Solution and properties of the resummed BK equation and KGBJS equation

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Krzysztof Kutak, D. T., arXiv:1306.3369 [hep-ph]

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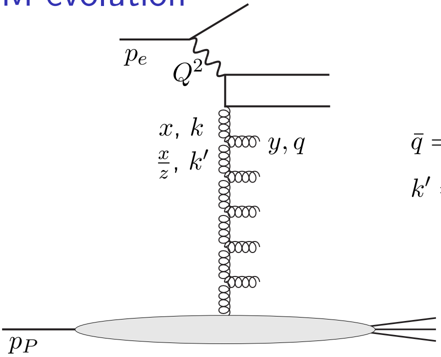
LIDER

CCFM evolution



$\bar{q} = \frac{q}{1-z}$

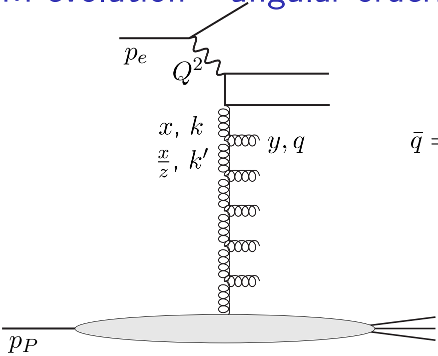
$k' = |\mathbf{k} + \mathbf{q}|$

$$\mathcal{A}(x, k^2, p) = \mathcal{A}_0(x, k^2, p) + \int \frac{d^2\bar{\mathbf{q}}}{\pi \bar{q}^2} \int_{x/x_0}^{1-Q_0/\bar{q}} dz\, \theta(p - z\bar{q}) P_{gg}(z, k, \bar{q}, p) \mathcal{A}\left(\frac{x}{z}, k'^2, \bar{q}\right)$$

$$P_{gg}(z, k, \bar{q}, p) = \bar{\alpha}_s \Delta_s(p, z\bar{q}) \left(\frac{\Delta_{ns}(z, k, \bar{q})}{z} + \frac{1}{1-z} \right)$$

Reviewed in: Avsar, Iancu, **Nucl. Phys. A 829** (2009) **31**

CCFM evolution – angular ordering



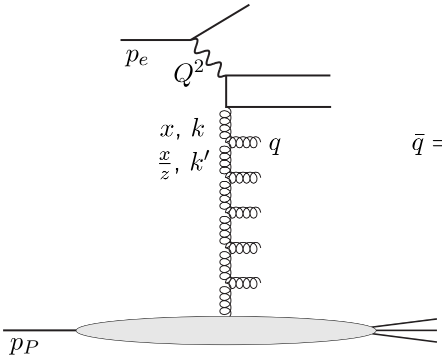
$$\bar{q} = \frac{q}{1-z} \qquad \xi = \frac{q}{yE}$$

$$E\xi = z\bar{q}$$

$$\mathcal{A}(x, k^2, p) = \mathcal{A}_0(x, k^2, p) + \int \frac{\mathrm{d}^2\bar{\mathbf{q}}}{\pi \bar{q}^2} \int_{x/x_0}^{1-Q_0/\bar{q}} \mathrm{d}z\, \theta(p - z\bar{q}) P_{gg}(z, k, \bar{q}, p) \mathcal{A}\left(\frac{x}{z}, k'^2, \bar{q}\right)$$

$$\bar{\xi} > \xi_{i+1} > \xi_i$$

CCFM evolution – low x limit



$\bar{q} = \frac{q}{1-z}$

$$\mathcal{A}(x, k^2, p) = \mathcal{A}_0(x, k^2, p) + \bar{\alpha}_s \int \frac{\mathrm{d}^2\bar{\mathbf{q}}}{\pi \bar{q}^2} \int_{x/x_0}^{1} \frac{\mathrm{d}z}{z}\, \theta(p - z\bar{q}) \Delta_{ns}(z, k, \bar{q}) \mathcal{A}\left(\frac{x}{z}, k'^2, \bar{q}\right)$$

