COHERENCE EFFECTS ON GLUON PRODUCTION IN A DENSE QCD MEDIUM

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To be published soon

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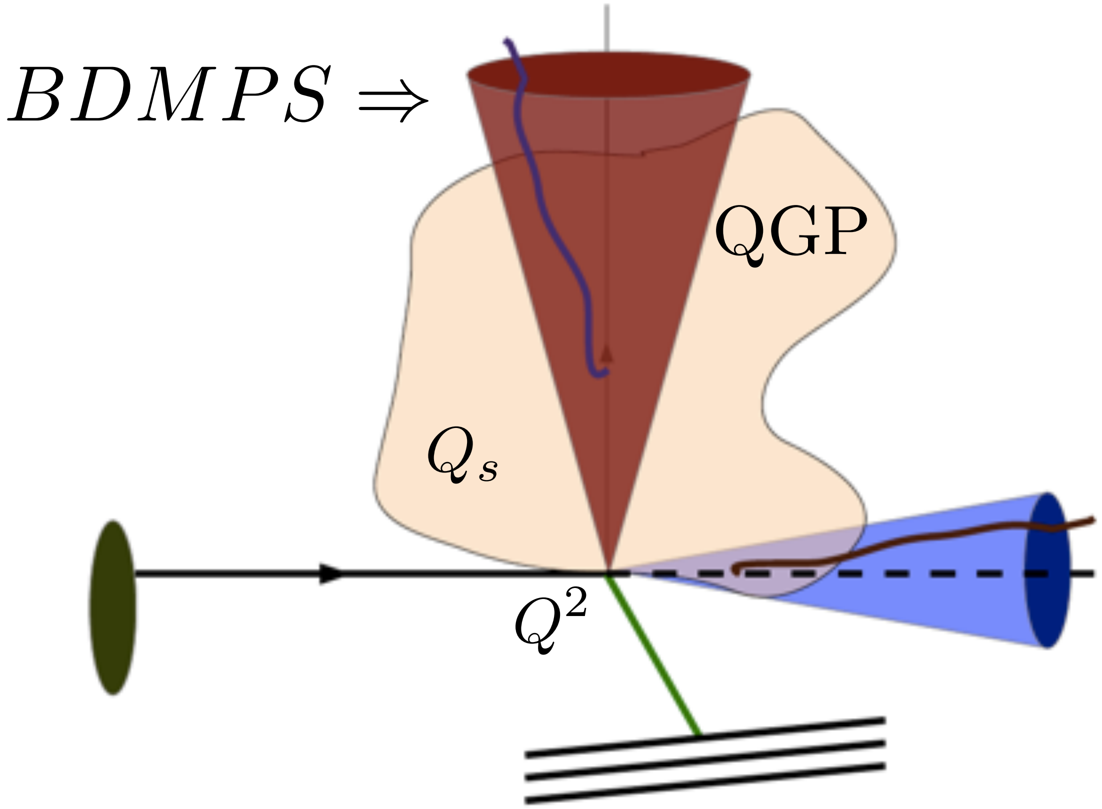
Medium of finite size

Collinear Factorization



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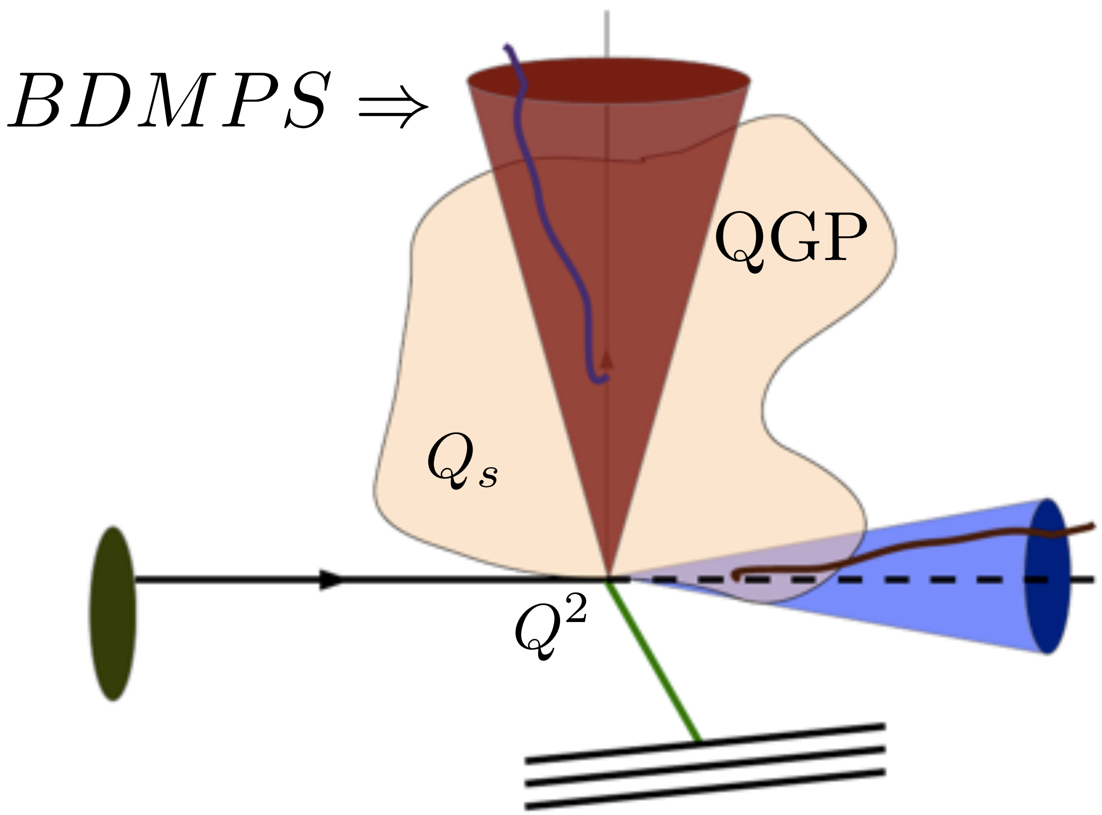
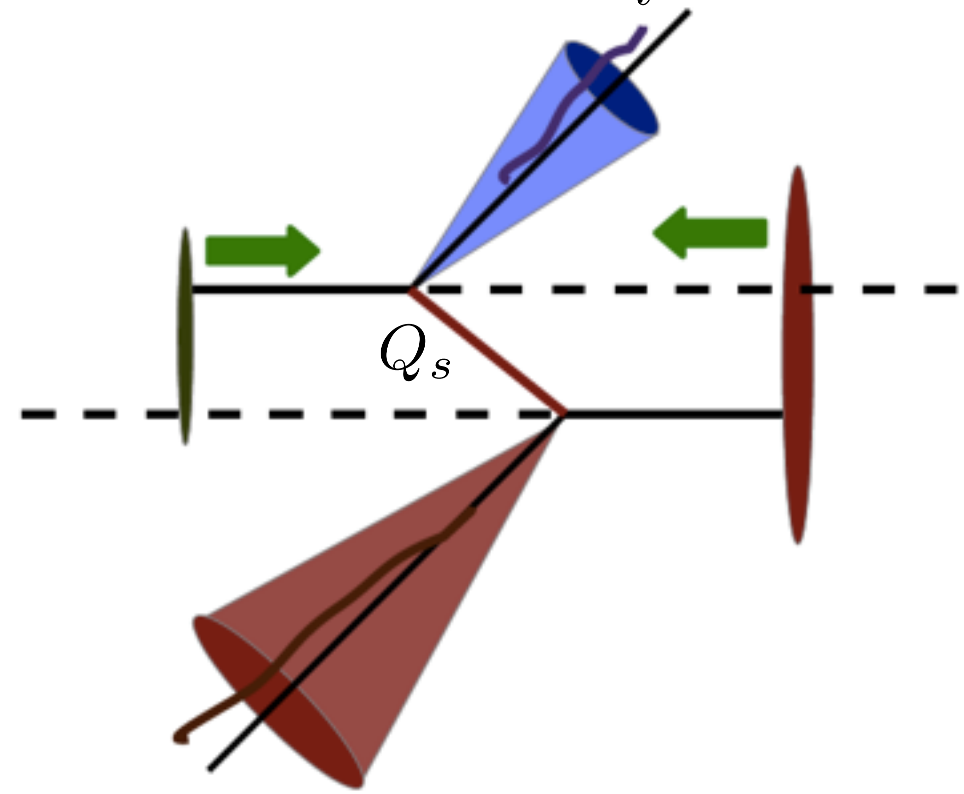


Shockwave

k_T factorization

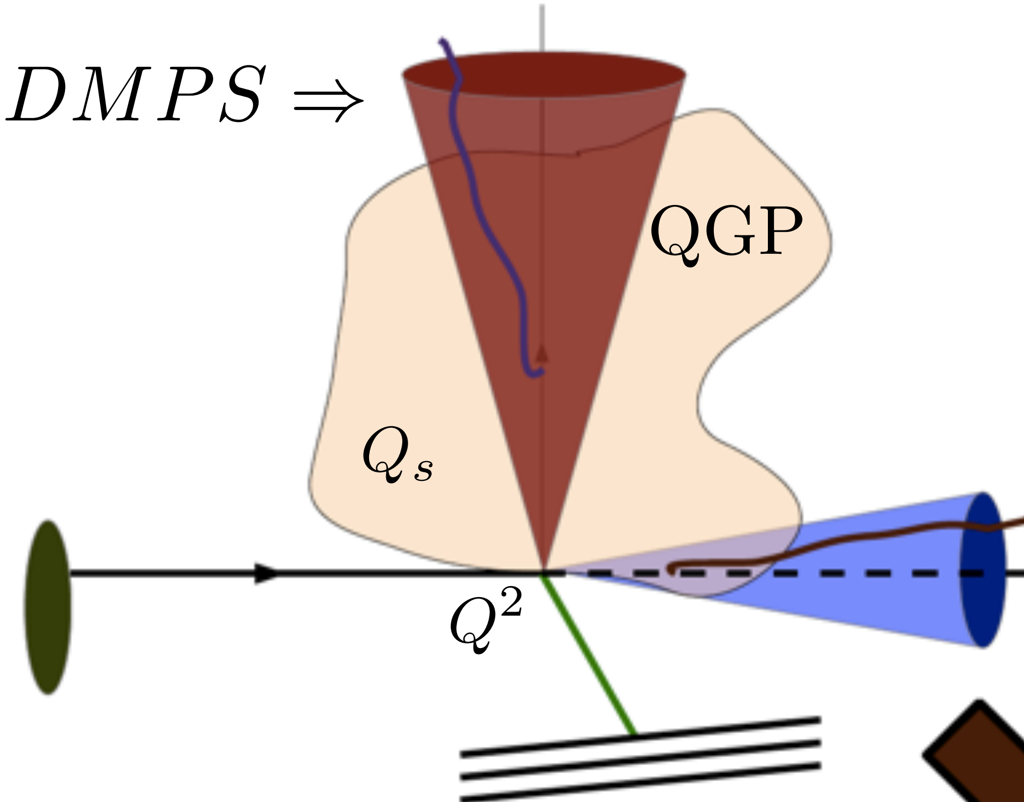
Hybrid formalism



Medium of finite size

Collinear Factorization

$BDMPS \Rightarrow$



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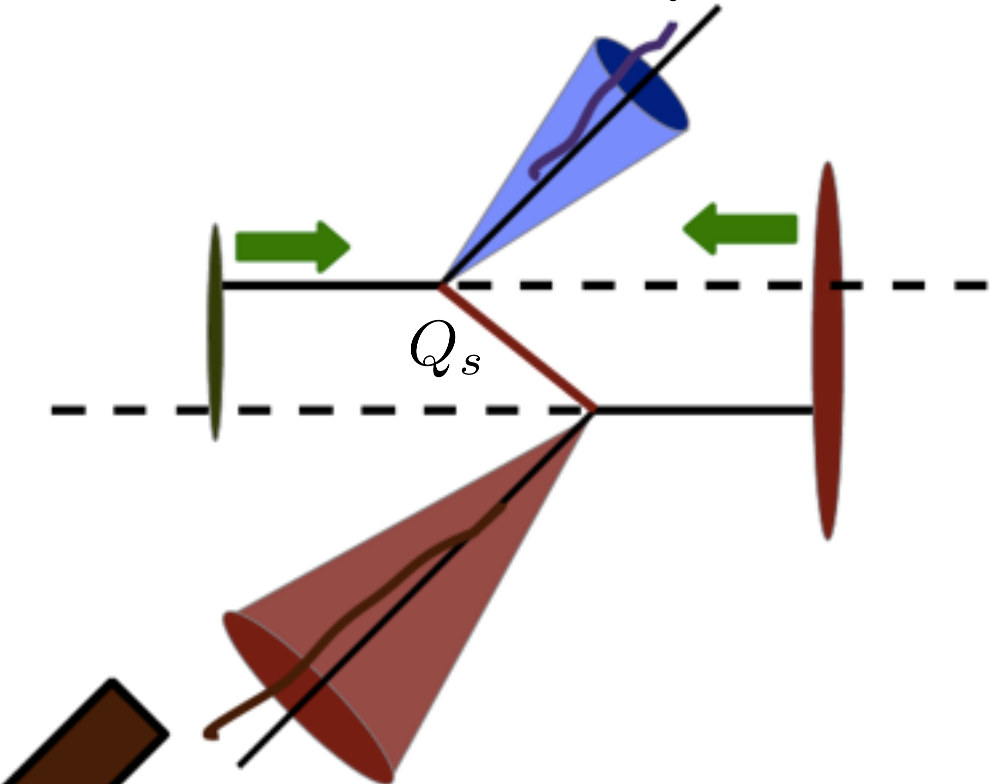
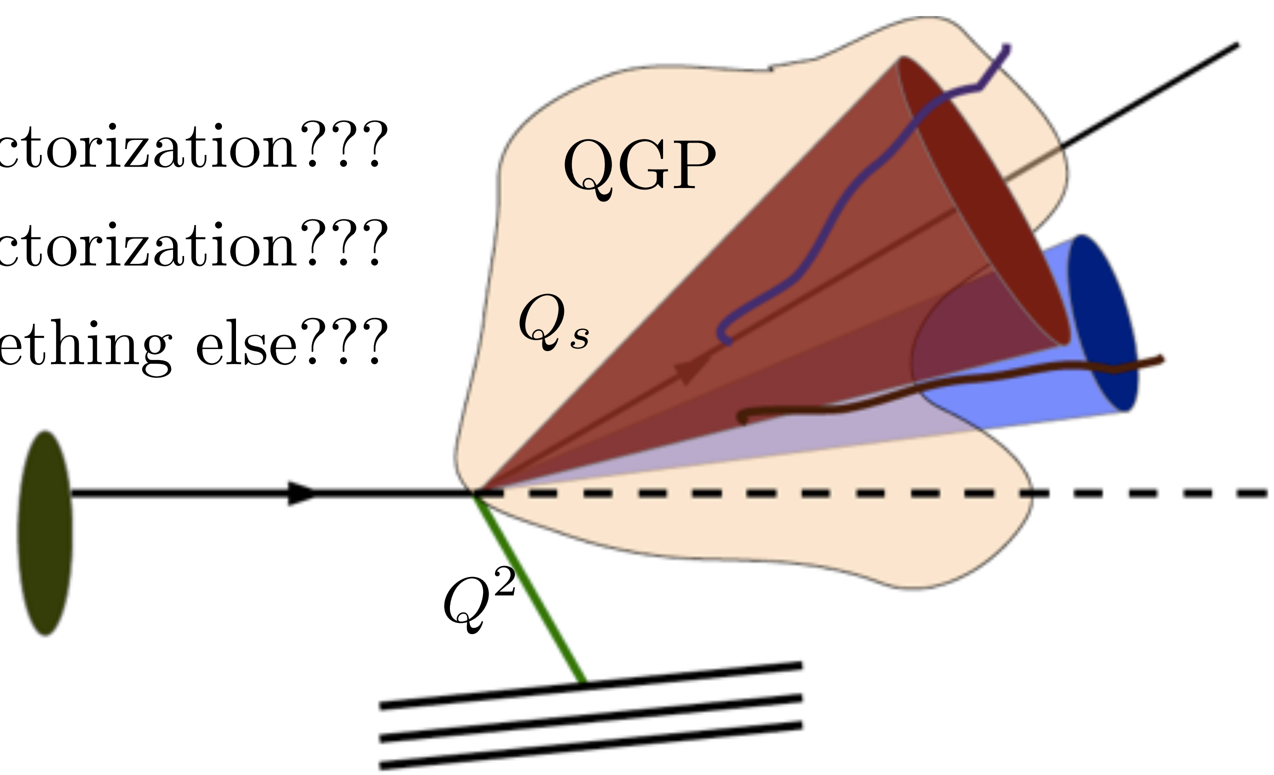


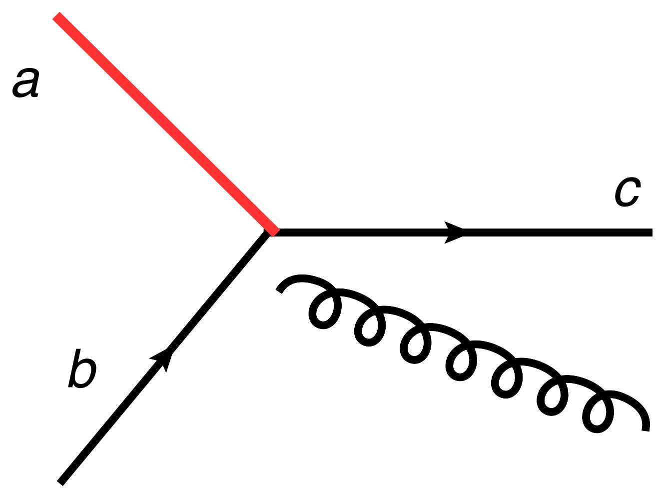
Collinear Factorization???

k_TFactorization???

