Corrections to Gaussian color charge fluctuations and their effect on the dipole and KNO scaling

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h3QCD

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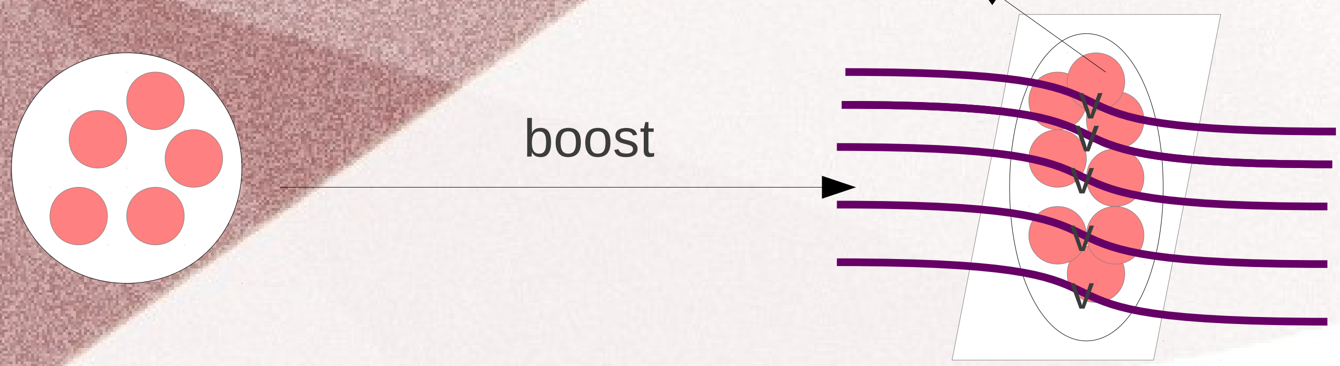
- High-energy limit of nuclear collisions;
  - Saturation scale $Q_s(x)$
    - The ***Color Glass Condensate*** is an effective theory describing the partonic behavior in the saturation region.

Confront CGC with experiment:

Transverse momentum integrated yields, single inclusive hadron yields, nuclear modification factors, azimuthal correlations.

- ***Dipole scattering amplitude;***

- ***Particle multiplicity distributions.***



Gaussian distribution of sources (McLerran-Venugopalan model):

L. D. McLerran and R. Venugopalan,

Phys. Rev. D49, 2233 (1994); 49, 3352 (1994)

$$W[\rho] = \exp\left[-\int d^2x_\perp \frac{\delta^{ab}\rho^a(x_\perp)\rho^b(x_\perp)}{2\mu^2}\right]$$

$$\mu^2 = \frac{g^2 A}{\pi R^2}$$

- Color charge squared per unit transverse area.

The model is valid for a large nucleus, $A^{1/3} \to \infty$

Higher order corrections to the MV model:

- Cubic term

S. Jeon and R. Venugopalan, Phys. Rev. D70, 105012 (2004); 71, 125003 (2005)

- Quartic action:

$$S[\rho(x)] \simeq \int d^2x \Big[\frac{\delta^{ab}\,\rho^a\rho^b}{2\mu^2} - \frac{d^{abc}\,\rho^a\rho^b\rho^c}{\kappa_3} + \frac{\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}}{\kappa_4}\rho^a\rho^b\rho^c\rho^d \Big]$$

$$\mu^2 \sim O(g^2 A^{1/3})$$

$$\kappa_3 \sim O(g^3 A^{2/3})$$

$$\kappa_4 \sim O(g^4 A)$$

A. Dumitru, J. Jalilian-Marian, E.P. Phys.Rev.D84 (2011) 014018

Apply quartic action for calculating:

- ***Dipole scattering amplitude***

Global fits to e+p require initial dipole scattering amplitude with steeper fall off than MV.

J. L. Albacete, N. Armesto, J. G. Milhano and C. A. Salgado, Phys. Rev. D80, 034031 (2009)

J. L. Albacete, N. Armesto, J.G. Milhano, P. Quiroga-Arias and C. A. Salgado, Eur. Phys. J. C71, 1705 (2011)

- ***Particle multiplicity distributions (KNO scaling)***

Dipole picture: The virtual photon fluctuates into a color dipole, which then probes the wavefunction of the target.

Dipole operator:

$$D(\mathbf{r}_\perp) \equiv \frac{1}{N_c} \left\langle \mathrm{tr} V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp) \right\rangle$$

Dipole size:

$$\mathbf{r}_\perp = \mathbf{x}_\perp - \mathbf{y}_\perp$$

Dipole scattering amplitude:

$$N(\mathbf{r}_\perp) = 1 - \frac{1}{N_c} \left\langle \mathrm{tr} V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp) \right\rangle$$

rcBK evolution equation:

$$\frac{\partial N_F(r,x)}{\partial \ln(x_0/x)} = \int d^2 r_1' K^{run}(r', r_1', r_2') \left[N_F(r_1,x) + N_F(r_2,x) - N_F(r,x) - N_F(r_1,x) N_F(r_2,x) \right]$$

MV result:

$$N(r) = 1 - \exp\left[-\frac{r^2\, Q_s^2(x_0)}{4}\, \ln\left(\frac{1}{\Lambda\, r} + e\right)\right]$$

