Turbulent thermalization of the Quark Gluon Plasma

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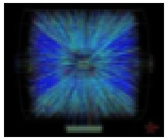
Heavy-ion collisions



- fluid-like behavior from very early time on
- very special transport properties, such as small η/s

How is local isotropization/thermalization achieved?

Progress in a first-principle understanding from two limiting cases

Holographic thermalization:

a) strong coupling? *Heller, Janik, Witaszczyk; Chesler, Yaffe ...*

Sizeable anisotropy at transition to hydrodynamic regime

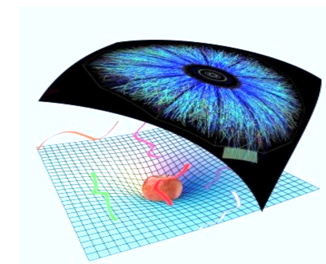


Fig. from strings.net.technion.ac.il

Turbulent thermalization:

b) weak coupling but highly occupied? *CGC: McLerran, Venugopalan ...*

Energy density of gluons with typical momentum Q_s (at time ~$1/Q_s$)

$\epsilon \sim \frac{Q_s^4}{\alpha_s}$ i.e. 'occupation numbers' $n(p \lesssim Q_s) \sim \frac{1}{\alpha_s}$

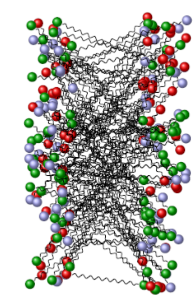


Fig. by T. Epelbaum

Non-equilibrium dynamics

Initial state:
Far from equilibrium

Non-equilibrium dynamics

Final state:
Thermal equilibrium

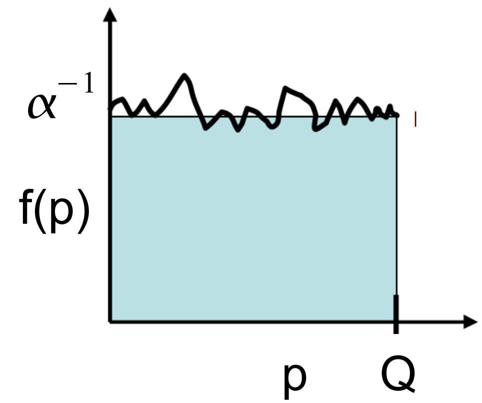
How is thermal equilibrium achieved?

Non-equilibrium dynamics

Solve ***Initial Value Problem*** in QCD

- ***Initial conditions:***

based on ***color glass condensate*** (CGC) description of heavy ion collisions ***(n(p)~1/α)***



- ***Non-equilibrium dynamics:***

- Classical-statistical lattice simulations (SU(2) for numerical studies) **(n(p)>>1)**

- Kinetic theory **(n(p)<~1/α)** (analytic discussion)

First principle

Intuitive picture

Turbulent Thermalization

Non-equilibrium phenomena may be shared by a large class of strongly correlated many-body systems

I) Thermalization in scalar field theory – Cosmology

(Micha, Tkachev PRD 70 (2004) 043538)

II) Thermalization in Yang-Mills theory in Minkowski space

(Berges,SS,Sexty PRD 86 (2012) 074006; SS PRD 86 (2012) 065008)

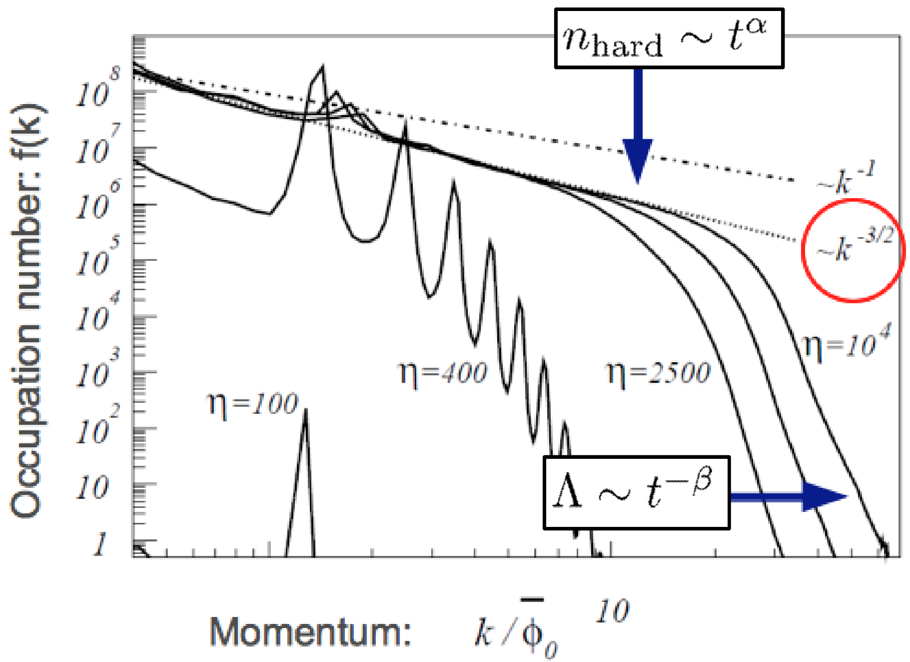
III) Thermalization in heavy-ion collisions at ultra-relativistic energies, weak coupling and large nuclei

(Berges,Boguslavski,SS,Venugopalan arXiv:1303.5650 [hep-ph])

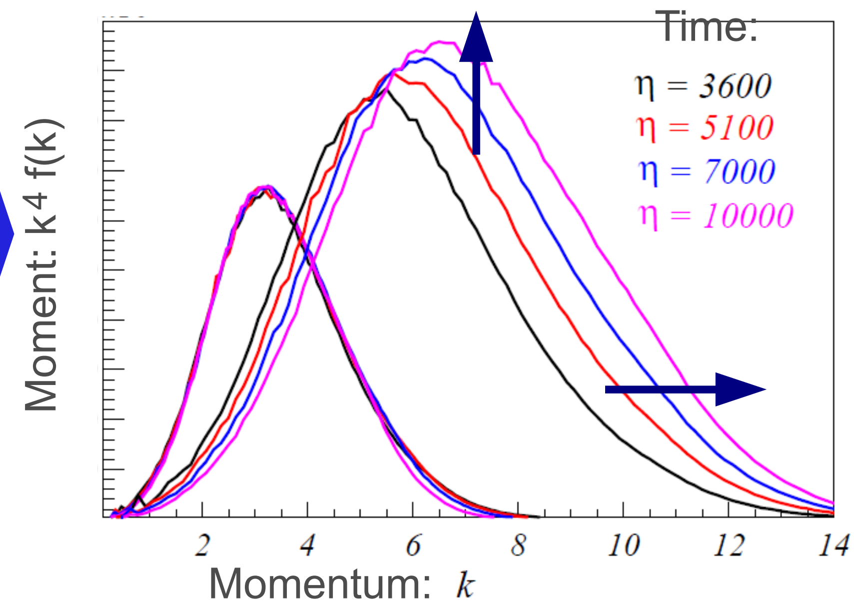
Turbulent thermalization - Cosmology

Model for thermalization of the early universe: Scalar field theory ($\lambda\Phi^4$)
Homogeneous background field Φ_0 + vacuum fluctuations

Non-thermal fixed point



Self-similar evolution



- The thermalization process is described by a ***quasi-stationary evolution*** with ***scaling exponents*** Dynamic: α=-4/5 β=-1/5 Spectral: κ=-3/2

