JIMWLK evolution for multi-particle production in Langevin form

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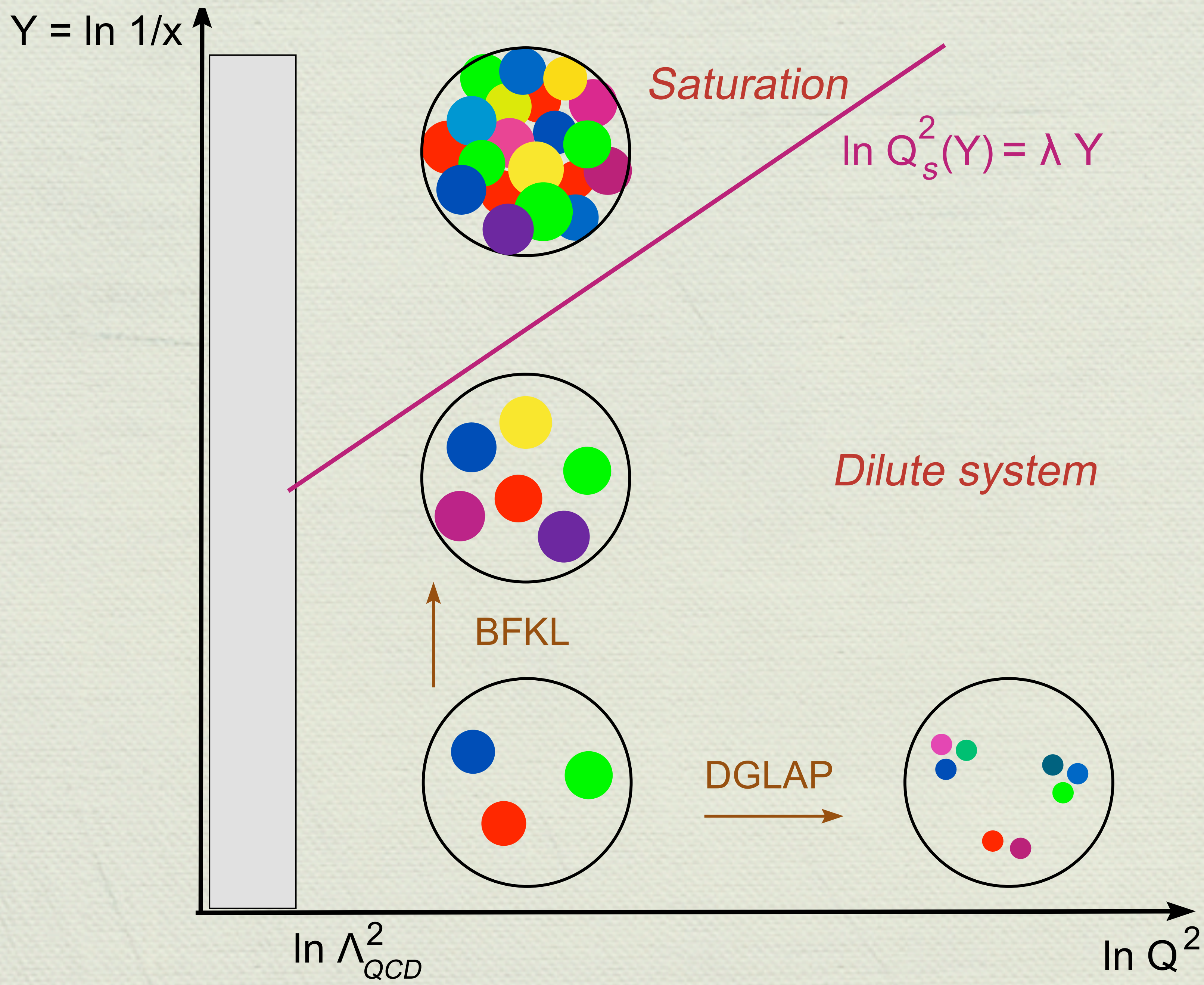
E. Iancu, DNT, arXiv:1306.nnnn

Outline

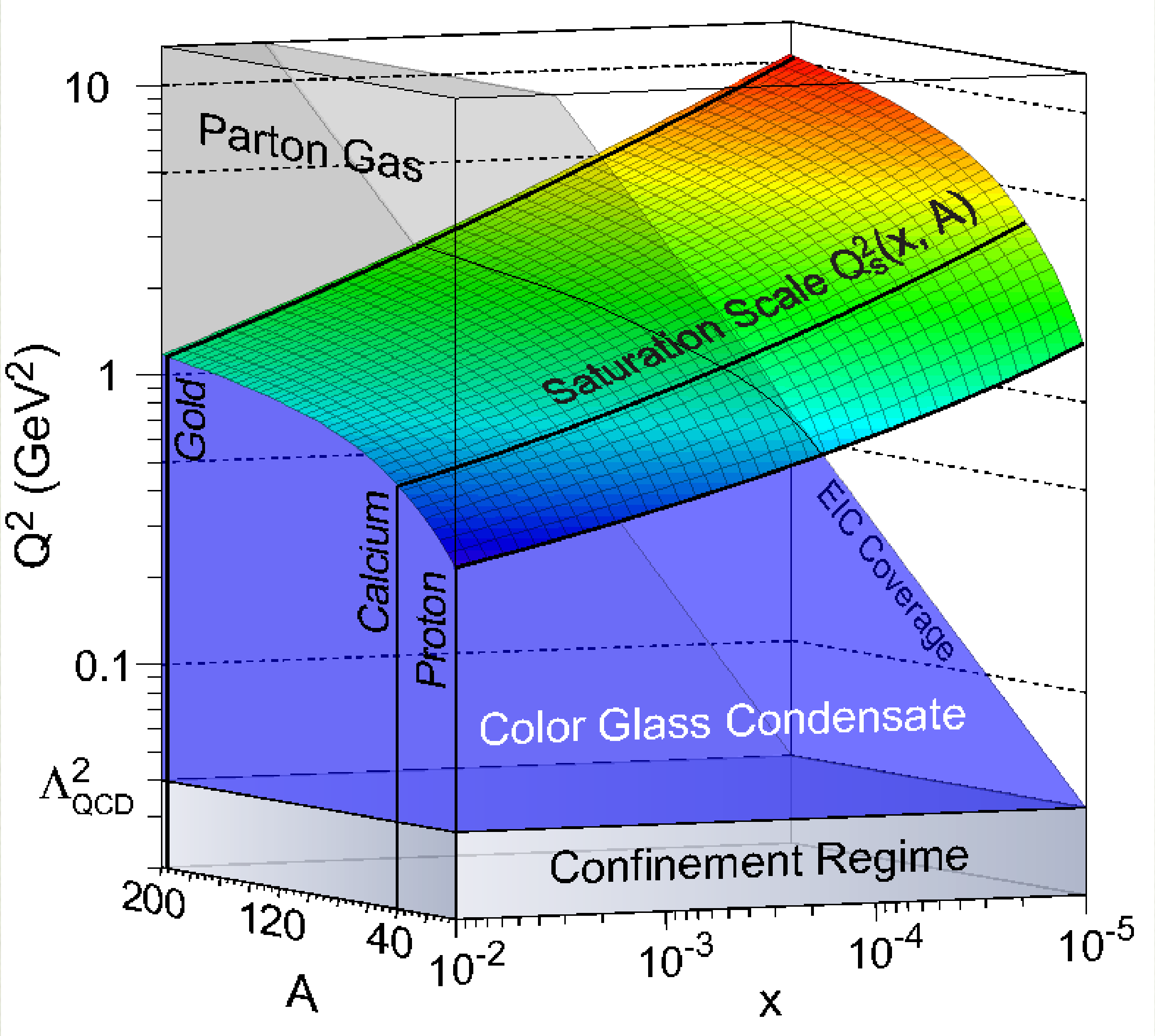
- JIMWLK evolution: dipoles, quadrupoles, ... or Langevin
- Multi-gluon production at a given rapidity
- Di-gluon production at different rapidities
- Evolution of weight-function squared
- Langevin for di-gluon production at different rapidities