AAMQS (Albacete-Armesto-Milhano-Quiroga-Salgado) fits:

$$N_{\mathrm{AAMQS}}(r, x_0 = 0.01) = 1 - \exp\left[-\frac{1}{4}\left(r^2 Q_s^2(x_0)\right)^{\gamma} \ln\left(e + \frac{1}{r\Lambda}\right)\right]$$

$$\gamma > 1$$

Dipole operator calculation

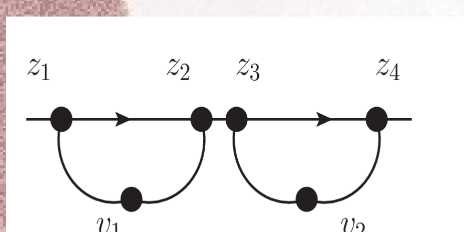
Perturbative expansion to first order in $1/\kappa_4$:

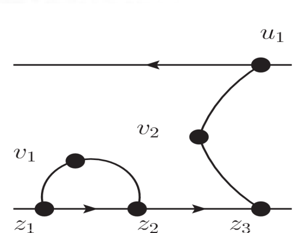
$$\langle O[\rho]\rangle \equiv \frac{\int \mathcal{D}\rho\, O[\rho]\, e^{-S_G[\rho]}\left[1-\frac{1}{\kappa_4}\int d^2u\ \rho_u^a\rho_u^a\rho_u^b\rho_u^b\right]}{\int \mathcal{D}\rho\, e^{-S_G[\rho]}\left[1-\frac{1}{\kappa_4}\int d^2u\ \rho_u^a\rho_u^a\rho_u^b\rho_u^b\right]}$$

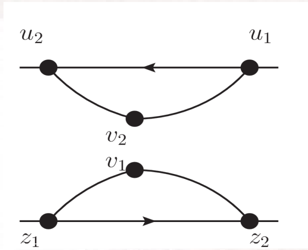
Expectation value of the Wilson lines order by order:

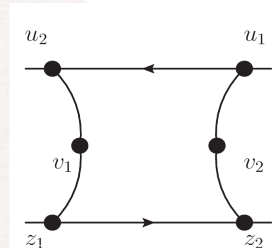
$$V(\mathbf{x}_\perp) = \mathcal{P}\exp\left\{-ig^2\int_{-\infty}^{\infty} dz^-\int d^2\mathbf{z}_\perp\, G_0(\mathbf{x}_\perp-\mathbf{z}_\perp)\,\rho_a(z^-,\mathbf{z}_\perp)\,t^a\right\}$$

Correction diagrams:









Quartic action result

$$N(r) = \frac{Q_s^2 r^2}{4} \log \frac{1}{r\Lambda} - \frac{C_F^2}{6\pi^3} \frac{g^8}{\kappa_4} \left[\int_{-\infty}^{\infty} dz^- \mu^4(z^-) \right]^2 r^2 \log^3 \frac{1}{r\Lambda}$$

Gaussian part

ρ^4 correction

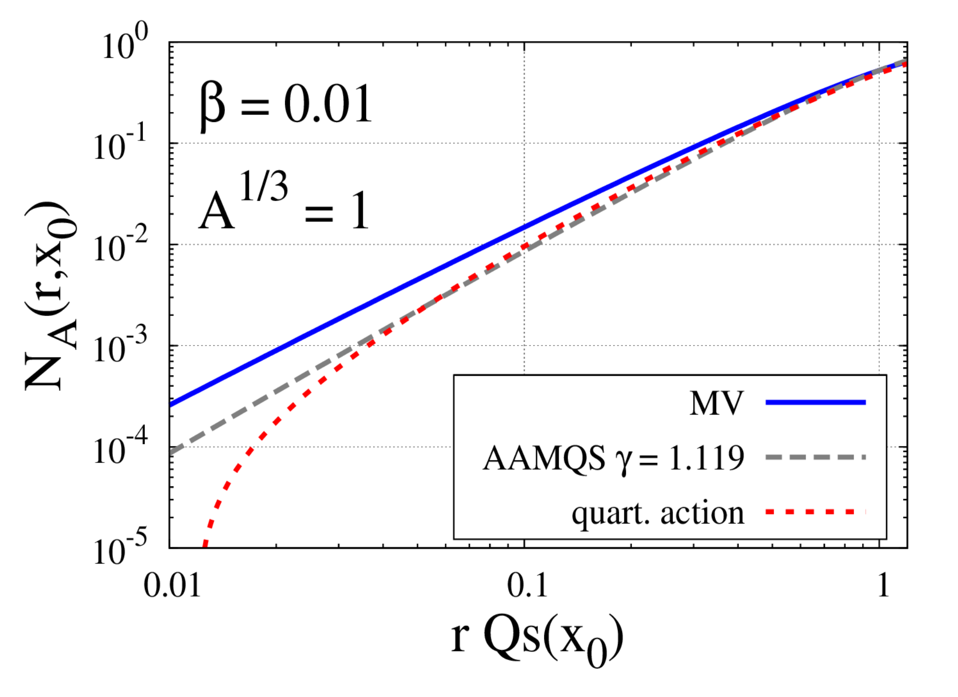
A. Dumitru, E.P.
Nucl.Phys. A879 (2012) 59-76

