Holographic thermalization at intermediate coupling

Aleksi Vuorinen

Bielefeld University

ECT*, Trento, 18.6.2013

R. Baier, S. Stricker, O. Taanila, AV, 1205.2998 (JHEP), 1207.1116 (PRD)
D. Steineder, S. Stricker, AV, 1209.0291 (PRL), 1304.3404 (JHEP)
**Table of contents**

1. Holography and heavy ion physics

2. Correlation functions in thermalizing SYM plasma
   - Basics of the duality
   - Off-equilibrium two-point functions

3. Results
   - Photon production at strong coupling
   - Beyond the $\lambda = \infty$ limit
   - Implications of the results

4. Conclusions
# Table of contents

1. **Holography and heavy ion physics**

2. **Correlation functions in thermalizing SYM plasma**
   - Basics of the duality
   - Off-equilibrium two-point functions

3. **Results**
   - Photon production at strong coupling
   - Beyond the $\lambda = \infty$ limit
   - Implications of the results

4. **Conclusions**
Why holography?

Two surprising lessons (and nontrivial theory challenges) from comparison of RHIC data with hydro simulations:

1. Very small, yet nonzero shear viscosity $\eta/s = \mathcal{O}(0.1)$
2. Early onset of hydrodynamic behavior: $\tau_{\text{hydro}} \sim 0.5 \text{ fm/c}$, close to causal limit

Both results in glaring contradiction with perturbative estimates $\Rightarrow$

Standard conclusion: QGP (@ RHIC) a strongly coupled liquid, out of the realm of weak coupling methods

QGP a multi scale system — optimal approach probably an efficient combination of different methods. Clearly missing: Tool to access strongly coupled dynamics
All approaches to (thermal) QCD some types of \textit{systematically improvable} approximations: pQCD, lattice QCD, effective theories, ...

Why not consider a different expansion point: SU($N_c$) gauge theory w/ 
- $N_c$ taken to infinity 
- Large ’t Hooft coupling $\lambda = g^2 N_c$ 
- Conformal invariance 
- Additional adjoint fermions and scalars to make theory $\mathcal{N} = 4$ supersymmetric

AdS/CFT conjecture (Maldacena, 1997): 
- String theory in AdS$_5$ exactly dual to $\mathcal{N} = 4$ Super Yang-Mills theory on 4d boundary 
- Strongly coupled, $N_c \to \infty$ SYM $\leftrightarrow$ Classical supergravity in AdS$_5$
Theory very different from QCD at $T = 0$. However:

- Most of the above limits systematically improvable
- At finite $T$, systems much more similar
  - Conformality and SUSY broken
  - Deconfinement, Debye screening, finite static correlation length, ...
- Very nontrivial field theory problems mapped to solvable classical gravity calculations
By now several results extending our qualitative understanding of strongly coupled non-Abelian plasmas:

- \( \lambda \rightarrow \infty \) behavior of many bulk thermodynamic quantities and transport constants
- Insights into thermalization process at strong coupling — cf. the ‘early thermalization puzzle’
- ... 

However, direct quantitative application of holographic results to heavy ion physics problematic because of (even qualitative) issues related to \( N_c \rightarrow \infty \) and \( \lambda \rightarrow \infty \) limits

- Example: At infinite coupling, thermalization pattern always top-down, i.e. most energetic field modes reach equilibrium first independent of initial conditions
Clearly, trying to bring solvable gravity setup closer to the real life heavy ion system is a worthwhile task.

Rest of the talk: Attempt to relax $\lambda = \infty$ limit in the context of holographic thermalization.
Table of contents

1. Holography and heavy ion physics

2. Correlation functions in thermalizing SYM plasma
   - Basics of the duality
   - Off-equilibrium two-point functions

3. Results
   - Photon production at strong coupling
   - Beyond the $\lambda = \infty$ limit
   - Implications of the results

4. Conclusions
AdS/CFT duality: $T = 0$

- Original conjecture relates $\text{SU}(N_c) \mathcal{N} = 4$ SYM theory living in $\mathbb{R}^{1,3}$ to type IIB String Theory in $\text{AdS}_5 \times S_5$
  
  \[
  \begin{align*}
  \text{"center" of AdS} & \quad \text{boundary} \\
  r = 0 & \quad r = \infty
  \end{align*}
  \]

- Pure AdS metric corresponds to vacuum state of the CFT

\[
ds^2 = L^2 \left( -r^2 dt^2 + \frac{dr^2}{r^2} + r^2 d\mathbf{x}^2 \right)
\]

- Dictionary: CFT operators $\leftrightarrow$ bulk fields, with identification

\[
(L/l_s)^4 = \lambda, \quad g_s = \lambda/(4\pi N_c)
\]