Partonic "phase diagram"



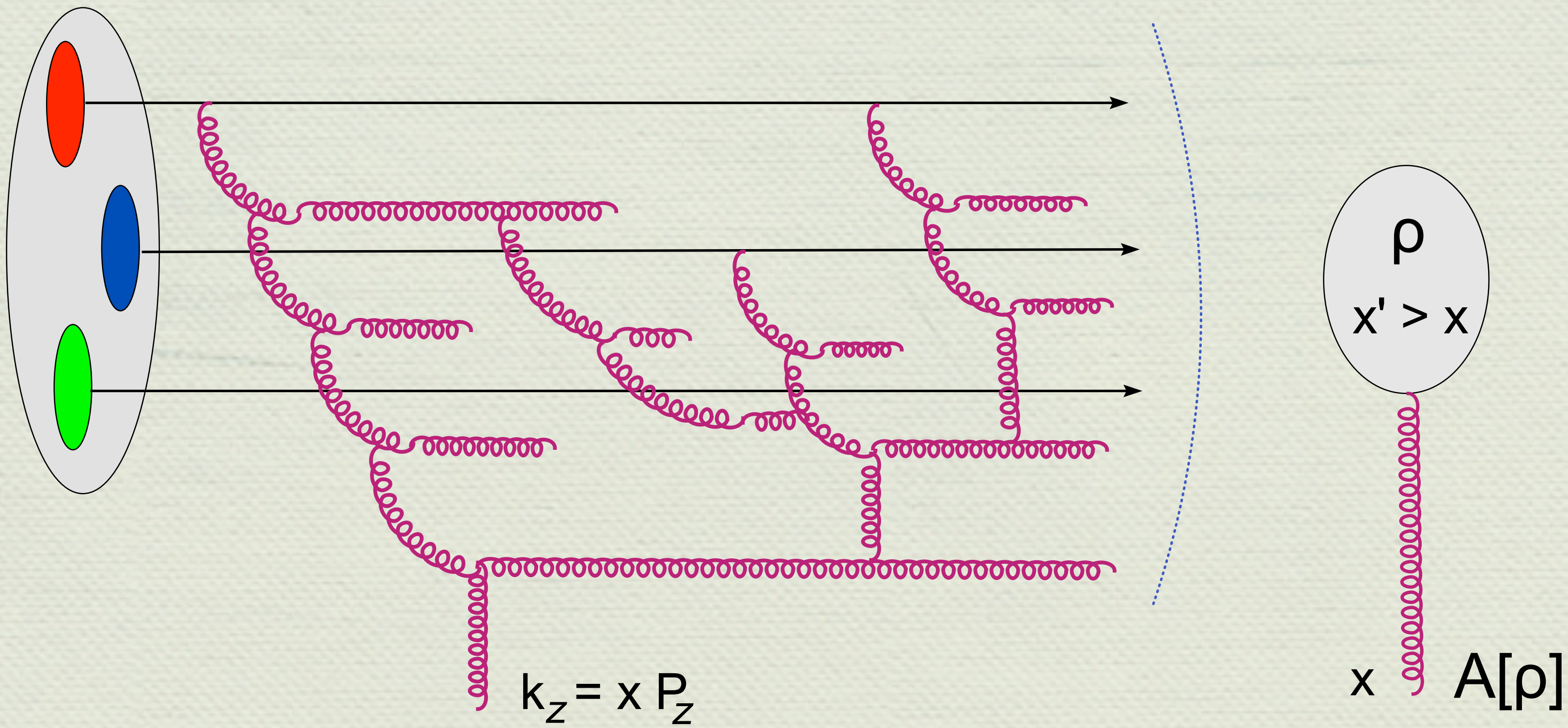
Saturation momentum

- Saturation when $\dfrac{xg(x,Q_s^2)}{Q_s^2 R^2} \sim \dfrac{1}{\alpha_s}$
- $Q_s^2(x,A) \sim Q_0^2 A^{1/3} \left(\dfrac{x_0}{x}\right)^\lambda$ with $\lambda = 0.2 \div 0.3$



The CGC

□ Integrate ``fast modes'' with large lifetime



□ Classical average : $\langle \hat{\mathcal{O}} \rangle_Y = \int [DU]\, W_Y[U]\, \hat{\mathcal{O}}$

□ Proper degrees of freedom: Wilson lines for scattering of propagating partons: $U^\dagger_{\boldsymbol{x}} = \mathrm{P}\exp\left[ig \int \mathrm{d}x^+ \alpha^a_{\boldsymbol{x}}(x^+) T^a\right]$