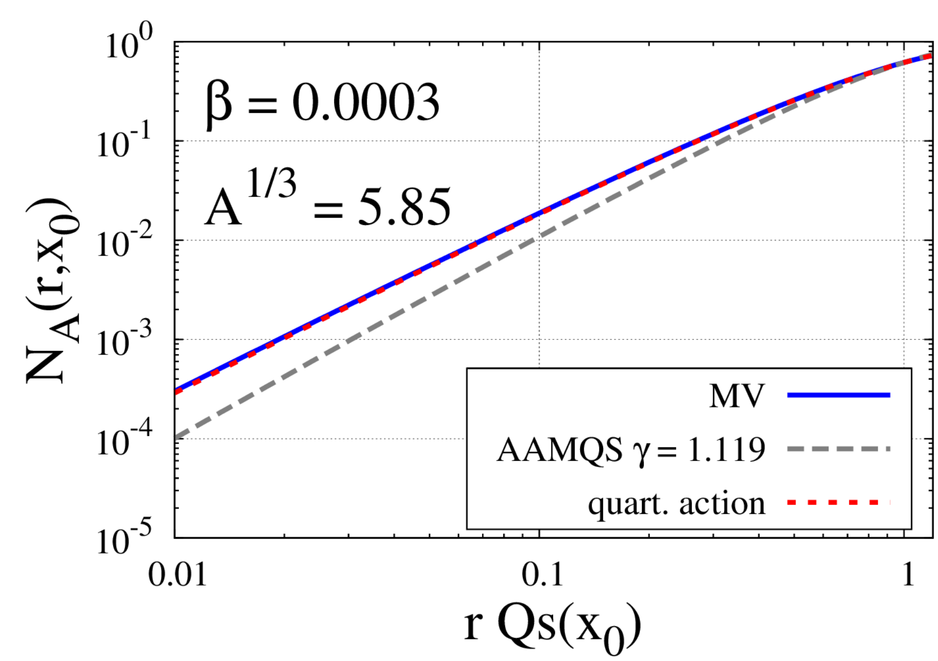
Comparison of results

$$N(r) = \frac{Q_s^2 r^2}{4} \log\frac{1}{r\Lambda} - \beta Q_s^2 r^2 \log^3\frac{1}{r\Lambda}$$

$$\beta_A \sim A^{-2/3}$$

$$\beta \simeq \frac{1}{100} \quad , \ (A=1)$$





ρ^4 operator may explain AAMQS model.

Apply quartic action for calculating:

- ***Dipole scattering amplitude***
- ***Particle multiplicity distributions (KNO scaling)***

Particle multiplicity distributions in the central region of inelastic pp collisions follow a **negative binomial distribution (NBD)** and exhibit **Koba-Nielsen-Olesen (KNO) scaling.**

NBD:

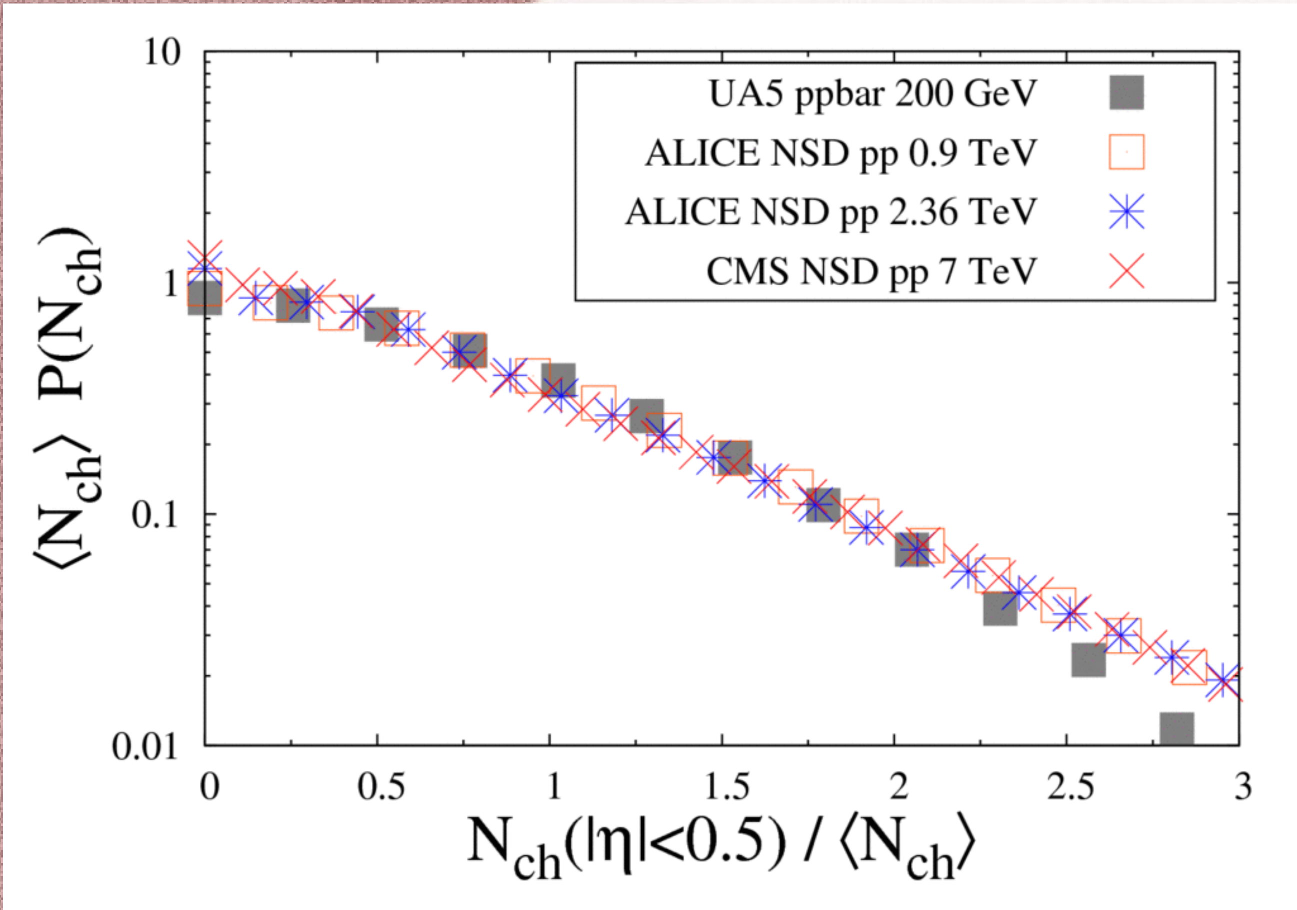
$$P(n) = \frac{\Gamma(k+n)}{\Gamma(k)\Gamma(n+1)} \frac{\bar{n}^n k^k}{(\bar{n}+k)^{n+k}}$$