Something else???



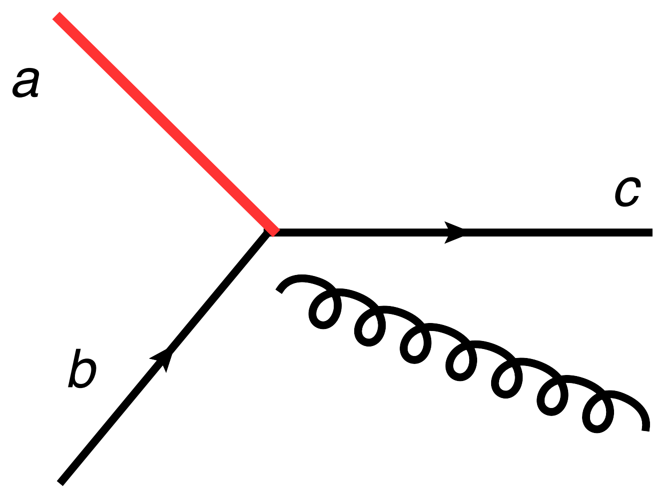
Angular ordering in a DIS-like process



$$Q_a = 0 \Rightarrow Singlet$$
$$Q_a \neq 0 \Rightarrow Octet$$

Playground to understand the properties of coherence in vacuum

Angular ordering in a DIS-like process



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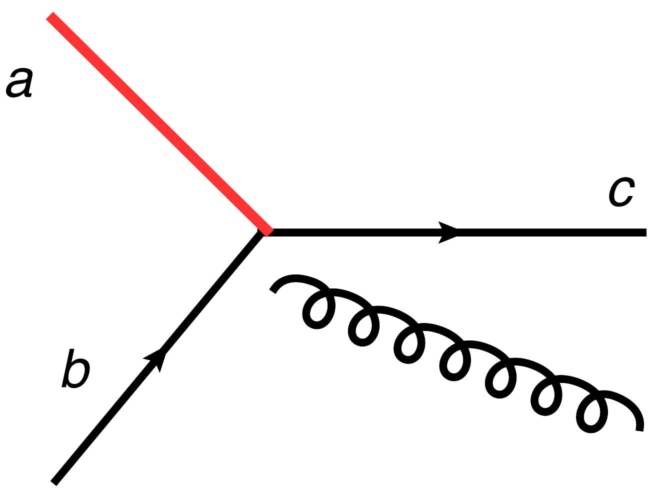

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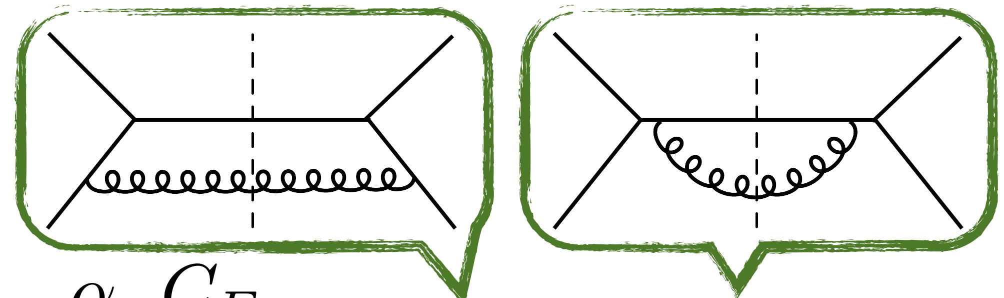

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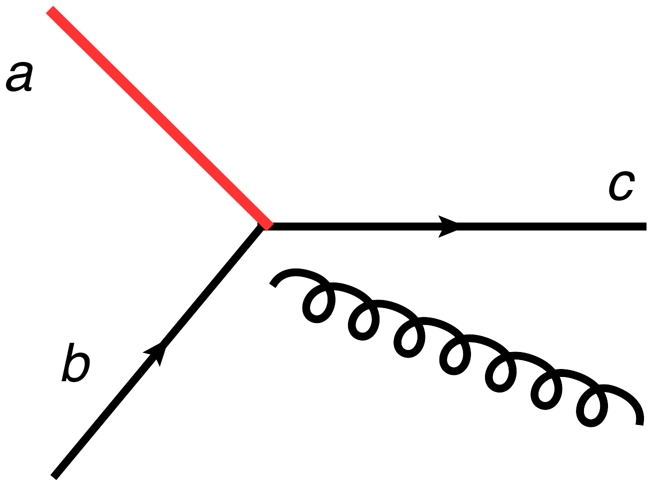
Coherent radiation

Angular ordering in a DIS-like process



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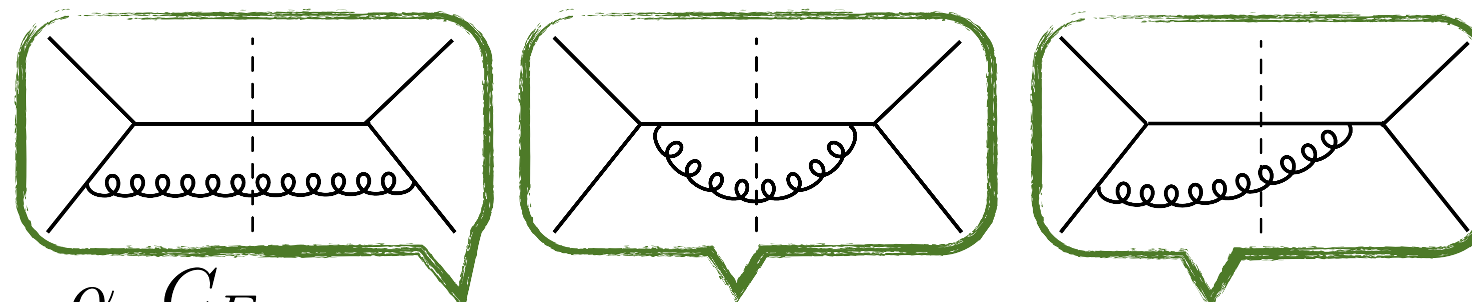

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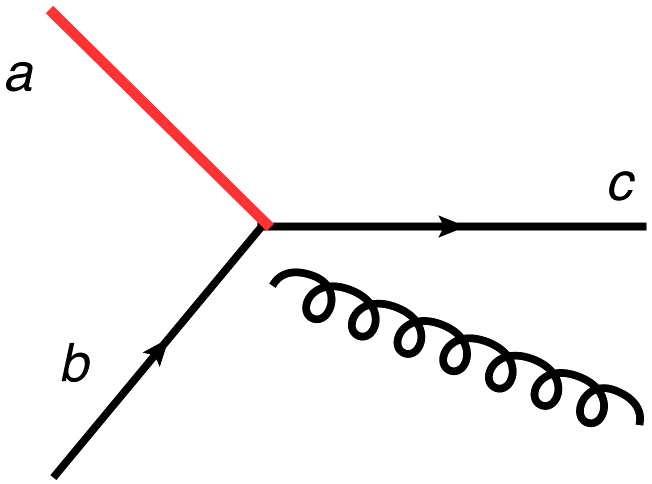
Coherent radiation

Total charge

Angular ordering in a DIS-like process



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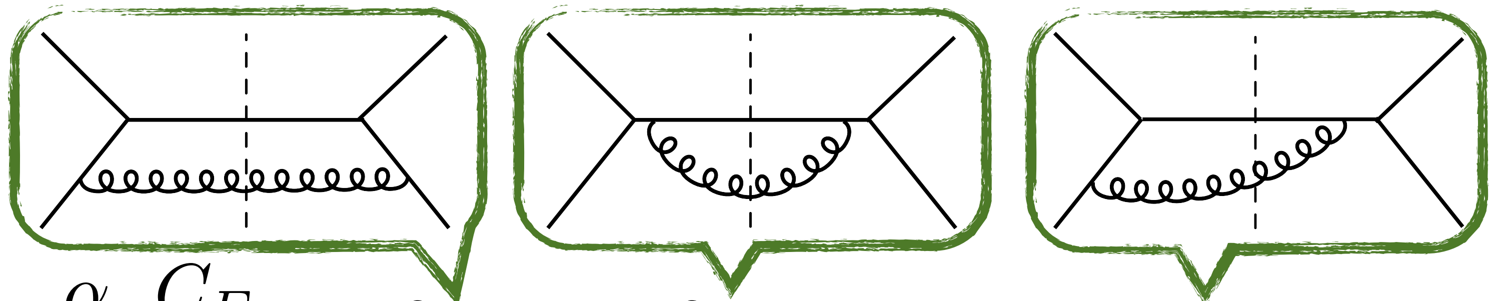

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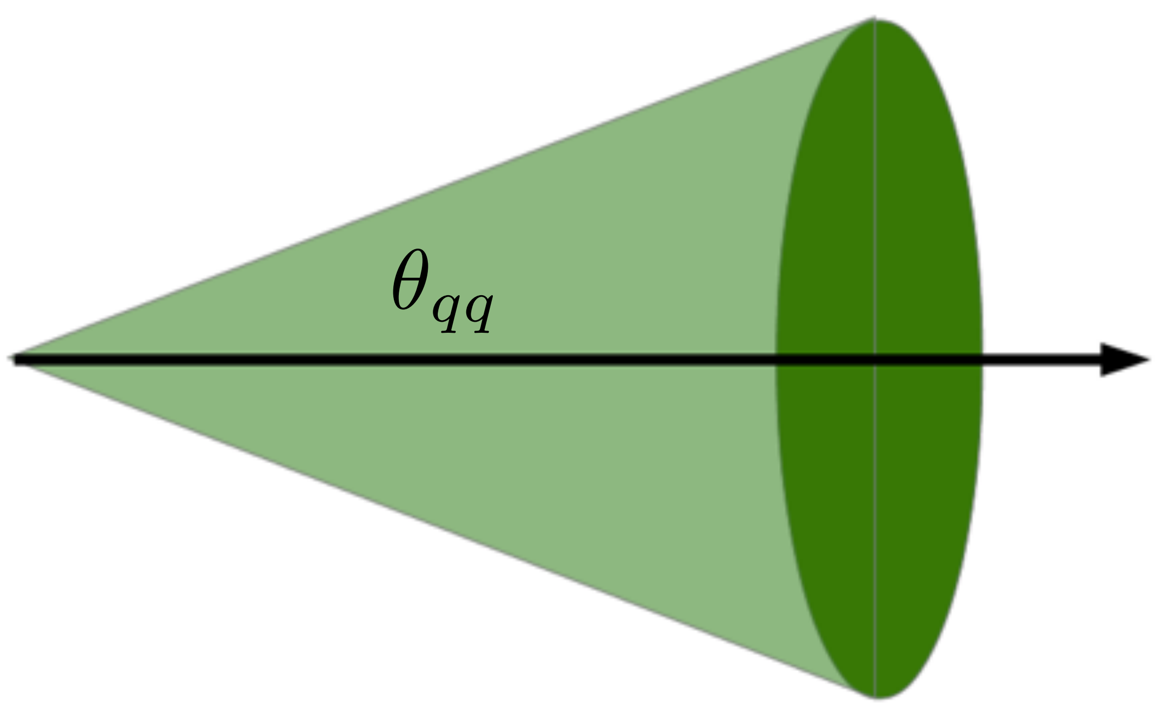

Coherent spectrum

$$\langle dN_i \rangle_\phi = \frac{\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta_i}{\theta_i} \Theta\left(\theta_{qq} - \theta_i\right)$$

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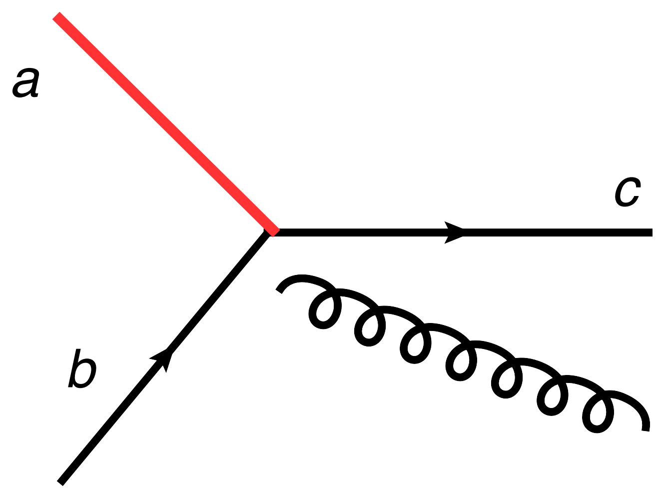
Angular ordering in a DIS-like process



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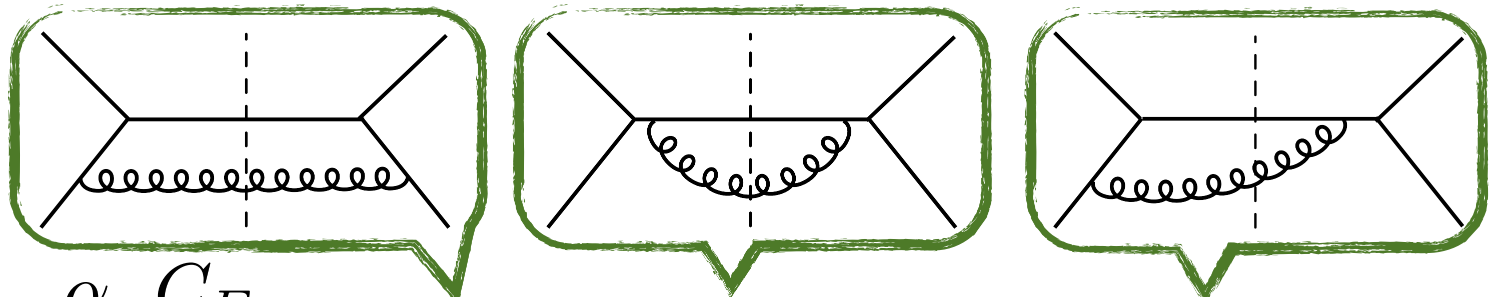