The JIMWLK equation

□ Quantum evolution: $$\partial W_Y[U]/\partial Y = H W_Y[U]$$

$$H = \frac{1}{8\pi^3} \int_{\boldsymbol{uvz}} \mathcal{K}^i_{\boldsymbol{uz}} \mathcal{K}^i_{\boldsymbol{vz}} \left[L^a_{\boldsymbol{u}} - U^{\dagger ab}_{\boldsymbol{z}} R^b_{\boldsymbol{u}}\right] \left[L^a_{\boldsymbol{v}} - U^{\dagger ac}_{\boldsymbol{z}} R^c_{\boldsymbol{v}}\right]$$

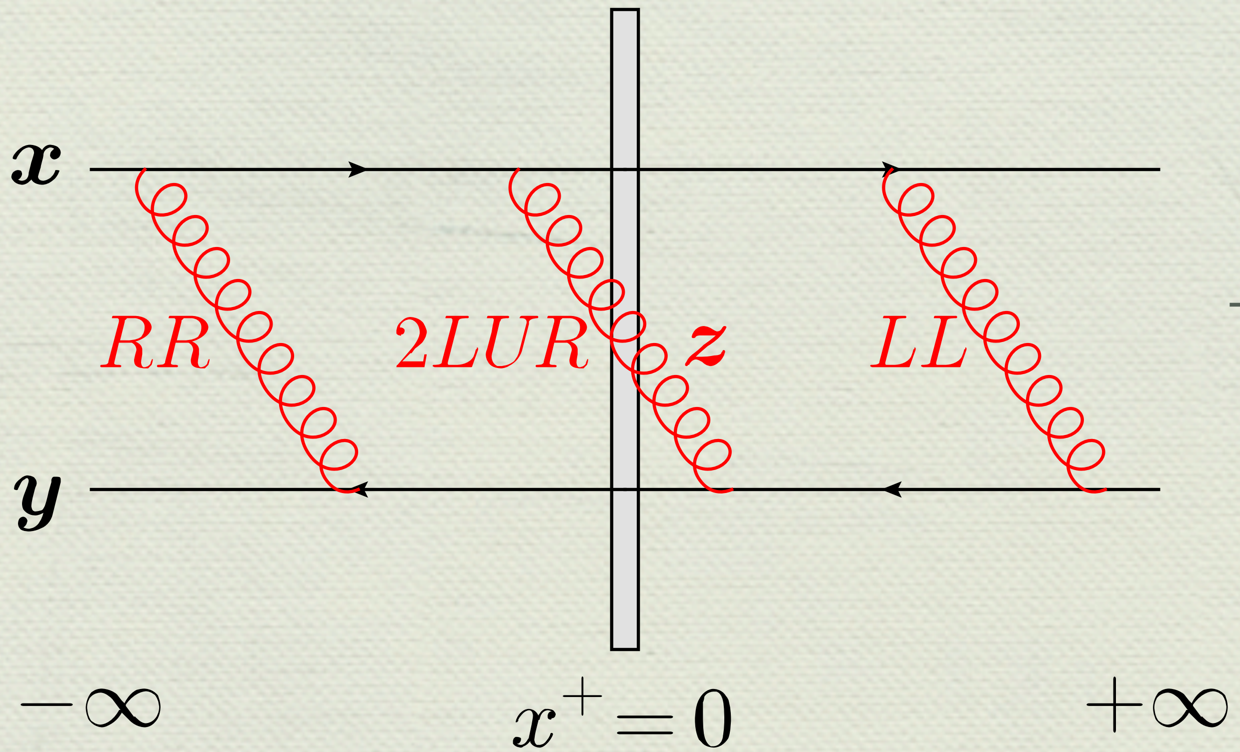
$K^i{}_{uz}$: Weizsacker-Williams kernel for emission of a soft gluon

□ Left and right derivatives act at largest and smallest x^+, after and before scattering.

$$L^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{x}} = \mathrm{i} g \delta_{\boldsymbol{ux}} T^a U^\dagger_{\boldsymbol{x}}, \qquad R^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{x}} = \mathrm{i} g \delta_{\boldsymbol{ux}} U^\dagger_{\boldsymbol{x}} T^a$$

1^{st} approach: projectile evolution

- IBP and act on observable, e.g. forward scattering of color dipole off nuclear shockwave $\hat{S}^F(\boldsymbol{xy}) = \frac{1}{N_g}\,\mathrm{Tr}[V_{\boldsymbol{y}} V_{\boldsymbol{x}}^\dagger]$



+ same quark attachments

- Leads to dipole (BK) equation. Similarly for quadrupole, etc.

- Hierarchy. Take large-N_c and/or Gaussian approximation.

2nd approach: Langevin

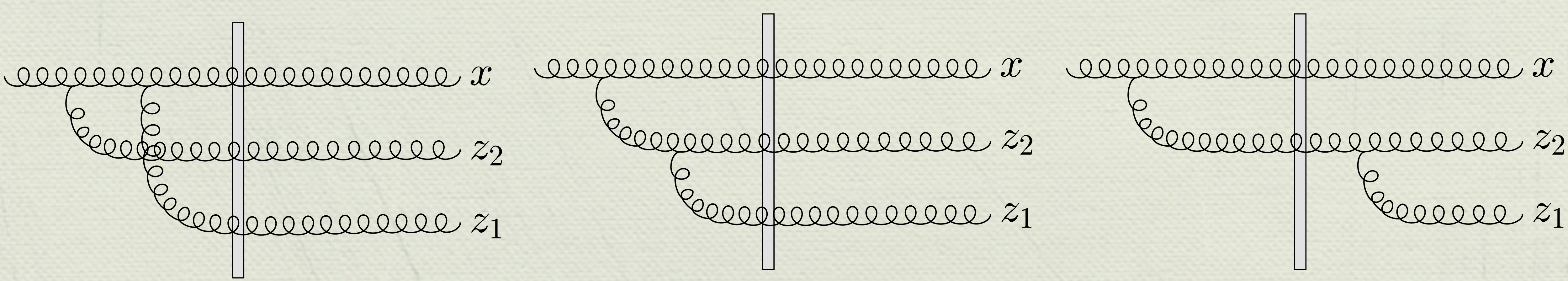
- Random walk in space of Wilson lines. Each step adds left and right layer to the Wilson line

