On a relation between production processes and total cross sections

Stéphane Munier

CPHT, École Polytechnique, CNRS

QCD at high density

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At very high energies, parton densities may become so large that they ***saturate****, which means that the evolution equations become* ***nonlinear*** *(BK, JIMWLK equations), and predict the emergence of a new hard, energy-dependent, momentum scale called the* ***saturation scale*** Q_s .

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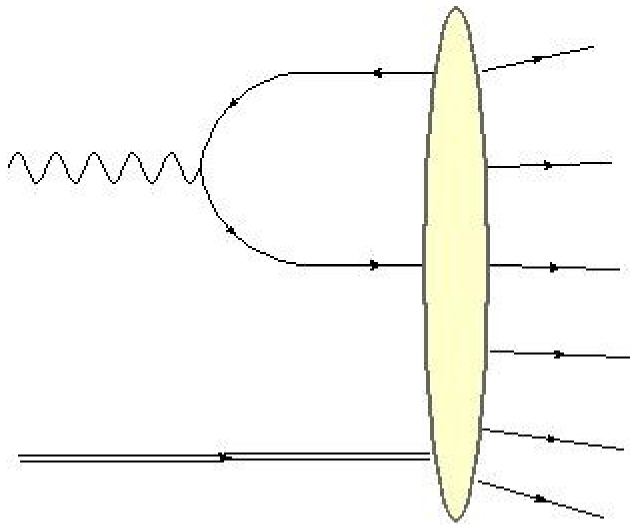
This regime is very interesting theoretically. Parton saturation may also have important phenomenological consequences at the LHC.

QCD at high density: How to test it?

At an **electron-hadron collider**:



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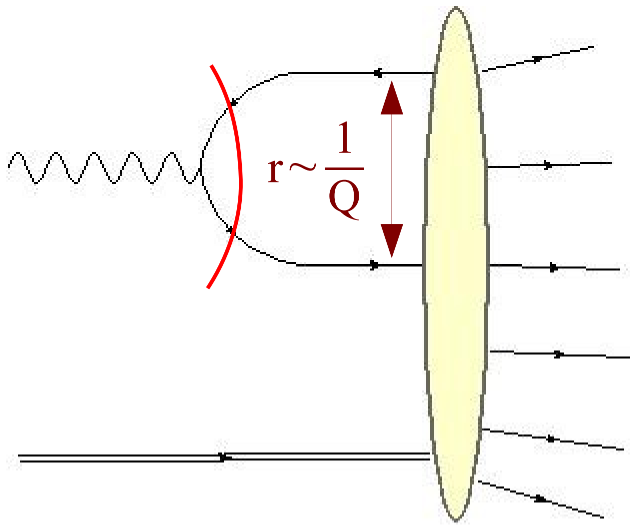
DIS can be understood as a dipole of "tunable" size r interacting with the target (p or A), that gets denser at higher energies.



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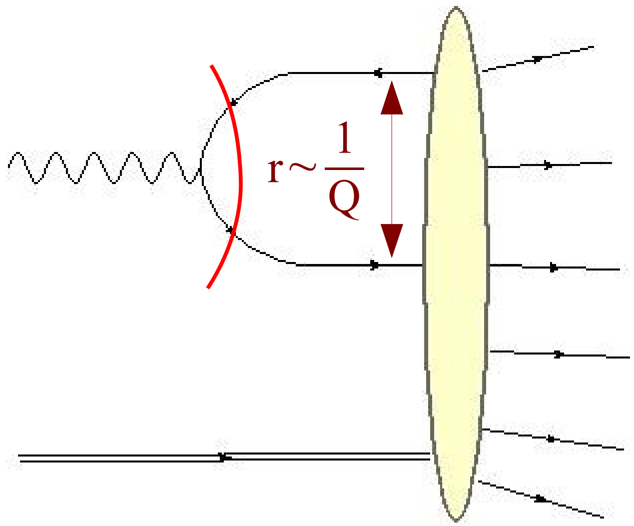

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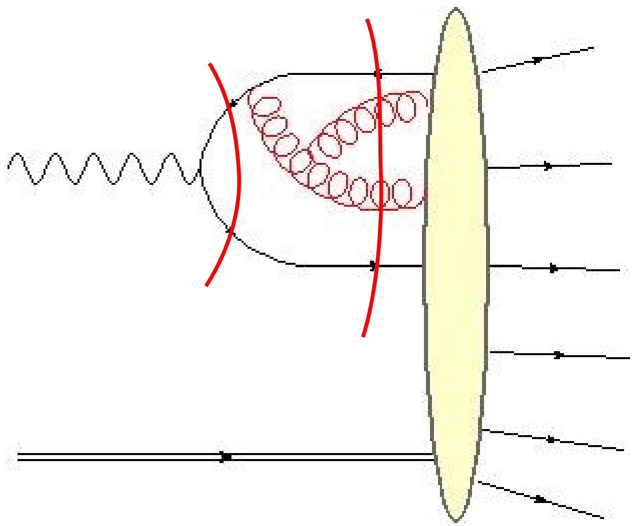

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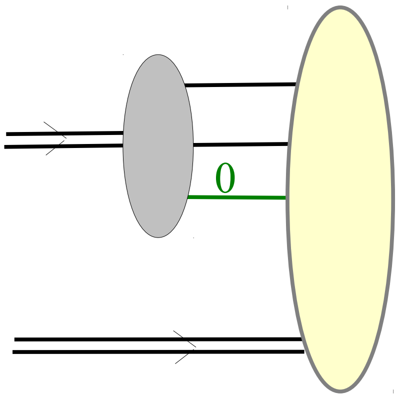
On the theoretical size, it is "easy" to formulate the QCD evolution of the dipole amplitude with the energy as ***radiative corrections to the dipole wave function****.*

BFKL (at low density), BK, JIMWLK equations (accounting for high-density effects)

QCD at high density: How to test it?

At a **hadron collider**, we need to find appropriate production processes:



QCD at high density: How to test it?

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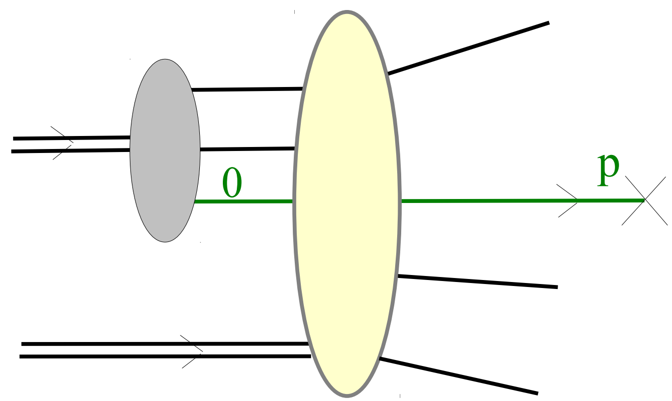
- p_T *-broadening:*



Observe a jet of transverse momentum
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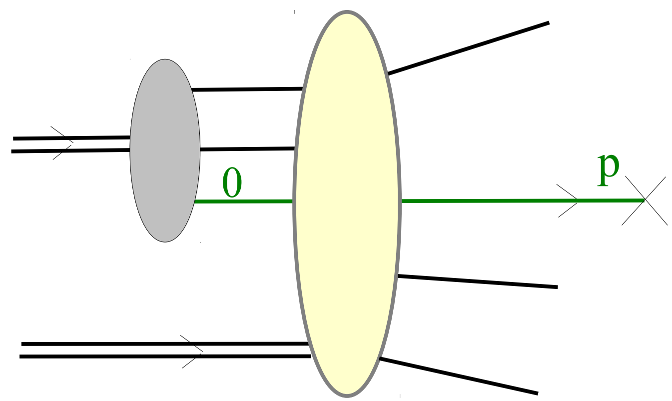

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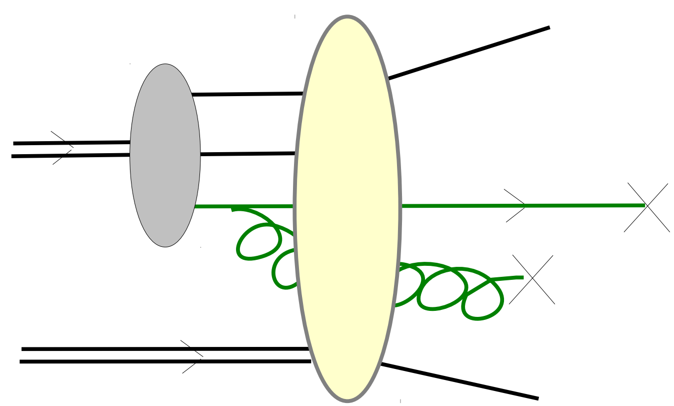
* *Forward dijet azimuthal correlations:*



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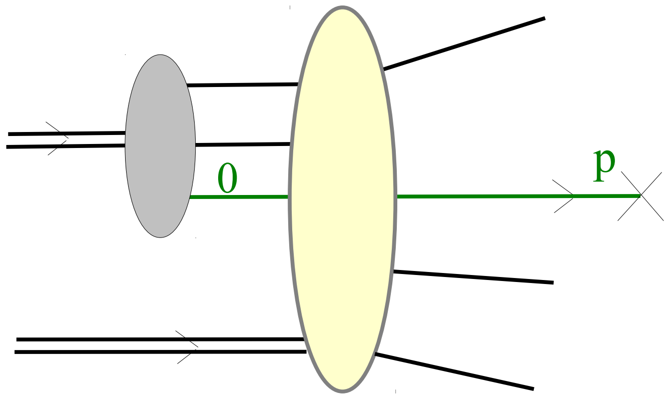

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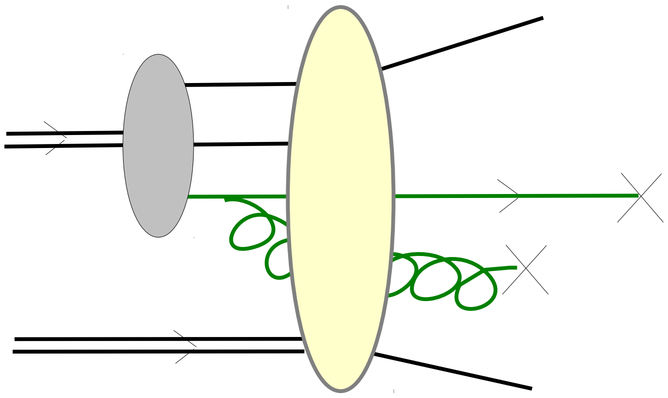

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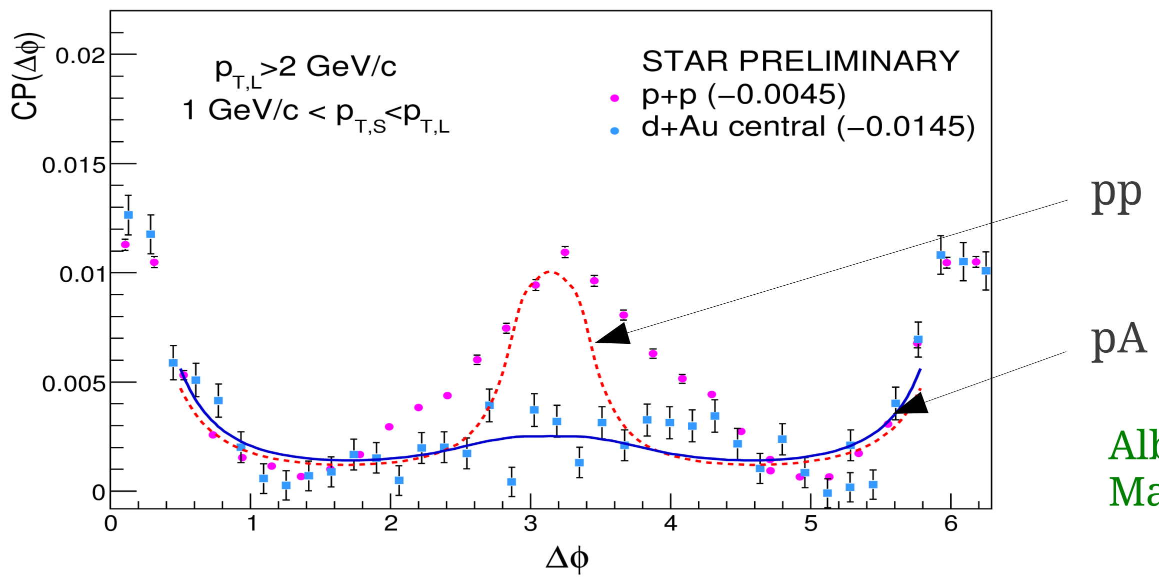
These observables are more tricky to formulate in QCD!