$P(n)$ - *Probability to produce n particles;*

$\bar{n}$ - *Mean multiplicity;*

k - *Fluctuation parameter.*

KNO scaling



$$\langle N_{ch} \rangle P(N_{ch}) \equiv \Psi(z)$$

$$z \equiv \frac{N_{ch}}{\langle N_{ch} \rangle}$$

$\Psi(z)$- Energy independent

- Requires explanation in terms of small-x gluons;

  - pT-integrated multiplicities (no external hard scale) involve saturation dynamics.

How to get *KNO* from *NBD*?

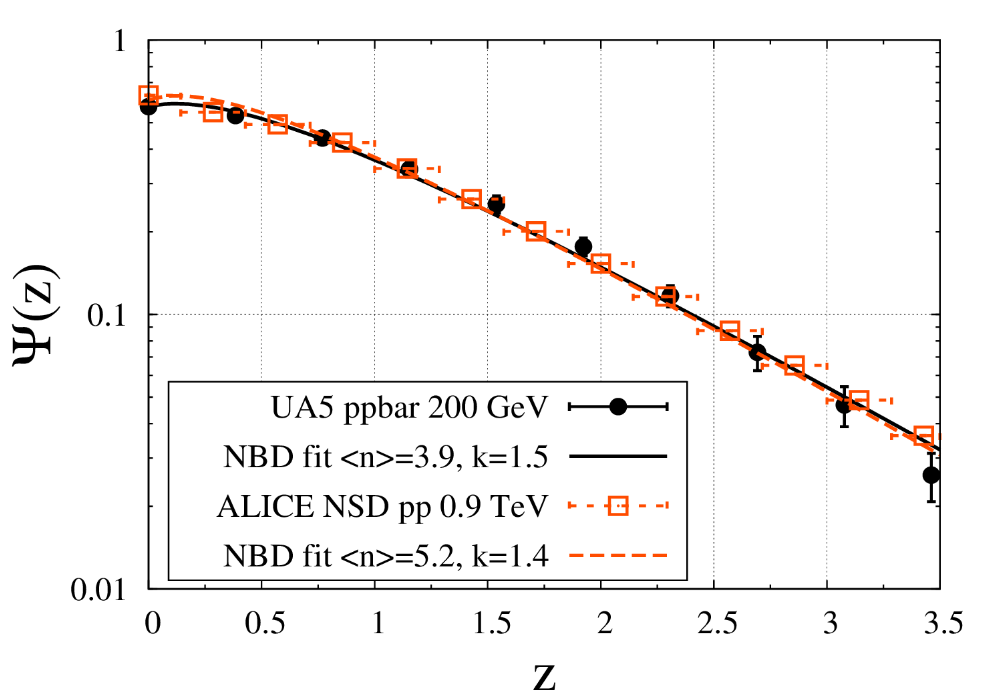
For k constant and $k \ll \bar{n}$

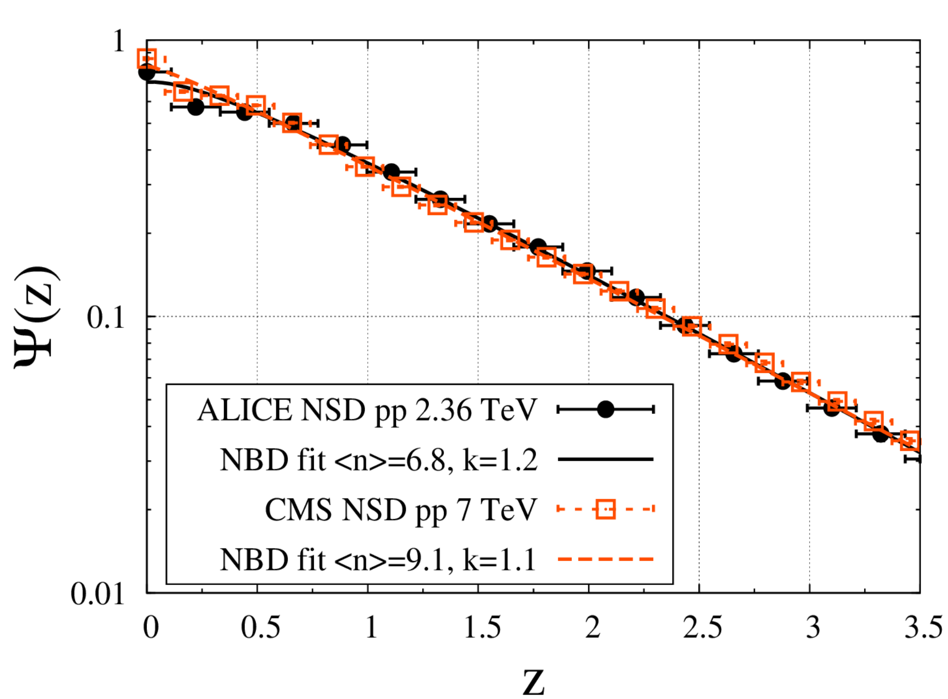
NBD leads to KNO scaling:

$$\bar{n}P(n)dz \sim z^{k-1}e^{-kz}dz, \qquad z \equiv \frac{n}{\bar{n}}$$

- k not exactly constant;
- Why is $k \ll \bar{n}$?

$\frac{\bar{n}}{k}$ increases with energy





$$\langle N_{ch} \rangle P(N_{ch}) \equiv \Psi(z)$$

$$z \equiv \frac{N_{ch}}{\langle N_{ch} \rangle}$$

NBD from MV model in the Glasma flux tube picture

F. Gelis, T. Lappi and L. McLerran, Nucl. Phys. A828, 149 (2009)

- Factorial cumulants for NBD:

$$m_n = \frac{(n-1)!}{k^{n-1}}\;\bar{n}^n$$

n - number of produced gluons

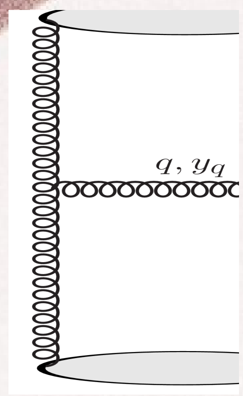