$\Rightarrow$ Strongly coupled, large-$N_c$ QFT $\leftrightarrow$ Classical sugra
AdS/CFT duality: $T \neq 0$

- Strongly coupled large-$N_c$ SYM plasma in thermal equilibrium $\leftrightarrow$ Classical gravity in AdS black hole background

<table>
<thead>
<tr>
<th>center</th>
<th>horizon</th>
<th>boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r = 0$</td>
<td>$r = r_h$</td>
<td>$r = \infty$</td>
</tr>
</tbody>
</table>

- Metric now features event horizon at $r = r_h$ ($L \equiv 1$ from now on)

$$ds^2 = -r^2(1 - r_h^4/r^4)dt^2 + \frac{dr^2}{r^2(1 - r_h^4/r^4)} + r^2 d\mathbf{x}^2$$

- Identification of field theory temperature with Hawking temperature of black hole fixes $T = r_h/\pi$
AdS/CFT duality: Thermalizing system

- Simplest way to take system out of equilibrium: Begin with a thin massive shell at \( r = r_s > r_h \) and let it collapse towards \( r_s = r_h \)

<table>
<thead>
<tr>
<th>center</th>
<th>horizon</th>
<th>shell</th>
<th>boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r = 0 )</td>
<td>( r = r_h )</td>
<td>( r = r_s )</td>
<td>( r = \infty )</td>
</tr>
</tbody>
</table>

- Shell fills entire three-space \( \Rightarrow \) System translationally and rotationally invariant
- Corresponds to rapid, spatially homogenous injection of energy to the field theory system
- Model proposed by Danielsson, Keski-Vakkuri, Kruczenski (1999); analyzed in more detail by e.g. Lin, Shuryak (2008) and Erdmenger, Lin (2011, 2012)
Can be realized by turning on a spatially homogenous scalar source in the 5d bulk, coupled to

- A marginal composite operator on the boundary
- Metric components through Einstein equations

$$ds^2 = \frac{1}{u^2} \left( -f(u, t) e^{-2\delta(u,t)} \, dt^2 + \frac{1}{f(u,t)} \, du^2 + dx^2 \right), \quad u = \frac{r_n^2}{r^2}$$

Special feature: In the limit of a slowly moving thin shell, off-equilibrium Green’s functions straightforwardly available
### Holographic Green’s functions

Off-equilibrium correlators offer a useful window to thermalization:

- Probe how fast different length (energy) scales equilibrate
- Related to measurable quantities, e.g. particle production rates
Holographic Green’s functions

Off-equilibrium correlators offer a useful window to thermalization:

- Probe how fast different length (energy) scales equilibrate
- Related to measurable quantities, e.g. particle production rates

Example: **Photon production rate** in quasistatic limit (valid for large $\omega$) obtainable from spectral function of an external U(1) gauge field, i.e. imaginary part of a retarded correlator

\[
k^0 \frac{d\Gamma_{\gamma}}{d^3 k} = \frac{\alpha_{\text{EM}}}{4\pi^2} \eta^{\mu\nu} \Pi_{\mu\nu}^<(k_0 \equiv \omega, k) = \frac{\alpha_{\text{EM}}}{4\pi^2} n_B(\omega) \chi(\omega, k)
\]

Evaluating $\chi$, we can both study photon production out of equilibrium and investigate *relative deviation* of spectral function from thermal limit

\[
R(\omega, k) \equiv \frac{\chi(\omega, k) - \chi_{\text{therm}}(\omega, k)}{\chi_{\text{therm}}(\omega, k)}
\]
Off-equilibrium correlators offer a useful window to thermalization:

- Probe how fast different length (energy) scales equilibrate
- Related to measurable quantities, e.g. particle production rates

Equilibrium limit worked out at both weak and strong coupling; Caron-Huot et al. (2006) & Hassanain et al. (2011)
Beyond infinite coupling: $\alpha'$ corrections

Recall key relation from AdS/CFT dictionary: $(L/l_s)^4 = L^4/\alpha'^2 = \lambda$, with $\alpha'$ the inverse string tension

- To go beyond $\lambda = \infty$ limit, need to add $\alpha'$ terms to supergravity action, i.e. first non-trivial terms in a small-curvature expansion
- Leading order corrections $\mathcal{O}(\alpha'^3) = \mathcal{O}(\lambda^{-3/2})$

End up dealing with $\mathcal{O}(\alpha'^3)$ improved type IIB sugra action

$$S_{IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left( R_{10} - \frac{1}{2}(\partial \phi)^2 - \frac{F_5^2}{4 \cdot 5!} + \gamma e^{-\frac{3}{2} \phi}(C + \mathcal{T})^4 \right),$$

$$T_{abcdef} \equiv i \nabla_a F^+_{bcdef} + \frac{1}{16} \left( F^+_{abcdef} F^+_{mn} - 3 F^+_{abfmn} F^+_{dec} \right),$$

$$F^+ \equiv \frac{1}{2}(1 + *) F_5, \quad \gamma \equiv \frac{1}{8} \zeta(3) \lambda^{-3/2}$$