Forward dijet azimuthal correlations

Approximate formulation

Marquet (2007)

*Prediction that some **"jet" correlations** are different between pA and pp:*



Albacete,
Marquet PRL (2010)

- *Qualitative agreement with experimental results*
- *A better description needs more theoretical work*

See also Dominguez, Marquet, Xiao, Yuan (2011)

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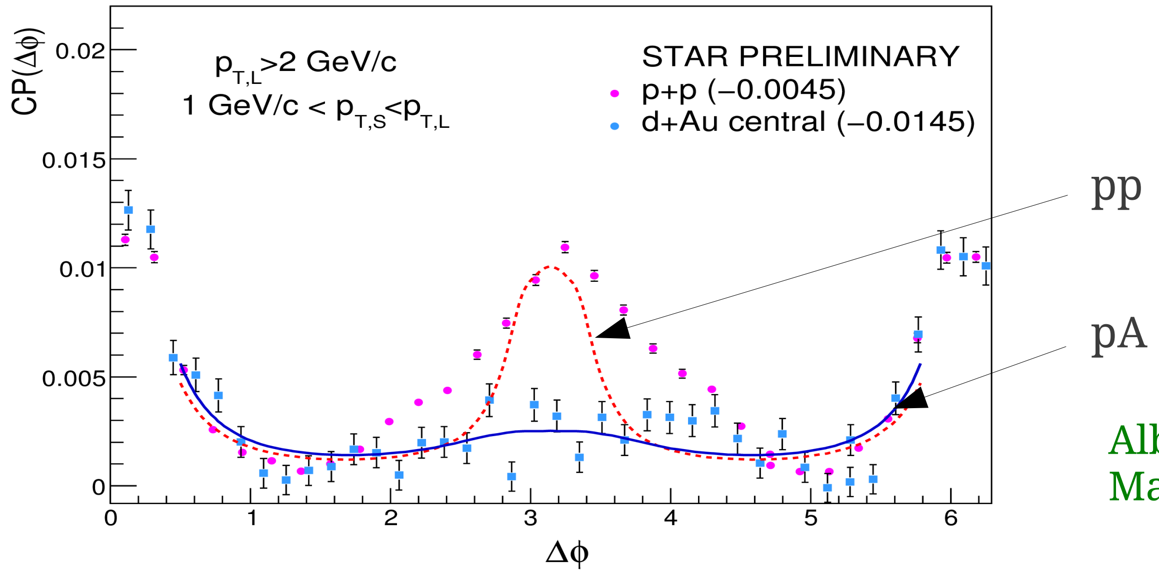

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This talk: ***report on a rigorous link between DIS total cross sections and this kind of semi-inclusive observables in pp and pA collisions.***

A. H. Mueller, S. Munier, Nucl. Phys. A (2012)

Outline

* Formulation of a production process in pA
* Quantum corrections: leading order
* Next-to-leading order

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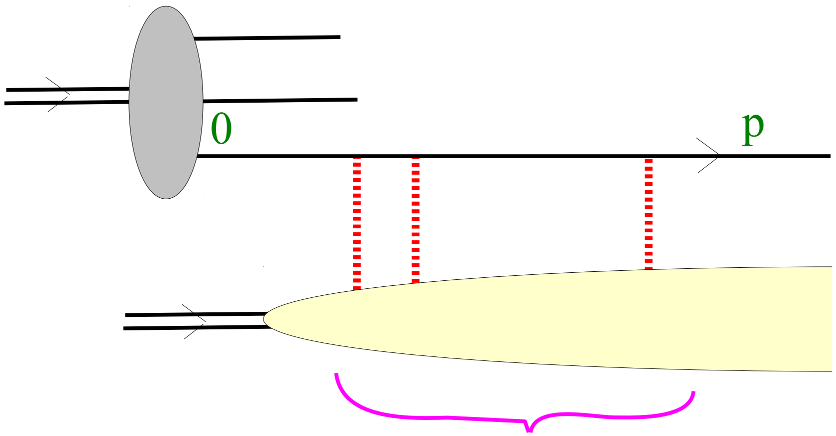
- *Formulation of a production process in pA*
- *Quantum corrections: leading order*

  $$\alpha_s \log s \rightarrow \sum (\alpha_s \log s)^n$$

- *Next-to-leading order*

  $$\alpha_s^2 \log s \rightarrow \sum \alpha_s (\alpha_s \log s)^n$$

Formulation of p_T *-broadening*



time $t=-\infty$ $t=0$ $t=+\infty$

Initial state *Final state*

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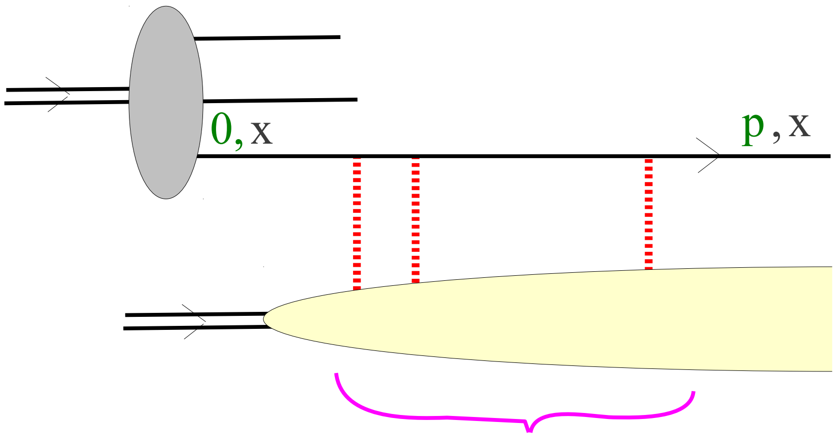

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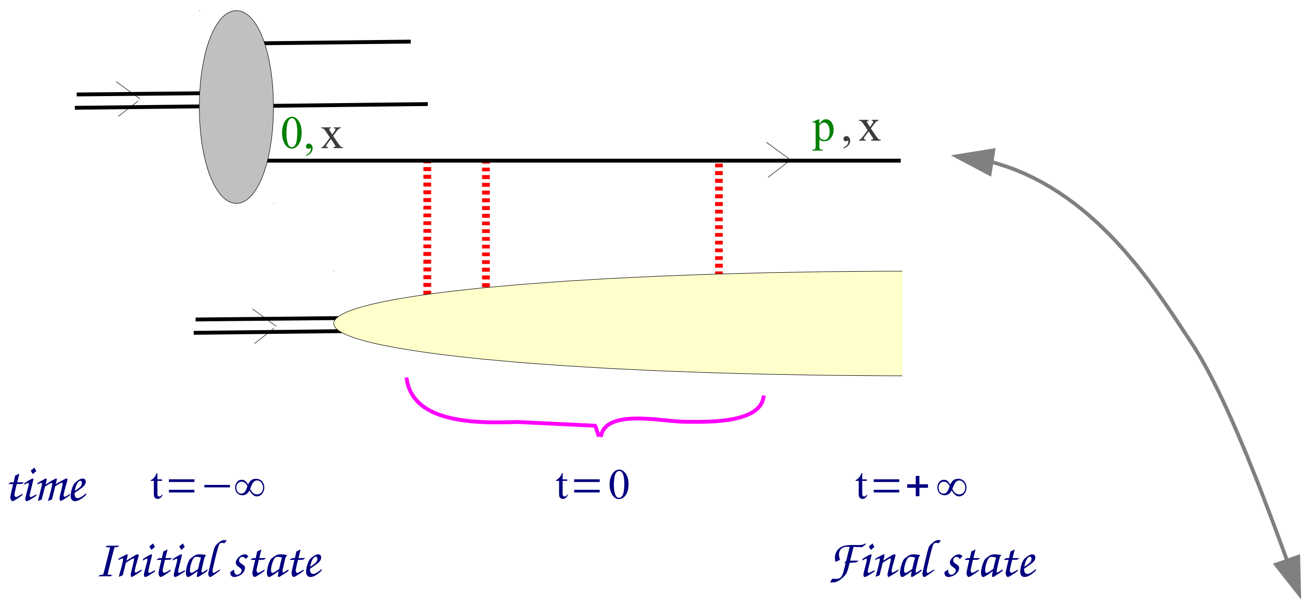
$$\int \frac{d^2x}{(2\pi)^2} e^{-ipx}$$