$$\bar{n} = \left\langle \frac{dN}{d^2\mathbf{p}_\perp dy_p} \right\rangle$$ - mean multiplicity

- Cross section for producing q gluons:

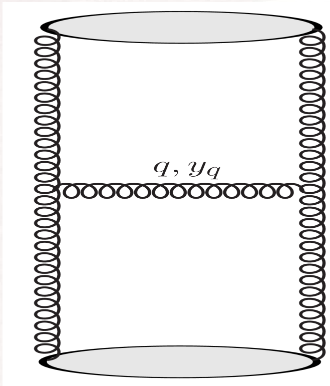
$$\left\langle \frac{dN}{dy_1 \cdots dy_n} \right\rangle_{\text{conn.}} = \beta_n \left\langle \frac{dN}{dy_1} \right\rangle \cdots \left\langle \frac{dN}{dy_n} \right\rangle$$

Calculation

- Single inclusive production (first cumulant, m_1):

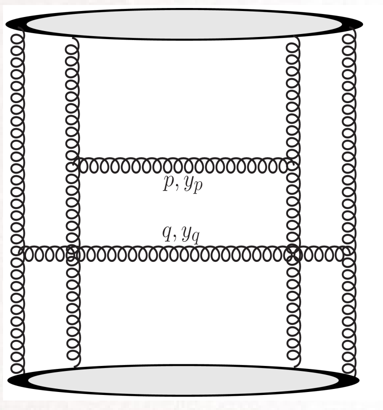


- Amplitude to produce one gluon



$$= \left\langle \frac{dN}{d^2\mathbf{p}_\perp dy_p} \right\rangle$$

- Two gluon production (second cumulant, m_2) :



$$= \left\langle \frac{dN}{d^2\mathbf{p}_{\perp 1} dy_1 \mathbf{p}_{\perp 2} dy_2} \right\rangle$$

MV model result

$$\left\langle \frac{dN}{dy_1 \cdots dy_n} \right\rangle_{\text{conn.}} = \beta_n \left\langle \frac{dN}{dy_1} \right\rangle \cdots \left\langle \frac{dN}{dy_n} \right\rangle$$

$$\beta_n = (n-1)!\ k^{1-n}$$ → NBD

$$\bar{n} \sim \frac{N_c(N_c^2-1)}{\alpha_s} Q_s^2 \pi R^2$$

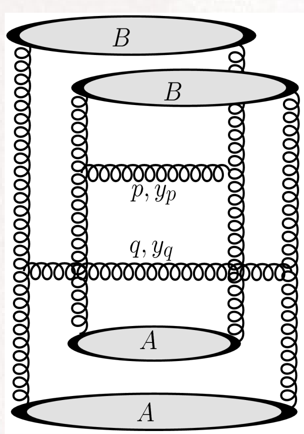
$$k \sim \frac{N_c^2-1}{2\pi} Q_s^2 \pi R^2$$

