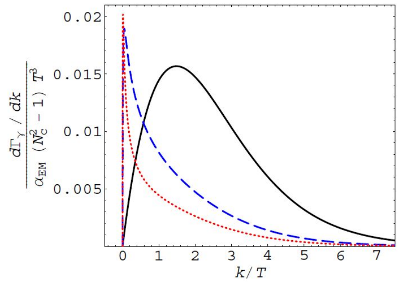

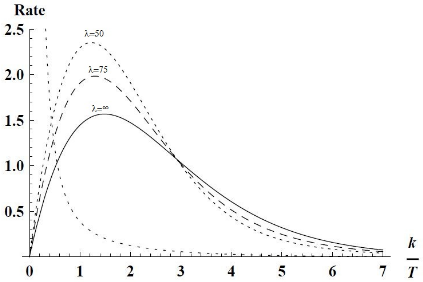

Equilibrium limit worked out at both weak and strong coupling; Caron-Huot *et al.* (2006) & Hassanain *et al.* (2011)

Beyond infinite coupling: α' corrections

Recall key relation from AdS/CFT dictionary: $(L/l_s)^4 = L^4/\alpha'^2 = \lambda$, with α' the inverse string tension

- To go beyond $\lambda = \infty$ limit, need to add α' terms to supergravity action, i.e. first non-trivial terms in a small-curvature expansion
- Leading order corrections $\mathcal{O}(\alpha'^3) = \mathcal{O}(\lambda^{-3/2})$

End up dealing with $\mathcal{O}(\alpha'^3)$ improved type IIB sugra action

$$
\begin{aligned}
S_{IIB} &= \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}\left(R_{10} - \frac{1}{2}(\partial\phi)^2 - \frac{F_5^2}{4\cdot 5!} + \gamma e^{-\frac{3}{2}\phi}(C+\mathcal{T})^4\right), \\
\mathcal{T}_{abcdef} &\equiv i\nabla_a F^+_{bcdef} + \frac{1}{16}\left(F^+_{abcmn}{F^+_{def}}^{mn} - 3F^+_{abfmn}{F^+_{dec}}^{mn}\right), \\
F^+ &\equiv \frac{1}{2}(1+*)F_5, \quad \gamma \equiv \frac{1}{8}\zeta(3)\lambda^{-3/2}
\end{aligned}
$$

$\Rightarrow \gamma$-corrected metric and EoMs for different fields

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Start from the $\lambda = \infty$ limit. Investige, how

1. The photon spectral function and photon production rate approach their equilibrium values when $r_s/r_h \to 1$
2. This process depends on the energy (frequency) and virtuality of the photon

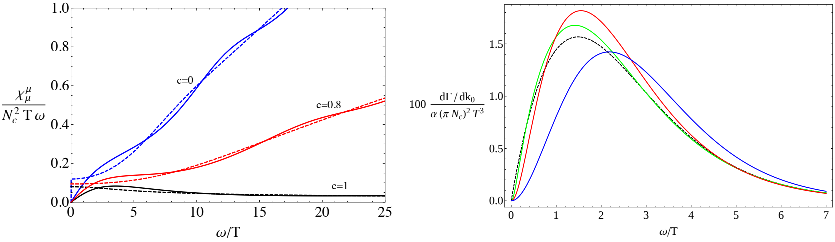
Recall also that:

- We work in the *quasistatic approximation*: No real dynamics, evolution of the system parameterized through r_s/r_h
- Quantity used to monitor the approach to equilibrium:

$$R(\omega, k) \equiv \frac{\chi(\omega, k) - \chi_{\text{therm}}(\omega, k)}{\chi_{\text{therm}}(\omega, k)}$$

- Notation for virtuality: $K = (\omega, \mathbf{k})$, $k \equiv |\mathbf{k}| = c\omega$, $c \in [0, 1]$

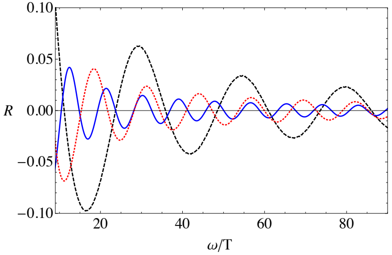
Photon spectral function and production rate

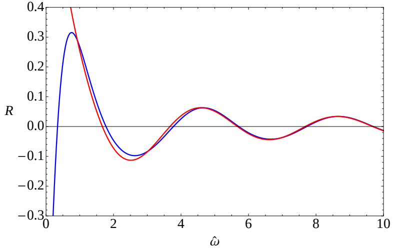


Left: Photon spectral functions for different virtualities in thermal equilibrium and at $r_s/r_h = 1.1$