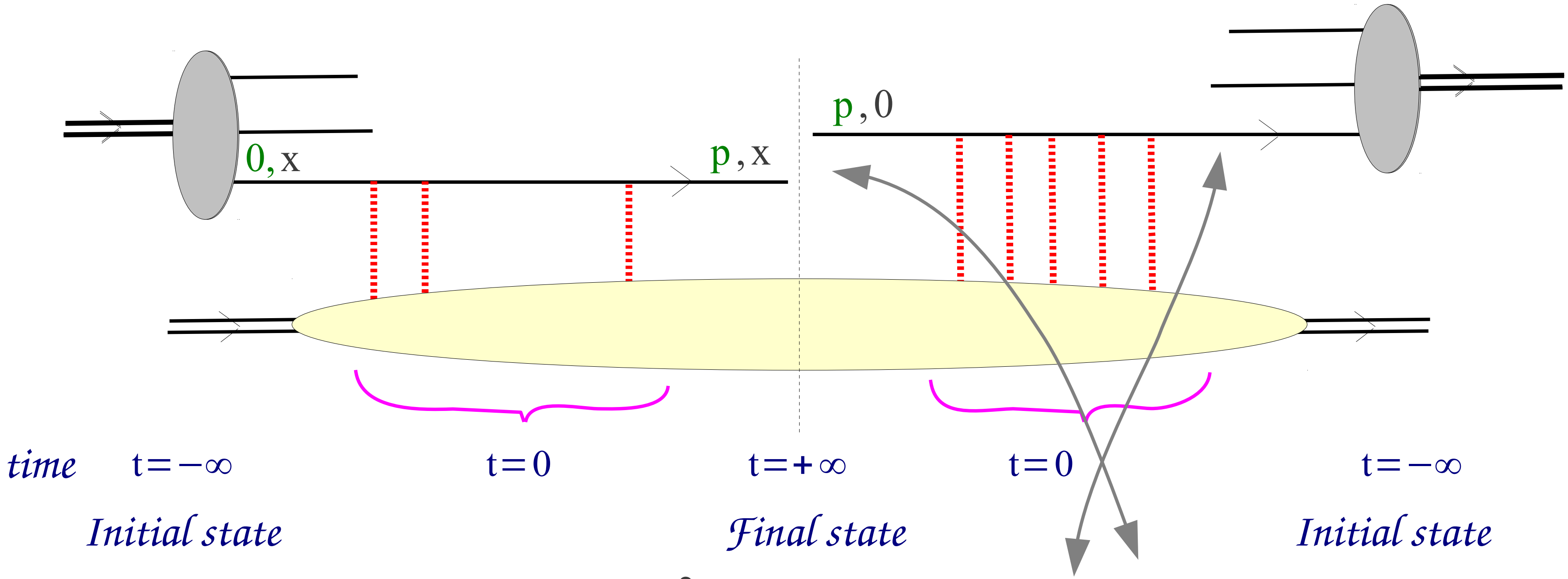
Formulation of p_T-broadening



$$\int \frac{d^2x}{(2\pi)^2} e^{-ipx} \quad V_x$$

Wilson line

Formulation of p_T-broadening

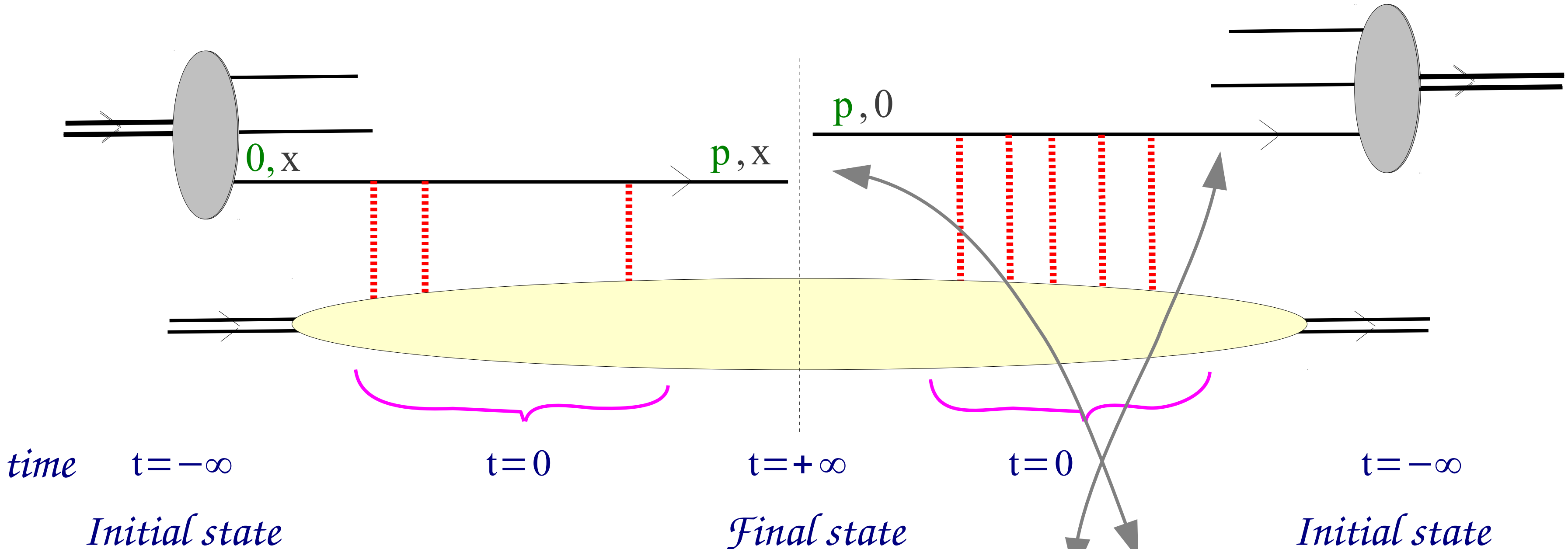


$$\frac{dN}{d^2 p} \sim \int \frac{d^2 x}{(2\pi)^2} e^{-ipx}$$

$V_0^* V_x$

Wilson lines

Formulation of p_T-broadening



$$\frac{dN}{d^2 p} = \int \frac{d^2 x}{(2\pi)^2} e^{-ipx} \left\langle \frac{1}{N_c} \mathrm{Tr}\, V_0^* V_x \right\rangle$$

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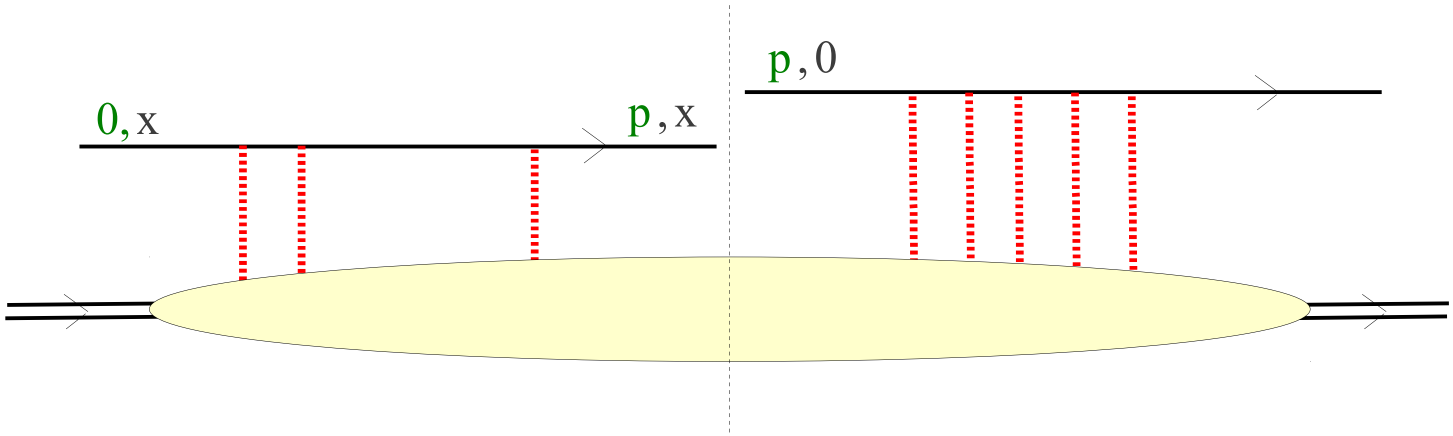

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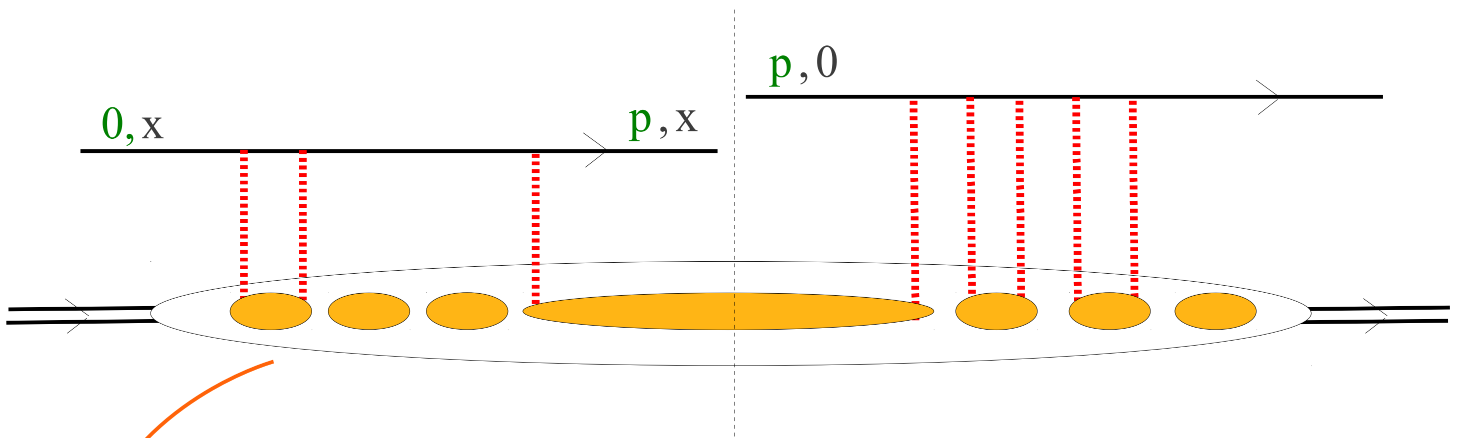

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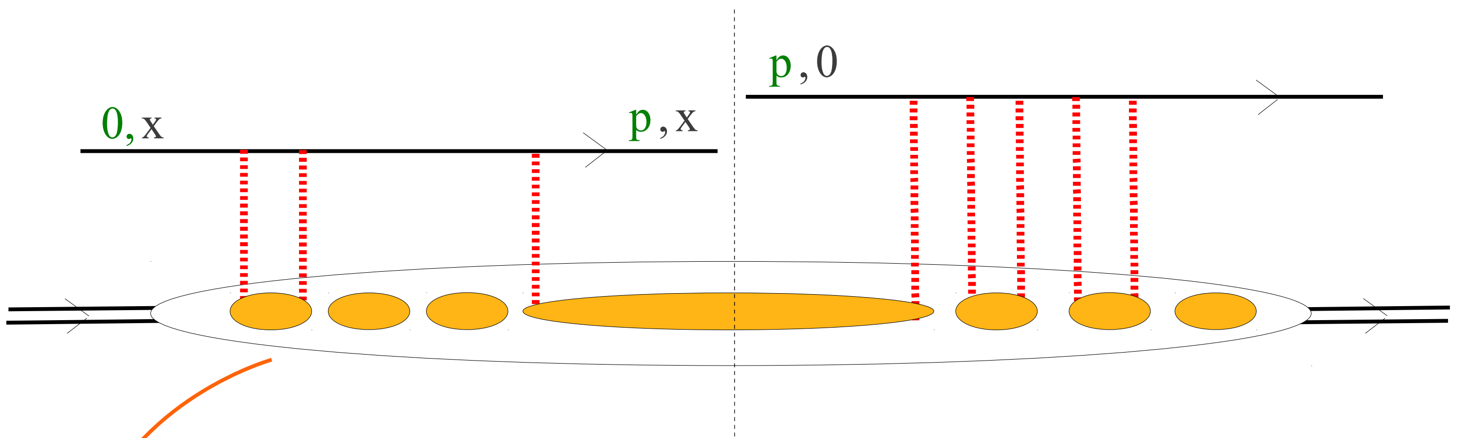
McLerran-Venugopalan model (assumes 2-gluon exchanges at most)

S-matrix element for the elastic scattering of a color dipole

Formulation of p_T-broadening



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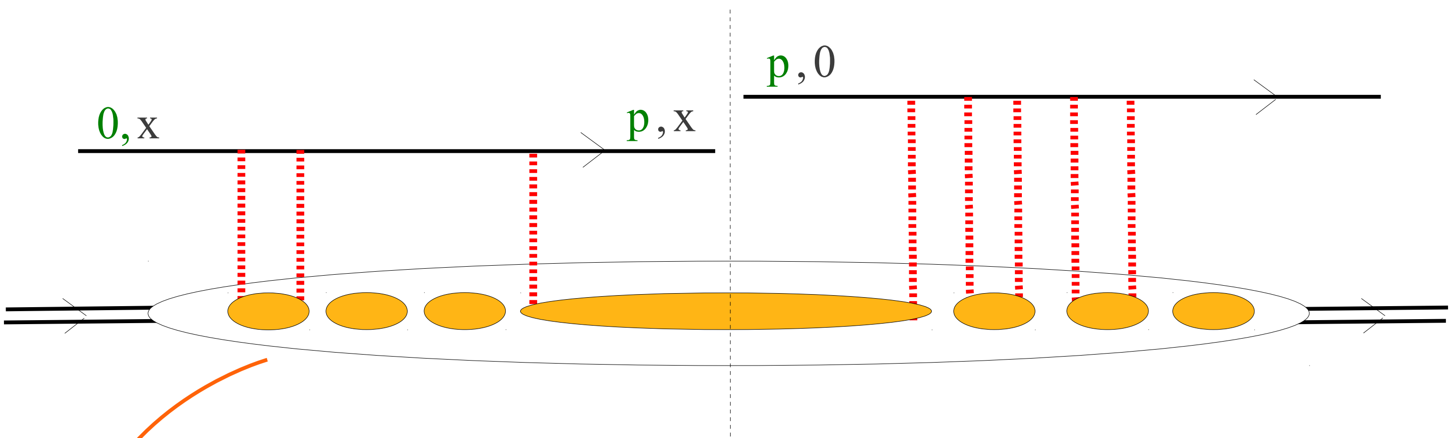

$$\frac{dN}{d^2p} = \int \frac{d^2x}{(2\pi)^2} e^{-ipx} \quad S_{dipole}(x) \sim e^{-p^2/Q_s^2}$$