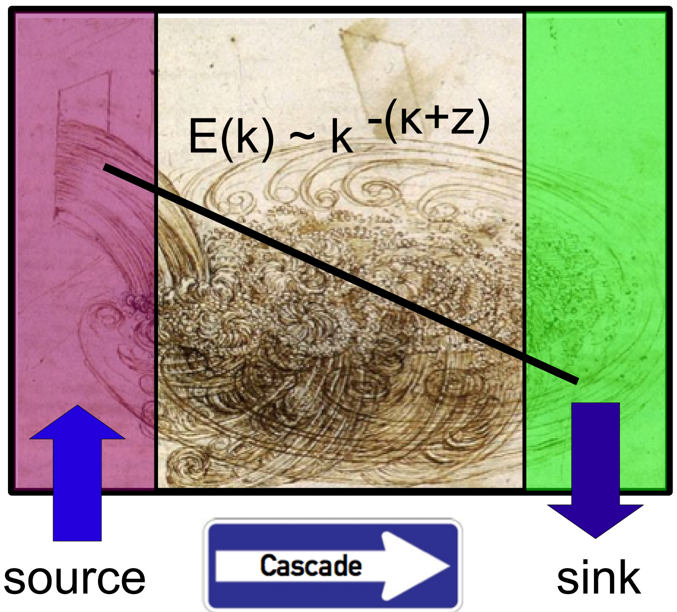
(Micha, Tkachev PRD 70 (2004) 043538)

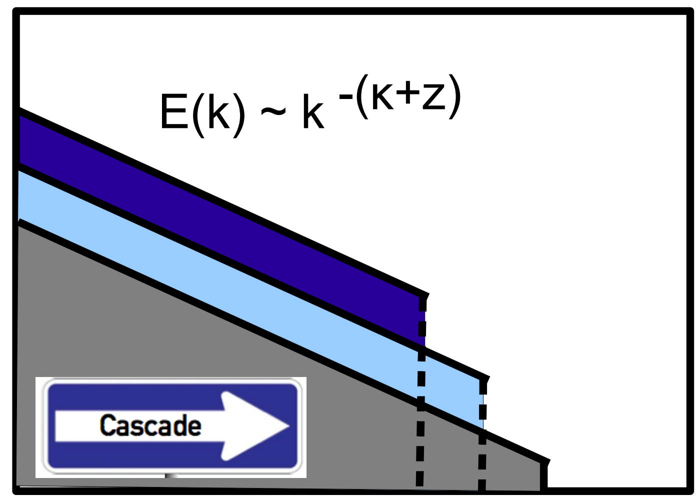
Wave turbulence

"Driven" Turbulence – Kolmogorov wave turbulence

vs.

"Free" Turbulence – Turbulent Thermalization





closed system

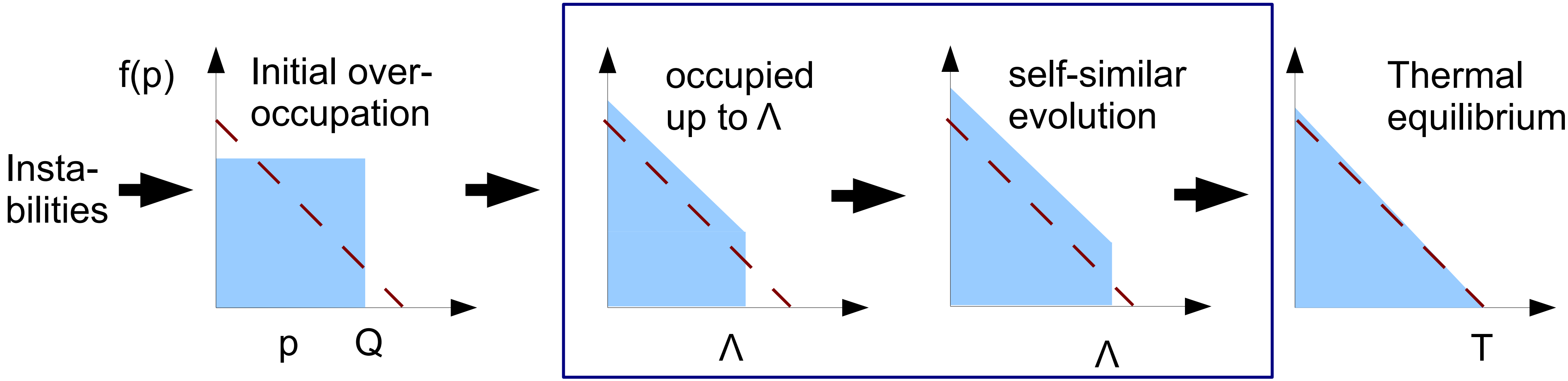
- ***Stationary solution*** with universal ***non-thermal*** scaling exponents

Uriel Frisch, *Turbulence. The Legacy of A. N. Kolmogorov.* (CUP, 1995)

- ***quasi-stationary solution*** with universal ***non-thermal spectral exponents***

- ***Self-similar evolution*** with universal ***dynamical scaling exponents***

Evolution in kinetic theory



Energy transport to the UV

- Search for ***self-consistent scaling solutions***

$$f(p,t) = t^{\alpha} \; f_S(t^{\beta} p) \qquad \partial_t f(p,t) = C[f](p,t)$$

- Fixed point equation + Scaling relation

$$\alpha f_S(p) + \beta \partial_p f_S(p) = C[f_S](p) \qquad \alpha - 1 = \mu(\alpha, \beta)$$

(Cosmology: Micha, Tkachev PRD 70 (2004) 043538)

Evolution in kinetic theory

- The self-similar evolution in time is characterized by ***universal scaling exponents*** fixed by

$$\alpha - 1 = \mu(\alpha, \beta) \qquad + \textit{ conservation laws}$$

Dynamic scaling relation

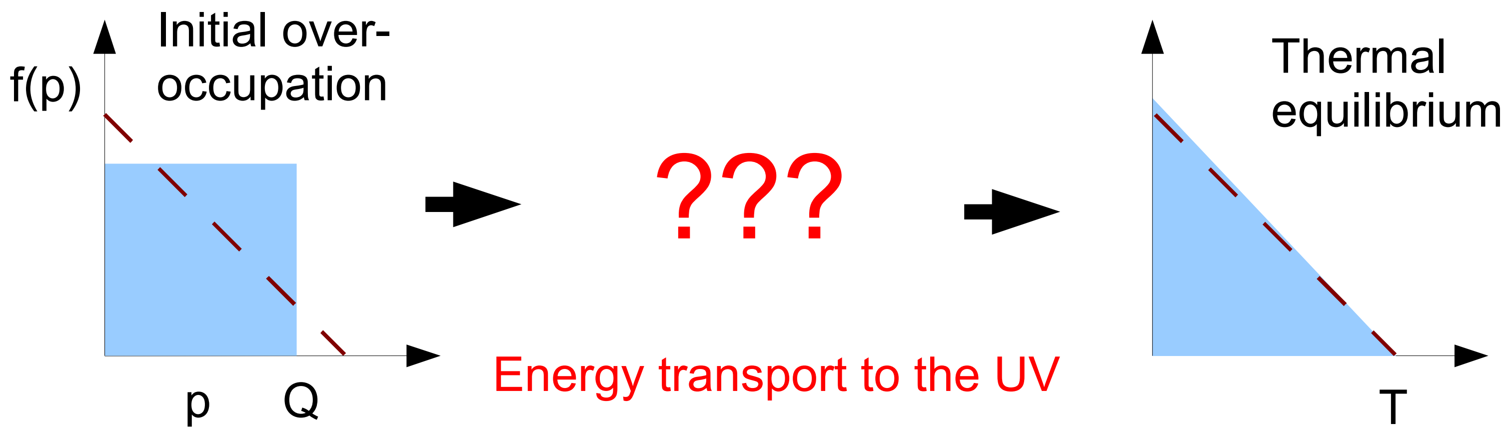
Interaction	*Spectral Shape (Exponent κ)*	*Λ evolution (Exponent β)*	*Occupancy evolution (Exponent α)*
2<->1+soft	***3/2***	***-1/5***	***-4/5***
2<->2	***4/3***	***-1/7***	***-4/7***
2<->3	***??***	***-1/7***	***-4/7***
(gauge theory)			

- ***Scalar theory:*** turbulent cascade is driven by ***2<->(1+soft)*** interaction and leads to a ***transient condensate*** formation