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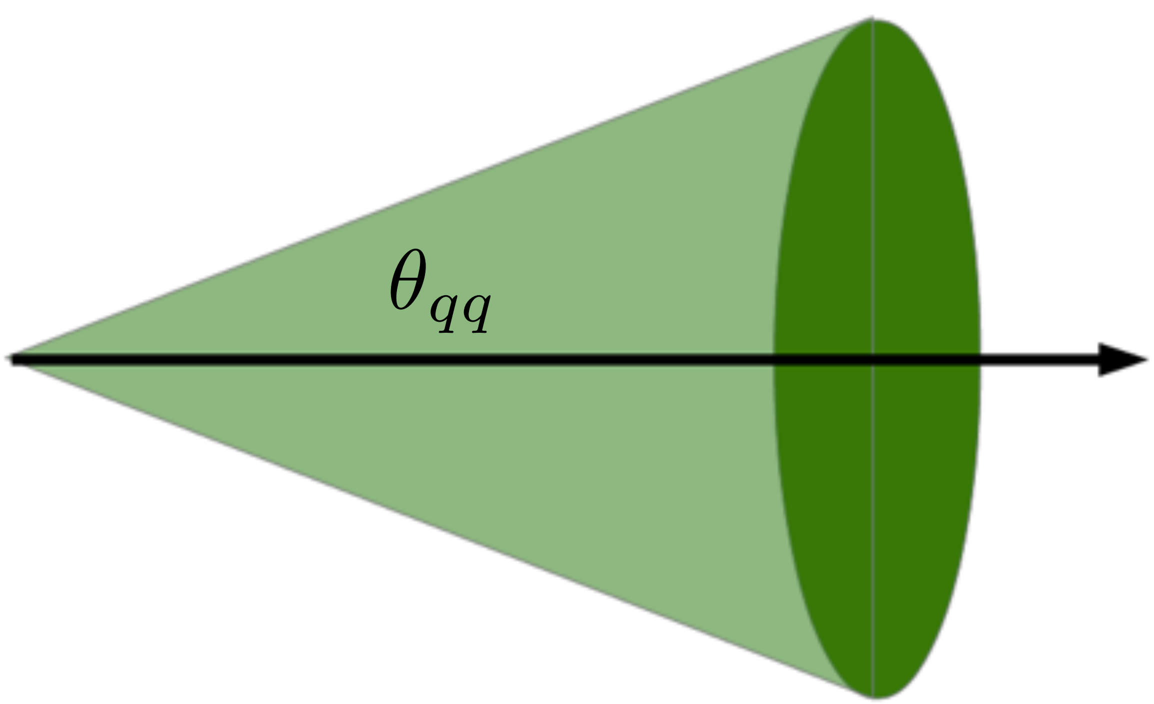

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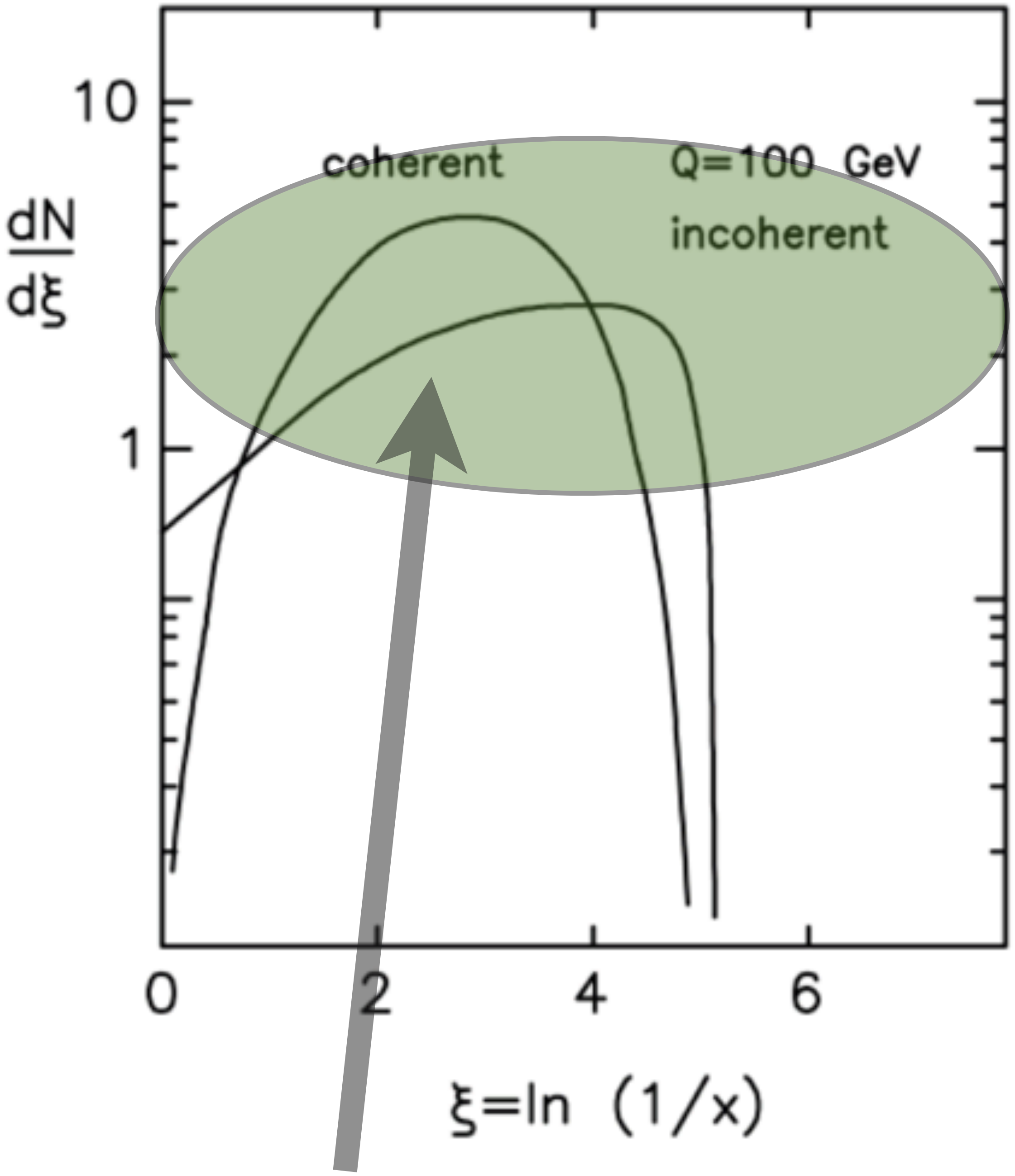
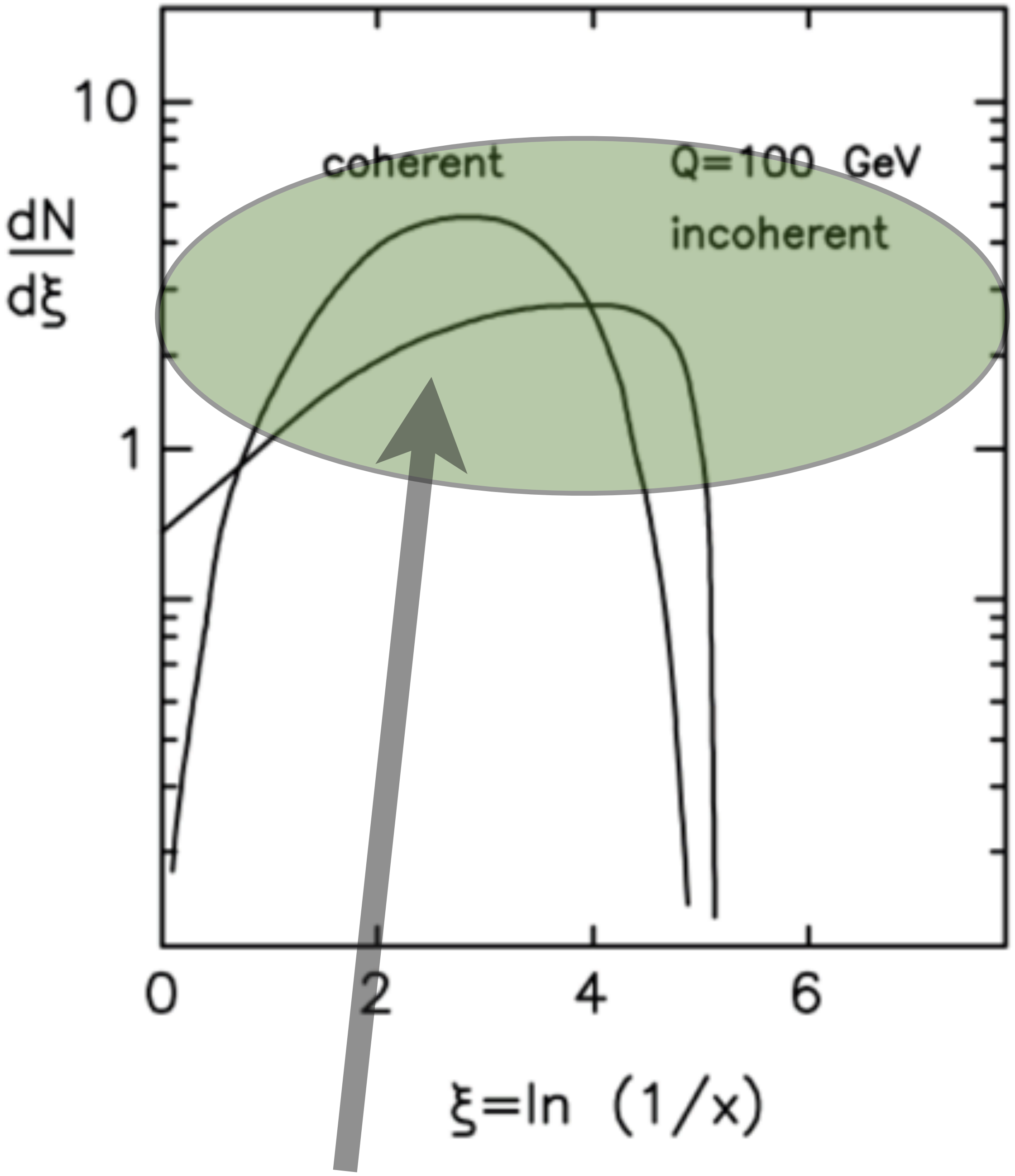
Collinear and soft divergence

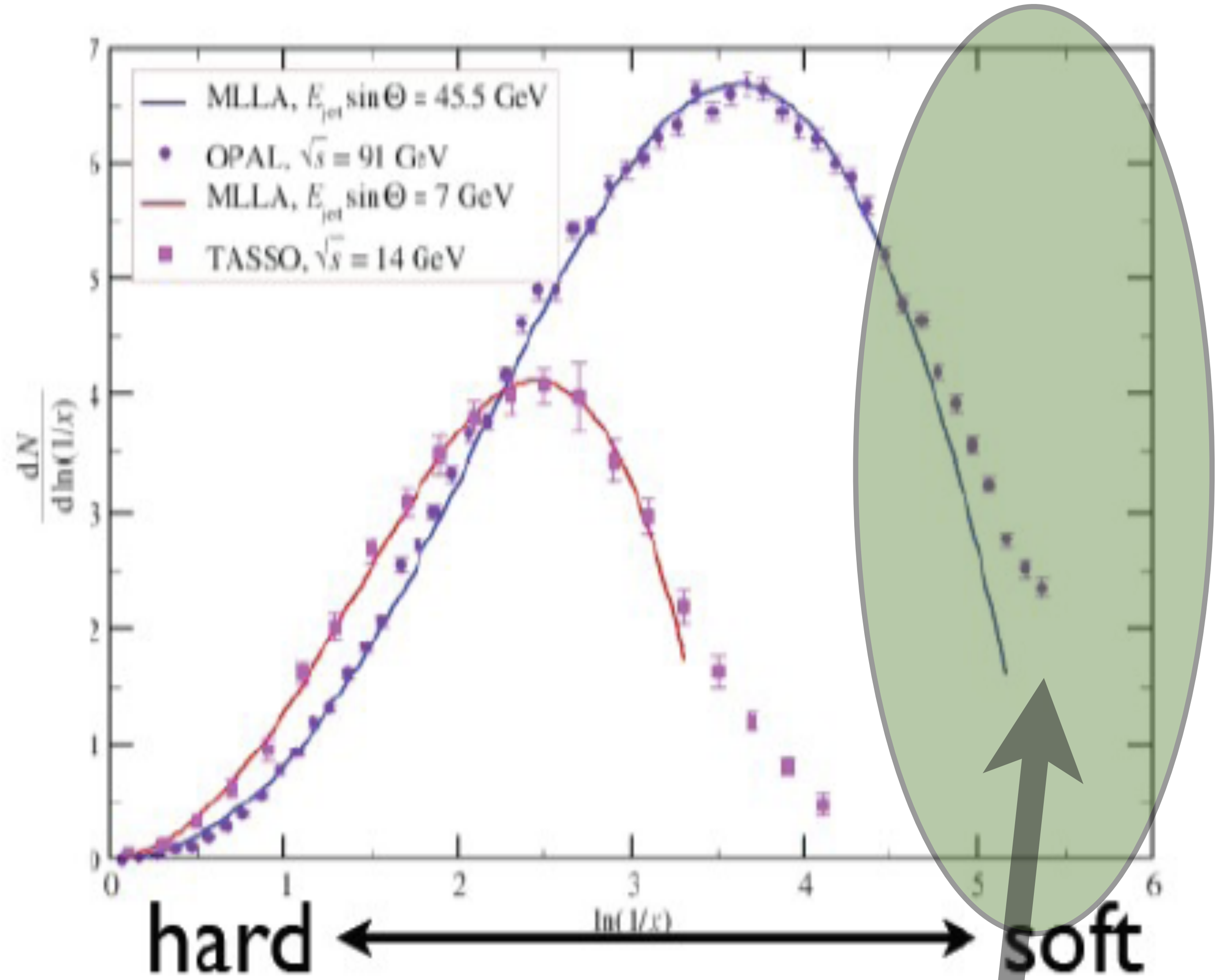
Angular ordering: experimental evidence



Sizable differences

Angular ordering: experimental evidence





TASSO Collaboration, Z. Phys. C 47 (1990) 187

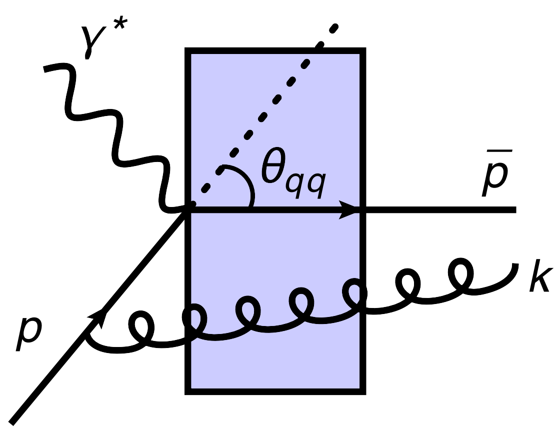
OPAL Collaboration, Phys. Lett. B 247 (1990) 617

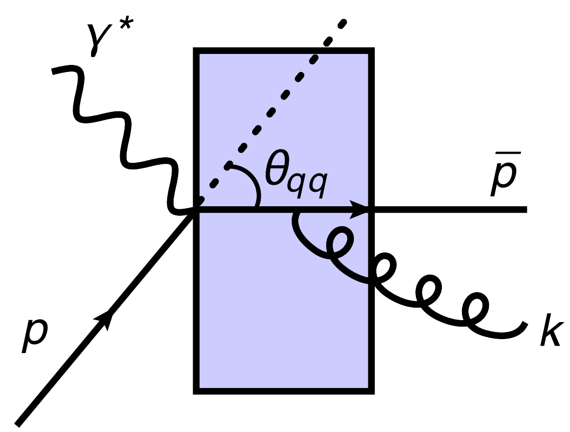
Sizable differences

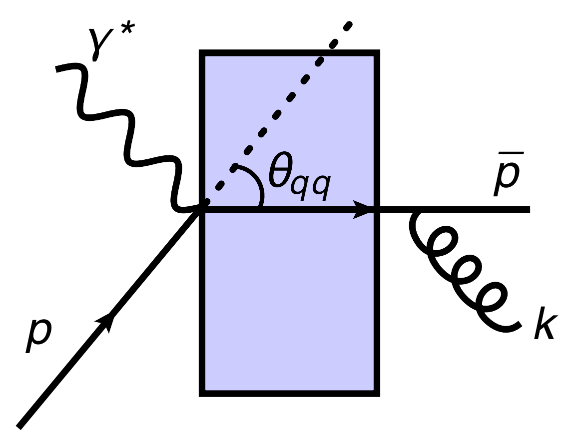
Suppression of soft gluons

Medium modifications to the initial and final state interference pattern

Dilute regime: M. Martinez et. al, Phys. Lett. B 717 (2012) 280-286
Dense regime: Soon to be published ⇒ *In this talk!!!*







GOALS
- ✯ Study another configuration relevant to HI collisions
- ✯ Playground to investigate medium modifications to the Initial State Radiation