- **Why KNO?**

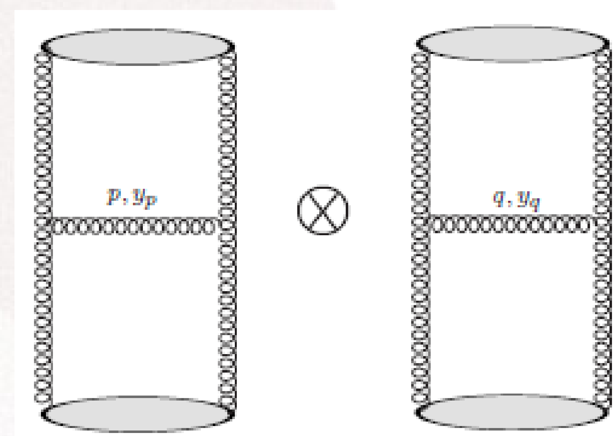
$$\frac{\bar{n}}{k} \sim \frac{N_c}{\alpha_s} \gg 1$$

- **Why is $k = \mathcal{O}(\alpha_s^0)$?**

Second cumulant: $m_2 = \frac{1}{k}\bar{n}^2$



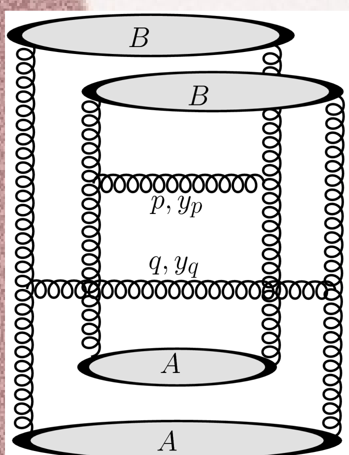
Same order in α_s



Calculation with quartic action

Two gluon production:

$$\left\langle \frac{dN}{dp\, dy_p\, dq\, dy_q} \right\rangle = \frac{1}{k} \left\langle \frac{dN}{dp\, dy_p} \right\rangle \left\langle \frac{dN}{dq\, dy_q} \right\rangle$$



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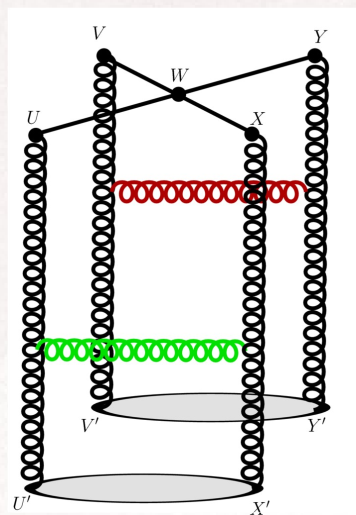
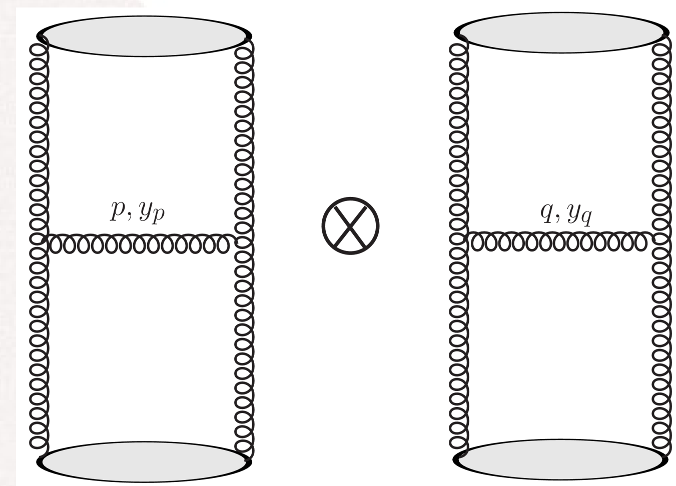


$= \frac{1}{k} \times$



Gaussian diagrams

Correction diagrams

Quartic action result

A. Dumitru, E.P. arXiv:1209.4105

$$\frac{\bar{n}}{k} \sim \frac{N_c}{\alpha_s} \left(1 - 3\,\beta\left(N_c^2 + 1\right)\right)$$

Correction

$$\beta \equiv \frac{C_F^2}{6\pi^3} \frac{g^8}{Q_s^2\,\kappa_4} \left[\int_{-\infty}^{\infty} dz^- \mu^4(z^-)\right]^2 \approx 0.01\,A^{-2/3}$$

$\beta > 0$ makes $\frac{\bar{n}}{k}$ smaller by a factor of 1.43 .

$\beta > 0$ makes $\frac{\bar{n}}{k}$ smaller.

- NBD fits to data show that $\frac{\bar{n}}{k}$ increases with energy

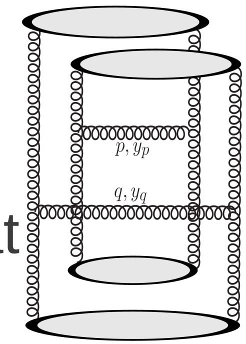
  $\Rightarrow$ might indicate flow towards a Gaussian theory;

- KNO scaling constrains the deviation of the small-x effective action from a Gaussian.