(Cosmology: Micha, Tkachev PRD 70 (2004) 043538)

SU(2) gauge theory – Static Box

- Consider homogenous and ***isotropic*** systems which are initially ***highly occupied*** and initially characterized by a single momentum scale Q

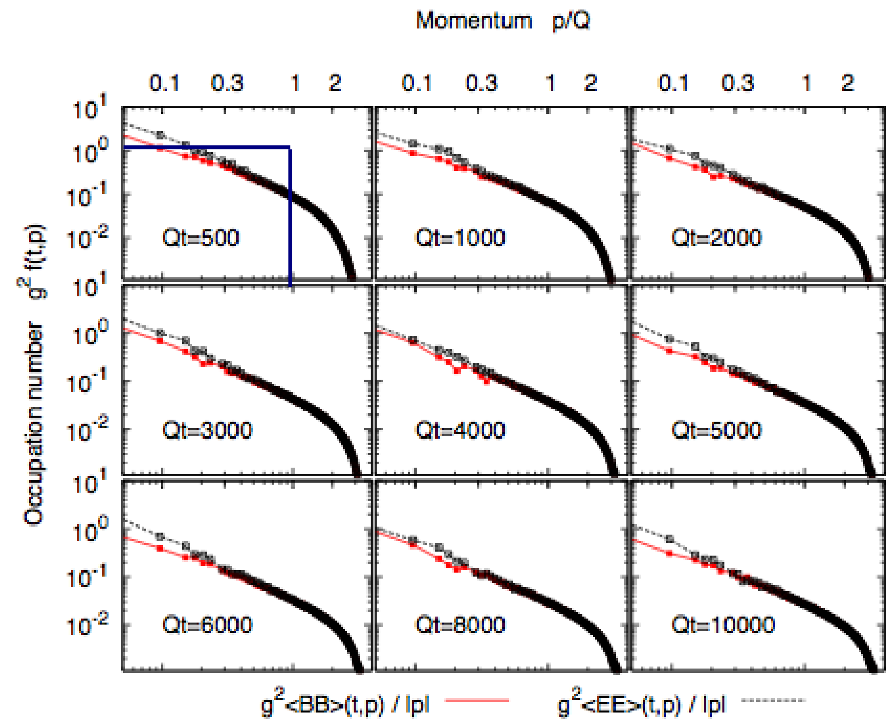


How does thermalization proceed? What are the relevant processes? Turbulence? Bose Einstein Condensation?

(c.f. Kurkela, Moore JHEP 1112 (2011) 044; Blaizot et al. Nucl.Phys. A873 (2012) 68-80)

Occupation number

- Non-perturbative calculation – occupation number is a ***gauge dependent*** quantity
- Chose ***temporal axial + Coulomb type gauge*** to fix the residual gauge freedom – interpretation of physical degrees of freedom
- Define occupation number from ***equal time correlation functions*** in temporal axial + Coulomb type gauge



(Berges, Scheffler, Sexty PLB 681 (2009) 362-366; Berges,SS,Sexty PRD 86 (2012) 074006 ; SS PRD 86 (2012) 065008; Kurkela, Moore PRD 86 (2012) 056008)

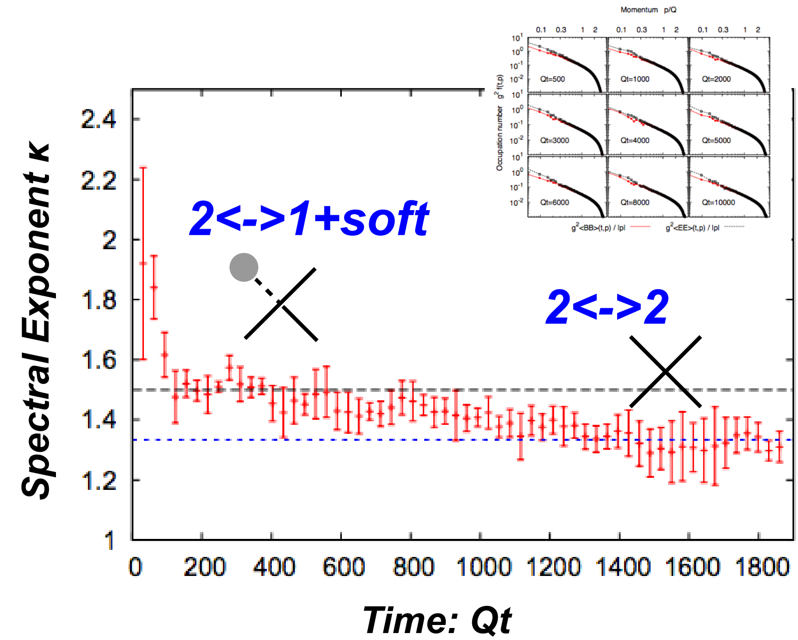
Turbulent Spectra

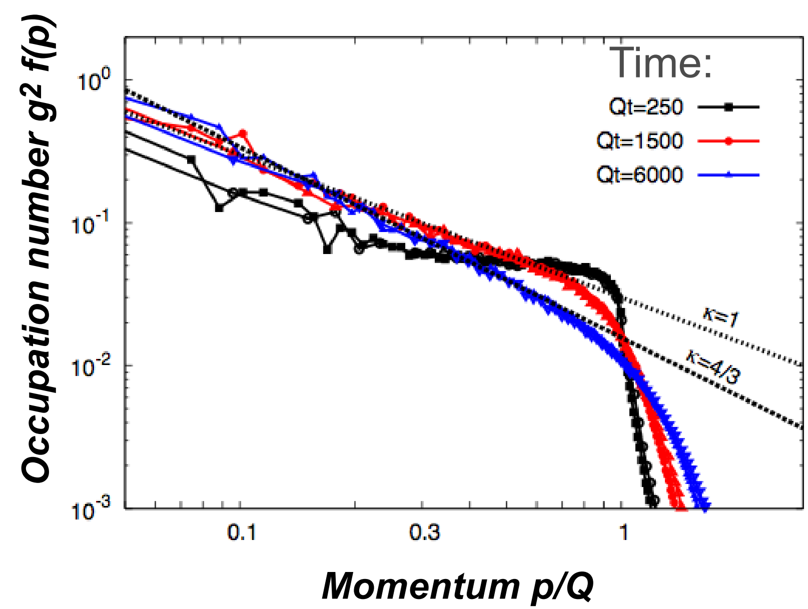
- Spectrum at ***early times*** consistent with ***ϰ=3/2*** as in the scalar theory – ***Possible sign of condensation***

- Spectrum at ***late times*** always closer to ***ϰ=4/3 - Inelastic processes much more efficient in gauge theory as compared to scalars***

- The appearance of the exponent ***ϰ=3/2*** also depends on initial conditions and ***disappears for smaller initial over-occupation***

(Berges, Scheffler, Sexty PLB 681 (2009) 362-366; Berges,SS,Sexty PRD 86 (2012) 074006 ; SS PRD 86 (2012) 065008; Kurkela, Moore PRD 86 (2012) 056008)