Classical Yang-Mills Eqs. I

Evolution of the gauge field: $\left[D_\mu, F^{\mu\nu}\right] = \mathcal{J}^\nu$

Color charge conservation: $\left[D_\mu, \mathcal{J}^\mu\right] = 0$

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Linearizing around a background field: $\mathcal{A}^\mu = A^\mu_{med} + a^\mu$

$$\Box_x a^i - 2ig\left[\mathcal{A}^-_{med}, \partial_- a^i\right] = \mathcal{J}^i - \partial^i\left(\frac{\mathcal{J}^+}{\partial_-}\right)$$

LC gauge

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Reduction formula: $\mathcal{M}^a_\lambda = \lim_{k^2\to 0} \int d^4x e^{ik\cdot x} \Box_x \mathcal{A}^a_\mu(x) \epsilon^\mu_\lambda(\vec{k})$

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Gluon spectrum:

$$(2\pi)^3 2k^+\frac{dN}{d^3k} = \sum_{\lambda=1,2}|\mathcal{M}^a_\lambda(\vec{k})|^2$$

Modeling the medium

Medium is described as a classical background field:

$$-\partial^2_{\mathbf{x}} \mathcal{A}^-_{med}(x^+, \mathbf{x}) = \rho(x^+, \mathbf{x})$$

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$$\langle \mathcal{A}_{med}^{a,-}(x^+,\boldsymbol{q})\mathcal{A}_{med}^{*b,-}(x'^+,\boldsymbol{q}')\rangle = \delta^{ab} n(x^+)\delta(x^+ - x'^+)\delta^{(2)}(\boldsymbol{q}-\boldsymbol{q}')\mathcal{V}^2(\boldsymbol{q})$$

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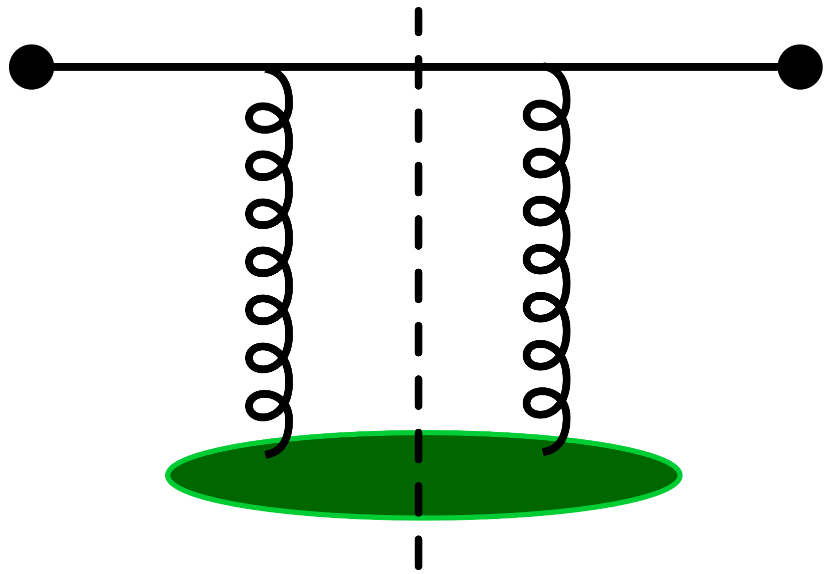


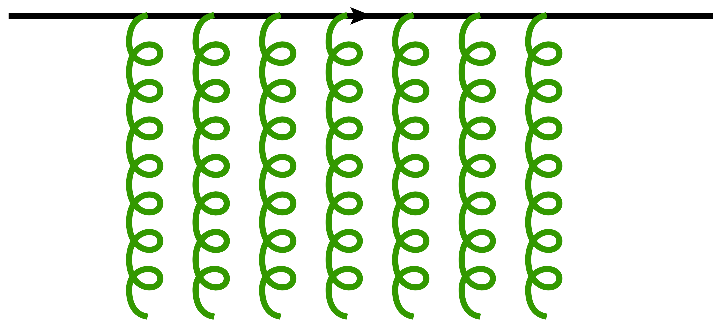
$$\lambda_{mfp} \gg m_D^{-1}$$

$$\mathcal{V}^2(\mathbf{q}) = \frac{m_D^2}{(2\pi)^2 (\mathbf{q}^2 + m_D^2)^2}$$

Gyulassy-Wang model: Nucl. Phys. B 420, (1994) 583

Classical Yang-Mills Eqs. II

Eikonal parton in a background field:



$$\mathcal{J}^{\mu}(x)_a = g\, v^{\mu}\, \mathcal{U}^{ab}(x^+, 0)\, \delta^3(\vec{x} - \vec{v}t)\, \theta(t)\, Q_b$$

$$\mathcal{U}^{ab}(x^+, y^+) = \mathcal{P} \exp \left[ig \int_{y^+}^{x^+} dz^+\, \mathcal{A}^-_{med}\left(z^+, \boldsymbol{r}(z^+)\right) \right]^{ab}$$

Classical Yang-Mills Eqs. II

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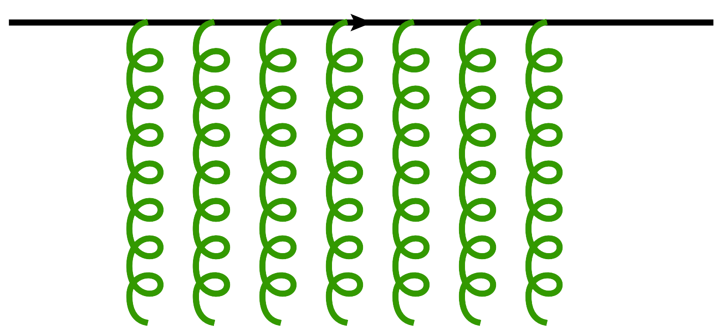

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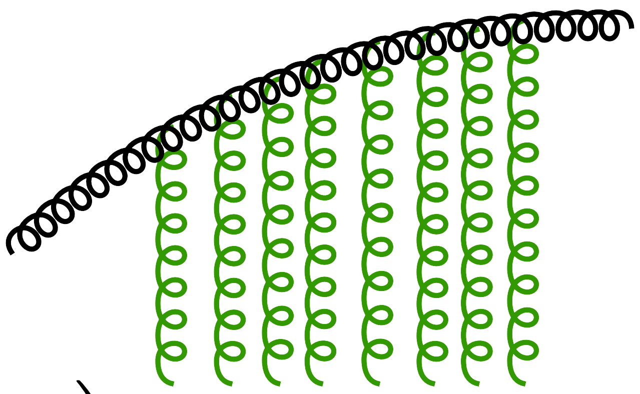
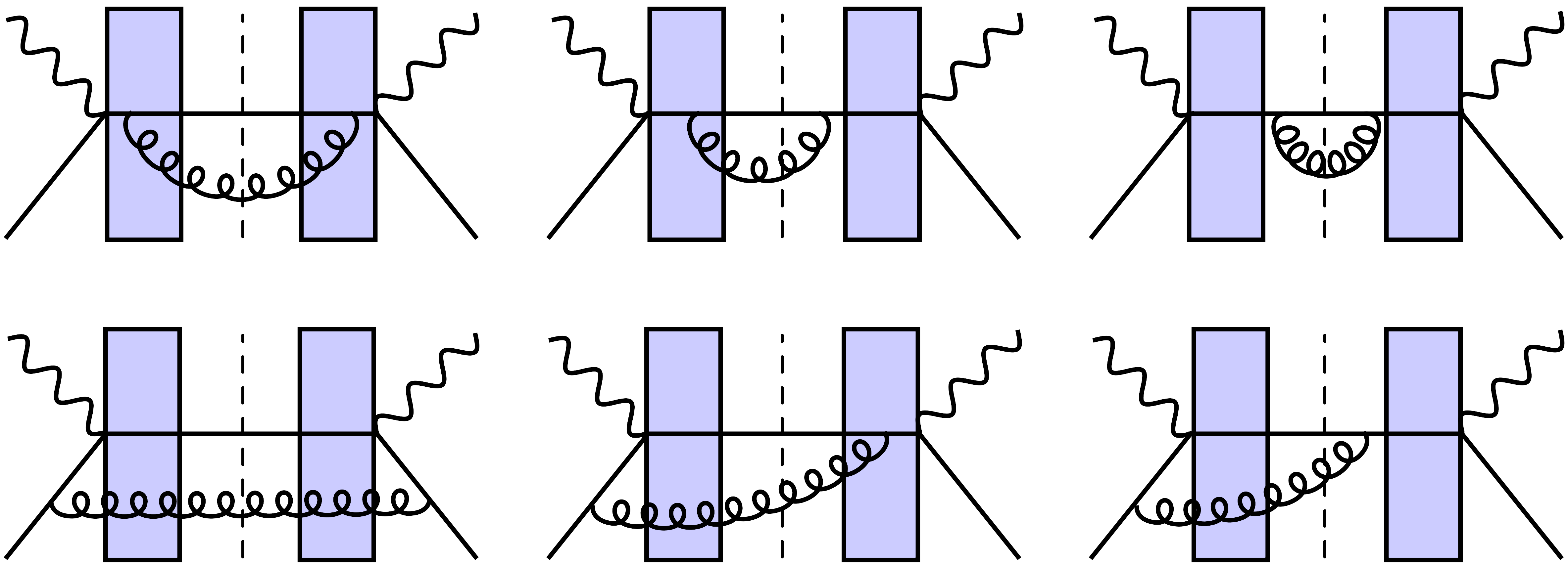
Soft gluon follows a non-eikonal trajectory



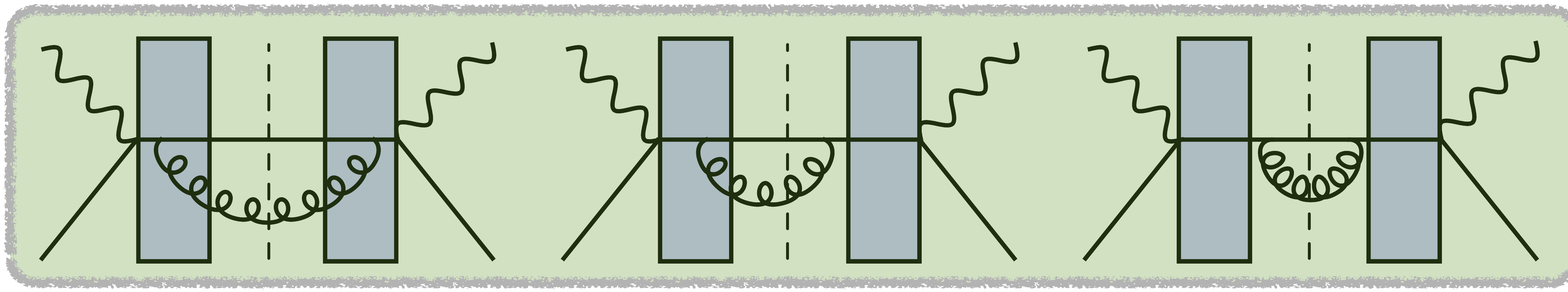
$$\mathcal{G}_{ab}(x^+, \mathbf{x}; y^+, \mathbf{y} | k^+) = \int_{\mathbf{r}(y^+)=\mathbf{y}}^{\mathbf{r}(x^+)=\mathbf{x}} \mathcal{D}\mathbf{r} \exp\left(i \frac{k^+}{2} \int_{y^+}^{x^+} dz\, \dot{\mathbf{r}}^2(z) \right) \mathcal{U}_{ab}(x^+, y^+)$$

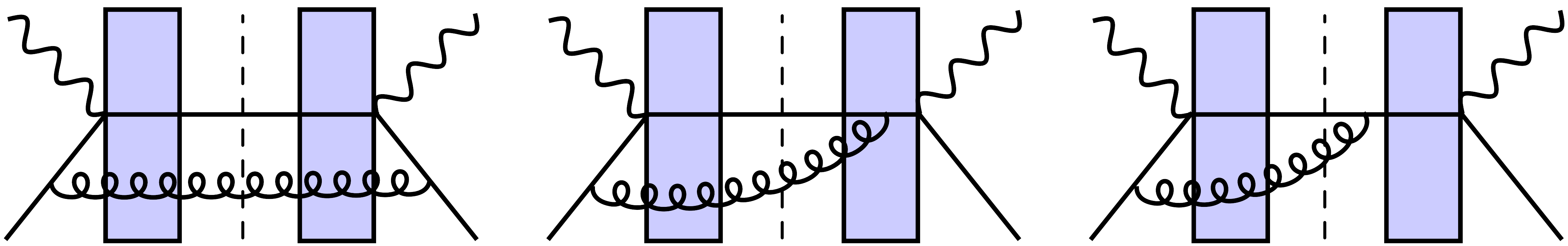
Gluon spectrum



Gluon spectrum

BDMPS-Z + vacuum





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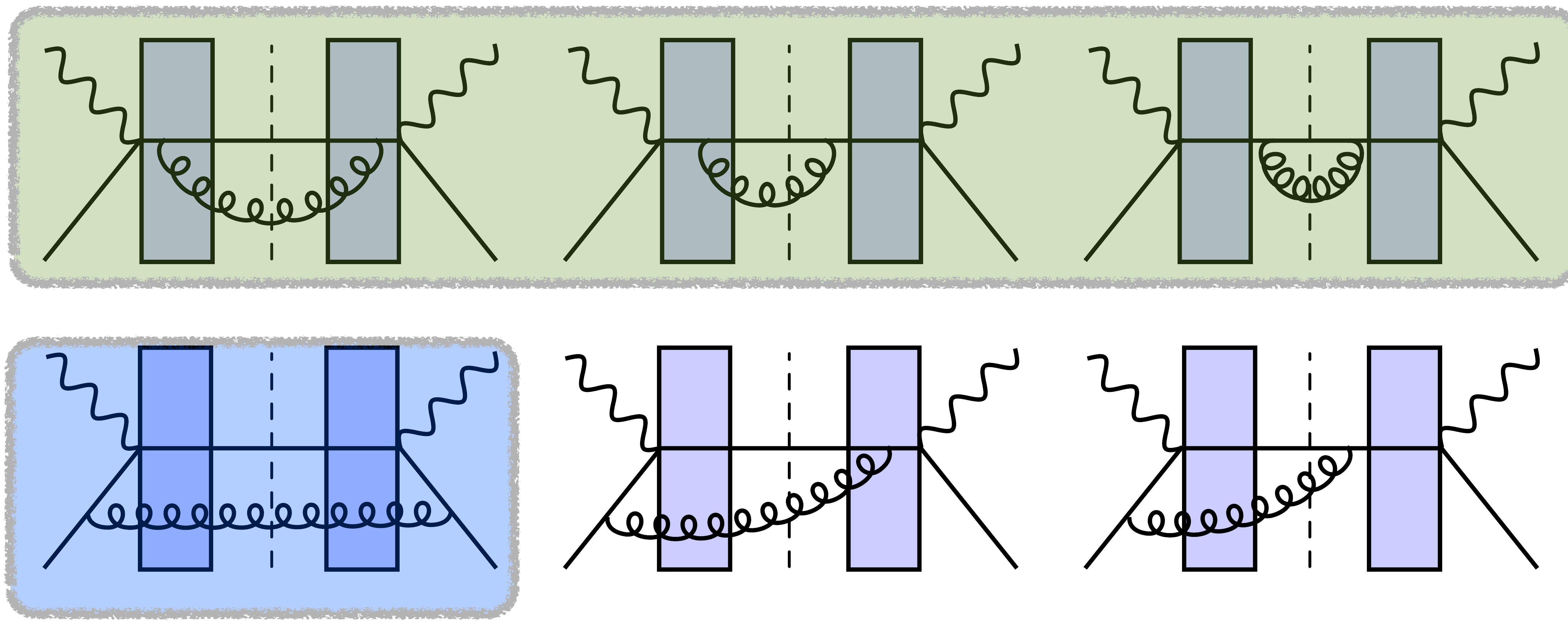
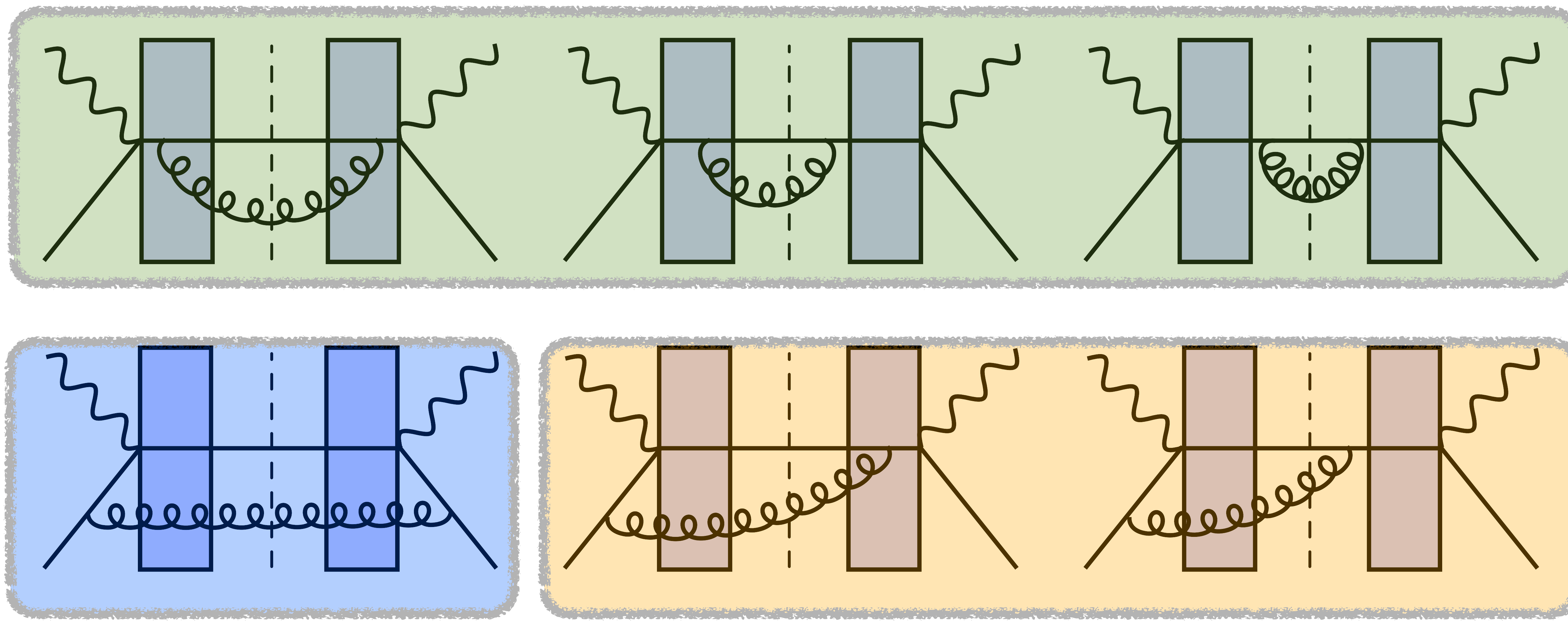