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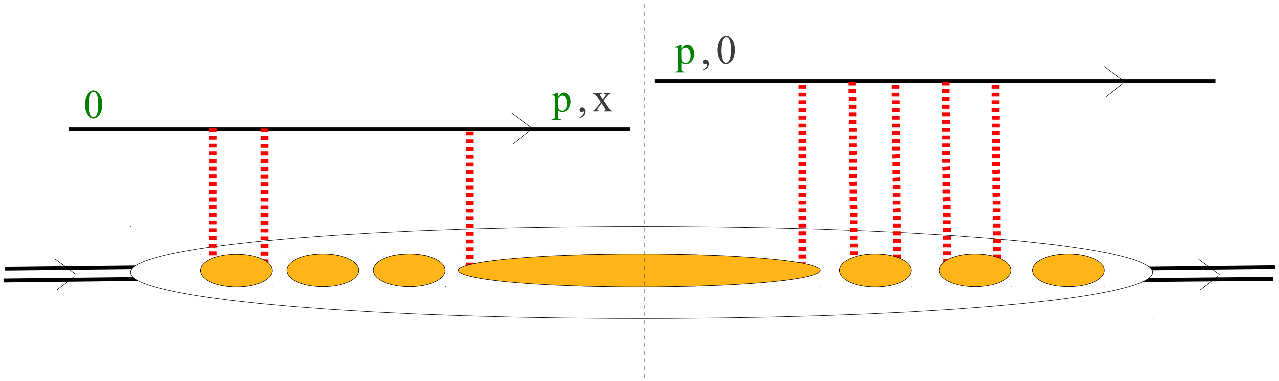
S-matrix element for the elastic scattering of a color dipole

$$S_{dipole}(x) = \exp\left(-\frac{x^2 Q_s^2}{4}\right)$$

Formulation of p_T-broadening

$$\frac{dN}{d^2p} =$$



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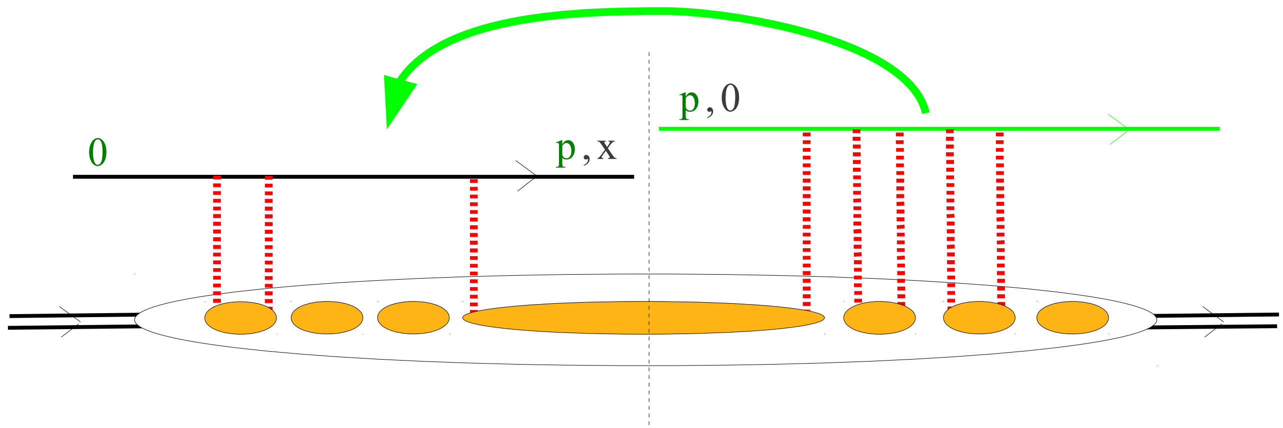

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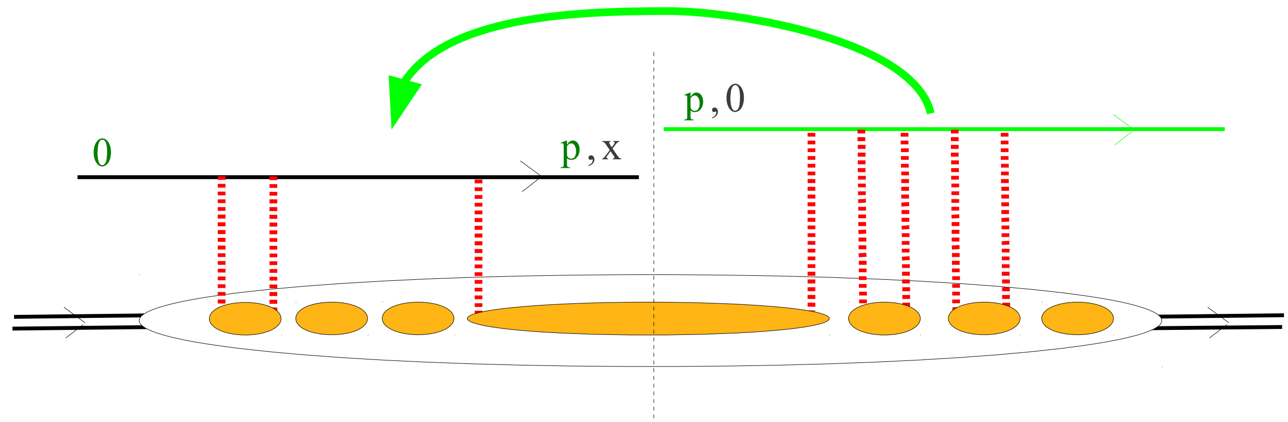

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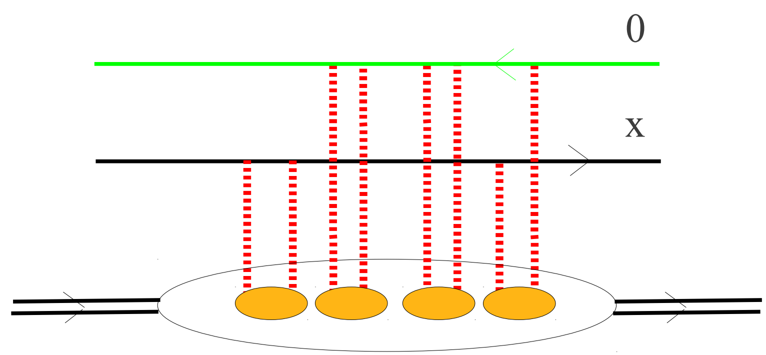

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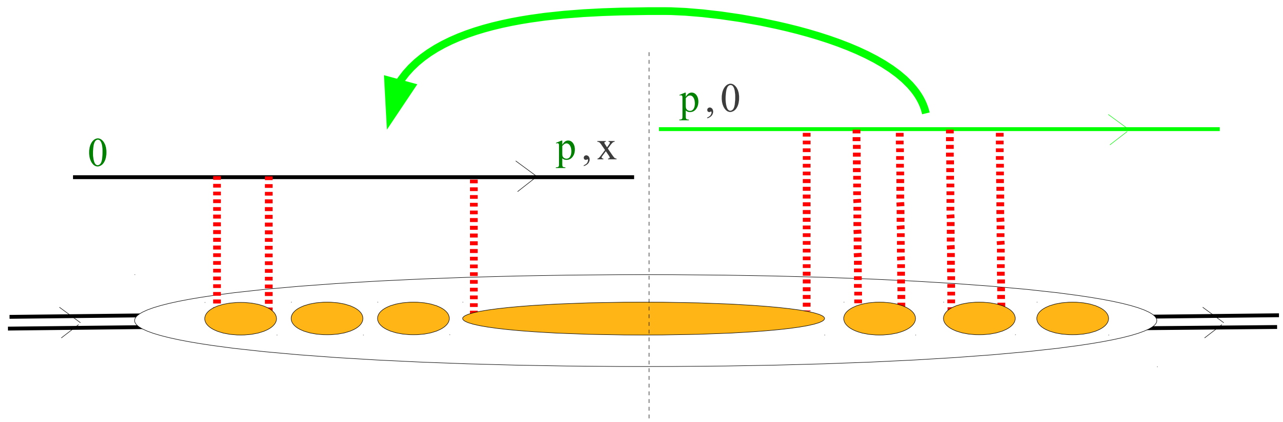

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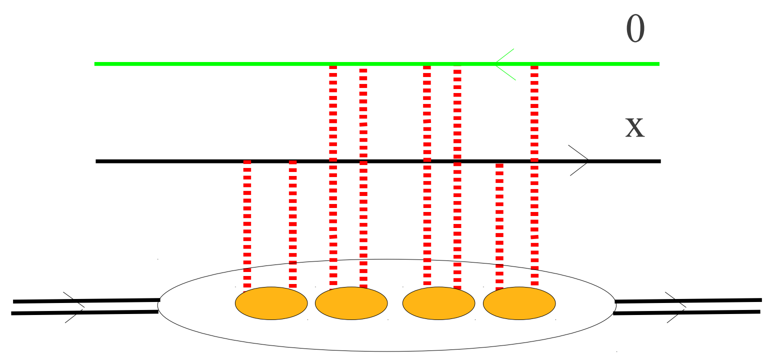


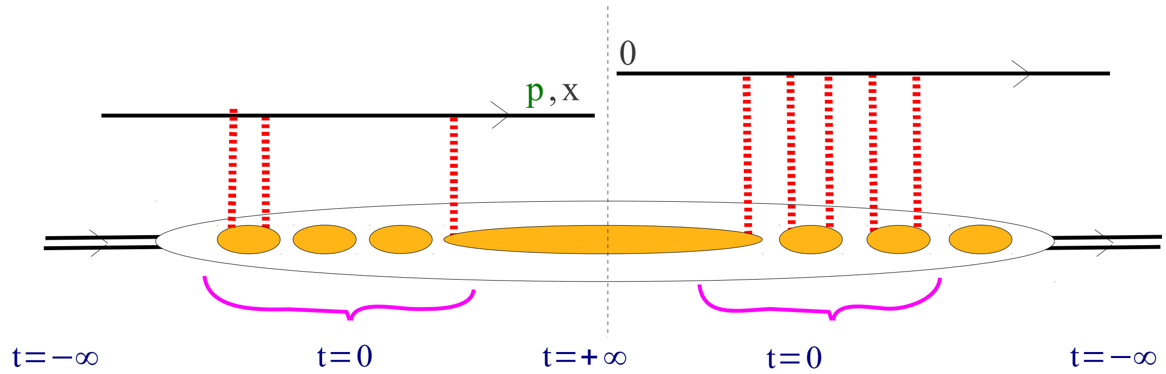
$$\frac{dN}{d^2p} = \int \frac{d^2x}{(2\pi)^2} e^{-ipx} S_{dipole}(x)$$

Intuitively: just bend the quark line in the complex conjugate amplitude to an antiquark line to transform it to a dipole amplitude!