Right: Approach of on-shell photon ($c = 1$) production rate towards the thermal limit: $r_s/r_h = 1.1, 1.01, 1.001, 1$

Approaching equilibrium: Relative deviation of χ





Left: Relative deviation $R(\omega)$ for $r_s/r_h = 1.1$ and $c = 0\,, 0.8,\ 1$

Right: Behavior of $R(\omega)$ for on-shell photons for $r_s/r_h = 1.1$ together with an analytic WKB-solution with amplitude $\sim 1/\omega$

NB: 1) Highly virtual field modes thermalize first

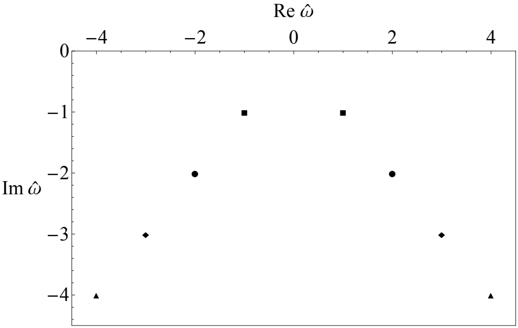
2) Clear top-down thermalization pattern (as always at $\lambda = \infty$)

So far, so good. Apparently no dramatic observable signatures in off-equilibrium photon production at infinite coupling.

Now take $\mathcal{O}(\lambda^{-3/2})$ corrections into account and study

- How the $\lambda = \infty$ quasinormal mode spectrum flows when λ is decreased
- What happens to the thermalization process at finite coupling

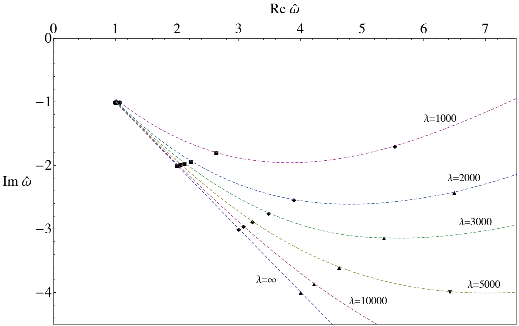
Quasinormal mode spectrum at finite coupling



Well-known fact: At $\lambda = \infty$, $\mathcal{N} = 4$ SYM doesn't have a quasiparticle spectrum $\omega_n(k) = E_n(k) + i\Gamma_n(k)$, $\Gamma_n \ll E_n$. Instead, solving for poles of retarded equilibrium correlator reveals quasinormal mode spectrum

$$\hat{\omega}_n|_{k=0} = \frac{\omega_n|_{k=0}}{2\pi T} = n\,(\pm 1 - i)$$

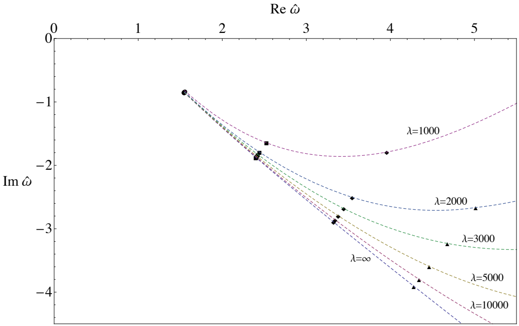
Quasinormal mode spectrum at finite coupling



Effect of decreasing λ: System flows towards quasiparticle spectrum at very large couplings (cf. photon production in equilibrium)

NB: Convergence of strong coupling expansion not guaranteed, when $\hat{\omega}_n|_{k=0} = n(\pm 1 - i) + \alpha_n/\lambda^{3/2}$ shifts by $\mathcal{O}(1)$ amount

Quasinormal mode spectrum at finite coupling

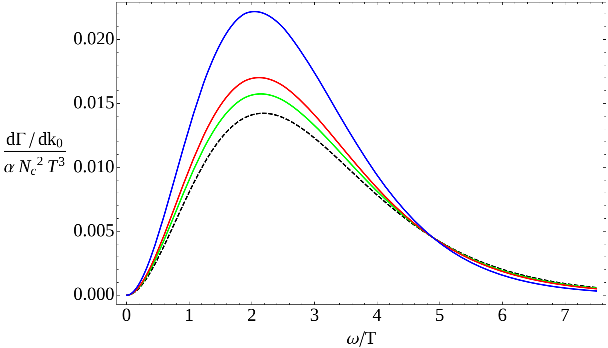


Similar shift at nonzero three-momentum: $k = 2\pi T$

Outside the $\lambda = \infty$ limit, the response of the strongly coupled plasma to infinitesimal perturbations appears to change, moving towards that of a weakly coupled quasiparticle system

What happens if we take the system further away from equilibrium?