P_T broadening of ISR

Gluon spectrum

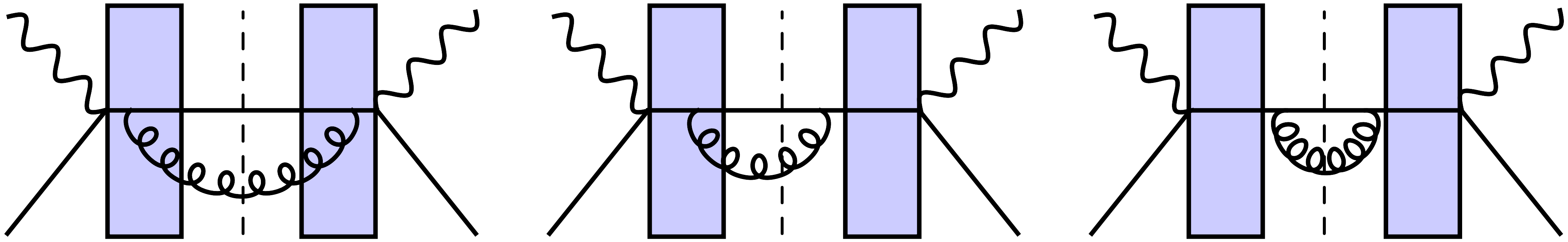
BDMPS-Z + vacuum



P_T broadening of ISR

Interferences in the medium: *New!!*

BDMPS-Z



$k_f = \sqrt{\omega \hat{q}}$

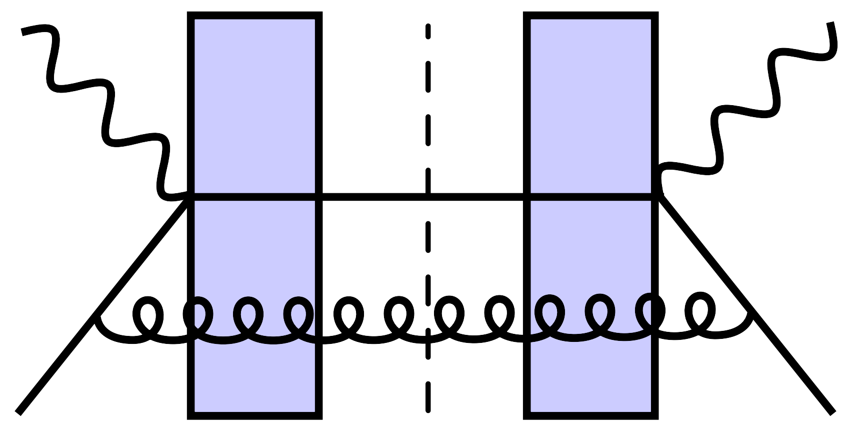
$$\sim \int_0^L dt' \int \frac{d^2\boldsymbol{k}'}{(2\pi)^2} \mathcal{P}(\boldsymbol{k}-\boldsymbol{k}', L-t') \sin\left(\frac{\boldsymbol{k}'^2}{2k_\mathrm{f}^2}\right) e^{-\frac{\boldsymbol{k}'^2}{2k_\mathrm{f}^2}}$$

$$\mathcal{P}(k,\xi) = \frac{4\pi}{\hat{q}\xi} e^{-\frac{k^2}{\hat{q}\xi}}$$

Medium induced radiation is a two step process

- Quantum emission + classical broadening
- Scales with the length of the medium

P_t broadening of ISR



$$\mathcal{P}(k,\xi) = \frac{4\pi}{\hat{q}\xi} e^{-\frac{k^2}{\hat{q}\xi}}$$

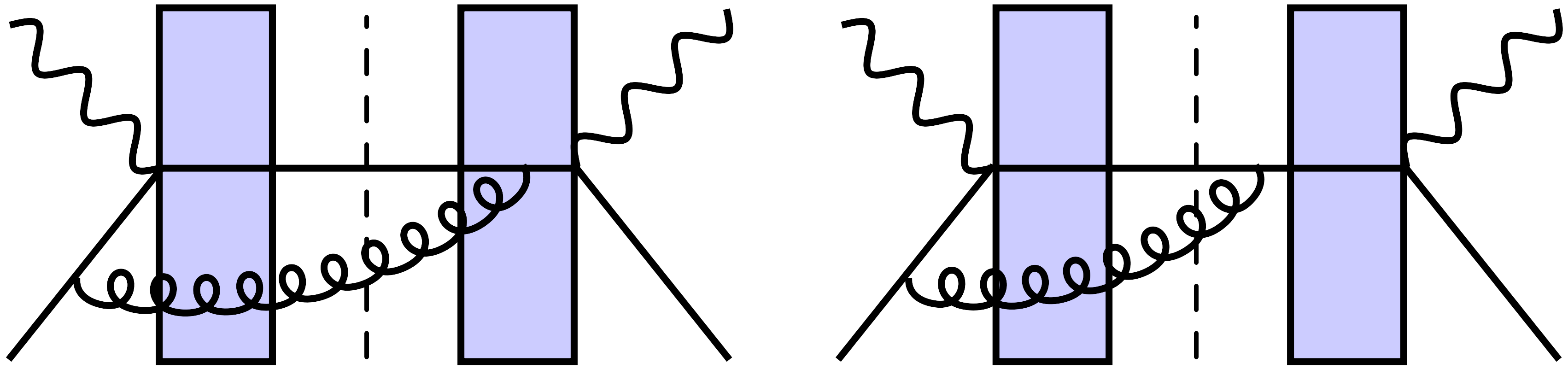
$$\sim \int \frac{d^2\boldsymbol{k}'}{(2\pi)^2} \frac{\mathcal{P}(\boldsymbol{k}' - \bar{\boldsymbol{\kappa}}, L^+)}{\boldsymbol{k}'^2}$$

P_T broadening of ISR is a two step process:

- Collinear Emission + classical broadening
- Classical broadening: reshuffle of the momentum of the gluon emissions

⇒ Typical value of the gluon momenta ~$Q_s = \hat{q}L$

Interferences



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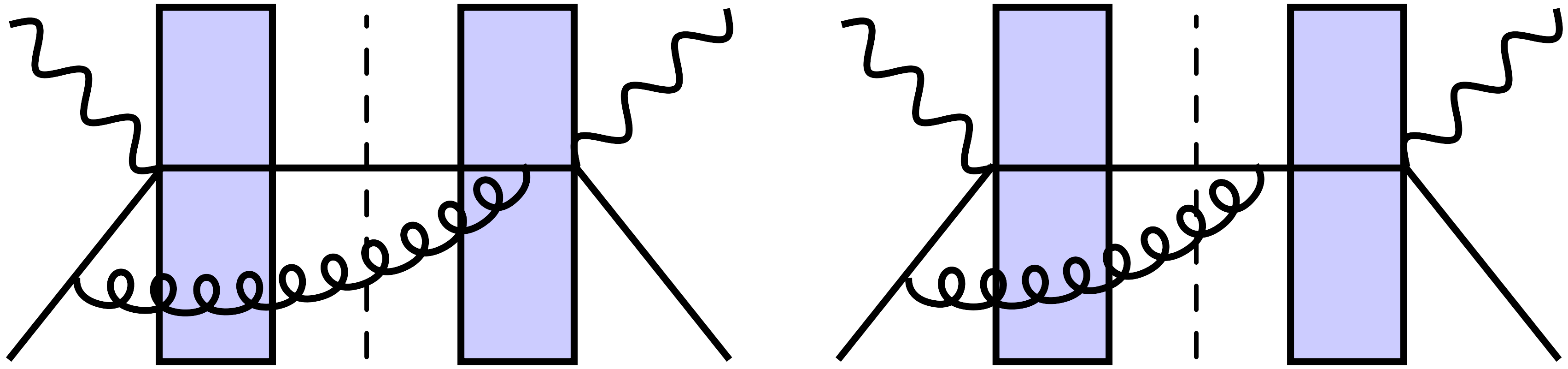
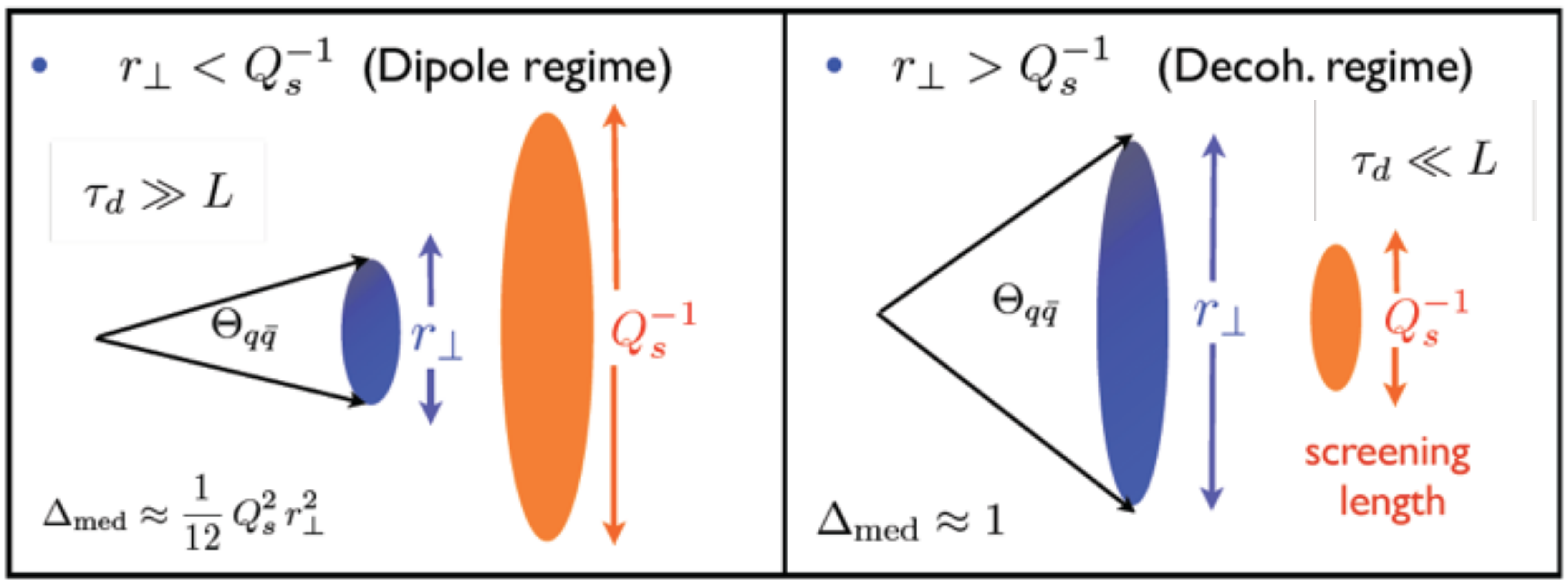


Warning: remember that in the antenna



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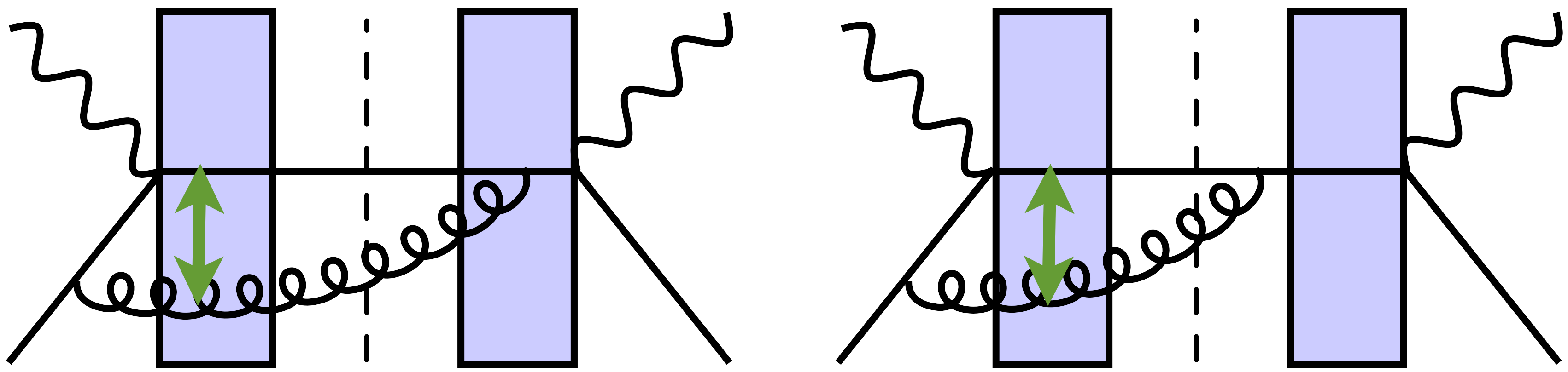