High energy factorization (k_T-factorization) applied to DIS

Working with BFKL/BK:

$$F_2(x, Q^2) = \sum_q e_q^2 \int \frac{\mathrm{d}k}{k} \sigma_{F_2}\left(k^2, Q^2, m_q^2\right) \mathcal{F}(x, k^2)$$

Working with CCFM/KGBJS:

$$F_2(x, Q^2) = \sum_q e_q^2 \int \frac{\mathrm{d}k}{k} \int_x^1 \frac{\mathrm{d}z}{z} \hat{\sigma}_{F_2}\left(z, k^2, Q^2, m_q^2\right) \mathcal{A}(\frac{x}{z}, k, p)$$

$$p^2 = \frac{Q^2}{z(1-z)}$$

Reformulated Balitsky-Kovchegov evolution equation

A new form introduces a new scale μ to sum low-q emissions:

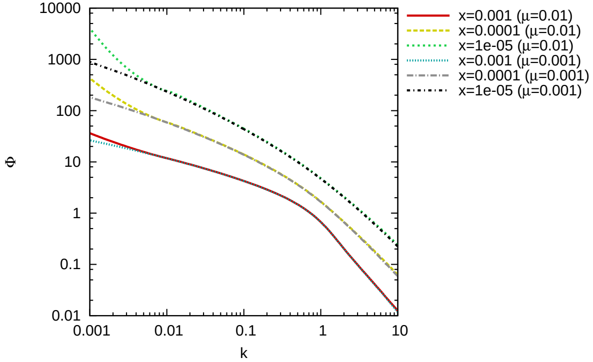
$$\phi(x,k) = \tilde{\phi}_0(x,k) + \bar{\alpha}_s \int_x^1 \frac{\mathrm{d}z}{z} \int \frac{\mathrm{d}^2 q}{\pi q^2} \theta(q^2-\mu^2) \Delta_R(z,k,\mu) \left(\phi(\frac{x}{z}, |k+q|^2) - \frac{q^2}{\pi R^2} \delta(q^2-k^2) \phi^2(\frac{x}{z}, q^2) \right)$$

with

$$\Delta_R(z,k,\mu) = \exp\left(-\bar{\alpha}_s \log\frac{1}{z} \log\frac{k^2}{\mu^2} \right)$$

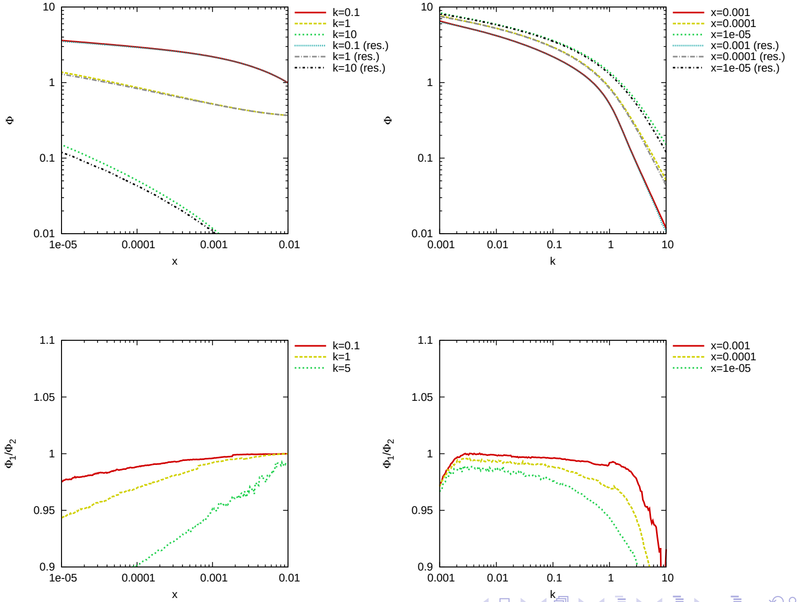
Kutak, Golec-Biernat, Jadach, Skrzypek, **JHEP 1202** (2012) **117**

Reformulated BK – sensitivity to μ



The new form works down to $k = \mu$.