$$\langle \hat{S}_{\boldsymbol{xy}} \rangle_Y = \frac{1}{N_g} \langle \mathrm{Tr}[U^\dagger_{N,\boldsymbol{x}} U_{N,\boldsymbol{y}}] \rangle_\nu \qquad U^\dagger_{n,\boldsymbol{x}} = \mathrm{e}^{\mathrm{i}\epsilon g \alpha^n_{L,\boldsymbol{x}}} \, U^\dagger_{n-1,\boldsymbol{x}} \mathrm{e}^{-\mathrm{i}\epsilon g \alpha^n_{R,\boldsymbol{x}}}$$

$$\alpha^n_{L,\boldsymbol{x}} = \frac{1}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{xz}} \, \nu^{ia}_{n,\boldsymbol{z}} \, T^a, \quad \alpha^n_{R,\boldsymbol{x}} = \frac{1}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{xz}} \, \nu^{ia}_{n,\boldsymbol{z}} U^{\dagger ab}_{n-1,\boldsymbol{z}} \, T^b$$

- ν: white noise $\quad \langle \nu^{ia}_{m,\boldsymbol{x}} \, \nu^{jb}_{n,\boldsymbol{y}} \rangle = \frac{1}{\epsilon} \delta^{ij} \delta^{ab} \delta_{mn} \delta_{\boldsymbol{xy}}$

- Only R-field depends on previous Wilson line → cascade



Wave-function squared (WFS)

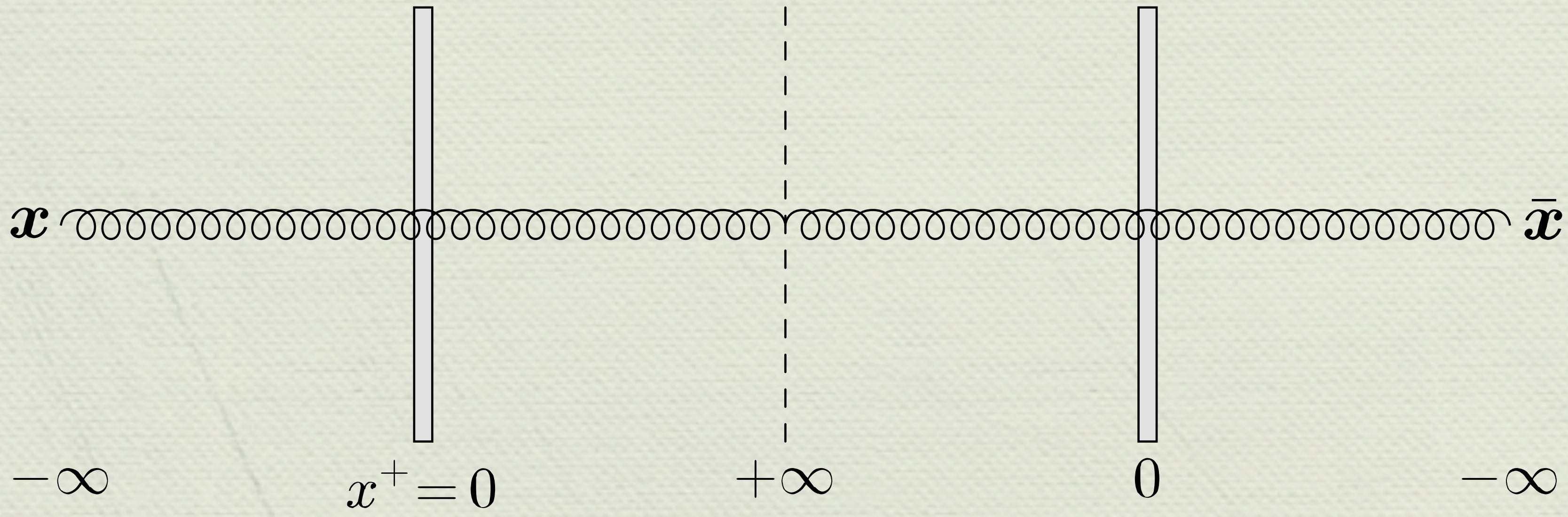
- "Physical" gluon: "mathematical" gluonic dipole

$$\hat{S}_{12}(\boldsymbol{x}\bar{\boldsymbol{x}}) = \frac{1}{N_c}\,\mathrm{Tr}[\bar{U}_{\bar{\boldsymbol{x}}}U^{\dagger}_{\boldsymbol{x}}]$$

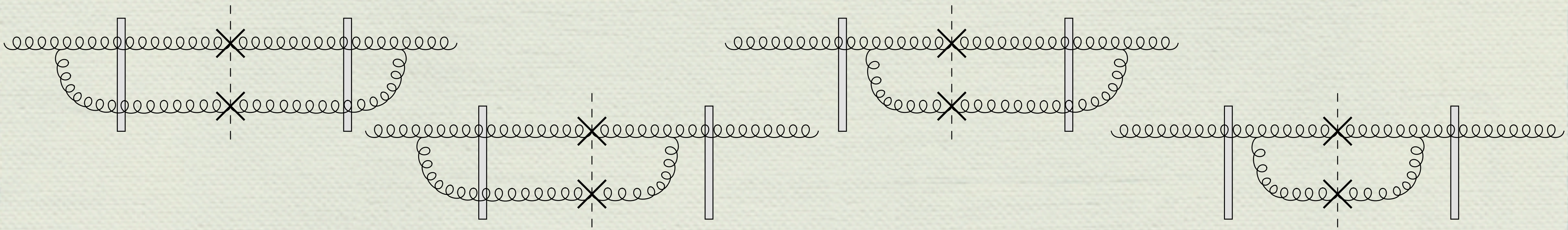
Different coordinates in DA and CCA: FT → "measure gluons"

"Barred" Wilson line to keep track of WF in the CCA

Will act as a generating functional for observables



Resolving same rapidity gluons



- For $\alpha(\eta_p - \eta_k) \ll 1$, no intermediate gluon radiation

$$\frac{\mathrm{d}\sigma_{2g}}{\mathrm{d}\eta_p \mathrm{d}^2\boldsymbol{p}\,\mathrm{d}\eta_k \mathrm{d}^2\boldsymbol{k}} = \frac{1}{(2\pi)^4}\int_{\boldsymbol{x}\bar{\boldsymbol{x}}} \mathrm{e}^{-\mathrm{i}\boldsymbol{p}\cdot(\boldsymbol{x}-\bar{\boldsymbol{x}})} \left\langle H_{\mathrm{prod}}(\boldsymbol{k}) \hat{S}_{12}(\boldsymbol{x}\bar{\boldsymbol{x}})\big|_{\bar{U}=U} \right\rangle_Y$$