Transverse size of the Quark-gluon system

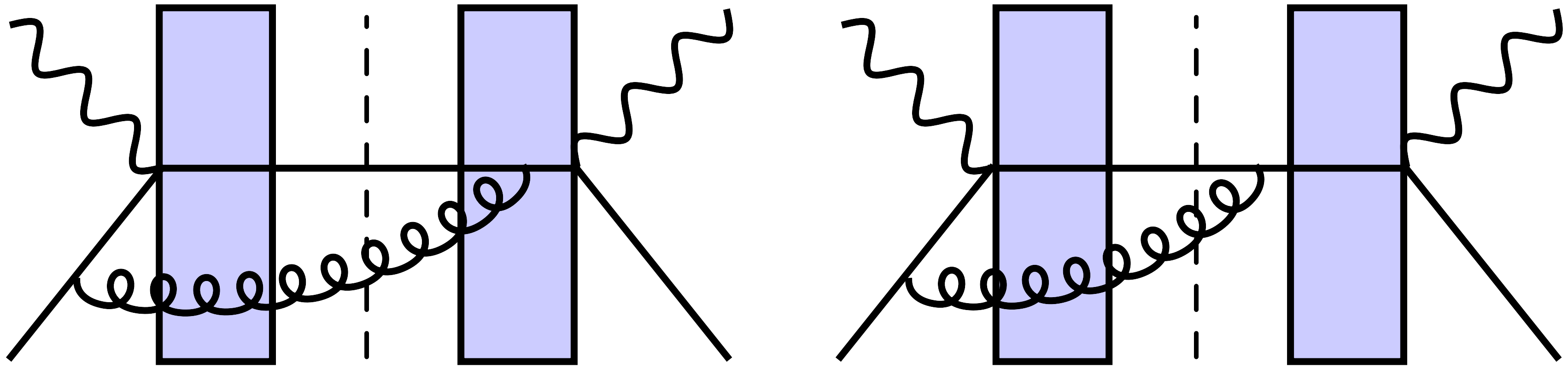
- If hard scattering is the largest scale:

⇒ Insensitive to the medium

- If typical medium induced momentum is the largest scale

⇒ Medium is able to resolve the QG system

Interferences



The Color correlation of the Quark-gluon system is measured by

$$\mathcal{K}(x^+,\boldsymbol{x};y^+,\boldsymbol{y}|k^+) = \int_{\boldsymbol{r}(y^+)=\boldsymbol{y}}^{\boldsymbol{r}(x^+)=\boldsymbol{x}} \mathcal{D}\boldsymbol{r}\,\exp\left[\int_{y^+}^{x^+} d\xi\left(i\frac{k^+}{2}\dot{\boldsymbol{r}}^2(\xi) - \frac{1}{2}n(\xi)\sigma\left(\boldsymbol{r}(\xi)\right)\right)\right]$$

- Describes the Brownian motion of the gluon
- Harmonic oscillator approximation: $n\sigma(\mathbf{r}) \approx \hat{q}\mathbf{r}^2$
- Two extreme limits

⇒Shockwave case (Deep LPM) $\tau_f \gg L$

⇒Infinite medium length $\tau_f \ll L$

Interferences: shockwave limit

- Medium acts as a unique scattering center

Interferences: shockwave limit

- Medium acts as a unique scattering center
- Gluon remains coherent while passing the medium

Interferences: shockwave limit

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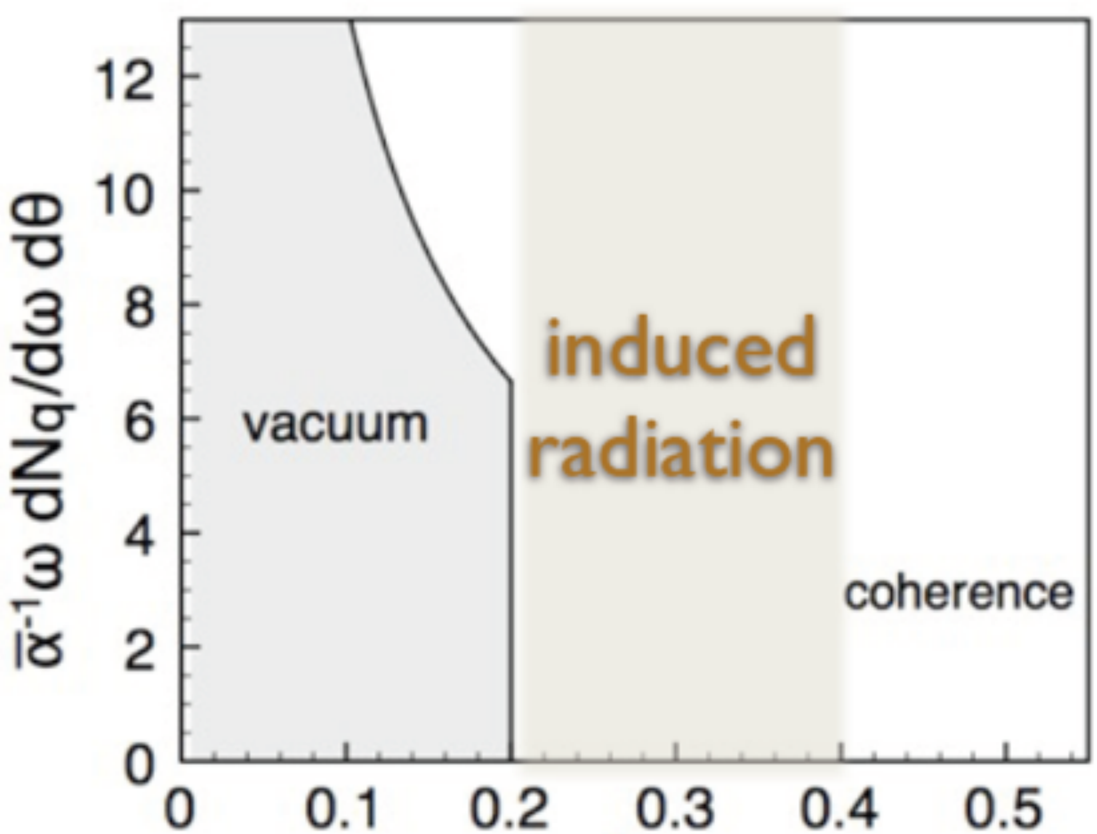

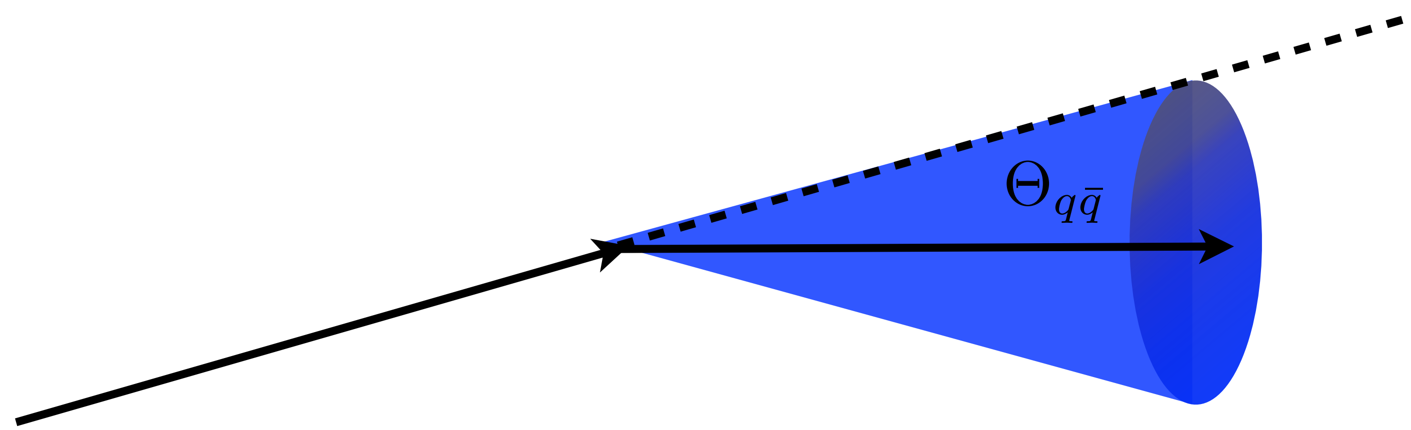

Interferences: shockwave limit

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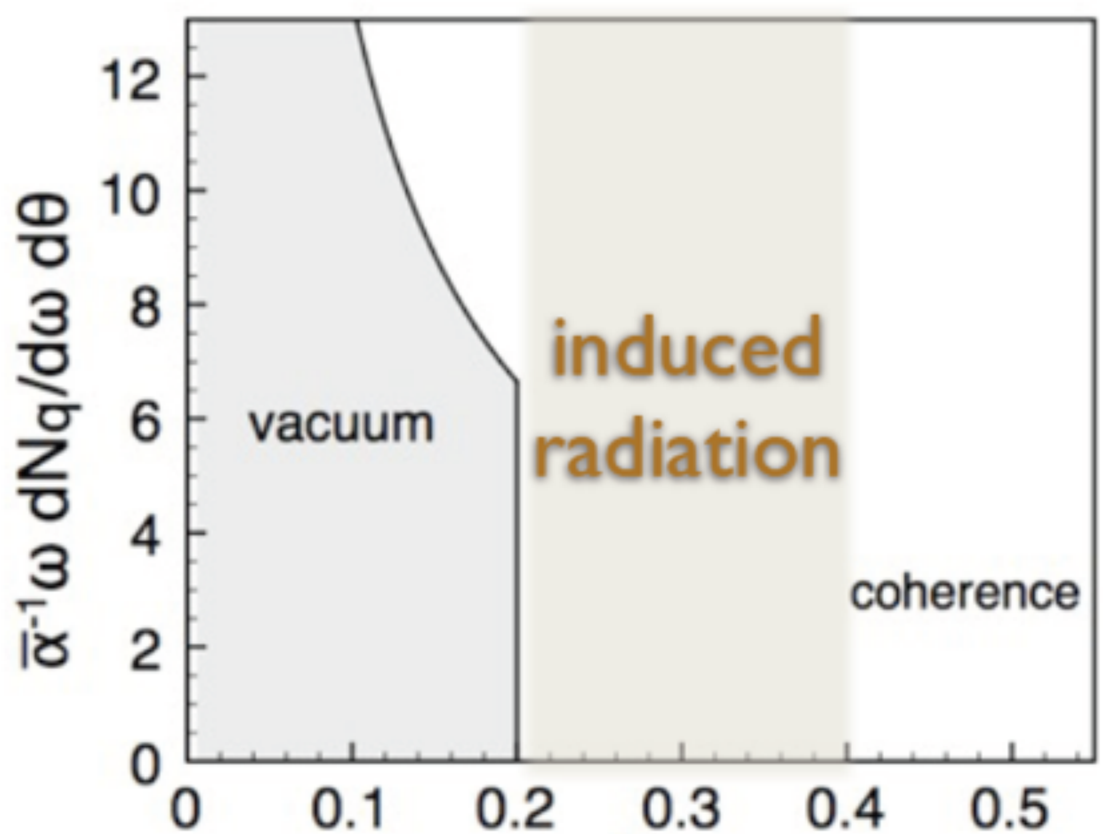
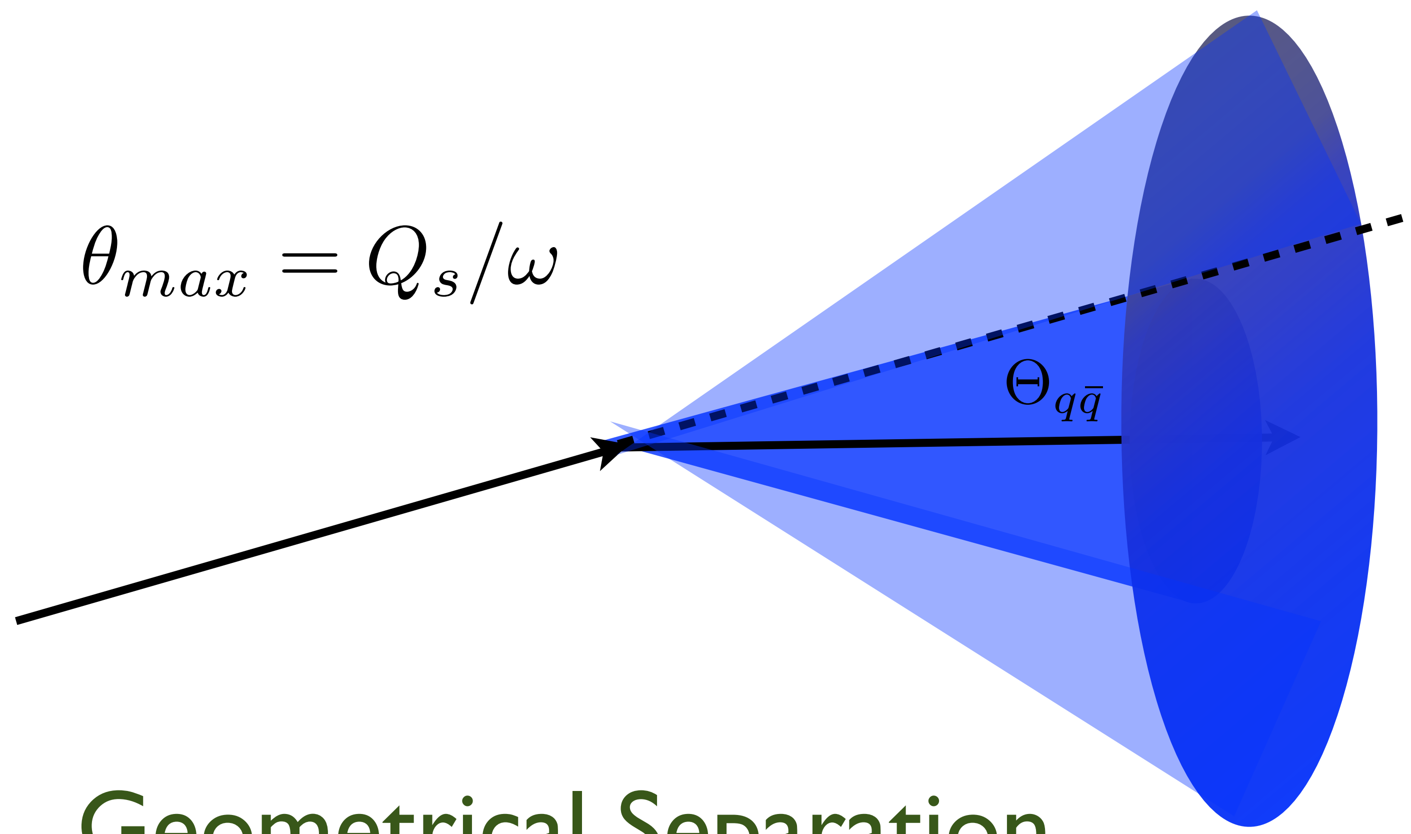


$$\theta_{max} = Q_s/\omega$$



Geometrical Separation

Interferences: shockwave limit

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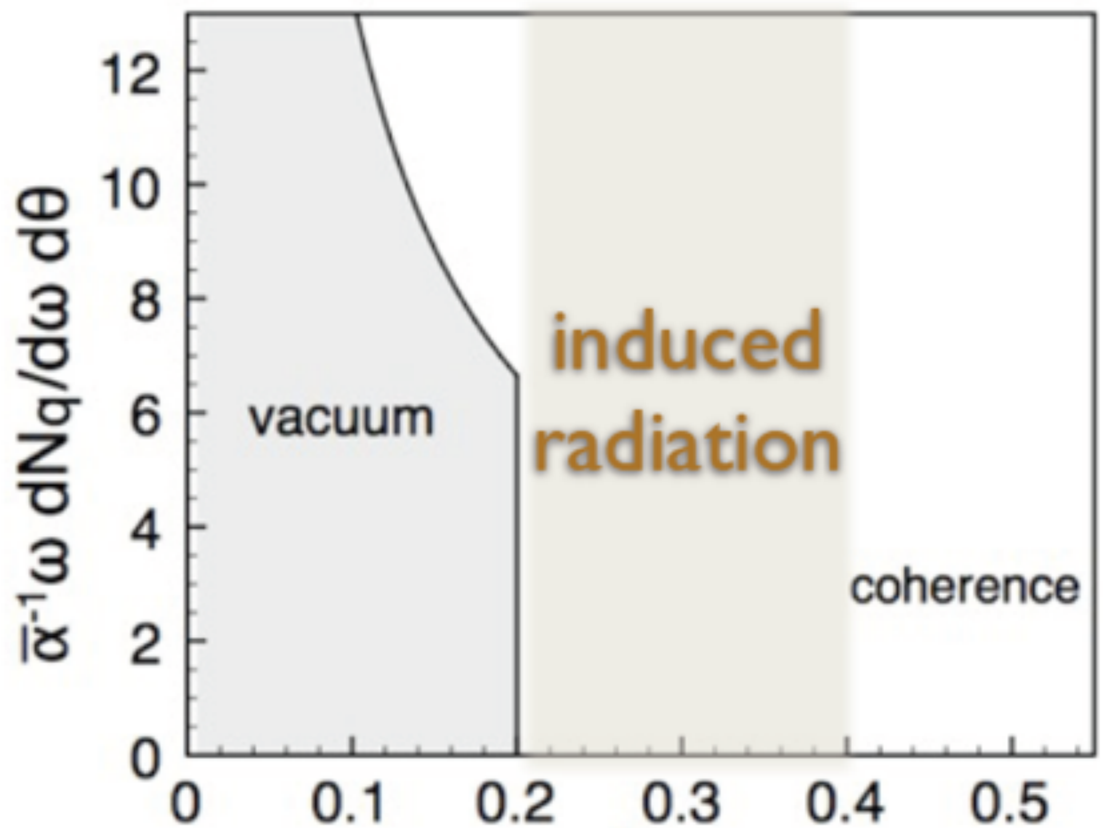