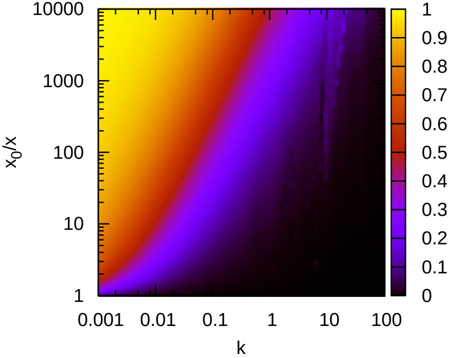
Reformulated BK – consistency check



BK evolution – nonlinear behavior

Following Kutak, Stasto, **Eur. Phys. J. C 41** (2005) **343**, define:

$$\beta(x,k) = \frac{|\Phi_{BFKL}(x,k) - \Phi_{BK}(x,k)|}{\Phi_{BFKL}(x,k)}$$



$\ln Q_s$ grows linearly with $C \ln \frac{1}{x}$

BK vs KGBJS

The BK equation:

$$\phi(x,k) = \tilde{\phi}_0(x,k) + \bar{\alpha}_s \int_{x/x_0}^{1} \frac{\mathrm{d}z}{z} \int \frac{\mathrm{d}^2\mathbf{q}}{\pi q^2} \theta(q^2-\mu^2)\Delta_R(z,k,\mu) \left(\phi(\frac{x}{z}, k'^2) - \frac{q^2}{\pi R^2} \delta(q^2-k^2) \phi^2(\frac{x}{z}, q^2) \right)$$

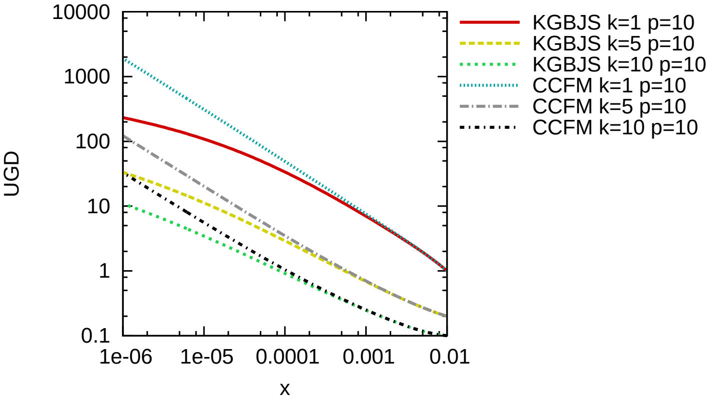
promoted to a nonlinear CCFM-based equation:

$$\mathcal{E}(x,k^2,p) = \mathcal{E}_0(x,k^2,p) + \bar{\alpha}_s \int_{x/x_0}^{1} \frac{\mathrm{d}z}{z} \int \frac{\mathrm{d}^2\bar{\mathbf{q}}}{\pi \bar{q}^2} \theta(p - z\bar{q})\Delta_{ns}(z,k,\bar{q}) \left(\mathcal{E}\left(\frac{x}{z}, k'^2, \bar{q}\right) - \frac{\bar{q}^2}{\pi R^2} \delta(\bar{q}^2-k^2)\, \mathcal{E}^2(\frac{x}{z}, \bar{q}^2, \bar{q}) \right)$$