Dynamic Scaling

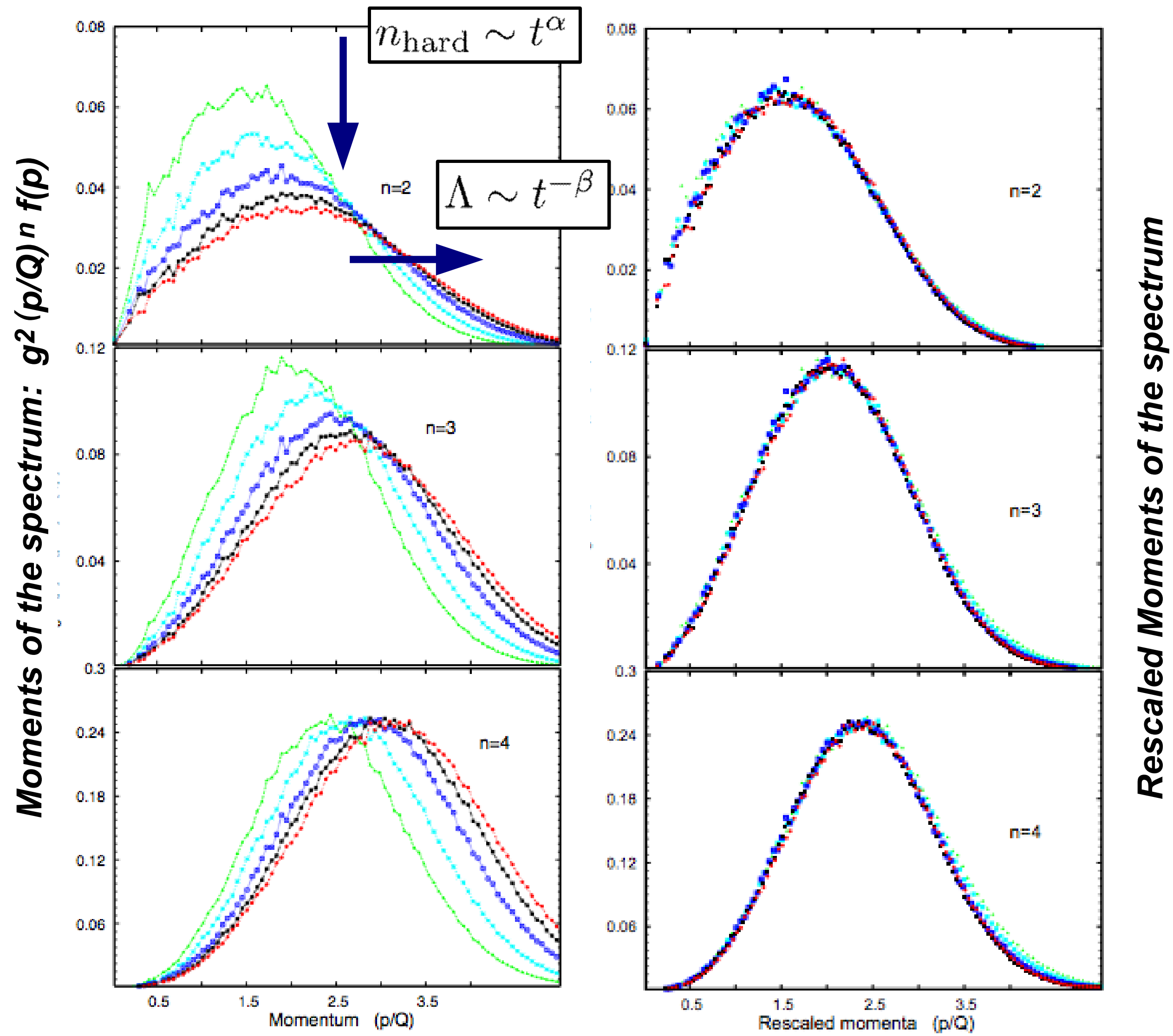
- Evolution at late times shows a ***self-similar*** behavior with dynamic scaling exponents

α = -4/7

β = -1/7

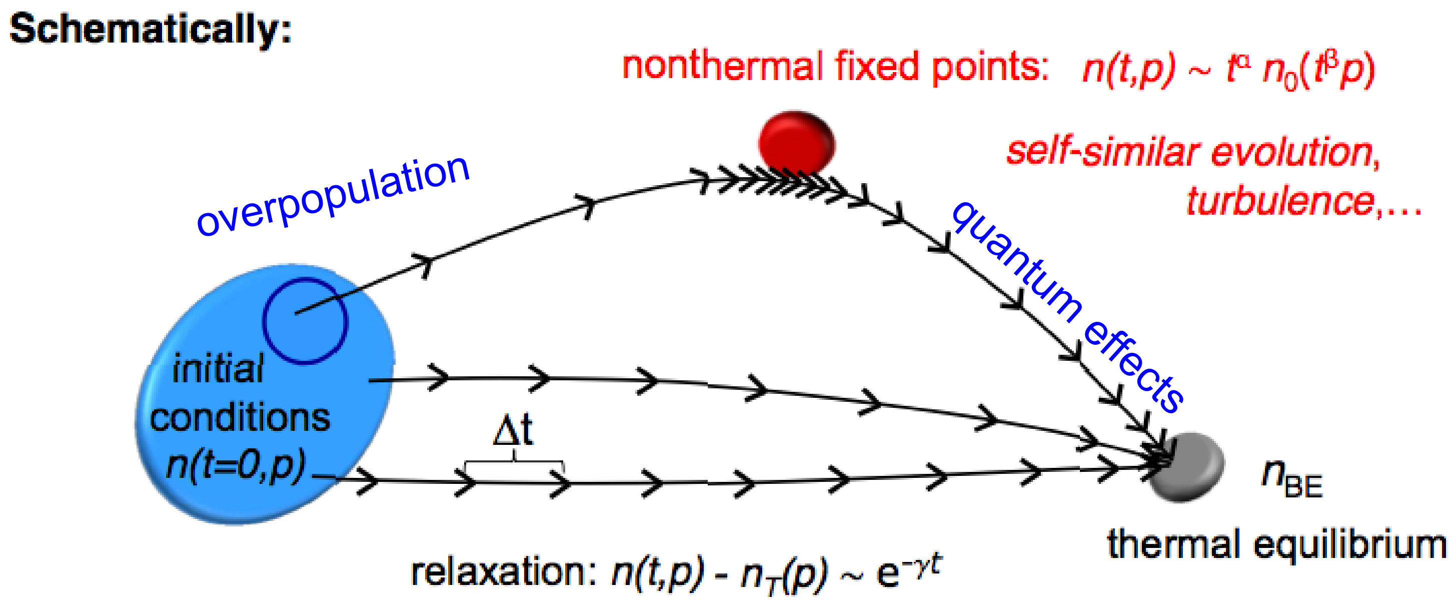
- Consistent with both ***elastic and inelastic*** scattering processes

(SS PRD 86 (2012) 065008; Kurkela, Moore PRD 86 (2012) 056008)



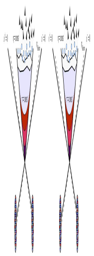
Turbulent thermalization

Thermalization of over-occupied systems proceeds as a ***turbulent cascade*** with a self-similar evolution associated to the presence of a ***non-thermal fixed point***

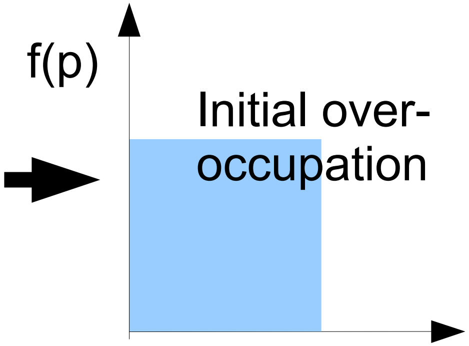


Does this picture hold for expanding systems as encountered in ultra-relativistic heavy-ion collisions?

Thermalization of expanding systems

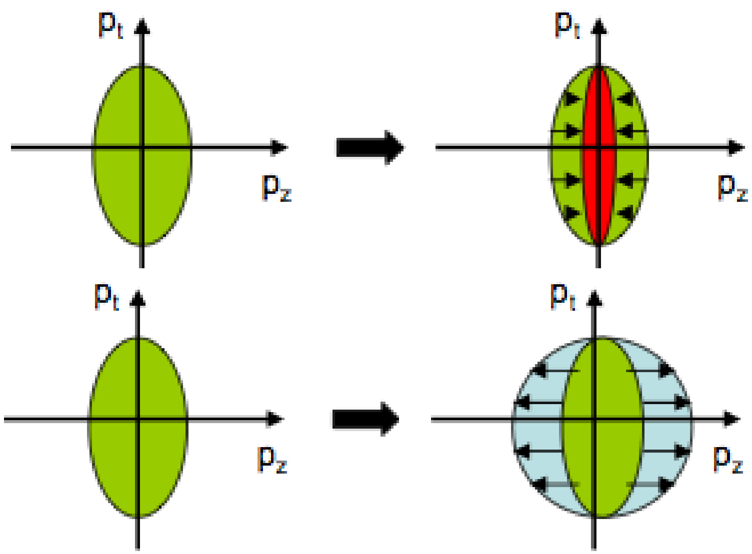


→ Glasma + vacuum fluctuations → Plasma Instabilities →



→ ???