Summary

- Higher dimensional operators in the effective action may not be highly suppressed for p+p collisions;
  - They may provide theoretical understanding of the AAMQS fits.
    - They bring corrections to the negative binomial distribution and are constrained by the KNO scaling.

Two-particle correlations



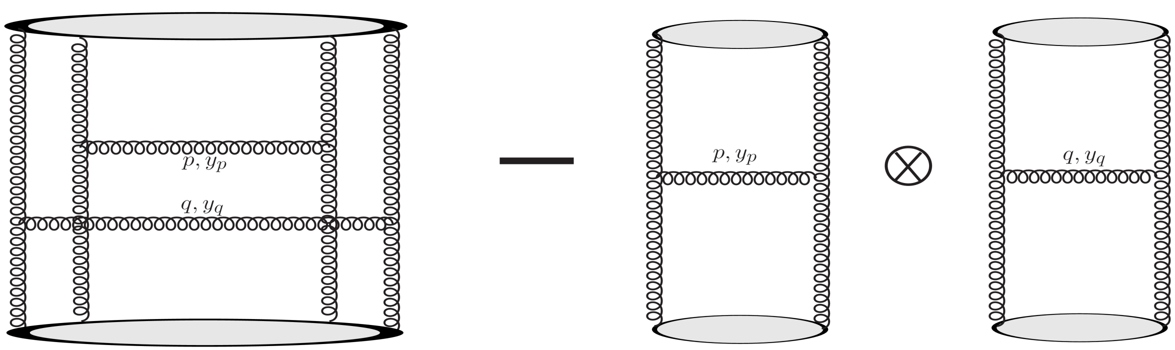
Two-gluon production diagram that gives an angular collimation al$\Delta\phi = 0$ over several units in

relat

$|y_p - y_q| \gtrsim 1$

rapidity (ridge). Dumitru et al. Phys. Lett. B 697, 21

(2011)



Correlated production of two gluons.

$$\frac{\left\langle \frac{dN_2}{d^2p\,dy_p\; d^2q\,dy_q} \right\rangle}{\left\langle \frac{dN}{d^2p\,dy_p} \right\rangle \left\langle \frac{dN}{d^2q\,dy_q} \right\rangle} - 1 \;\sim\; \frac{1}{N_c^2 - 1}$$

Non-Gaussian initial conditions for high-energy evolution

- Odderon operator $-d^{abc}\rho^a\rho^b\rho^c/\kappa_3$

S. Jeon and R. Venugopalan,
Phys. Rev. D70, 105012 (2004); 71, 125003 (2005)

- Effective action for a system of $k \gg 1$ valence quarks in SU(3);
- Random walk of SU(3) color charges in the space of representations (m,n);
- Probability $P(m,n)$ $\quad P(m,n) = e^{-S(m,n)}$

$$S(m,n;k) \simeq \frac{N_c}{k} C_2(m,n) - \frac{1}{3}\left(\frac{N_c}{k}\right)^2 C_3(m,n) + \frac{1}{6}\left(\frac{N_c}{k}\right)^3 C_4(m,n)$$

$C_2,\ C_3,\ C_4$ - Casimir operators for the representation (m,n)

- Define color charge per unit area $\rho^a \equiv g\,Q^a/\Delta^2 x$

where $|Q| = \sqrt{Q^a Q^a} \equiv \sqrt{C_2}$

Two-point and four-point functions

- Perturbative expansion in $\frac{1}{\kappa_4}$, expectation values are computed as:

$$\langle O[\rho]\rangle \equiv \frac{\int \mathcal{D}\rho O[\rho] e^{-S_G[\rho]} \left[1 - \frac{1}{\kappa_4}\int d^2u \rho^a_u\rho^a_u\rho^b_u\rho^b_u\right]}{\int \mathcal{D}\rho e^{-S_G[\rho]} \left[1 - \frac{1}{\kappa_4}\int d^2u \rho^a_u\rho^a_u\rho^b_u\rho^b_u\right]}$$

S_G - Gaussian action

- Two-point function

$$\langle \rho^{*a}(k)\rho^b(k')\rangle = \tilde{\mu}^2\delta^{ab}(2\pi)^2\delta(k-k')$$

$$\tilde{\mu}^2 \equiv \mu^2\left(1 - 4\frac{\mu^4}{\kappa_4}\frac{N_c^2+1}{\Delta^2 x}\right)$$

- Four point function

$$\langle \rho^{*\,a}(k_1)\rho^{*\,b}(k_2)\rho^c(k_3)\rho^d(k_4)\rangle =$$

$$(2\pi)^4\,\tilde{\mu}^4\Big[\delta^{ab}\delta^{cd}\delta(k_1+k_2)\delta(k_3+k_4) + \delta^{ac}\delta^{bd}\delta(k_1-k_3)\delta(k_2-k_4) + \delta^{ad}\delta^{bc}\delta(k_1-k_4)\delta(k_2-k_3)$$

$$+\frac{2}{\pi^2}\frac{\tilde{\mu}^4}{\kappa_4}\left(\delta^{ab}\delta^{cd}+\delta^{ac}\delta^{bd}+\delta^{ad}\delta^{bc}\right)\delta(k_1+k_2-k_3-k_4)\Big]$$