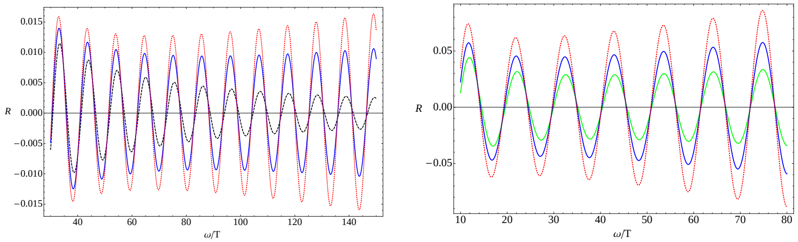
Photon production rate at intermediate λ



Emission rate of on-shell photons for $r_s/r_h = 1.01$ and $\lambda = \infty$, 120, 80, 40

Behavior qualitatively similar to equilibrium case; in particular, the result much less sensitive to λ than the QNM spectrum

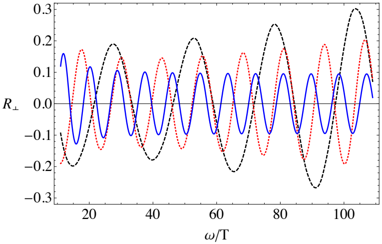
Relative deviation at intermediate λ

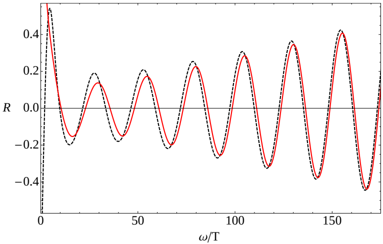


Relative deviation $R \equiv (\chi - \chi_{\rm th})/\chi_{\rm th}$ for $r_s/r_h = 1.01$ and $\lambda = \infty$, 500, 300 (left) and 150, 100, 75 (right), again for real photons ($c = 1$)

NB: Change of pattern with decreasing λ: UV modes no longer first to thermalize!

Relative deviation at intermediate λ





Left: Relative deviation for $r_s/r_h = 1.1$, $\lambda = 100$ and $c = 0$, 0.8 and 1. For maximally virtual photons ($c = 0$), amplitude approaches constant when $\omega \to \infty$

Right: Analytic WKB-calculation for the on-shell ($c = 1$) case confirms linear rise of amplitude at large ω

So what to make of all this? Evidence for holographic plasma starting to behave like a system of weakly coupled quasiparticles, or simply

- ... a peculiarity of photon production, but not a fundamental feature of the collective behavior of the plasma?
  - Apparently not the case. Equivalent calculations recently carried out for energy momentum tensor correlators, with qualitatively identical results (S. Stricker, 1307.xxxx)

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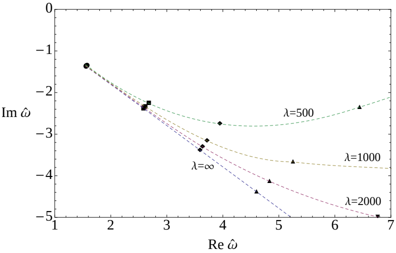

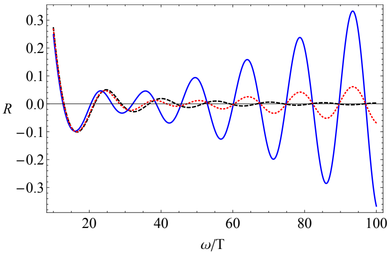

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Only time (and more work) will tell. Particularly important challenge: Generalize dynamical models of thermalization to $\lambda \neq \infty$

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Take home messages

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2. Solidish: Indications that the behavior of a holographic plasma starts to have weakly coupled characteristics within the realm of a strong coupling expansion
   - QNM poles flow in the direction of a quasiparticle spectrum
   - Top-down thermalization pattern weakens and shifts towards bottom-up
3. Tentative: To draw even qualitative conclusions about the physical heavy ion system ($\lambda = \mathcal{O}(20)$) using hologrpahy, accounting for strong coupling corrections imperative