Kutak, Golec-Biernat, Jadach, Skrzypek, **JHEP 02** (2012) **117**

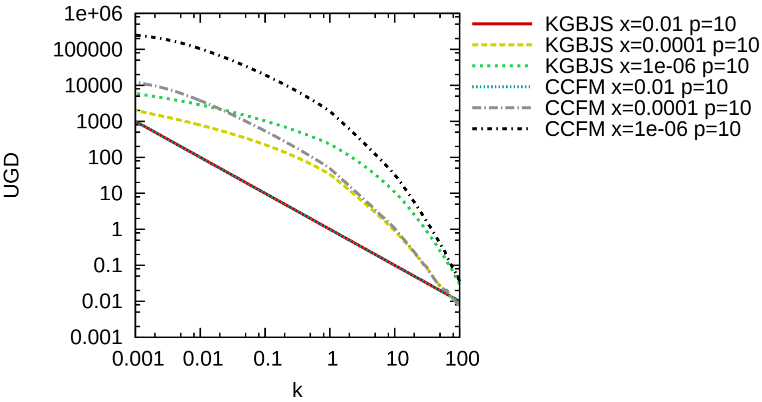
Kutak, **JHEP 12** (2012) **33**

KGBJS – solution

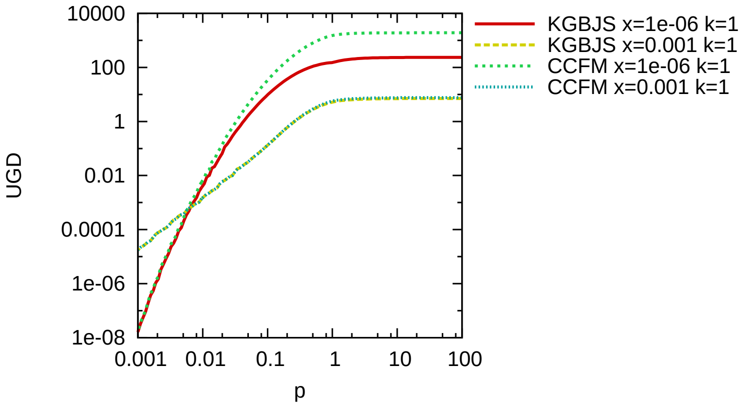


Krzysztof Kutak, D. T., arXiv:**1306.3369** [hep-ph]

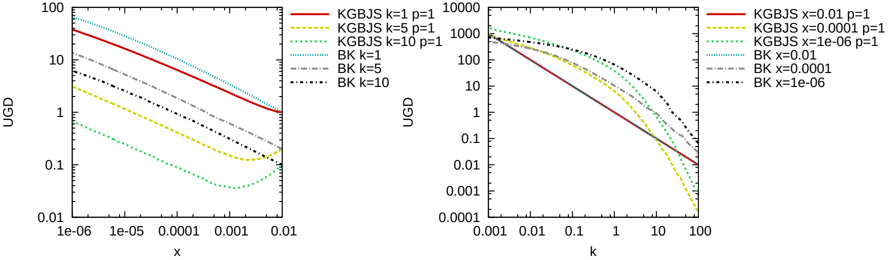
KGBJS – solution



KGBJS – solution

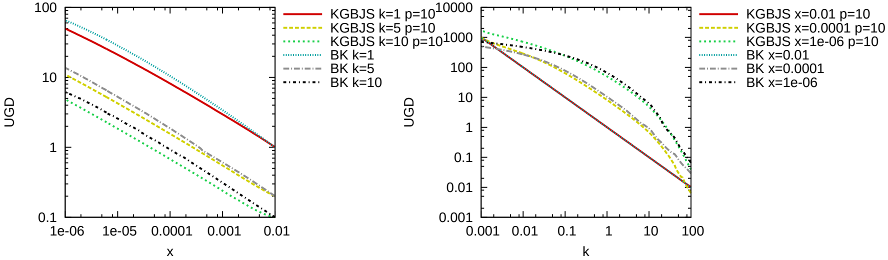


Solutions of KGBJS and BK



For $p \to \infty$, KGBJS becomes similar to BK.