Zakharov (1996...)

already mentioned in Bin Wu's talk yesterday

Quantum corrections



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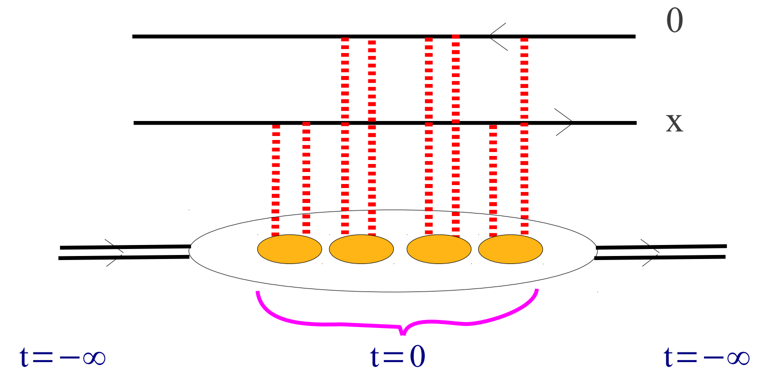

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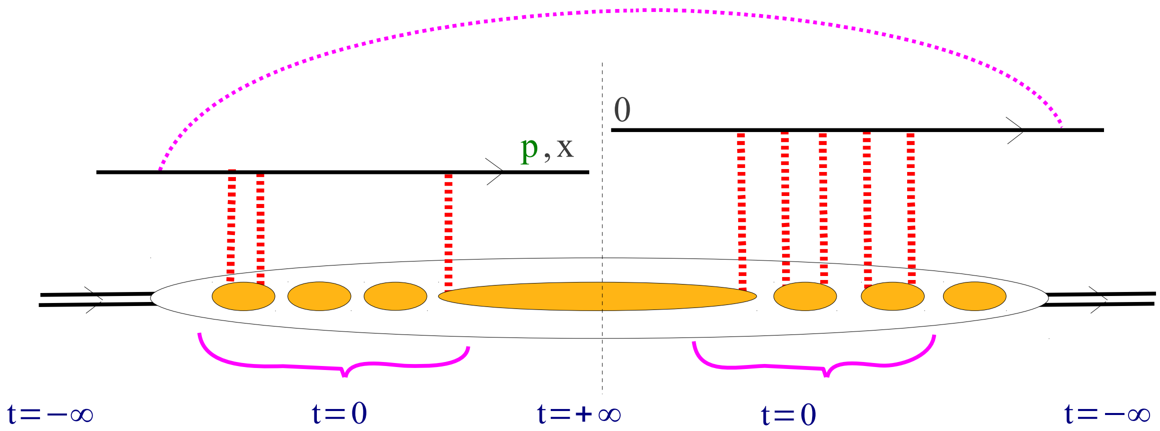

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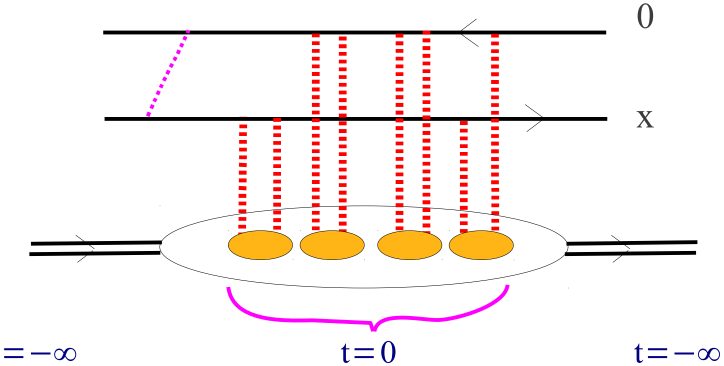

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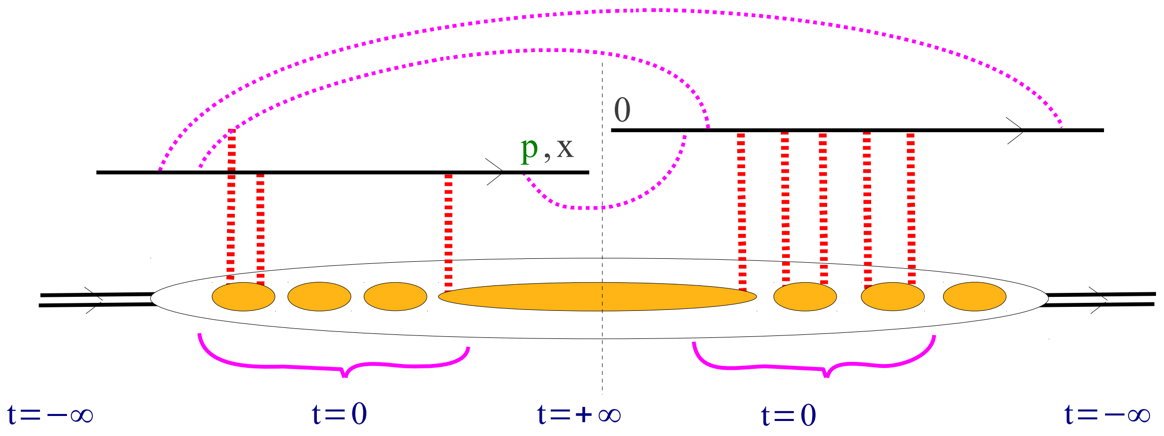

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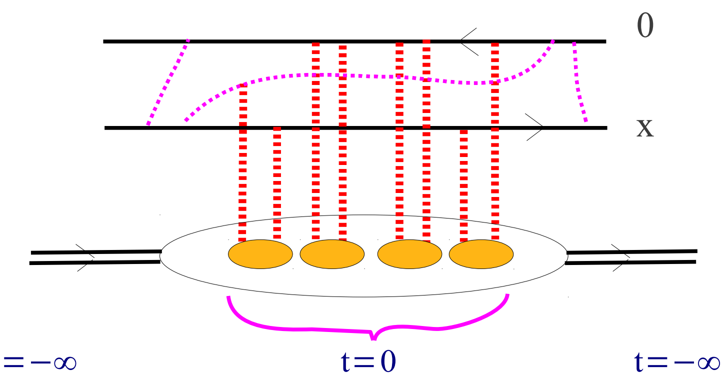
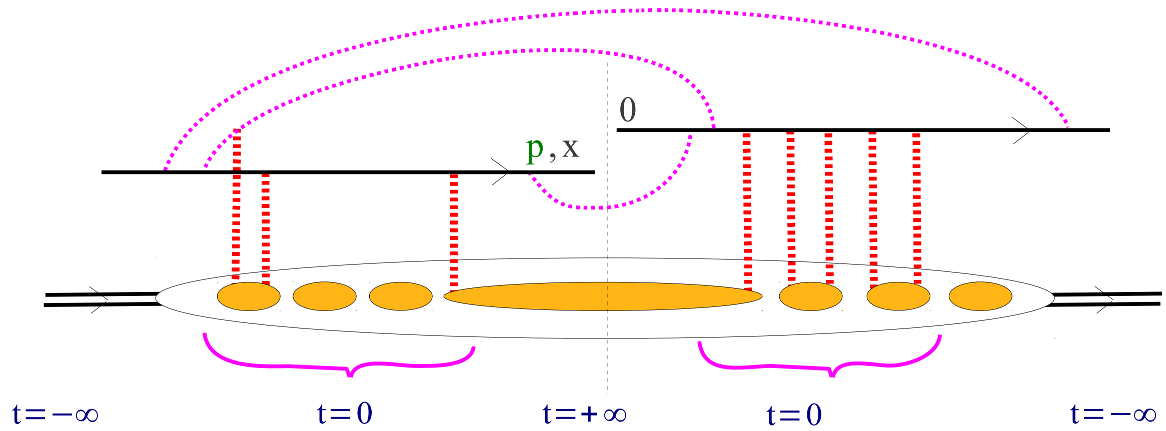
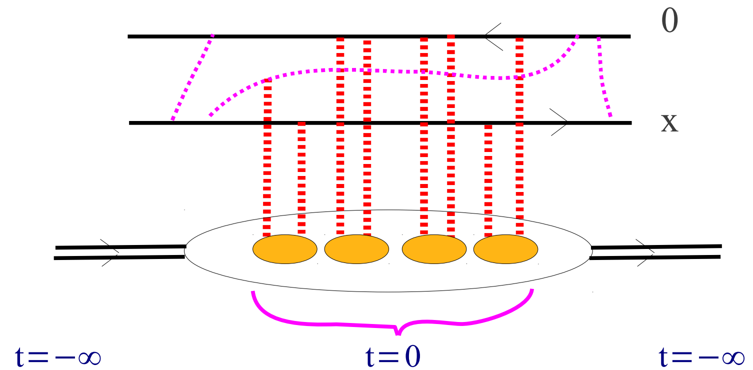


$$S_{\mathrm{dipole}}(x) = \left\langle \frac{1}{N_c} \mathrm{Tr}\left(V_0^* V_x \right) \right\rangle$$

Quantum corrections



$$\frac{dN}{d^2p} = \int \frac{d^2x}{(2\pi)^2} e^{-ipx} \left\langle \frac{1}{N_c} \mathrm{Tr}\left(T\, V_0^* e^{-i\int d^4y\, L_I} \right) \left(T\, V_x\, e^{i\int d^4y\, L_I} \right) \right\rangle$$



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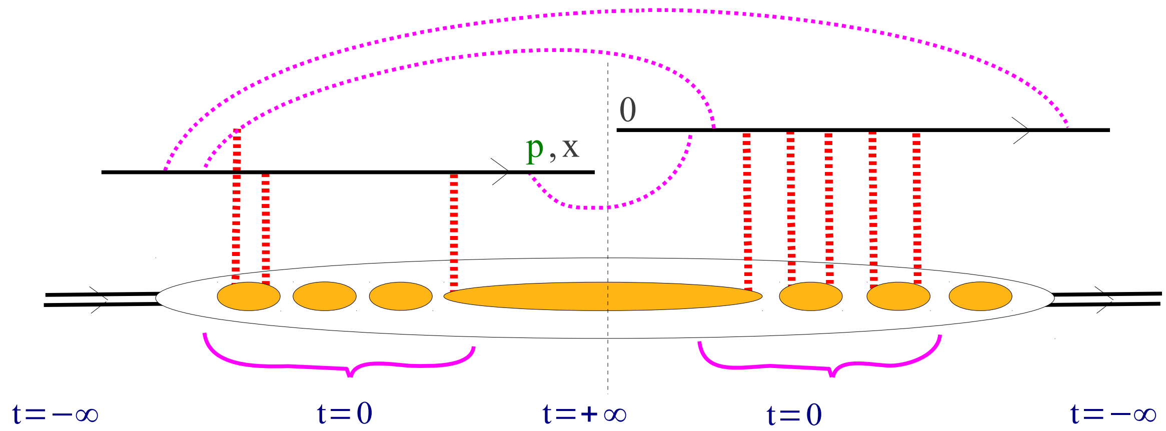
Quantum corrections



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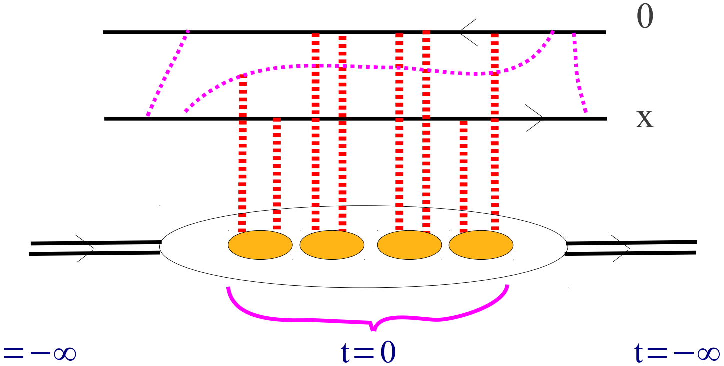

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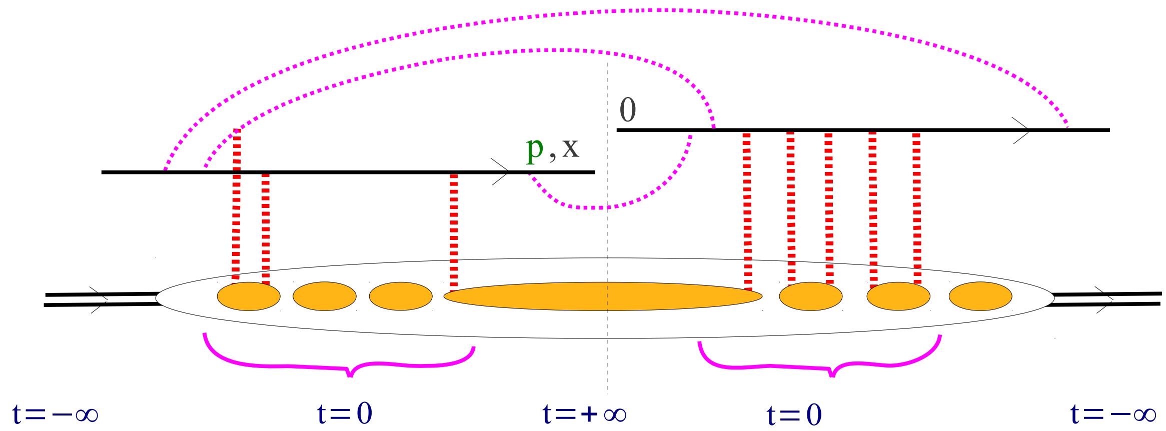
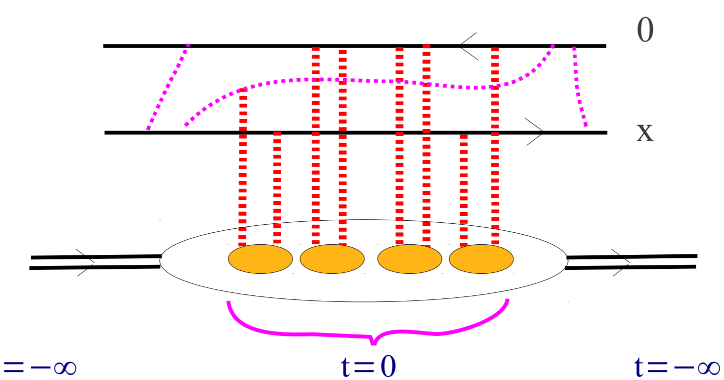
Quantum corrections



$$\frac{dN}{d^2p}=\int\frac{d^2x}{(2\pi)^2}e^{-ipx}\left\langle\frac{1}{N_c}\mathrm{Tr}\left(TV_0^*e^{-i\int d^4y\,L_I}\right)\left(TV_x\,e^{i\int d^4y\,L_I}\right)\right\rangle$$