- There is a natural ***competition*** between ***interactions*** and the ***longitudinal expansion*** which renders the system ***anisotropic*** on large time scales



Longitudinal Expansion:

- Red-shift of longitudinal momenta p_z
  → increase of anisotropy
- Dilution of the system

Interactions:

- Isotropize the system

Thermalization of expanding systems

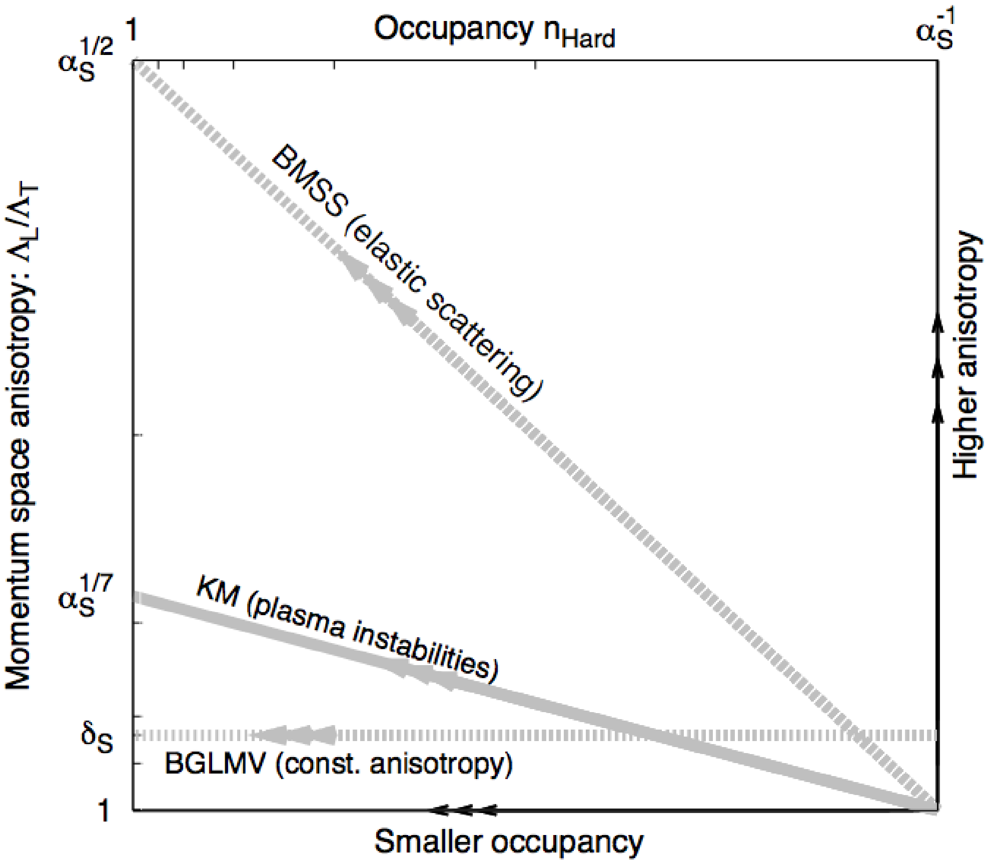
- Different scenarios of how thermalization proceeds have been proposed in the literature

Baier et al. (BMSS),
PLB 502 (2001) 51-58

Kurkela, Moore (KM),
JHEP 1111 (2011) 120

Blaizot et al. (BGLMV),
Nucl. Phys. A 873 (2012) 68-80

Classical-statistical lattice simulations can be used to determine which solution is realized!



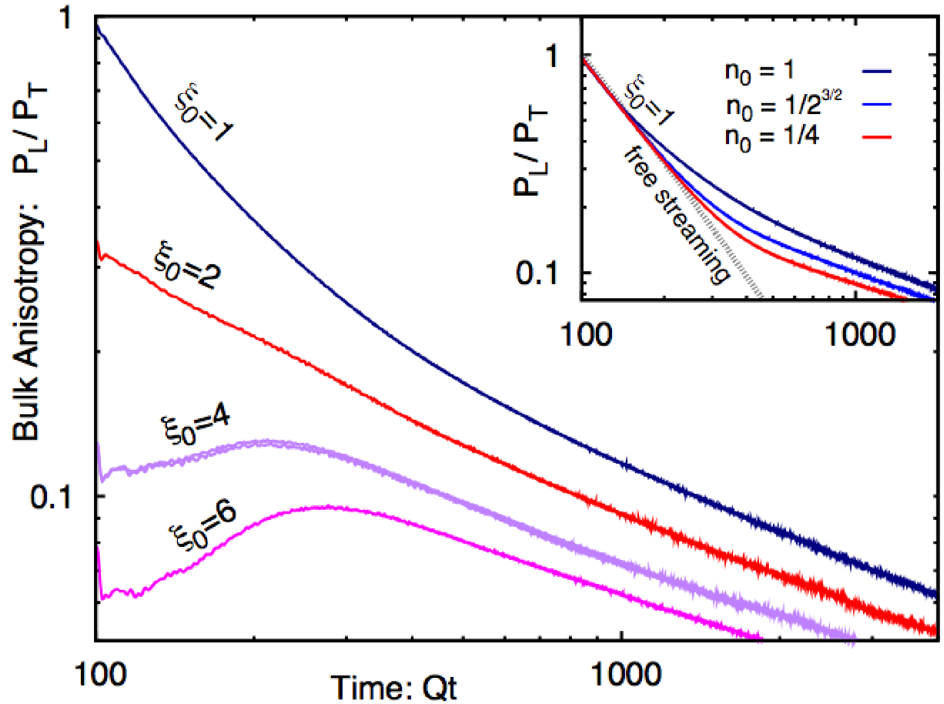
Expanding systems - Anisotropy

Consider a large class of different initial conditions

$$f(p_T, p_z, t_0) = \frac{n_0}{2g^2}\Theta\left(Q - \sqrt{p_T^2 + (\xi_0 p_z)^2}\right)$$

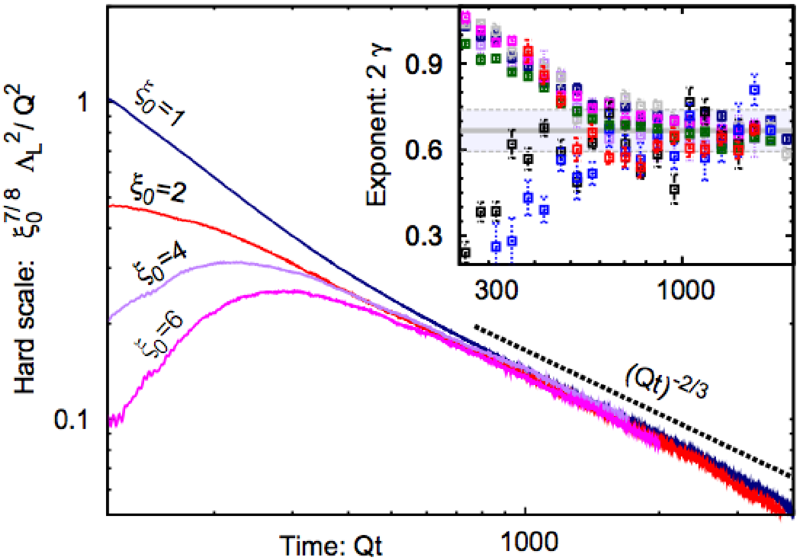
initial occupancy — initial anisotropy

- The ***anisotropy*** of the system ***increases*** due to the longitudinal expansion.
- The system remains ***strongly interacting*** throughout the entire evolution.
- At late times, the evolution becomes ***insensitive*** to the details of the ***initial conditions***