Generating function: $$F(z) \equiv \sum_{n=0}^{\infty} z^n P_n$$

Factorial cumulants:

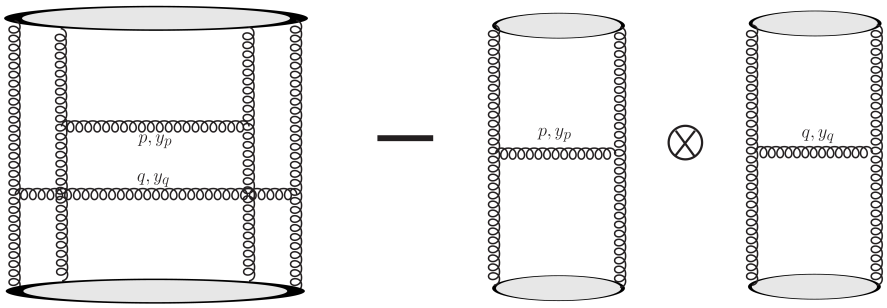
$$m_q \equiv \langle n(n-1)\cdots(n-q+1)\rangle - \text{disconnected} = \frac{d^q ln\, F(z)}{dz^q}$$

Factorial moments:

$$\langle n(n-1)\cdots(n-q+1)\rangle = \int \mathcal{D}\rho\, W[\rho]\, (n[\rho])^q$$

$$n[\rho] = \frac{dN}{d^2\mathbf{p}_\perp dy_p}$$ is multiplicity corresponding to fixed configuration of sources.

Two-gluon production:

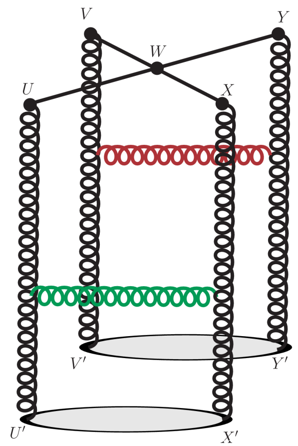


$$C(\mathbf{p},\mathbf{q}) = \left\langle \frac{dN}{d^2\mathbf{p}_{\perp 1}dy_1 p_{\perp 2}dy_2} \right\rangle - \left\langle \frac{dN}{d^2\mathbf{p}_{\perp}dy_p} \right\rangle \left\langle \frac{dN}{d^2\mathbf{q}_{\perp}dy_q} \right\rangle$$

$$C(\mathbf{p},\mathbf{q}) = \frac{g^{12}}{4(2\pi)^6} f_{gaa'} f_{g'bb'} f_{gcc'} f_{g'dd'} \int \prod_{i=1}^{4} \frac{d^2k_i}{(2\pi)_i^2 k_i^2} \frac{L_\mu(\mathbf{p},k_1)L^\mu(\mathbf{p},k_2)L_\nu(\mathbf{q},k_3)L^\nu(\mathbf{q},k_4)}{(\mathbf{p}-k_1)^2(\mathbf{p}-k_2)^2(\mathbf{q}-k_3)^2(\mathbf{q}-k_4)^2} \times$$

$$\langle \rho_1^{*a}(k_2)\rho_1^{*b}(k_4)\rho_1^c(k_1)\rho_1^d(k_3)\rangle \langle \rho_2^{*a'}(\mathbf{p}-k_2)\rho_2^{*b'}(\mathbf{q}-k_4)\rho_2^{c'}(\mathbf{p}-k_1)\rho_2^{d'}(\mathbf{q}-k_3)\rangle$$

Connected two-gluon diagrams from the *quartic action*:

$2\times$ 

$$= \frac{16 g^{12}}{(2\pi)^8 \pi^2 \kappa_4} \left[\int dv^- \tilde{\mu}^2(v^-)\right]^2 \left[\int dv^- \tilde{\mu}^4(v^-)\right]^2 \frac{S_\perp N_c^2 (N_c^2-1)(N_c^2+1)}{p^2 q^2} \times$$

$$\int \frac{d^2 k_1}{k_1^2} \frac{1}{(p-k_1)^2} \int \frac{d^2 k_2}{k_2^2} \frac{1}{(q-k_2)^2}$$

$$= \frac{g^{12}}{4\pi^8 \kappa_4} \left[\int dv^- \tilde{\mu}^2(v^-)\right]^2 \left[\int dv^- \tilde{\mu}^4(v^-)\right]^2 \frac{S_\perp N_c^2 (N_c^2-1)(N_c^2+1)}{p^4 q^4} \ln\frac{p}{Q_s} \ln\frac{q}{Q_s}$$

$$C(p,q) = \frac{2\pi - \frac{4Q_s^2(N_c^2+1)}{\kappa_4}\frac{\left[\int dv^- \tilde{\mu}^4\right]^2}{\left[\int dv^- \tilde{\mu}^2\right]^2}}{q_s^2(N_c^2-1)S_\perp}\left\langle \frac{dN}{dp\,dy_p}\right\rangle \left\langle \frac{dN}{dq\,dy_q}\right\rangle$$

Second cumulant: $m_2 = \frac{1}{k}\bar{n}^2$

$$C(p,q) = \frac{1}{k}\left\langle \frac{dN}{dp\,dy_p}\right\rangle \left\langle \frac{dN}{dq\,dy_q}\right\rangle$$