Two times

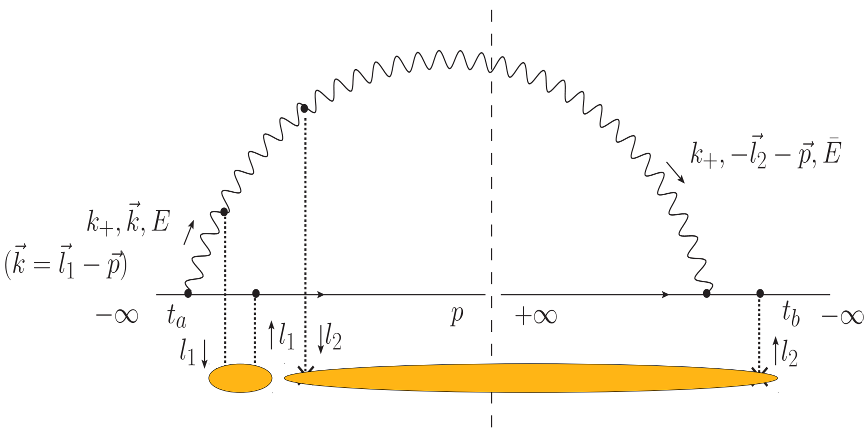


$$S_{dipole}(x)=\left\langle\frac{1}{N_c}\mathrm{Tr}\left(TV_0^*V_x\,e^{i\int d^4y\,L_I}\right)\right\rangle$$

One single time

?

Quantum corrections: leading order



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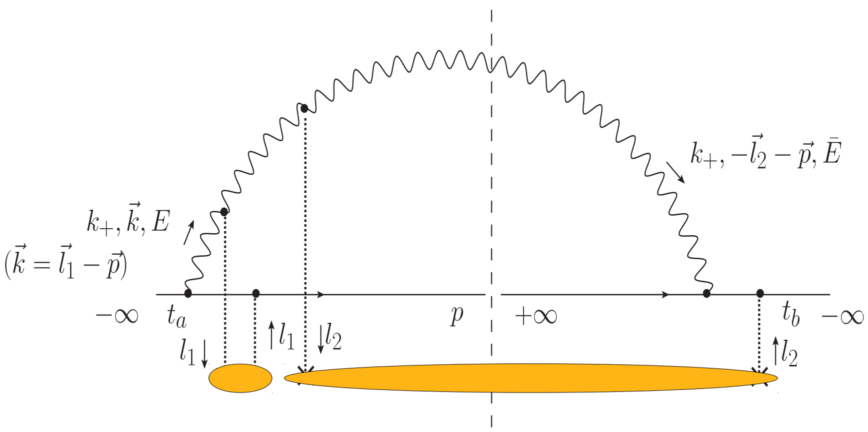

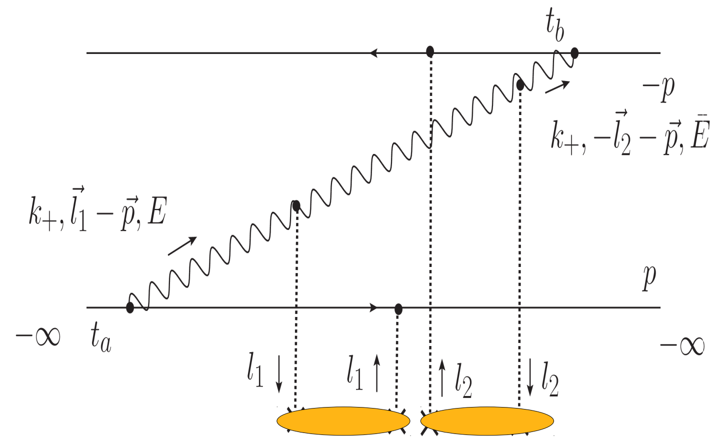

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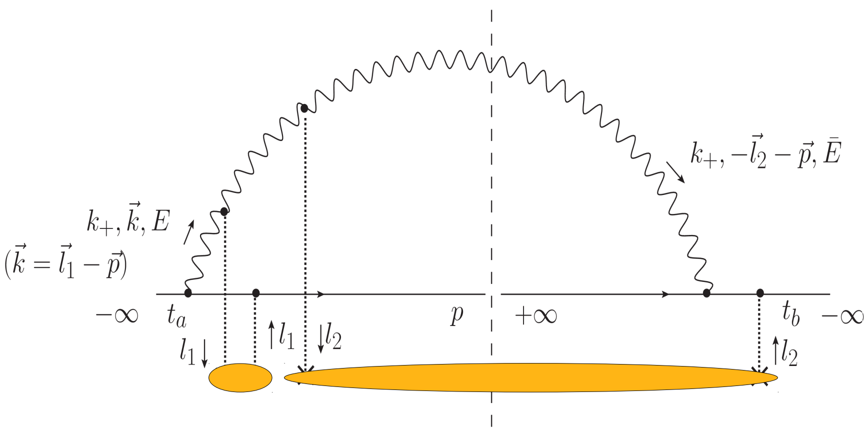
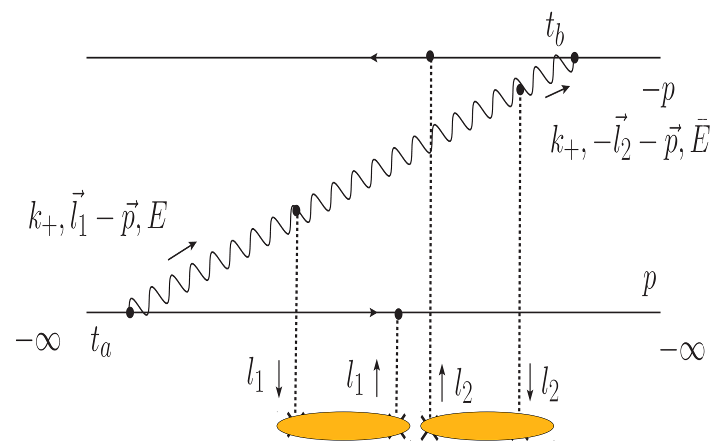


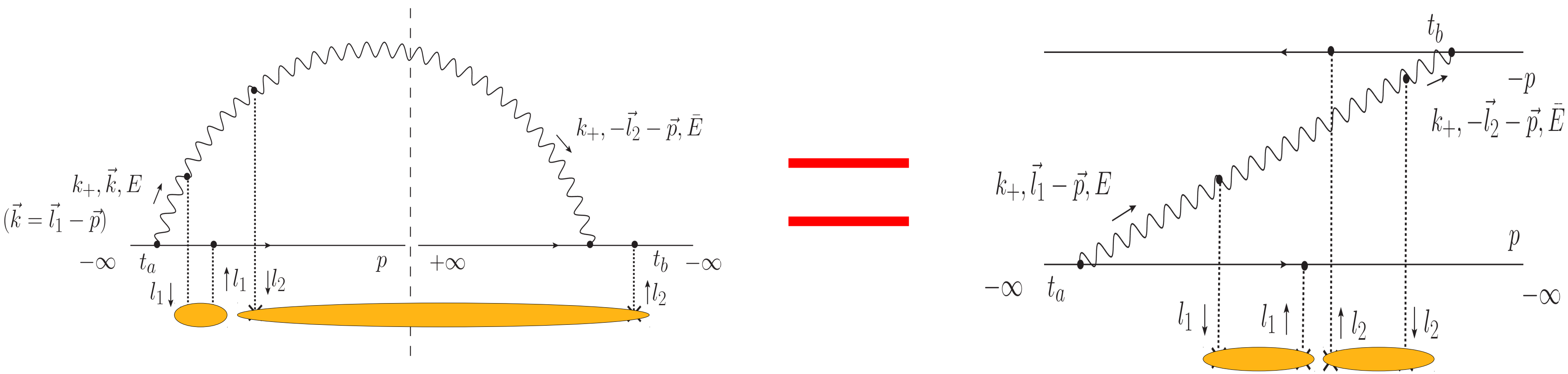


Contribution of this graph to $\dfrac{dN}{d^2 p}$

$$-\frac{\alpha_s N_c}{N_c^2-1}\int_0^{\sqrt{s}}\frac{dk_+}{k_+}\int\frac{d^2 l_1}{l_1^2}\,\frac{d^2 l_2}{l_2^2}\,\frac{(p-l_1)(p+l_2)}{(p-l_1)^2(p+l_2)^2}\big(\alpha_s\, xg(l_1)\big)\big(\alpha_s\, xg(l_2)\big)$$

Quantum corrections: leading order

Graph-to-graph, momentum-to-momentum correspondence



Contribution of this graph to $\frac{dN}{d^2p}$ ***and*** *to the dipole S-matrix element in momentum space:*

$$-\frac{\alpha_s N_c}{N_c^2-1}\int_0^{\sqrt{s}} \frac{dk_+}{k_+}\int \frac{d^2 l_1}{l_1^2}\frac{d^2 l_2}{l_2^2}\frac{(p-l_1)(p+l_2)}{(p-l_1)^2(p+l_2)^2}\big(\alpha_s xg(l_1)\big)\big(\alpha_s xg(l_2)\big)$$

Quantum corrections: leading order

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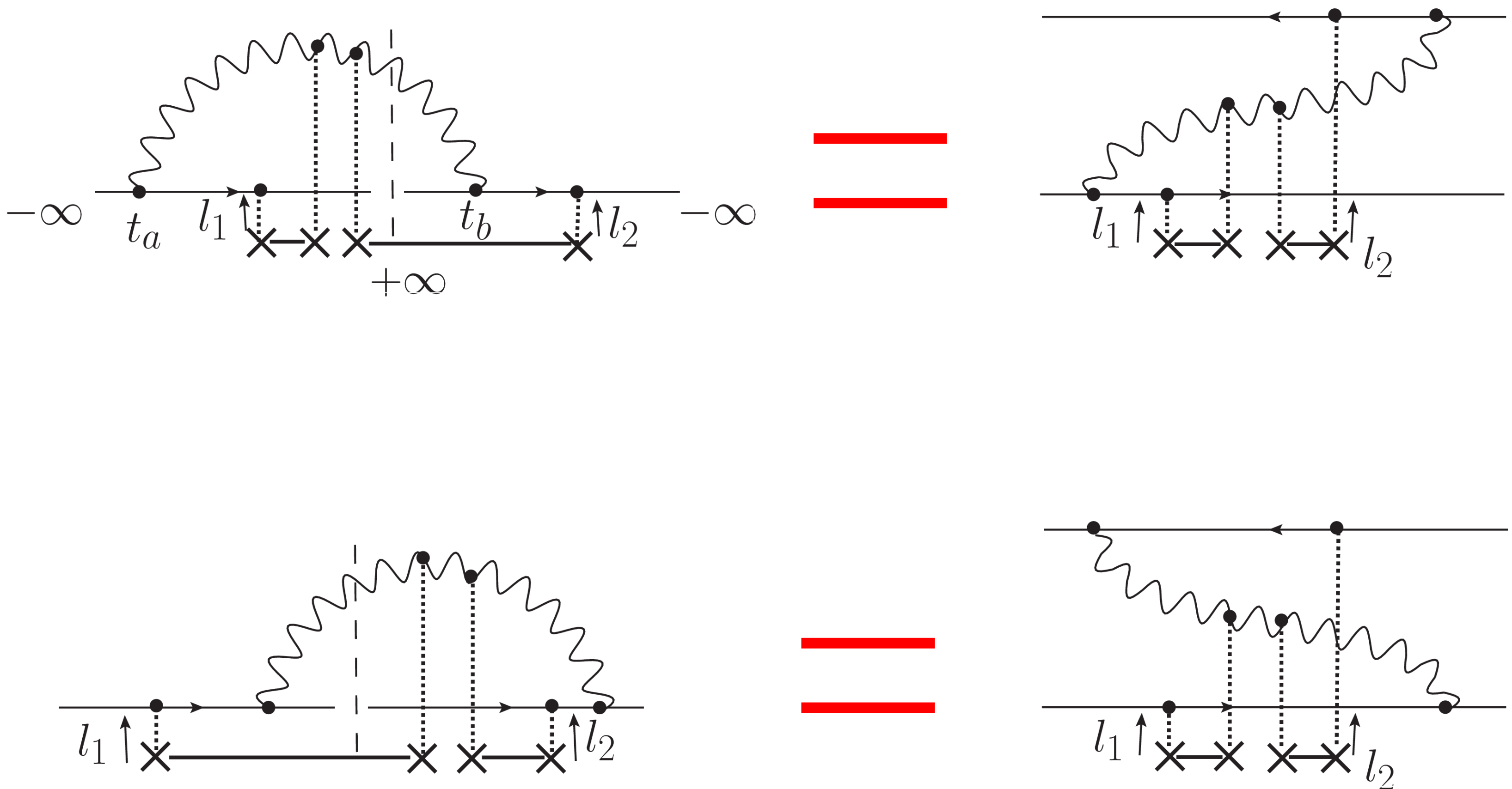