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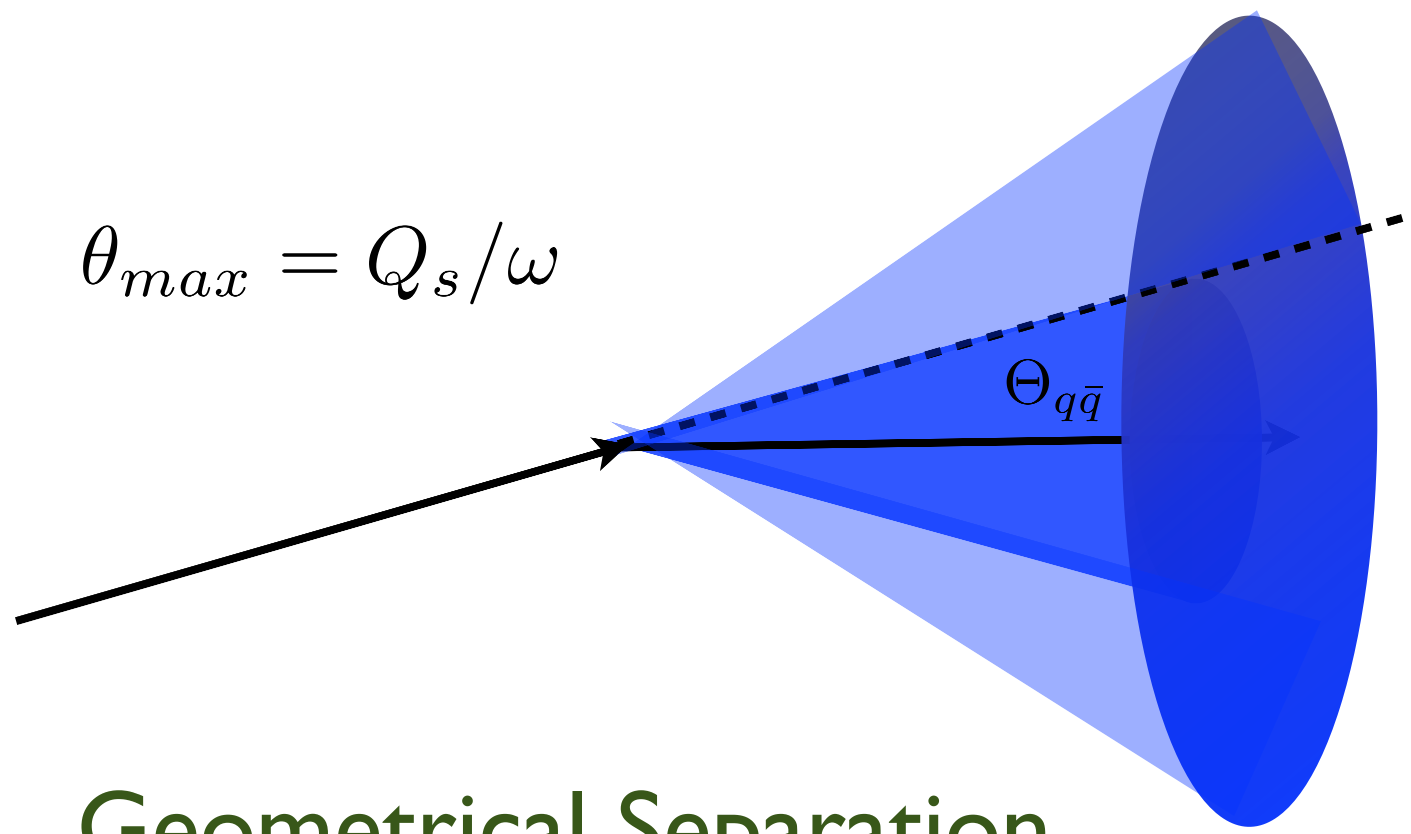


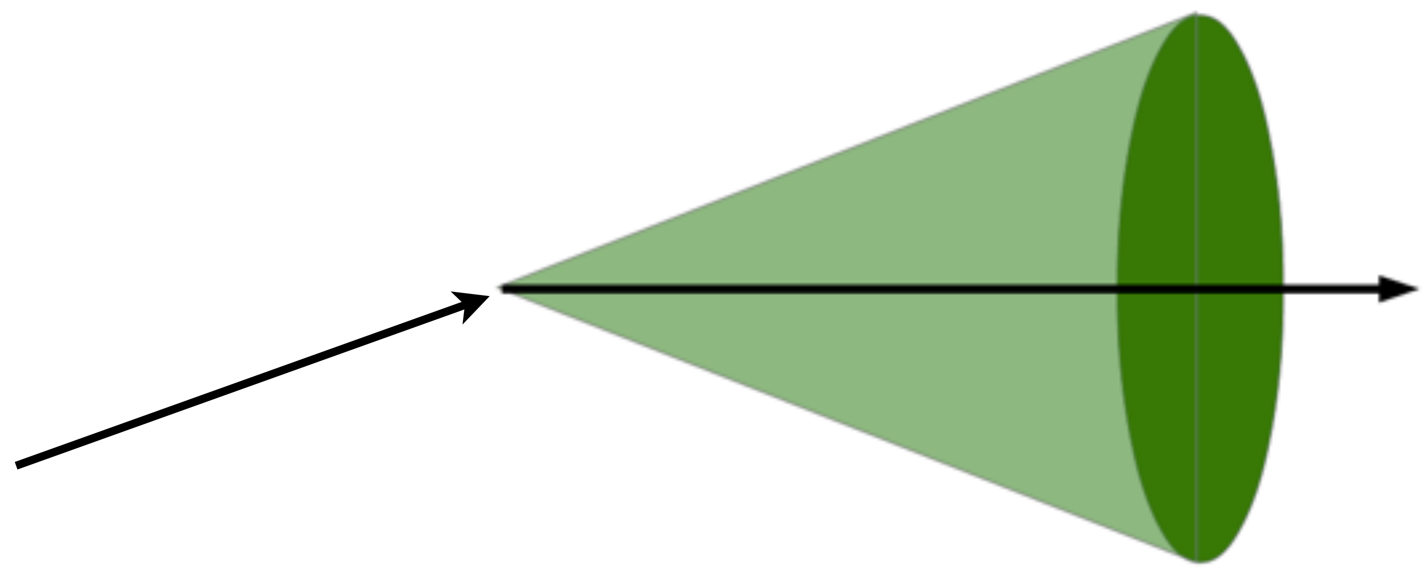
Geometrical Separation

Contact with high energy limit: Kovchegov-Mueller result

Interferences: Infinite medium length

- Interferences cancel asymptotically
- Gluon spectrum: BDMPS-Z+vacuum

No interferences



$$Q_s = \hat{q}L$$

Medium acts as a breaking vacuum coherence

⇒ Incoherent vacuum emissions at large angles.

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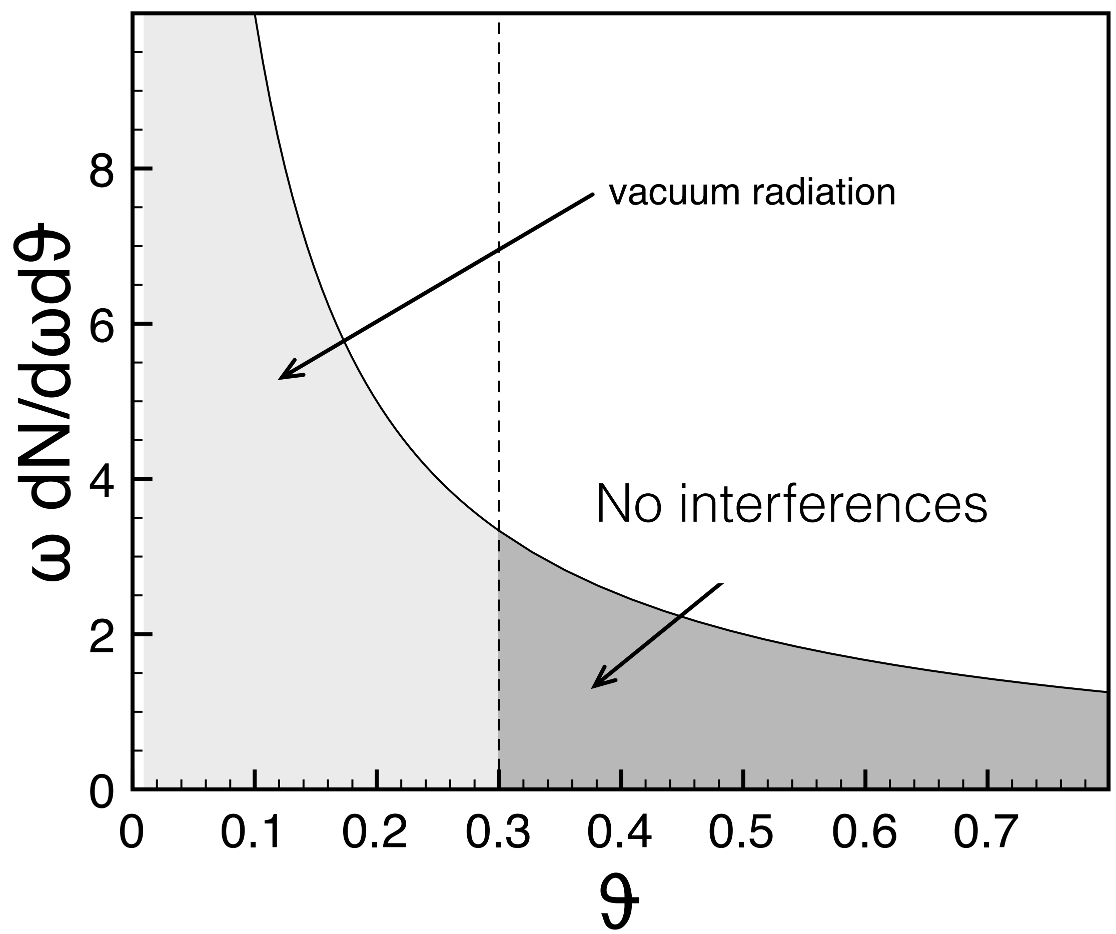

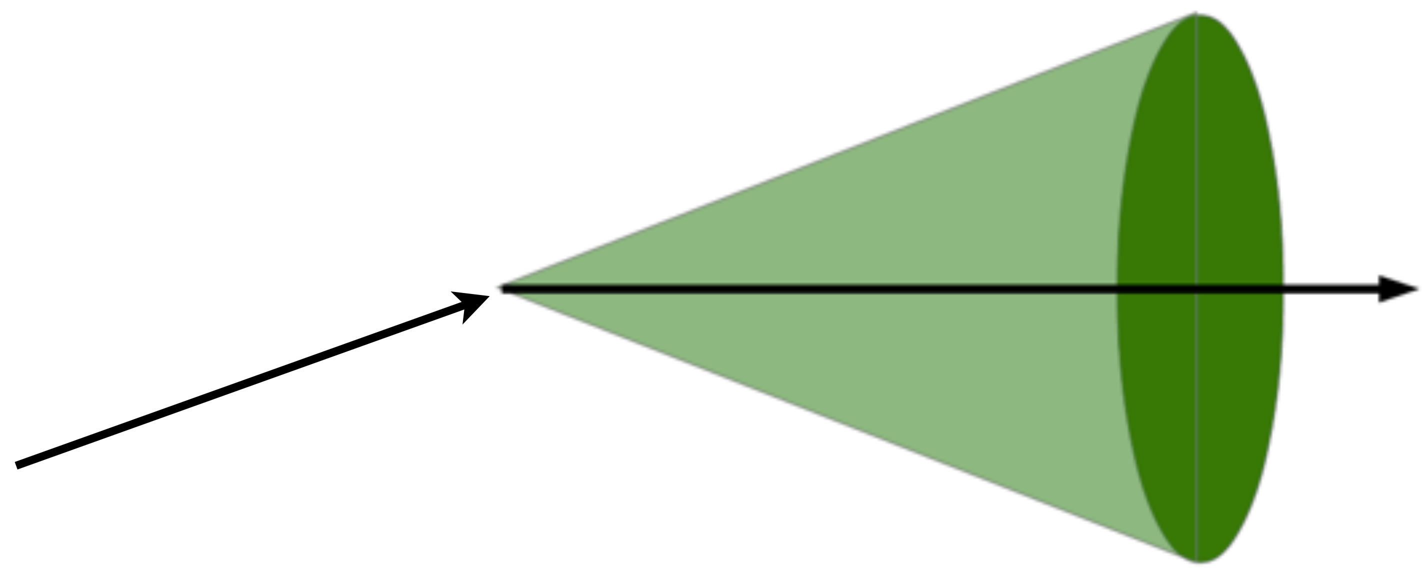

$$Q_s = \hat{q} L$$

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⇒ Incoherent vacuum emissions at large angles.

Conclusions

- Interference between the initial and final state radiation are indeed affected in the presence of a QCD medium.
- Use this setup for phenomenological consequences....

IS2013

International Conference on the Initial Stages in High Energy Nuclear Collisions

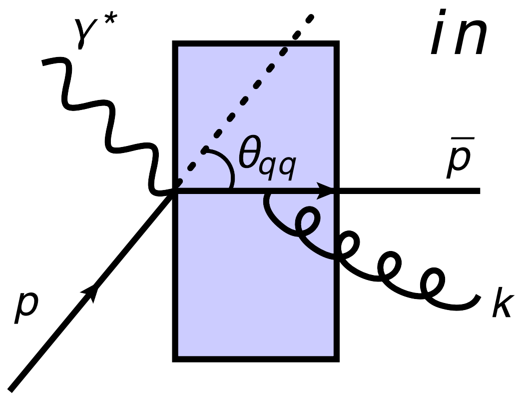
September 8-14 2013, Illa da Toxa, Galicia, Spain

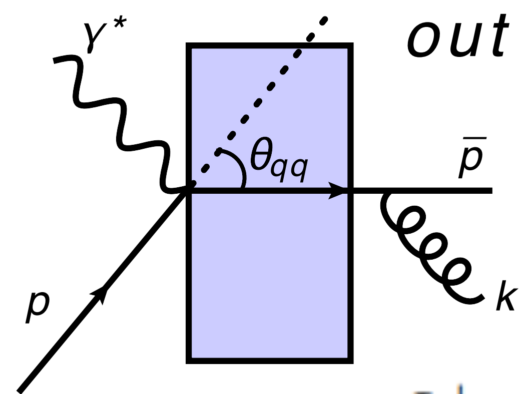
Abstract submission deadline: July 7th, 2013

http://igfae.usc.es/is2013

BACKUP SLIDES

Scattering amplitude from CYM Eqs.