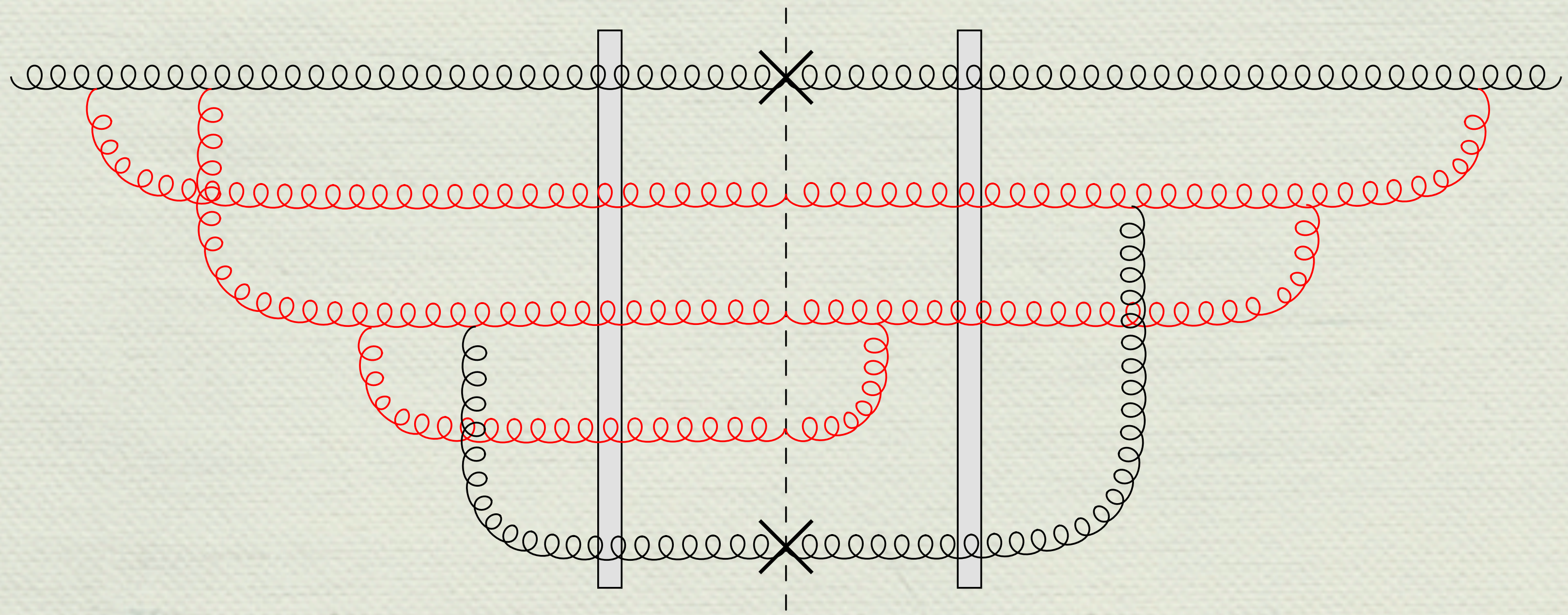
$$H_{\mathrm{prod}}(\boldsymbol{k}) = \frac{1}{4\pi^3}\int_{\boldsymbol{y}\bar{\boldsymbol{y}}} \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{y}-\bar{\boldsymbol{y}})} \int_{\boldsymbol{u}\boldsymbol{v}} \mathcal{K}^i_{\boldsymbol{y}\boldsymbol{u}} \mathcal{K}^i_{\bar{\boldsymbol{y}}\boldsymbol{v}} \left[L^a_{\boldsymbol{u}} - U^{\dagger ab}_{\boldsymbol{y}} R^b_{\boldsymbol{u}}\right]\left[\bar{L}^a_{\boldsymbol{v}} - \bar{U}^{\dagger ac}_{\bar{\boldsymbol{y}}} \bar{R}^c_{\boldsymbol{v}}\right]$$

- H_{prod} reduces to JIMWLK if $\int d^2k$ and remove bars

$$\left\langle R^b_{\boldsymbol{u}}\, \bar{R}^c_{\boldsymbol{v}}\, \hat{S}_{12}(\boldsymbol{x}\bar{\boldsymbol{x}})\big|_{\bar{U}=U} \right\rangle_Y = \int [DU] W_Y[U]\, \frac{1}{N_g}\,\mathrm{Tr}\left[\left(R^c_{\boldsymbol{v}} U_{\bar{\boldsymbol{x}}}\right)\left(R^b_{\boldsymbol{u}} U^\dagger_{\boldsymbol{x}}\right)\right]$$

- Bars disappeared, **new structures** *RU, LU*

Inserting unresolved gluons



- For $\alpha(Y\text{-}Y_A) \gtrsim 1$, intermediate gluon radiation
- Evolve (dress) gluon WFS from Y_A to Y
- Initial condition at Y_A: U_A and $\bar{U}_A$

$$\left\langle \hat{S}_{12}(\boldsymbol{x}\bar{\boldsymbol{x}}) \right\rangle_{Y-Y_A}^{A} = \int [DU\, D\bar{U}]\, W_{Y-Y_A}[U, \bar{U} | U_A, \bar{U}_A]\, \frac{1}{N_g}\, \mathrm{Tr}\left[\bar{U}_{\bar{\boldsymbol{x}}} U_{\boldsymbol{x}}^{\dagger}\right]$$