(Berges,Boguslavski,SS,Venugopalan arXiv:1303.5650 [hep-ph])

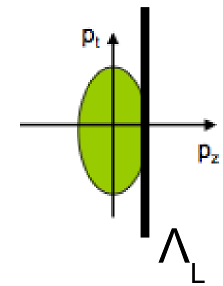
Expanding systems - Scaling

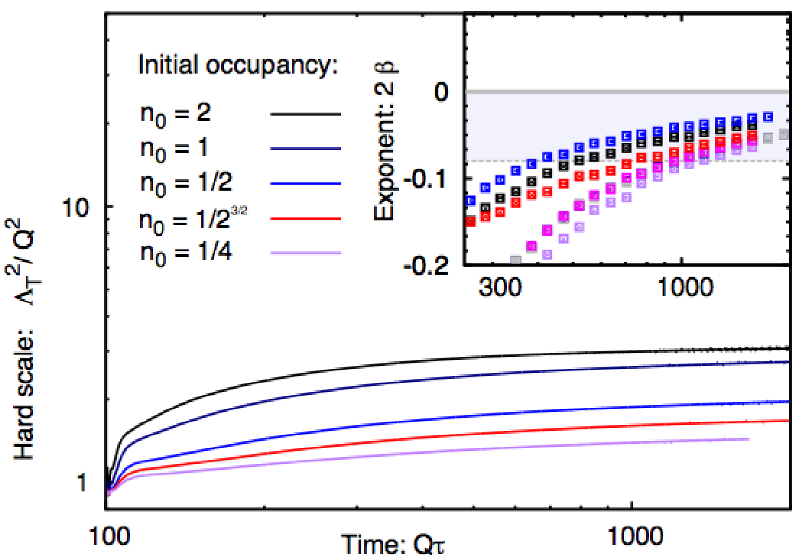


- The typical ***longitudinal momentum*** of hard excitations exhibits a ***universal scaling*** behavior

$$\Lambda_L^2/Q^2 \sim (Qt)^{-2\gamma}$$

$$2\gamma \quad = \quad 0.67 \pm 0.07$$

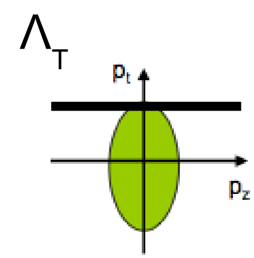




- The typical ***transverse momentum*** of hard excitations remains approximately ***constant***

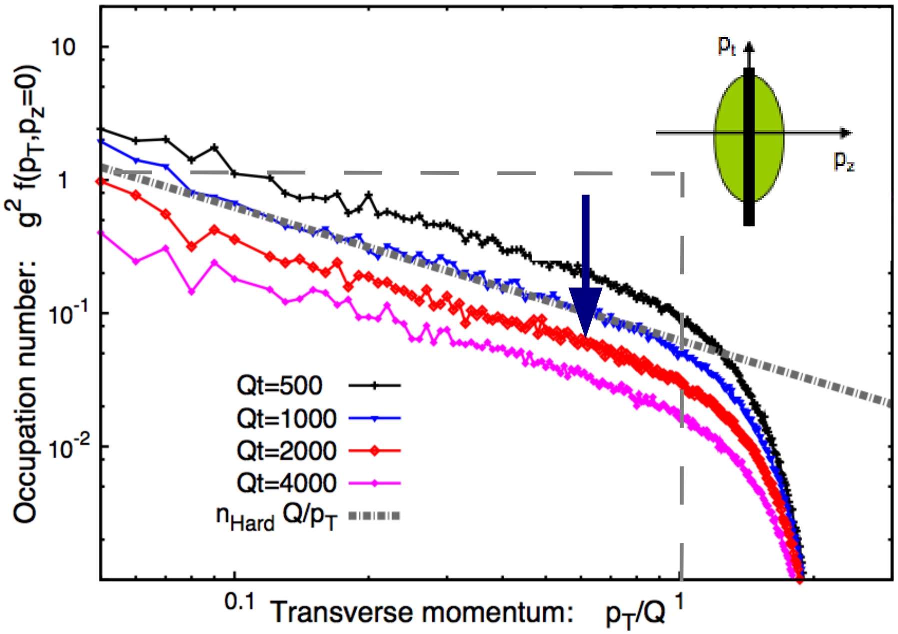
$$\Lambda_T^2/Q^2 \sim (Qt)^{-2\beta}$$

$$2\beta \simeq 0$$

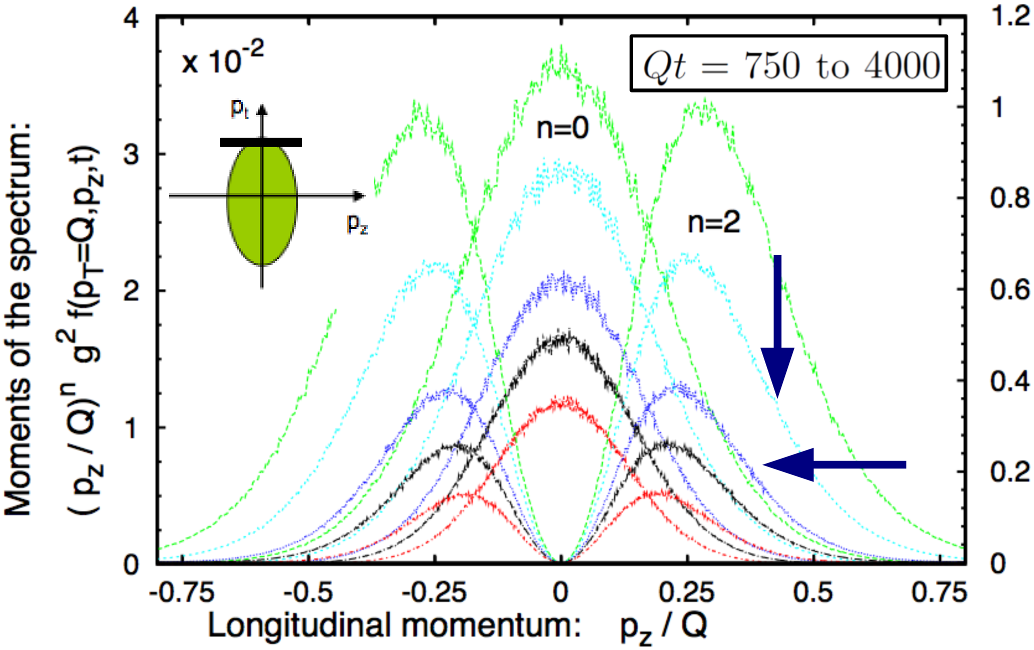


Expanding systems - Spectrum

Transverse spectrum
(Coulomb type gauge)



Longitudinal spectrum
(Coulomb type gauge)