Solutions of KGBJS and BK



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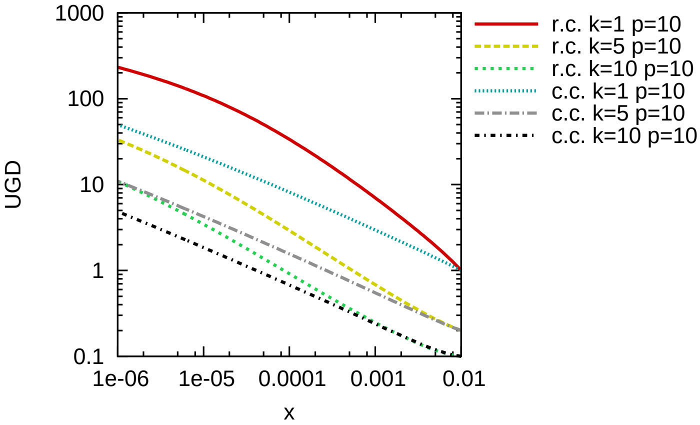
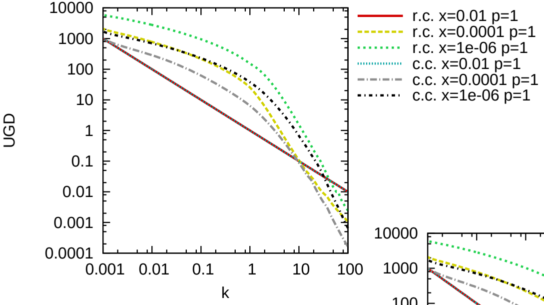
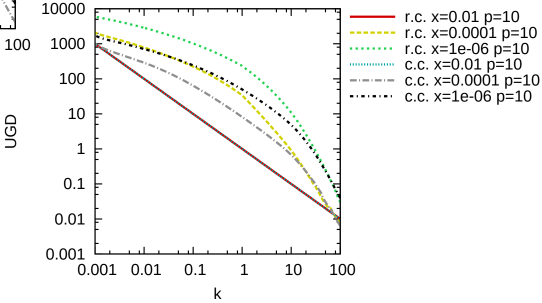
KGBJS – the effect of running coupling



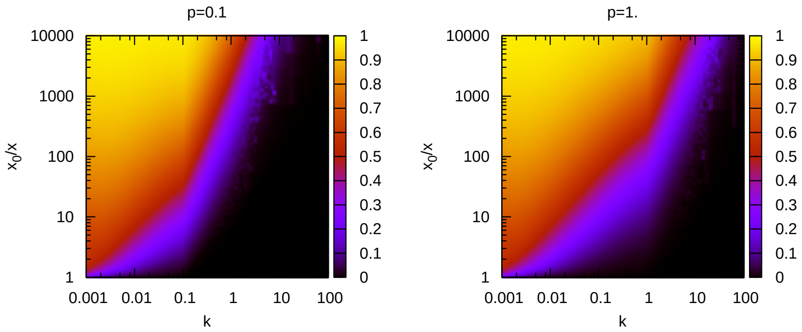
$$\alpha_s(k^2) = \frac{12\pi}{27} \frac{1}{\ln \frac{\max\{k^2, 1\text{GeV}^2\}}{(0.2\text{GeV})^2}} \qquad \alpha_s = \frac{\pi}{3} 0.2$$

KGBJS – the effect of running coupling





KGBJS – limits of linear behavior

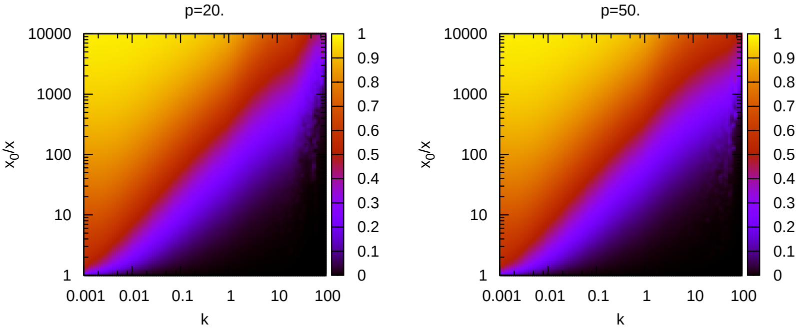


Contour lines indicate shape of the $Q_s(x,p)$ dependecy:

$$\beta(x, Q_s(x,p), p) = const.$$

Qualitatively similar effect to the saturation lines observed by Avsar, Stasto, **JHEP 1006** (2010) **112**.