- Production of gluon at Y_A

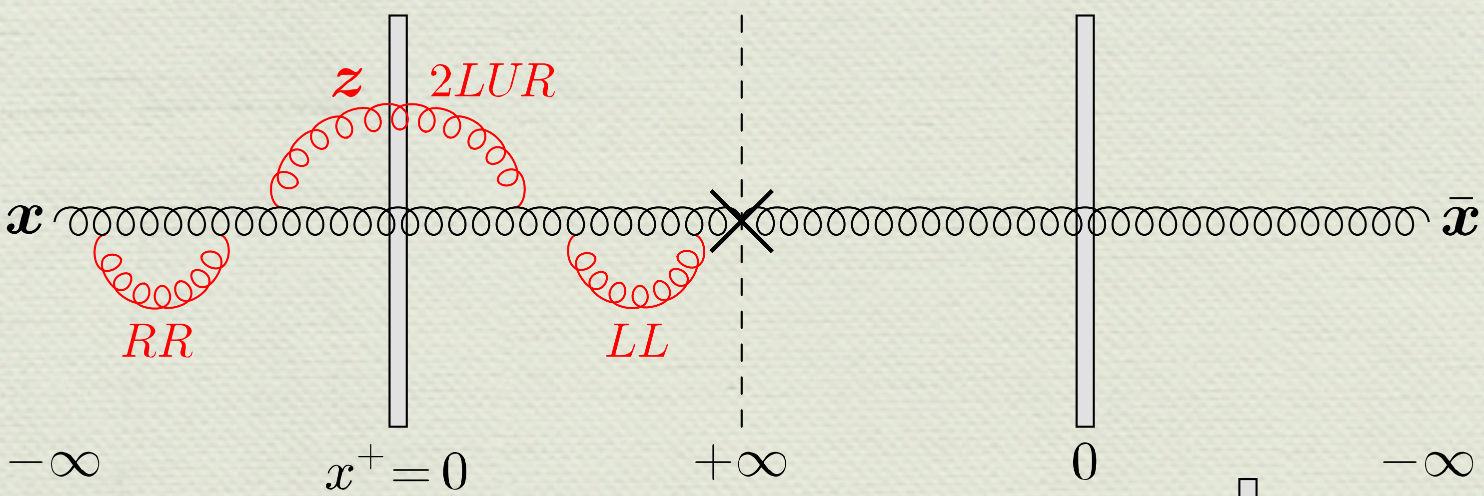
$$\frac{\mathrm{d}\sigma_{2g}}{\mathrm{d}Y\,\mathrm{d}^2\boldsymbol{p}\,\mathrm{d}Y_A\,\mathrm{d}^2\boldsymbol{k}_A} \propto \int [DU_A]\, W_{Y_A}[U_A] \left[H_{\mathrm{prod}}^{A} \left\langle \hat{S}_{12}(\boldsymbol{x}\bar{\boldsymbol{x}}) \right\rangle_{Y-Y_A}^{A} \right]_{\bar{U}_A = U_A}$$

Evolution Hamiltonian

- Conditional weight-function evolves as

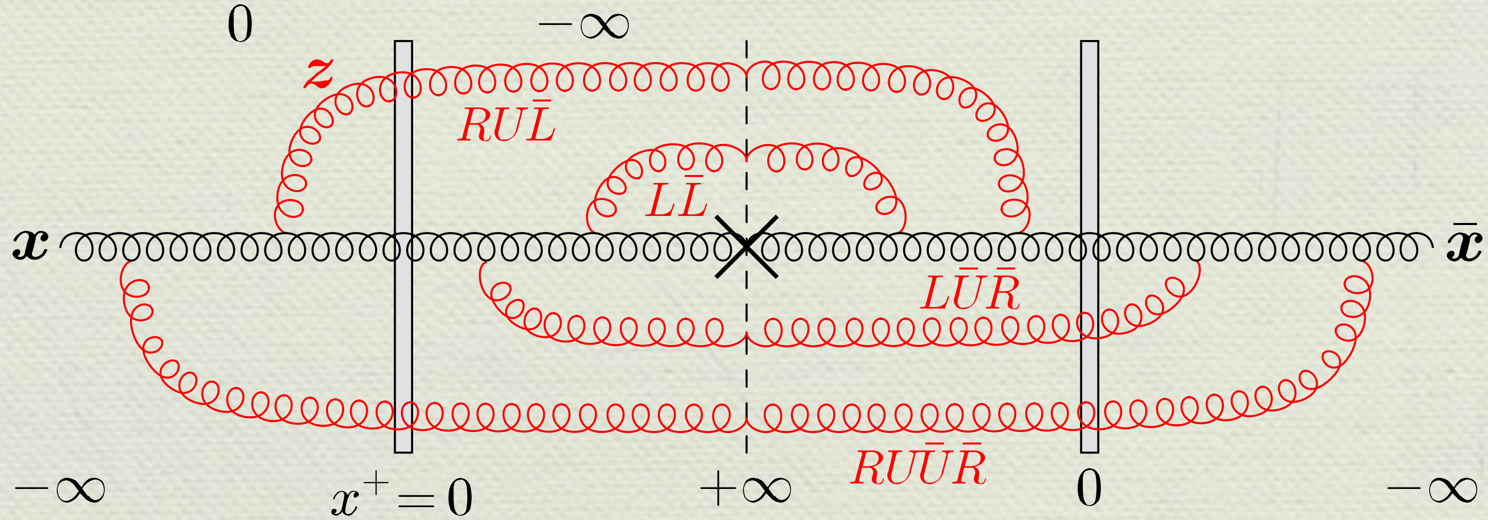
$$\frac{\partial}{\partial Y} W_Y[U,\bar{U}|U_A,\bar{U}_A] = \big(\underbrace{H_{11}+H_{22}+2H_{12}}_{H_{\rm evol}}\big) W_Y[U,\bar{U}|U_A,\bar{U}_A]$$