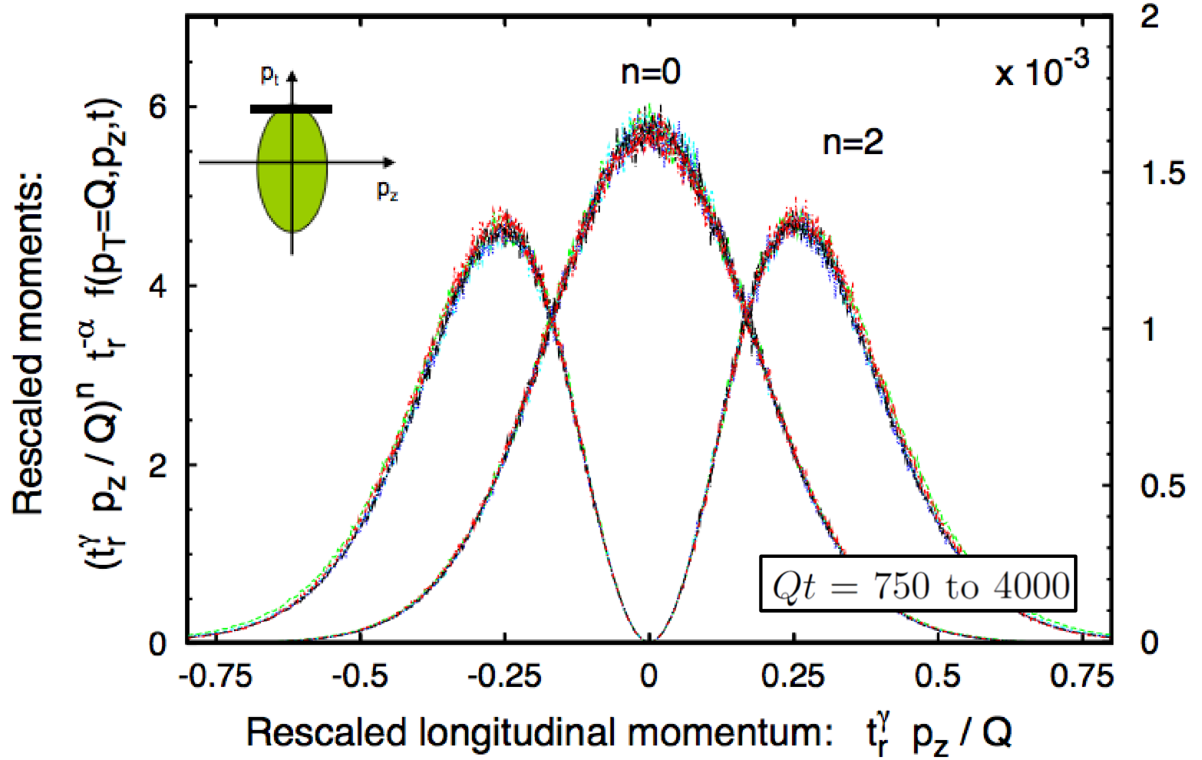


- "Thermal-like" T/p_T spectrum with decreasing amplitude

$$n_{\text{Hard}}(t) = f(p_T = Q, p_z = 0, t)$$

- Decrease of the characteristic longitudinal momentum

Expanding systems - Self-similarity



- The spectrum of hard excitations shows a ***self-similar evolution*** characteristic of ***wave turbulence*** $f(p_\perp, p_z, t) = t^\alpha \underbrace{f_S(t^\beta p_\perp, t^\gamma p_z)}$

stationary fixed point distribution

Non-thermal fixed point

Boltzmann equation with generic collision term for longitudinal expansion:

$$\left[\partial_t - \frac{p_z}{t}\partial_{p_z}\right] f(p_T,p_z,t) = C[p_T,p_z,t;f]$$

Self-similar evolution:

$$f(p_T,p_z,t) = t^{\alpha} f_S(t^{\beta}p_T, t^{\gamma}p_z) \ , \quad C[p_T,p_z,t;f] = t^{\mu}\, C[t^{\beta}p_T, t^{\gamma}p_z; f_S]$$

→ a) **fixed point equation for stationary distribution**:

$$\alpha f_S(p_T,p_z) + \beta p_T \partial_{p_T} f_S(p_T,p_z + (\gamma-1)\, p_z \partial_{p_z} f_S(p_T,p_z) = C[p_T,p_z;f_S]$$

→ b) **scaling condition**:

$$\boxed{\alpha - 1 = \mu(\alpha,\beta,\gamma)}$$

Non-thermal fixed point

Interpret scaling condition with **energy/number conserving*** Fokker-Planck-type dynamics for the collisional broadening of longitudinal momentum distribution:

$$C^{(\mathrm{elast})}[p_T, p_z; f] = \hat{q}\ \partial^2_{p_z} f(p_T, p_z, t)$$

with **momentum diffusion parameter:** $\hat{q} \sim \alpha_S^2 \int \frac{d^2 p_T}{(2\pi)^2} \int \frac{dp_z}{2\pi}\ f^2(p_T, p_z, t)$

→ 1) $\mu = 3\alpha - 2\beta + \gamma$ $\xrightarrow{\alpha - 1 = \mu(\alpha,\beta,\gamma)}$ $2\alpha - 2\beta + \gamma + 1 = 0$

2) number conservation $\longrightarrow$ $\alpha - 2\beta - \gamma + 1 = 0$

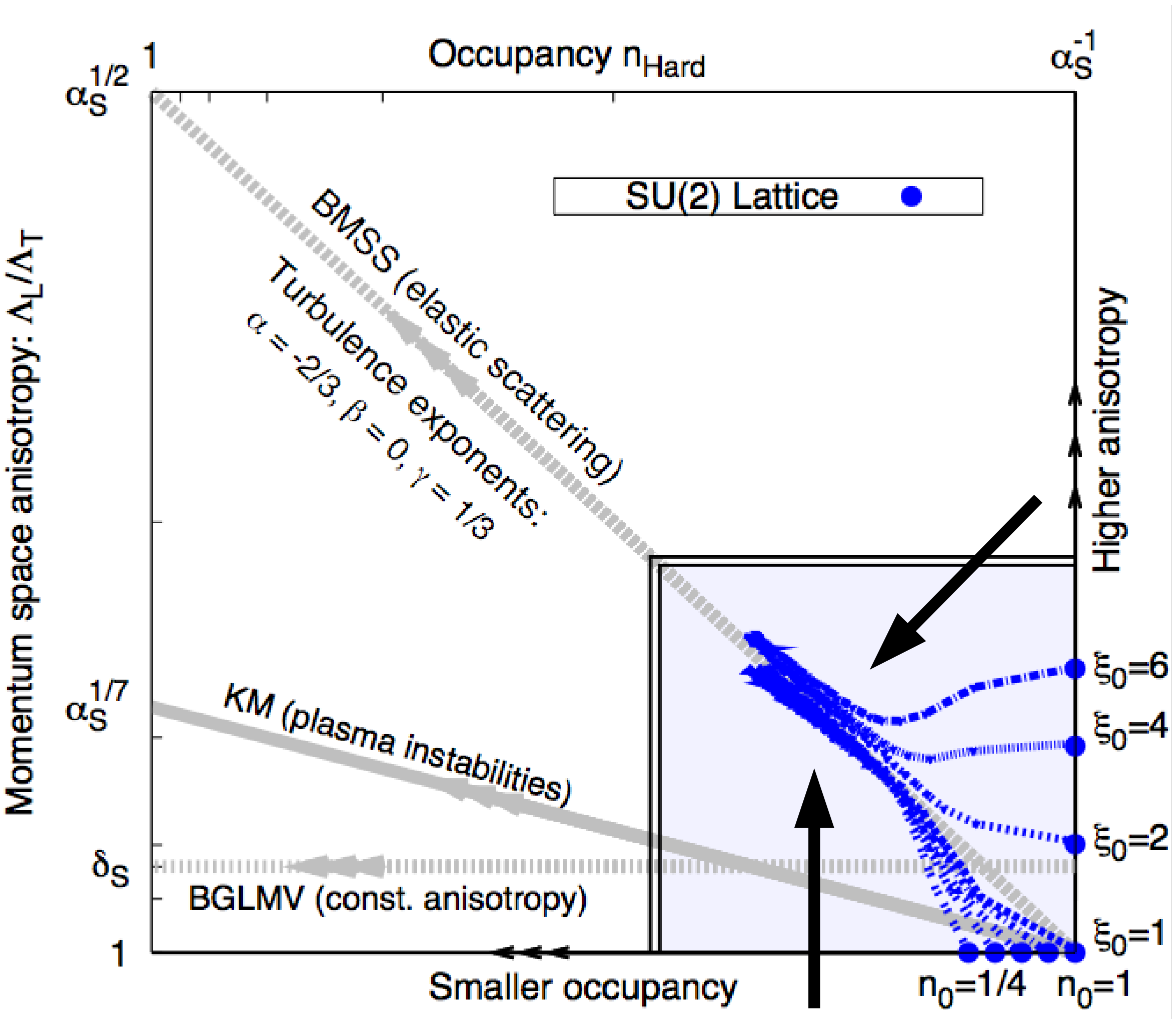
3) energy conservation $\longrightarrow$ $\alpha - 3\beta - \gamma + 1 = 0$

$$\alpha = -2/3\ , \qquad \beta = 0\ , \qquad \gamma = 1/3$$

remarkable agreement with lattice data!