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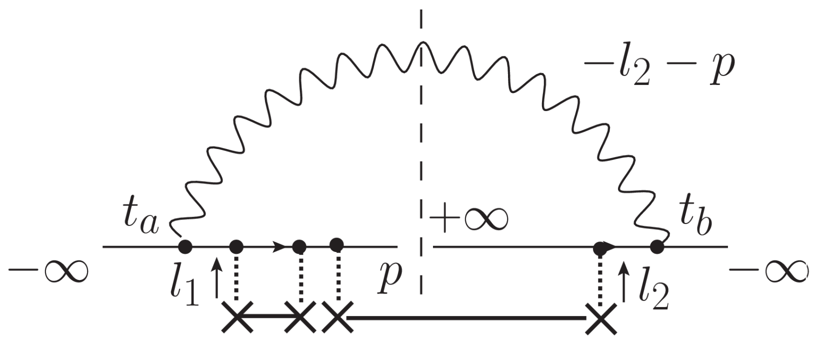

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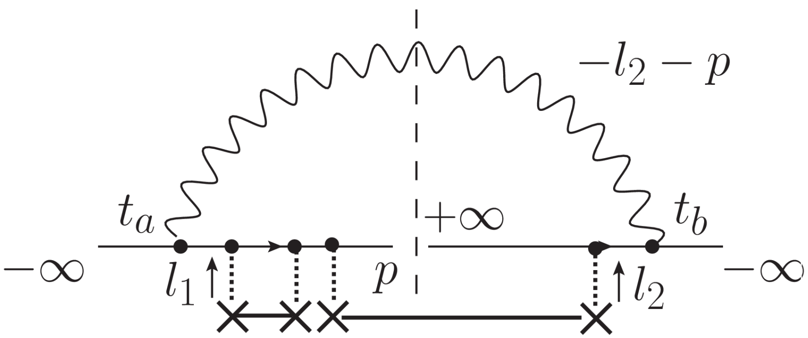

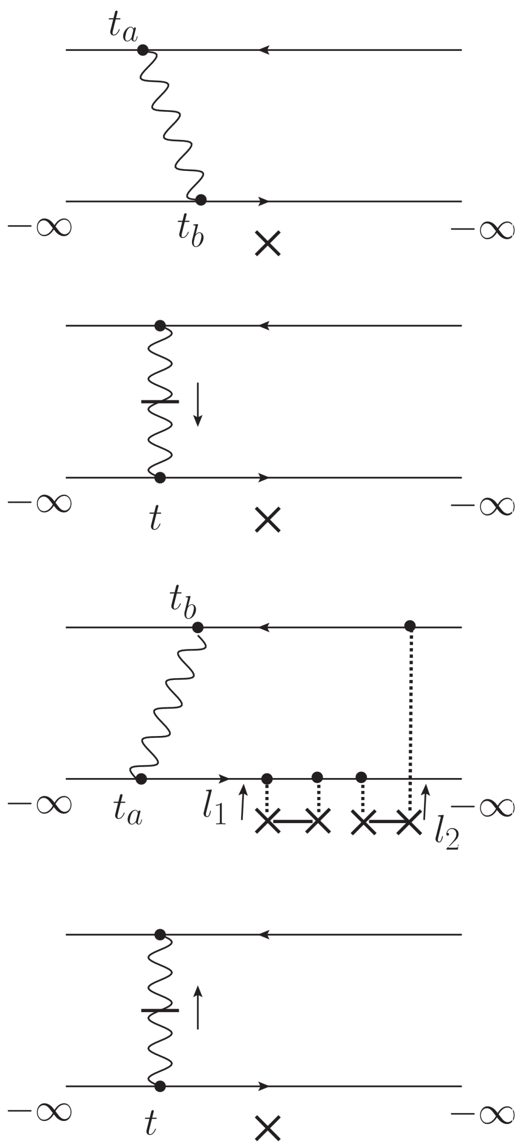

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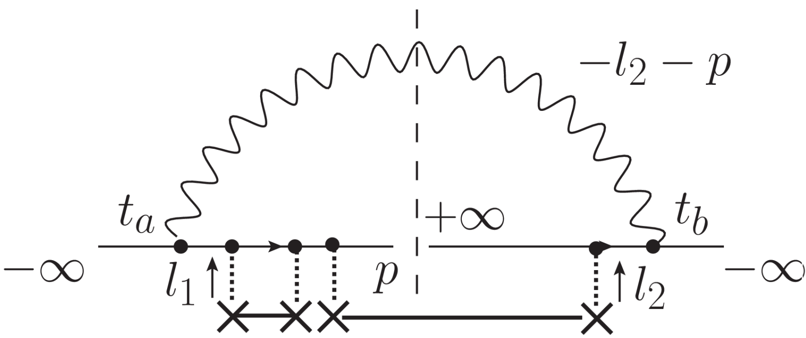
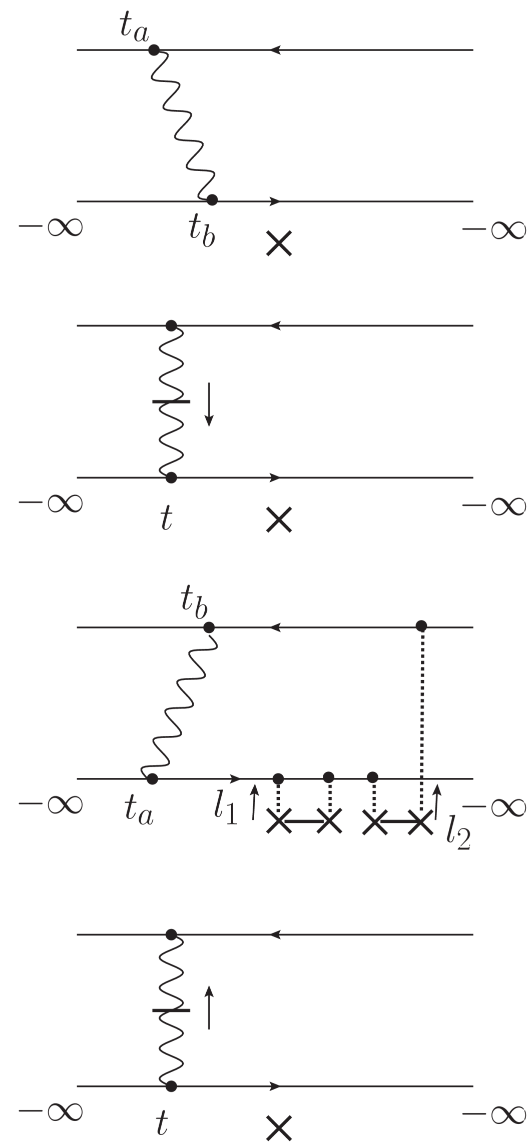


Some graphs on the "broadening side" that have no obvious counterpart on the dipole side turn out to "miraculously" cancel among themselves...



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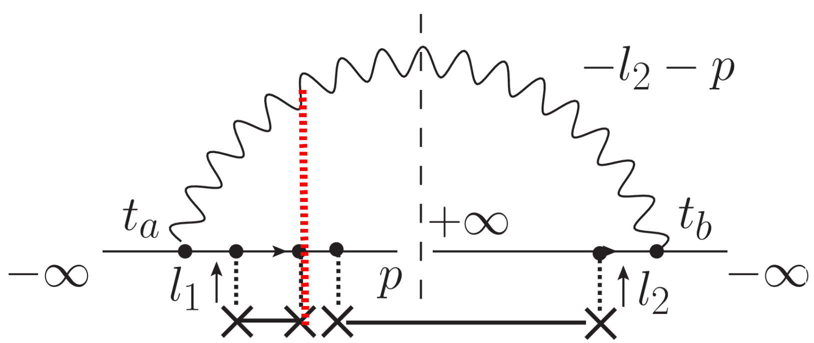

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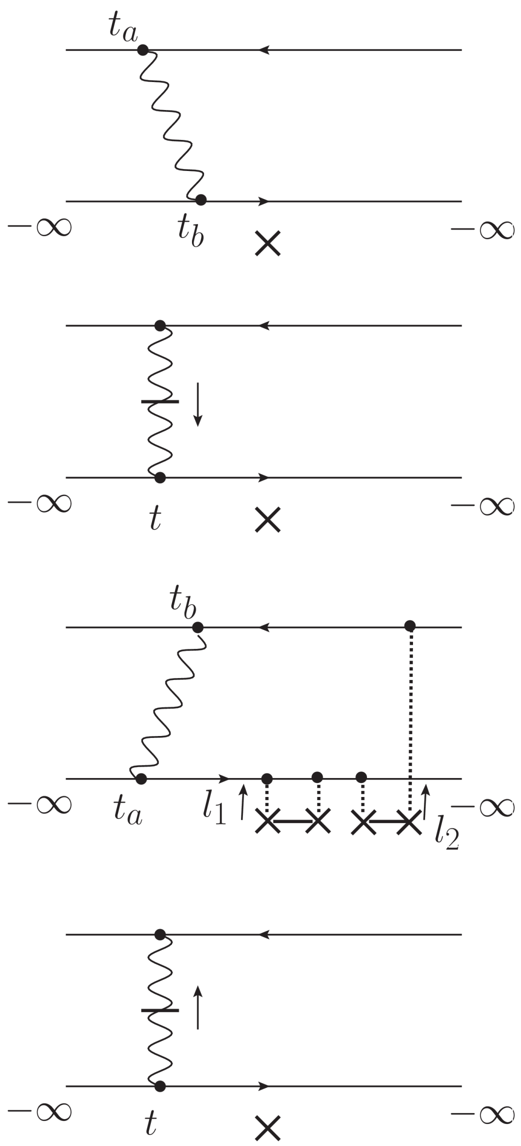
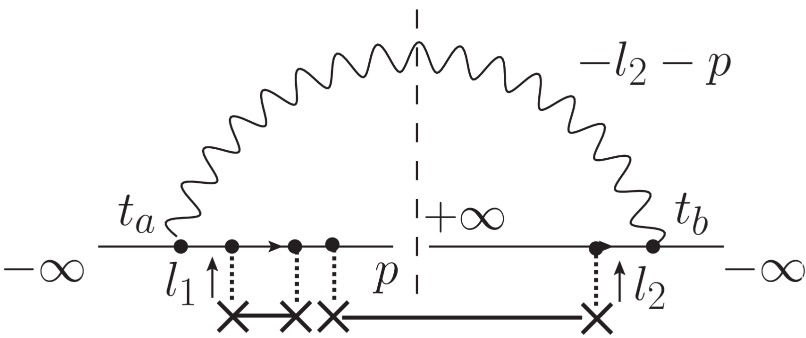