- Viewed as projectile evolution



$= H_{11}$

$H_{12} =$



1st approach: evolution of projectile WFS

- Large-N_c : closed diagonal system of two equations
  Stating the result up to a FT

$$\frac{\partial\langle N_{\boldsymbol{k}}(\boldsymbol{x}\bar{\boldsymbol{x}})\rangle_Y}{\partial Y} = \frac{\bar{\alpha}}{2\pi}\int_{\boldsymbol{z}}\Big\{(\mathcal{K}_{\boldsymbol{x}\boldsymbol{x}\boldsymbol{z}} - \mathcal{K}_{\boldsymbol{x}\bar{\boldsymbol{x}}\boldsymbol{z}})\big[\langle\hat{S}^F(\boldsymbol{x}\boldsymbol{z})\rangle_Y\langle N_{\boldsymbol{k}}(\boldsymbol{z}\bar{\boldsymbol{x}})\rangle_Y - \langle N_{\boldsymbol{k}}(\boldsymbol{x}\bar{\boldsymbol{x}})\rangle_Y\big]$$
$$+(\mathcal{K}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}\boldsymbol{z}} - \mathcal{K}_{\boldsymbol{x}\bar{\boldsymbol{x}}\boldsymbol{z}})\big[\langle\hat{S}^F(\boldsymbol{z}\bar{\boldsymbol{x}})\rangle_Y\langle N_{\boldsymbol{k}}(\boldsymbol{x}\boldsymbol{z})\rangle_Y - \langle N_{\boldsymbol{k}}(\boldsymbol{x}\bar{\boldsymbol{x}})\rangle_Y\big]$$
$$+\mathcal{K}_{\boldsymbol{x}\bar{\boldsymbol{x}}\boldsymbol{z}}\big[\langle\hat{S}^F(\boldsymbol{x}\bar{\boldsymbol{x}})\rangle_Y\langle N_{\boldsymbol{k}}(\boldsymbol{z}\boldsymbol{z})\rangle_Y - \langle N_{\boldsymbol{k}}(\boldsymbol{x}\bar{\boldsymbol{x}})\rangle_Y + \langle N^{(2)}_{\boldsymbol{k}}(\boldsymbol{x}\bar{\boldsymbol{x}}\boldsymbol{z}\boldsymbol{z})\rangle_Y\big]\Big\}$$

with S^F solving BK and where ...

$$\frac{\partial\langle N^{(2)}_{\boldsymbol{k}}(\boldsymbol{x}\bar{\boldsymbol{x}}\boldsymbol{z}\boldsymbol{z})\rangle_Y}{\partial Y} = \frac{\bar{\alpha}}{4\pi}\int_{\boldsymbol{y}}(\mathcal{M}_{\boldsymbol{x}\boldsymbol{z}\boldsymbol{w}} + \mathcal{M}_{\boldsymbol{x}\bar{\boldsymbol{x}}\boldsymbol{w}} - \mathcal{M}_{\bar{\boldsymbol{x}}\boldsymbol{z}\boldsymbol{w}})\langle\hat{S}^F(\boldsymbol{x}\boldsymbol{w})\rangle_Y\langle N^{(2)}_{\boldsymbol{k}}(\boldsymbol{w}\bar{\boldsymbol{x}}\boldsymbol{z}\boldsymbol{z})\rangle_Y$$
$$+(\mathcal{M}_{\bar{\boldsymbol{x}}\boldsymbol{z}\boldsymbol{w}} + \mathcal{M}_{\boldsymbol{x}\bar{\boldsymbol{x}}\boldsymbol{w}} - \mathcal{M}_{\boldsymbol{x}\boldsymbol{z}\boldsymbol{w}})\langle\hat{S}^F(\boldsymbol{w}\bar{\boldsymbol{x}})\rangle_Y\langle N^{(2)}_{\boldsymbol{k}}(\boldsymbol{x}\boldsymbol{w}\boldsymbol{z}\boldsymbol{z})\rangle_Y$$
$$-(\mathcal{M}_{\boldsymbol{x}\boldsymbol{z}\boldsymbol{w}} + \mathcal{M}_{\bar{\boldsymbol{x}}\boldsymbol{z}\boldsymbol{w}} + \mathcal{M}_{\boldsymbol{x}\bar{\boldsymbol{x}}\boldsymbol{w}})\langle N^{(2)}_{\boldsymbol{k}}(\boldsymbol{x}\bar{\boldsymbol{x}}\boldsymbol{z}\boldsymbol{z})\rangle_Y$$
$$-(\mathcal{M}_{\bar{\boldsymbol{x}}\boldsymbol{z}\boldsymbol{w}} + \mathcal{M}_{\boldsymbol{x}\boldsymbol{z}\boldsymbol{w}} - \mathcal{M}_{\boldsymbol{x}\bar{\boldsymbol{x}}\boldsymbol{w}})\langle N^{(2)}_{\boldsymbol{k}}(\boldsymbol{x}\bar{\boldsymbol{x}}\boldsymbol{w}\boldsymbol{w})\rangle_Y$$
$$-(\mathcal{M}_{\bar{\boldsymbol{x}}\boldsymbol{z}\boldsymbol{w}} + \mathcal{M}_{\boldsymbol{x}\boldsymbol{z}\boldsymbol{w}} - \mathcal{M}_{\boldsymbol{x}\bar{\boldsymbol{x}}\boldsymbol{w}})\langle\hat{S}^F(\boldsymbol{x}\bar{\boldsymbol{x}})\rangle_Y\langle N^{(2)}_{\boldsymbol{k}}(\boldsymbol{w}\boldsymbol{w}\boldsymbol{z}\boldsymbol{z})\rangle_Y.$$

$2^{\rm nd}$ approach: Langevin (finite-N_c)

- Building the generating functional

$$\left\langle \hat{S}_{12}(\boldsymbol{x}\bar{\boldsymbol{x}})\right\rangle^{A}_{Y-Y_A} = \frac{1}{N_g}\left\langle \mathrm{Tr}\big[\bar{U}_{N_A,\bar{\boldsymbol{x}}} U^\dagger_{N_A,\boldsymbol{x}}\big]\right\rangle_\nu$$

In DA same as standard JIMWLK. In CCA

$$\bar{U}^\dagger_{n,\boldsymbol{x}} = \exp[\mathrm{i}\epsilon g \bar{\alpha}^n_{L,\bar{\boldsymbol{x}}}]\, \bar{U}^\dagger_{n-1,\bar{\boldsymbol{x}}} \exp[-\mathrm{i}\epsilon g \bar{\alpha}^n_{R,\bar{\boldsymbol{x}}}]$$

Left and right fields

$$\bar{\alpha}^n_{L,\boldsymbol{x}} = \frac{1}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{x}\boldsymbol{z}}\, \nu^{ia}_{n,\boldsymbol{z}}\, T^a, \qquad \bar{\alpha}^n_{R,\boldsymbol{x}} = \frac{1}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{x}\boldsymbol{z}}\, \nu^{ia}_{n,\boldsymbol{z}} \bar{U}^{\dagger ab}_{n-1,\boldsymbol{z}}\, T^b$$

Same noise, only initial condition differs

- Not enough for cross section

2nd approach: Langevin (finite-N_c)