*cf. early stages of Baier, Mueller, Schiff, Son, PLB 502 (2001) 51 ('BMSS')

The attractor solution



- Universal scaling behavior for different initial conditions
- Qualitative agreement with “bottum-up” thermalization scenario

Conclusions & Outlook

Classical-statistical lattice simulations demonstrate the existence of a **non-thermal fixed point** also for the expanding plasma at high energies

- self-similar evolution with **universal scaling exponents/functions** characteristic for wave turbulence (novel universality class because of expansion)

- **zero longitudinal pressure** fixed point with 'thermal' **$1/p_T$ spectrum**

- **self-similar evolution** leads to **BMSS attractor** in the classical regime

- after an initial transient regime dominated by plasma instabilities the evolution becomes **independent of initial condition** details

What happens in the quantum regime? Does the weak coupling behavior survive at realistic values of the coupling constant?

Backup - Definitions

Gauge invariant definition of Λ_L^2 :

$$\Lambda_L^2(\tau) = \frac{\langle \mathcal{H}^x{}_x(\tau)\rangle + \langle \mathcal{H}^y{}_y(\tau)\rangle - \langle \mathcal{H}^\eta{}_\eta(\tau)\rangle}{\epsilon(\tau)}$$

$$\mathcal{H}^\mu{}_\mu(\tau) = \frac{1}{V_\perp L_\eta} \int d^2\vec{x}_\perp \; d\eta \; D^{ab}_\alpha(x) \mathcal{F}^{\alpha\mu}_b(x) \; D^\beta_{ac}(x) \mathcal{F}^c_{\beta\mu}(x)$$

Perturbative expression:

$$\Lambda_L^2(\tau) \simeq \frac{\int d^2\vec{p}_T \; dp_z \; p_z^2 \; \omega_p \; f(\vec{p}_T, p_z, \tau)}{\int d^2\vec{p}_T \; dp_z \; \omega_p \; f(\vec{p}_T, p_z, \tau)}$$

Fock Schwinger + Coul. Gauge:

$$A_\tau = 0 \quad + \quad \tau^{-2}\partial_\eta A_\eta(x) + \sum_i \partial_i A_i(x) \Big|_{\tau} = 0$$

Particle Number:

$$f(\vec{p}_T, p_z, \tau) = \frac{\tau^2}{N_g V_\perp L_\eta} \sum_{a=1}^{N_c^2-1} \sum_{\lambda=1,2} \left\langle \left| \, g^{\mu\nu} \left[\left(\xi_\mu^{(\lambda)\vec{p}_T \nu +}(\tau) \right)^* \overleftrightarrow{\partial_\tau} \; A^a_\nu(\tau, \vec{p}_T, \nu) \right] \right|^2 \right\rangle$$

Backup - Definitions

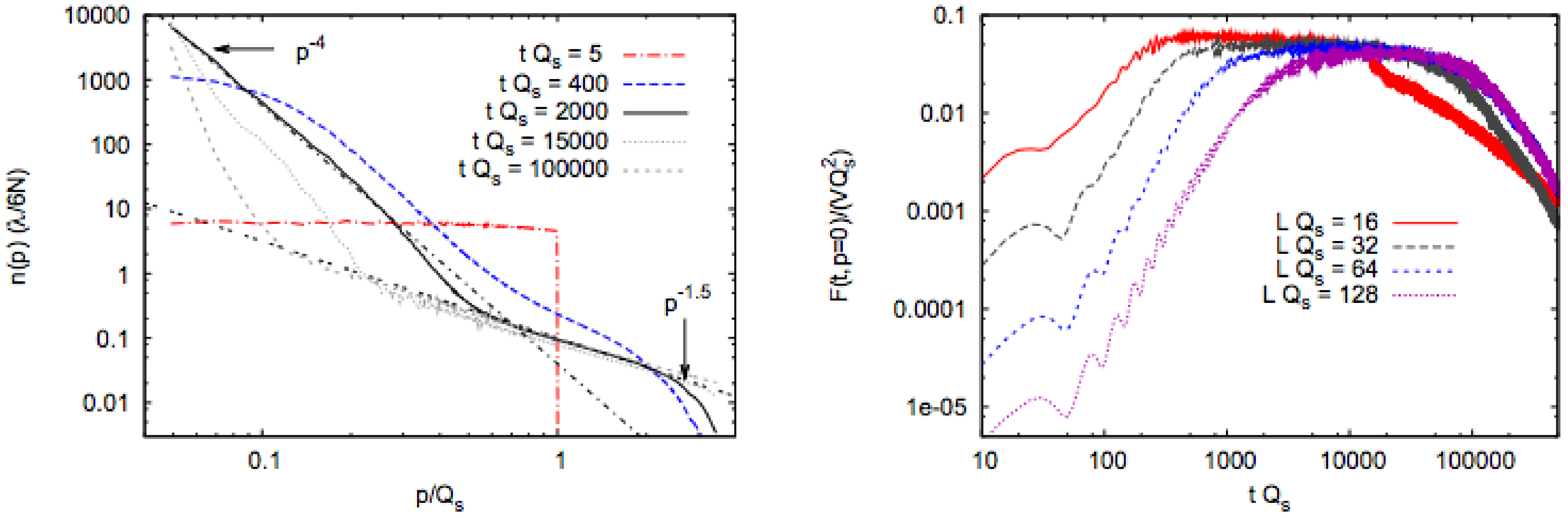
Mode functions in Fock Schwinger + Coul. Gauge:

$$\xi_\mu^{(1)\vec{p}_T\nu\pm}(x) = \frac{\sqrt{\pi}\, e^{\pm\pi\nu/2}}{2p_\perp}\begin{pmatrix} -p_y \\ p_x \\ 0 \end{pmatrix} H_{i\nu}^{(2/1)}(p_\perp\tau) \qquad \xi_\mu^{(2)\vec{p}_T\nu\pm}(x) = \begin{pmatrix} \nu p_x/(p_\perp^2\tau_0^2)\ R_\perp^{\vec{p}_T\nu\pm}(\tau) \\ \nu p_y/(p_\perp^2\tau_0^2)\ R_\perp^{\vec{p}_T\nu\pm}(\tau) \\ -R_\eta^{\vec{p}_T\nu\pm}(\tau) \end{pmatrix}$$

$$R_\perp^{\vec{p}_T\nu\pm}(\tau) = -\frac{p_\perp\tau_0}{\nu^2+p_\perp^2\tau_0^2}\ c_{2/1}^{\pm}\ H_{i\nu}^{\prime(2/1)}(p_\perp\tau_0) + \int_{\tau_0}^{\tau} d\tau'\, \tau'^{-1} c_{2/1}^{\pm} H_{i\nu}^{(2/1)}(p_\perp\tau')\ ,$$

$$R_\eta^{\vec{p}_T\nu\pm}(\tau) = -\frac{p_\perp\tau_0}{\nu^2+p_\perp^2\tau_0^2}\ c_{2/1}^{\pm}\ H_{i\nu}^{\prime(2/1)}(p_\perp\tau_0) + \int_{\tau_0}^{\tau} d\tau'\, \frac{\tau'}{\tau_0^2} c_{2/1}^{\pm} H_{i\nu}^{(2/1)}(p_\perp\tau')\ .$$

Backup – Bose Condensation



(Berges, Sexty Phys.Rev.Lett. 108 (2012) 161601)