Quantum corrections: leading order

Graph-to-"group of graphs", momentum-to-momentum correspondence



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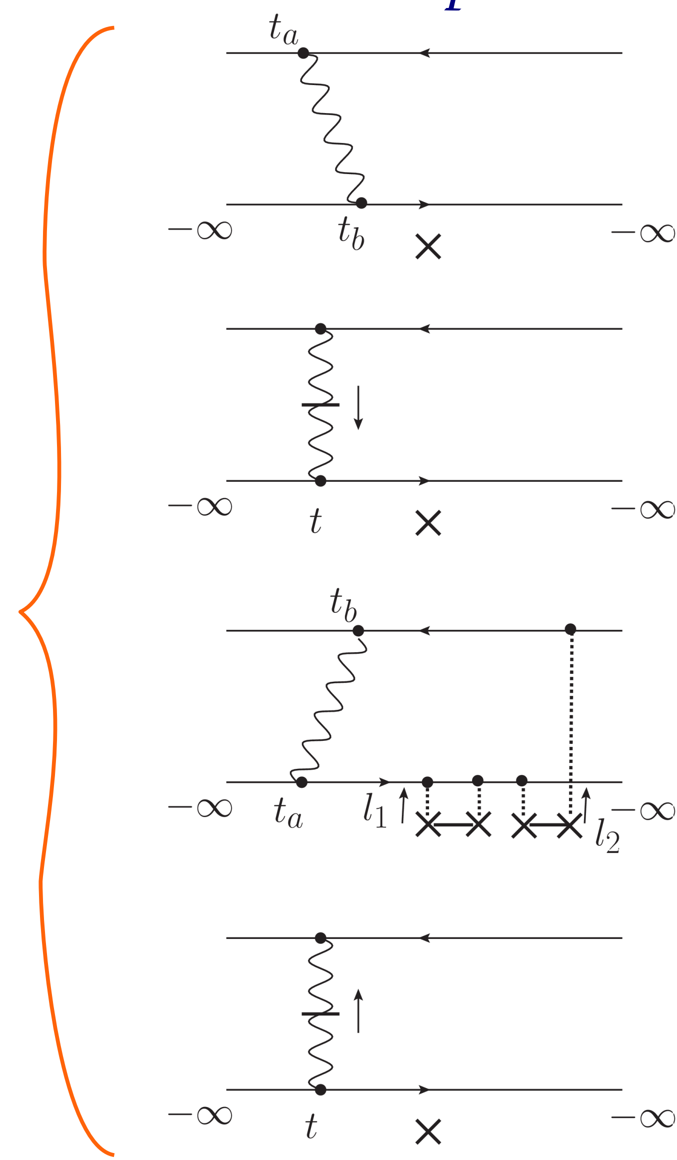

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Leading-order quantum corrections are identical in the cases of broadening and dipole scattering!

Kovchegov et al (2002...)

Quantum corrections: next-to-leading order

Hundreds of graphs on both *(broadening and dipole)* ***sides!***

even though we afford to take the large-N limit...

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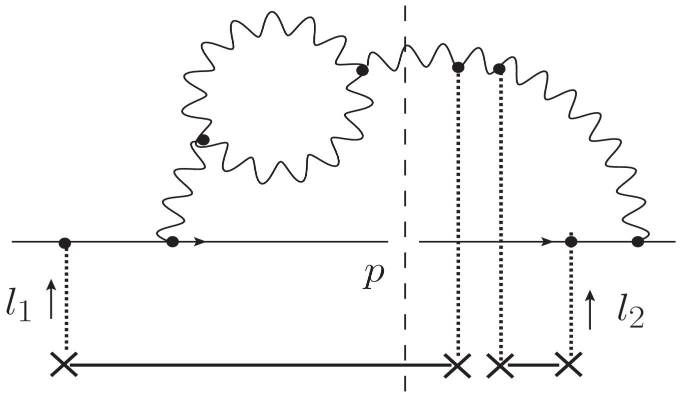

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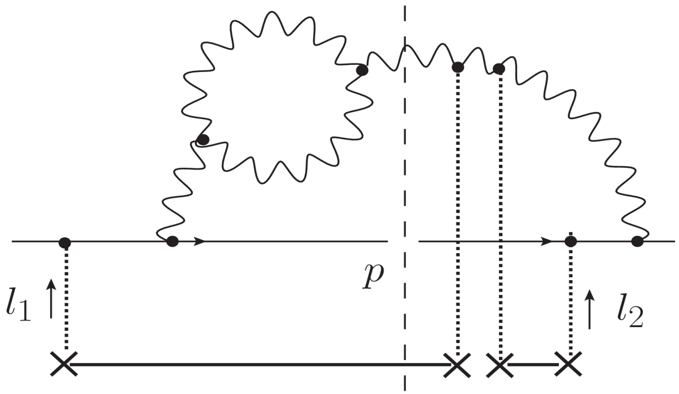

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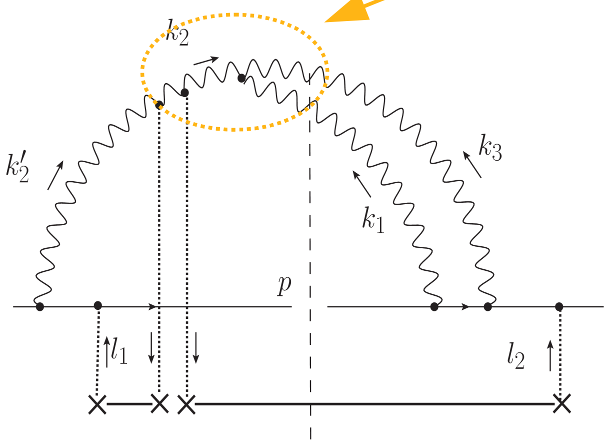

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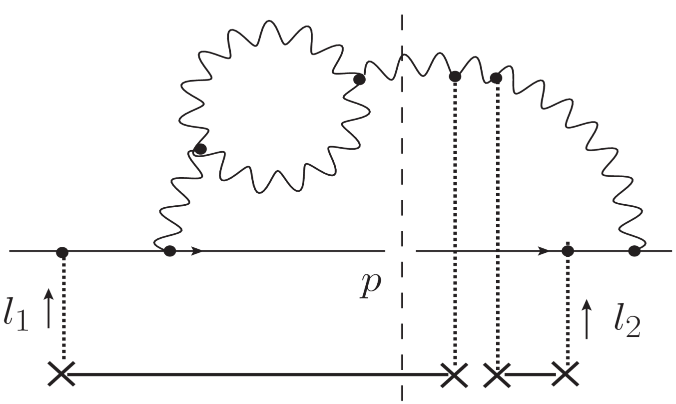

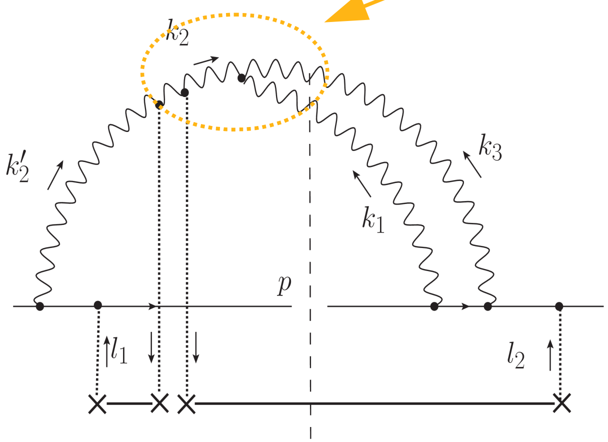

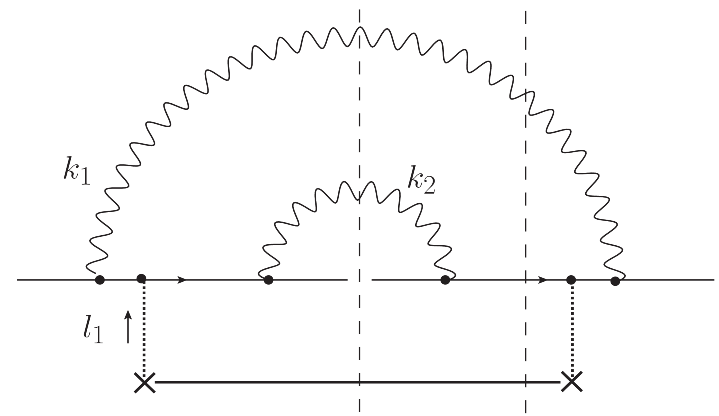

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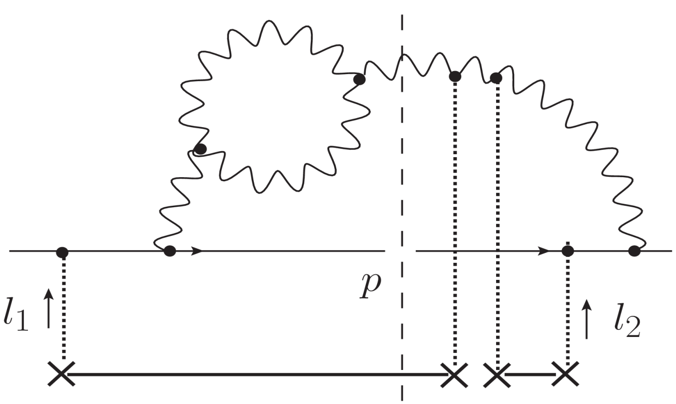

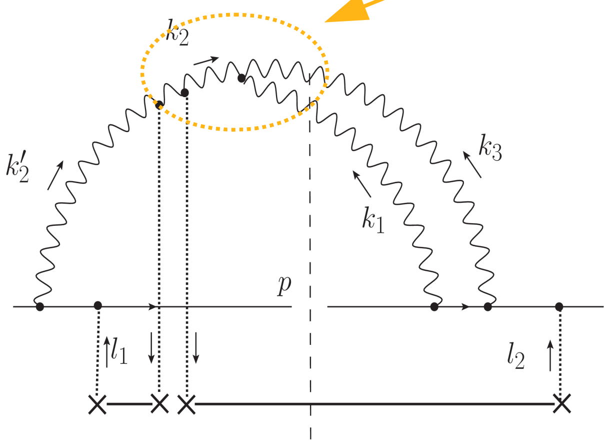

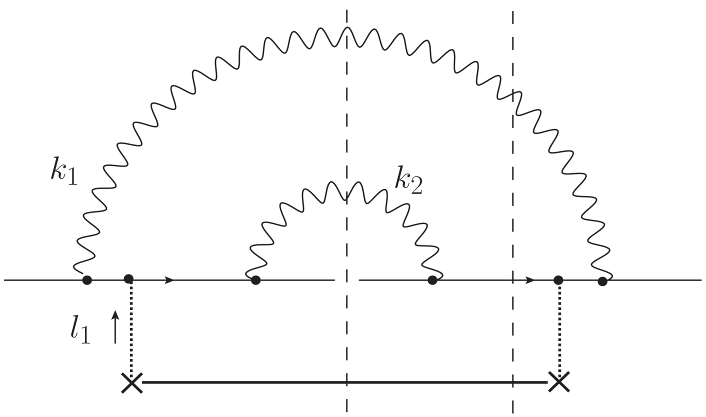

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<u>Our conclusion:</u> the correspondence between broadening and dipole scattering is preserved at NLO!

Summary and outlook

- *We proved that the broadening cross section and the dipole elastic amplitude are indeed related by a simple Fourier transform when quantum corrections are included* ***to next-to-leading order****:*

$$\frac{dN}{d^2p} = \int \frac{d^2x}{(2\pi)^2} e^{-ipx} S_{dipole}(x)$$

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- The method we used was "brute force" inspection of all relevant graphs. Is there a deeper way to understand the identity? Too many "miracles" happen to believe that this correspondence is an accident...

- Is the identity valid beyond NLO? (We actually think that it is not).