□ Building the cross section. For instance, one of the 4 terms

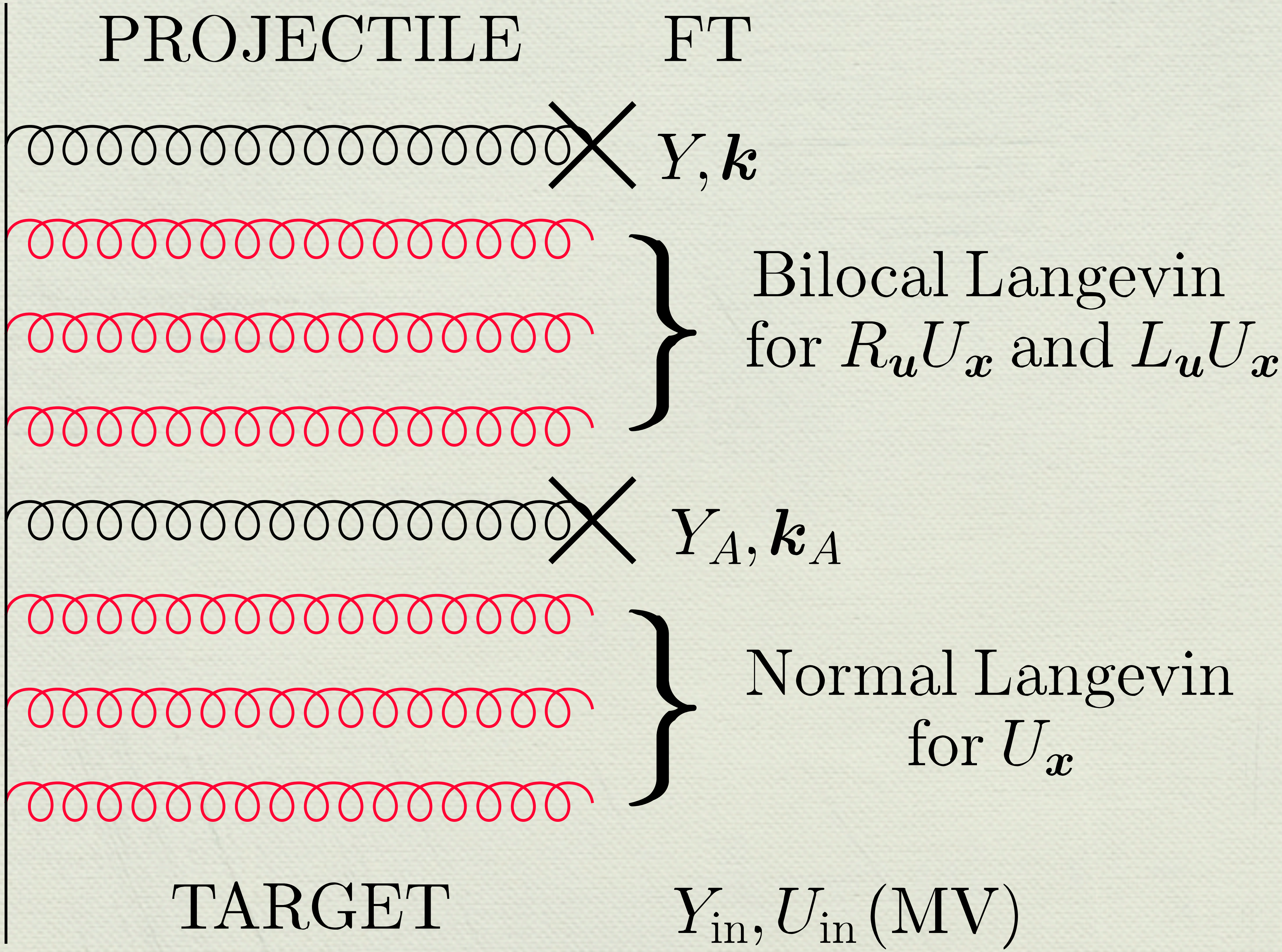
$$R^a_{A,\boldsymbol{u}}\,\bar{R}^b_{A,\boldsymbol{v}}\,\langle \hat{S}_{12}(\boldsymbol{x}\bar{\boldsymbol{x}})\rangle^A_{Y-Y_A}\big|_{\bar{U}_A=U_A} = \frac{1}{N_g}\,\big\langle \mathrm{Tr}\big[\big(R^b_{A,\boldsymbol{v}}U_{N_A,\bar{\boldsymbol{x}}}\big)\big(R^a_{A,\boldsymbol{u}}U^\dagger_{N_A,\boldsymbol{x}}\big)\big]\big\rangle_\nu$$

□ R-derivatives act at U_A and $\bar{U}_A$ at Y_A. Need a recurrence formula

$$\begin{aligned} R^a_{A,\boldsymbol{u}}U^\dagger_{n,\boldsymbol{x}} =\ & \exp[\mathrm{i}\epsilon g\alpha^n_{L,\boldsymbol{x}}]\big(R^a_{A,\boldsymbol{u}}U^\dagger_{n-1,\boldsymbol{x}}\big)\exp[-\mathrm{i}\epsilon g\alpha^n_{R,\boldsymbol{x}}] \\ & -\frac{\mathrm{i}\epsilon g}{\sqrt{4\pi^3}}\int_{\boldsymbol{z}}\mathcal{K}^i_{\boldsymbol{x}\boldsymbol{z}}\nu^{ic}_{n,\boldsymbol{z}}\big(R^a_{A,\boldsymbol{u}}U^\dagger_{n-1,\boldsymbol{z}}\big)^{cb}\exp[\mathrm{i}\epsilon g\alpha^n_{L,\boldsymbol{x}}]U^\dagger_{n-1,\boldsymbol{x}}T^b \end{aligned}$$

□ Initially local $R^a_{A,\boldsymbol{u}}U^\dagger_{A,\boldsymbol{x}} = \mathrm{i}g\delta_{\boldsymbol{u}\boldsymbol{x}}U^\dagger_{A,\boldsymbol{x}}T^a$
becomes bi-local already after 1st step.

Putting all together



Conclusion

- Di-gluon production at different rapidities in pA collisions, e.g. forward-central

- Langevin equation with bi-local structures. Looks feasible. No alternative...

- Generalization: three separated gluons $\rightarrow$ tri-local structures, ...