$\Rightarrow$ $\gamma$-corrected metric and EoMs for different fields
Table of contents

1. Holography and heavy ion physics

2. Correlation functions in thermalizing SYM plasma
   - Basics of the duality
   - Off-equilibrium two-point functions

3. Results
   - Photon production at strong coupling
   - Beyond the $\lambda = \infty$ limit
   - Implications of the results

4. Conclusions
Start from the $\lambda = \infty$ limit. Investigate how

1. The photon spectral function and photon production rate approach their equilibrium values when $r_s/r_h \to 1$
2. This process depends on the energy (frequency) and virtuality of the photon

Recall also that:

- We work in the \textit{quasistatic approximation}: No real dynamics, evolution of the system parameterized through $r_s/r_h$
- Quantity used to monitor the approach to equilibrium:

$$R(\omega, k) \equiv \frac{\chi(\omega, k) - \chi_{\text{therm}}(\omega, k)}{\chi_{\text{therm}}(\omega, k)}$$

- Notation for virtuality: $K = (\omega, k), \ k \equiv |k| = c\omega, \ c \in [0, 1]$
Photon spectral function and production rate

Left: Photon spectral functions for different virtualities in thermal equilibrium and at $r_s/r_h = 1.1$

Right: Approach of on-shell photon ($c = 1$) production rate towards the thermal limit: $r_s/r_h = 1.1, 1.01, 1.001, 1$
Approaching equilibrium: Relative deviation of $\chi$

Left: Relative deviation $R(\omega)$ for $r_s/r_h = 1.1$ and $c = 0, 0.8, 1$

Right: Behavior of $R(\omega)$ for on-shell photons for $r_s/r_h = 1.1$ together with an analytic WKB-solution with amplitude $\sim 1/\omega$

NB: 1) Highly virtual field modes thermalize first
2) Clear top-down thermalization pattern (as always at $\lambda = \infty$)
So far, so good. Apparently no dramatic observable signatures in off-equilibrium photon production at infinite coupling.

Now take $\mathcal{O}(\lambda^{-3/2})$ corrections into account and study

- How the $\lambda = \infty$ quasinormal mode spectrum flows when $\lambda$ is decreased
- What happens to the thermalization process at finite coupling
Quasinormal mode spectrum at finite coupling

Well-known fact: At $\lambda = \infty$, $\mathcal{N} = 4$ SYM doesn’t have a quasiparticle spectrum $\omega_n(k) = E_n(k) + i\Gamma_n(k)$, $\Gamma_n \ll E_n$. Instead, solving for poles of retarded equilibrium correlator reveals quasinormal mode spectrum

$$\hat{\omega}_n|_{k=0} = \frac{\omega_n|_{k=0}}{2\pi T} = n(\pm 1 - i)$$
Quasinormal mode spectrum at finite coupling

Effect of decreasing $\lambda$: System flows towards quasiparticle spectrum at very large couplings (cf. photon production in equilibrium)

NB: Convergence of strong coupling expansion not guaranteed, when $\hat{\omega}_n|_{k=0} = n (\pm 1 - i) + \alpha_n/\lambda^{3/2}$ shifts by $\mathcal{O}(1)$ amount
Quasinormal mode spectrum at finite coupling

Similar shift at nonzero three-momentum: $k = 2\pi T$
Outside the $\lambda = \infty$ limit, the response of the strongly coupled plasma to infinitesimal perturbations appears to change, moving towards that of a weakly coupled quasiparticle system.

What happens if we take the system further away from equilibrium?
Emission rate of on-shell photons for $r_s/r_h = 1.01$ and $\lambda = \infty, 120, 80, 40$

Behavior qualitatively similar to equilibrium case; in particular, the result much less sensitive to $\lambda$ than the QNM spectrum
Relative deviation at intermediate $\lambda$

Relative deviation $R \equiv (\chi - \chi_{th})/\chi_{th}$ for $r_s/r_h = 1.01$ and $\lambda = \infty$, 500, 300 (left) and 150, 100, 75 (right), again for real photons ($c = 1$)

NB: Change of pattern with decreasing $\lambda$: UV modes no longer first to thermalize!
Relative deviation at intermediate $\lambda$

Left: Relative deviation for $r_s/r_h = 1.1$, $\lambda = 100$ and $c = 0$, 0.8 and 1. For maximally virtual photons ($c = 0$), amplitude approaches constant when $\omega \to \infty$