KGBJS – limits of linear behavior



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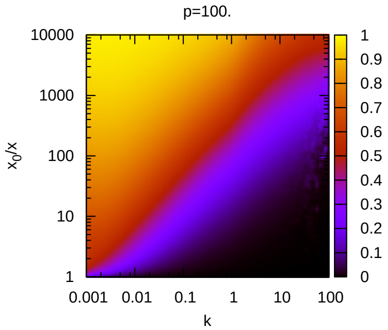
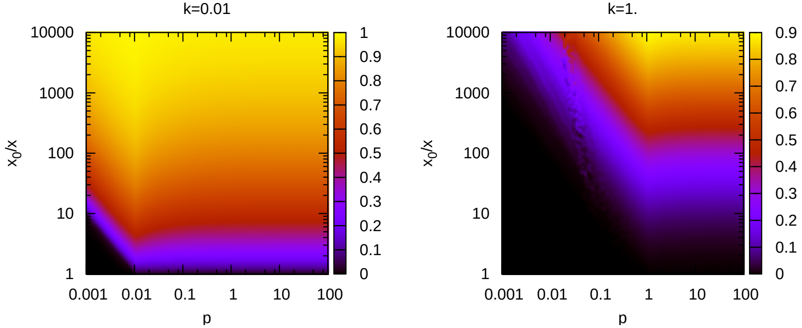


For large p, the growth of of the saturation scale is "liberated".

KGBJS – limits of linear behavior



One may also consider the $P_s(x,k)$ dependecy:

$$\beta\left(x,k,P_s(x,k)\right) = const$$

Note that the quadratic term of KGBJS is:

$$-\frac{1}{\pi R^2}\int_x^{x_0}\frac{\mathrm{d}w}{w}\theta(p-zk)P_{gg}(z,k,k)\mathcal{E}^2\left(w,k,k\right)$$

Thank you.

Future work:

- include Δ_s and full splitting function in the calculations
- test variants of kinematical constraints
- evolve high-energy-factorizable gluon density directly
- study phenomenological implications

The Balitsky-Kovchegov evolution equation

Integral form in momentum space:

$$\phi(x,k) = \phi_0(x,k) + \bar{\alpha}_s \int_x^1 \frac{\mathrm{d}z}{z} \int_0^\infty \frac{\mathrm{d}l^2}{l^2} \left(\frac{l^2 \phi(\frac{x}{z}, l^2) - k^2 \phi(\frac{x}{z}, k^2)}{|k^2 - l^2|} + \frac{k^2 \phi(\frac{x}{z}, k)}{\sqrt{4l^4 + k^4}} \right) - \bar{\alpha}_s \int_x^1 \frac{\mathrm{d}z}{z} \phi^2(\frac{x}{z}, k)$$

CCFM evolution – Δ_{ns}

$$\mathcal{A}(x,k^2,p) = \mathcal{A}_0(x,k^2,p) + \bar{\alpha}_s \int \frac{\mathrm{d}^2\bar{\mathbf{q}}}{\pi\bar{q}^2} \int_{x/x_0}^{1} \frac{\mathrm{d}z}{z}\, \theta(p - z\bar{q}) \Delta_{ns}(z,k,\bar{q}) \mathcal{A}\left(\frac{x}{z}, k'^2, \bar{q}\right)$$

$$\Delta_{ns} = \exp\left(-\bar{\alpha}_s \ln\frac{1}{z} \ln\frac{k^2}{zq^2}\right) \quad \text{for} \quad k^2 > zq^2$$

In this work, calculations were done with Δ_{ns} after Kwiecinski, Martin, Sutton, **PRD 52** (1995) **1445**:

$$\Delta_{ns}(z,k,q) = \exp\left(-\bar{\alpha}_s \ln\frac{z_0}{z} \ln\frac{k^2}{z_0 z q^2}\right)$$

with

$$z_0 = \begin{cases} z & \text{for } \frac{k}{q} < z \\ \frac{k}{q} & \text{otherwise} \\ 1 & \text{for } \frac{k}{q} > 1 \end{cases}$$