Right: Analytic WKB-calculation for the on-shell ($c = 1$) case confirms linear rise of amplitude at large $\omega$
So what to make of all this? Evidence for holographic plasma starting to behave like a system of weakly coupled quasiparticles, or simply

- ... a peculiarity of photon production, but not a fundamental feature of the collective behavior of the plasma?
  - Apparently not the case. Equivalent calculations recently carried out for energy momentum tensor correlators, with qualitatively identical results (S. Stricker, 1307.xxxx)
So what to make of all this? Evidence for holographic plasma starting to behave like a system of weakly coupled quasiparticles, or simply

- ... a peculiarity of photon production, but not a fundamental feature of the collective behavior of the plasma?
- Apparently not the case. Equivalent calculations recently carried out for energy momentum tensor correlators, with qualitatively identical results (S. Stricker, 1307.xxx).

![Graph showing Re w versus Im w for different values of lambda.](image1.png)

![Graph showing R versus omega/T.](image2.png)
So what to make of all this? Evidence for holographic plasma starting to behave like a system of weakly coupled quasiparticles, or simply

- ... a peculiarity of photon production, but not a fundamental feature of the collective behavior of the plasma?
  - Apparently not the case. Equivalent calculations recently carried out for energy momentum tensor correlators, with qualitatively identical results (S. Stricker, 1307.xxxx)
- ... due to the breakdown of some approximation?
  - Quasistatic limit good approximation as long as $\omega / T \gg 1$
  - Strong coupling exp. applied with care: $(\text{NLO-LO})/\text{LO} \lesssim O(1/10)$
So what to make of all this? Evidence for holographic plasma starting to behave like a system of weakly coupled quasiparticles, or simply

- a peculiarity of photon production, but not a fundamental feature of the collective behavior of the plasma?
  - Apparently not the case. Equivalent calculations recently carried out for energy momentum tensor correlators, with qualitatively identical results (S. Stricker, 1307.xxxx)

- due to the breakdown of some approximation?
  - Quasistatic limit good approximation as long as $\omega / T \gg 1$
  - Strong coupling exp. applied with care: $(\text{NLO-LO})/\text{LO} \lesssim O(1/10)$

- a sign of the unphysical nature of collapsing shell model?
  - Perhaps; certainly warrants more work. However, at least QNM flow results universal
So what to make of all this? Evidence for holographic plasma starting to behave like a system of weakly coupled quasiparticles, or simply

- ... a peculiarity of photon production, but not a fundamental feature of the collective behavior of the plasma?
  - Apparently not the case. Equivalent calculations recently carried out for energy momentum tensor correlators, with qualitatively identical results (S. Stricker, 1307.xxxx)
- ... due to the breakdown of some approximation?
  - Quasistatic limit good approximation as long as $\omega / T \gg 1$
  - Strong coupling exp. applied with care: $(\text{NLO-LO})/\text{LO} \lesssim \mathcal{O}(1/10)$
- ... a sign of the unphysical nature of collapsing shell model?
  - Perhaps; certainly warrants more work. However, at least QNM flow results universal

Only time (and more work) will tell. Particularly important challenge: Generalize dynamical models of thermalization to $\lambda \neq \infty$
# Table of contents

1. Holography and heavy ion physics

2. Correlation functions in thermalizing SYM plasma
   - Basics of the duality
   - Off-equilibrium two-point functions

3. Results
   - Photon production at strong coupling
   - Beyond the $\lambda = \infty$ limit
   - Implications of the results

4. Conclusions
Solid: Taking holographic (thermalization) calculations away from $\lambda = \infty$ limit not only possible, but potentially a very fruitful exercise.
Conclusions

Take home messages

1. Solid: Taking holographic (thermalization) calculations away from $\lambda = \infty$ limit not only possible, but potentially a very fruitful exercise.

2. Solidish: Indications that the behavior of a holographic plasma starts to have weakly coupled characteristics within the realm of a strong coupling expansion.
   - QNM poles flow in the direction of a quasiparticle spectrum.
   - Top-down thermalization pattern weakens and shifts towards bottom-up.
Take home messages

1 Solid: Taking holographic (thermalization) calculations away from $\lambda = \infty$ limit not only possible, but potentially a very fruitful exercise

2 Solidish: Indications that the behavior of a holographic plasma starts to have weakly coupled characteristics within the realm of a strong coupling expansion
   - QNM poles flow in the direction of a quasiparticle spectrum
   - Top-down thermalization pattern weakens and shifts towards bottom-up

3 Tentative: To draw even qualitative conclusions about the physical heavy ion system ($\lambda = \mathcal{O}(20)$) using holography, accounting for strong coupling corrections imperative