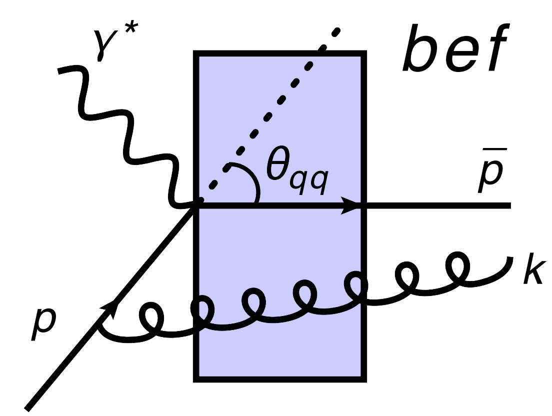


Outcoming parton

$$\mathcal{M}^a_{\lambda,in}(\vec{k}) = \frac{g}{k^+}\int d^2\boldsymbol{x} e^{i(k^- L^+ - \boldsymbol{k}\cdot\boldsymbol{x})}\int_0^{L^+} dy^+ e^{ik^+\bar{u}^- y^+}$$
$$\times\ \boldsymbol{\epsilon}_\lambda\cdot(i\boldsymbol{\partial}_y + k^+\bar{\boldsymbol{u}})\mathcal{G}_{ab}\big(L^+,\boldsymbol{x},y^+,\boldsymbol{y}=\bar{\boldsymbol{u}}y^+|k^+\big)\,\mathcal{U}_{bc}(y^+,0)Q^{out}_c$$
$$\mathcal{M}^a_{\lambda,out}(\vec{k}) = -2i\frac{\boldsymbol{\epsilon}_\lambda\cdot\bar{\boldsymbol{\kappa}}}{\bar{\boldsymbol{\kappa}}^2}\,e^{i(k\cdot\bar{u})L^+}\,\mathcal{U}_{ab}(L^+,0)Q^{out}_b\ ,$$

Incoming parton

$$\mathcal{M}^a_{\lambda,bef}(\vec{k}) = \frac{g}{k^+}\int_{x^+=\infty} d^2\boldsymbol{x}\,e^{i(k^- x^+ - \boldsymbol{k}\cdot\boldsymbol{x})}\int_{-\infty}^0 dy^+ e^{ik^+u^- y^+}$$
$$\times\ \boldsymbol{\epsilon}_\lambda\cdot(i\boldsymbol{\partial}_y + k^+\boldsymbol{u})\mathcal{G}_{ab}\big(x^+,\boldsymbol{x},y^+,\boldsymbol{y}=\boldsymbol{u}y^+|k^+\big)\,Q^{in}_b$$



Correlators

Quark-gluon dipole

$$\frac{1}{N_c^2-1}\langle \mathcal{G}(x^+,\mathbf{x};y^+,\mathbf{y})\mathcal{U}^\dagger(x^+,y^+)\rangle = \mathcal{K}(x^+,\mathbf{x};y^+,\mathbf{y}|k^+)$$

$$\mathcal{K}(x^+,\boldsymbol{x};y^+,\boldsymbol{y}|k^+) = \int_{\boldsymbol{r}(y^+)=\boldsymbol{y}}^{\boldsymbol{r}(x^+)=\boldsymbol{x}} \mathcal{D}\boldsymbol{r}\, \exp\left[\int_{y^+}^{x^+} d\xi \left(i\frac{k^+}{2}\dot{\boldsymbol{r}}^2(\xi) - \frac{1}{2}n(\xi)\sigma\left(\boldsymbol{r}(\xi)\right)\right)\right]$$

Gluon dipole

$$\int d\mathbf{x}\, d\mathbf{x}' e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\frac{1}{N_c^2-1}\langle \mathcal{G}(x^+,\mathbf{x};y^+,\mathbf{y})\mathcal{G}^\dagger(x^+,y^+)\rangle = \mathcal{S}(x^+,y^+,\mathbf{x}-\mathbf{y})$$

$$\mathcal{S}(x^+,y^+;\boldsymbol{x}-\boldsymbol{y}) = \exp\left[-\frac{1}{2}\int_{y^+}^{x^+} d\xi\, n(\xi)\,\sigma(\boldsymbol{x}-\boldsymbol{y})\right]$$

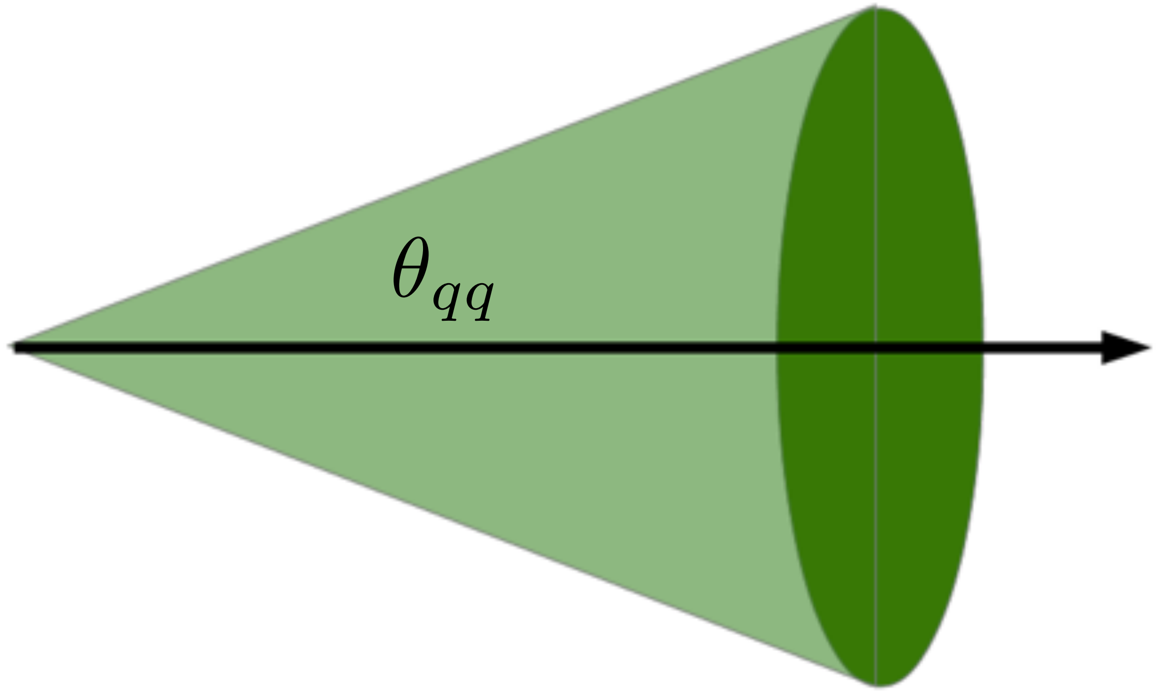
Dipole cross section

$$\sigma(\boldsymbol{r}) = \int \frac{d^2\boldsymbol{q}}{(2\pi)^2}\mathcal{V}(\boldsymbol{q})\left[1-\cos(\boldsymbol{r}\cdot\boldsymbol{q})\right]$$

Leading logs and AO

$$\kappa^{\mathbf{2}} < \delta\mathbf{k}^{\mathbf{2}} \qquad \delta\mathbf{k}^{\mathbf{2}} \equiv \mathbf{Q}^{\mathbf{2}}$$

$$\omega\frac{dN}{d\omega d\kappa^2} = \frac{1}{\kappa^{\mathbf{2}}} \qquad \text{(DGLAP)}$$

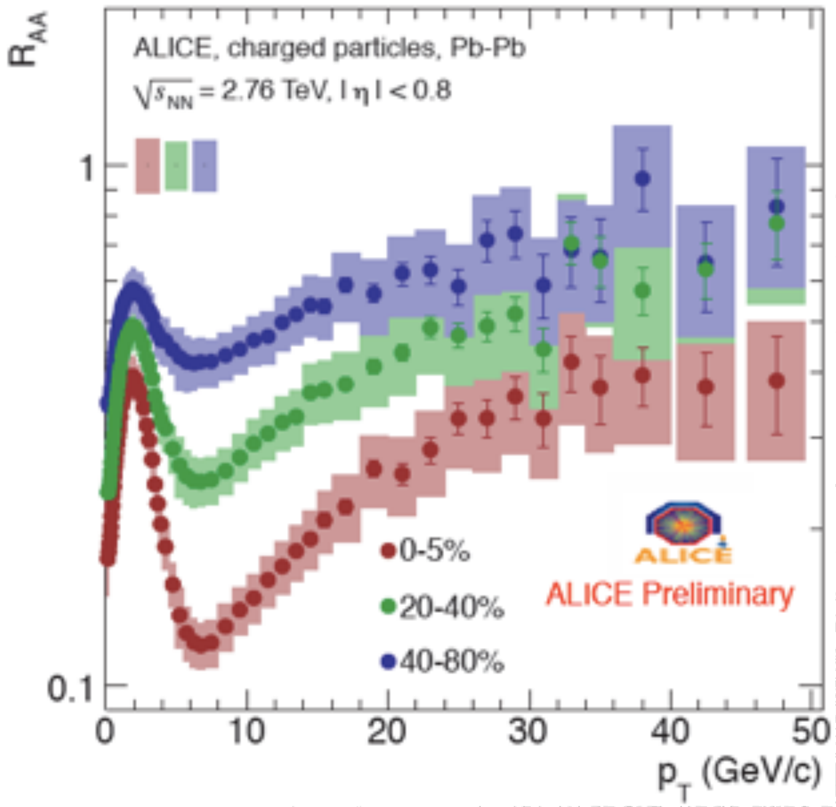


$$\omega\frac{dN}{d\omega} = \int_{Q_0^2}^{\delta\mathbf{k}^{\mathbf{2}}} \frac{d\kappa^{\mathbf{2}}}{\kappa^{\mathbf{2}}} = \log\left(\frac{\delta\mathbf{k}^{\mathbf{2}}}{Q_0^2}\right) \sim \log\left(\frac{Q^2}{Q_0^2}\right) \qquad \text{L. L.}$$

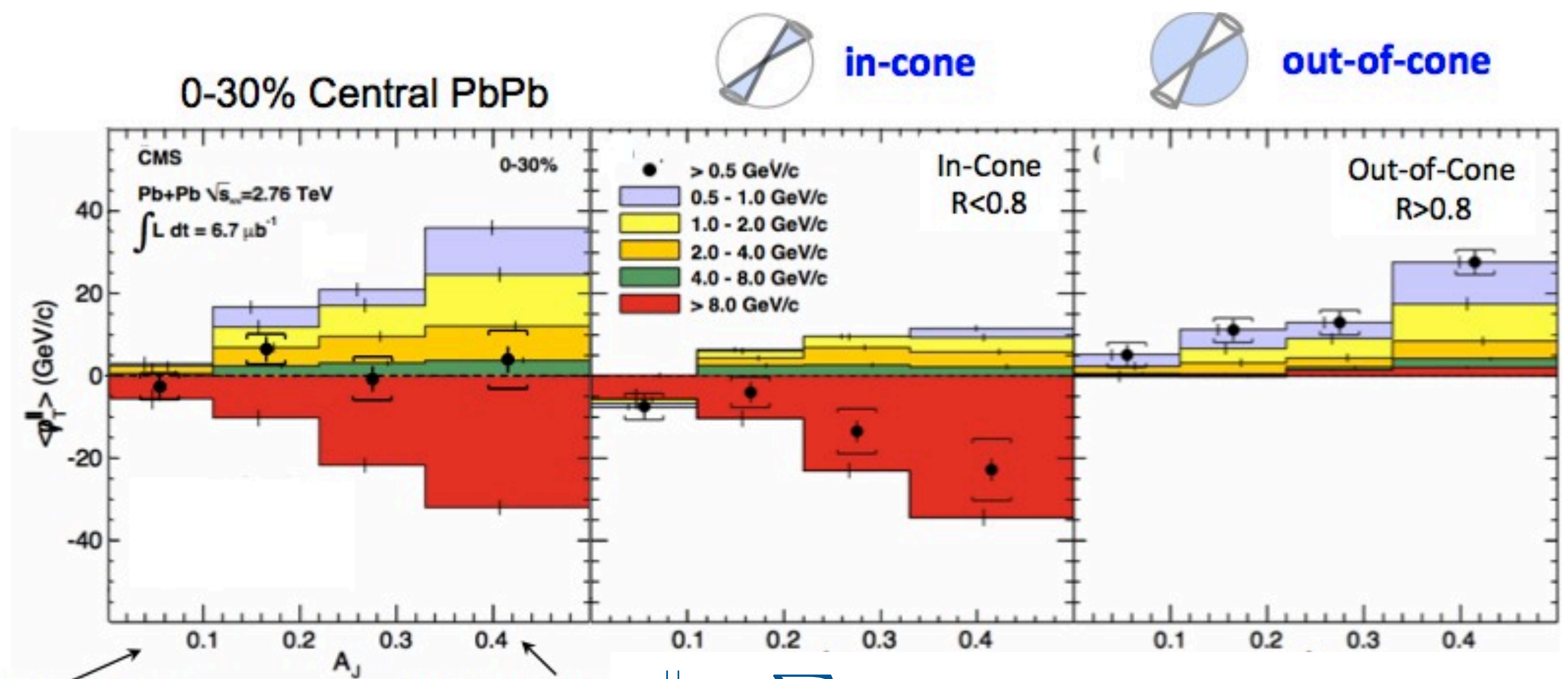
$$N \propto \int_{Q_0^2}^{Q^2} \omega\frac{dN}{d\omega} = \frac{1}{2}\left[\log\left(\frac{Q^2}{Q_0^2}\right)\right]^2 \qquad \text{D. L. L.}$$

Jets in HIC @ LHC

(i) Suppression of high-p_T hadrons



(ii) Soft large angle emissions



$$p_T^{||} \equiv \sum_{\text{tracks}} -p_{T,\text{track}} \cos\left(\phi_{\text{track}} - \phi_{\text{leading jet}}\right)$$

$$\Delta\phi > \pi/2$$

$$A_J = \frac{E_{T1} - E_{T2}}{E_{T1} + E_{T2}}$$

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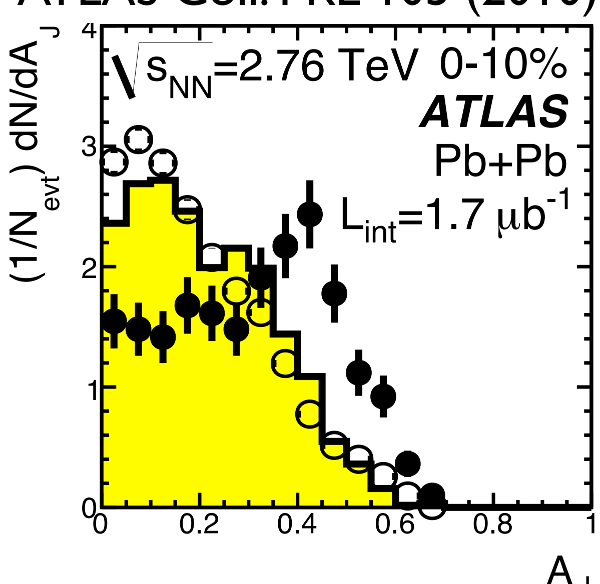
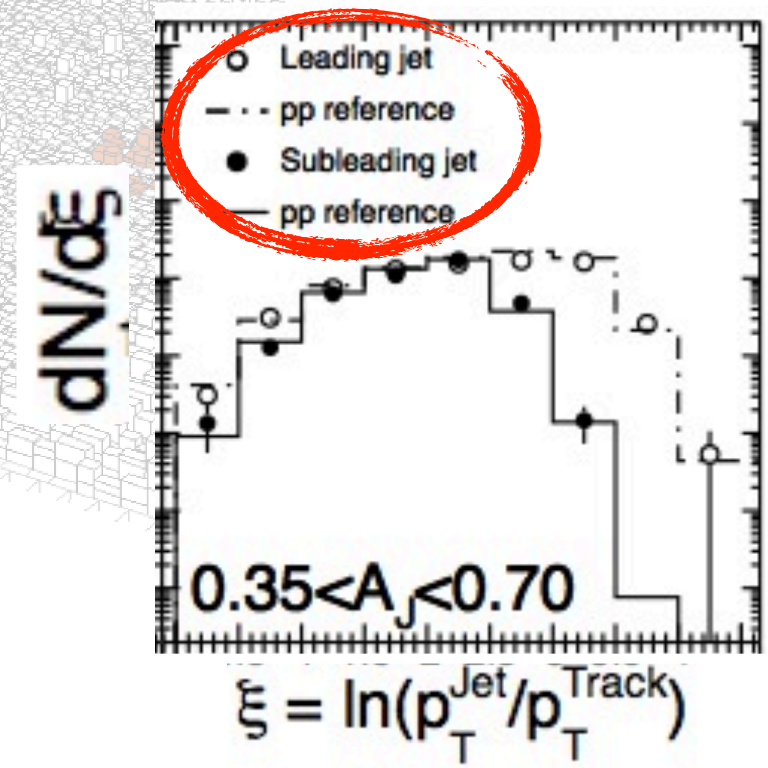
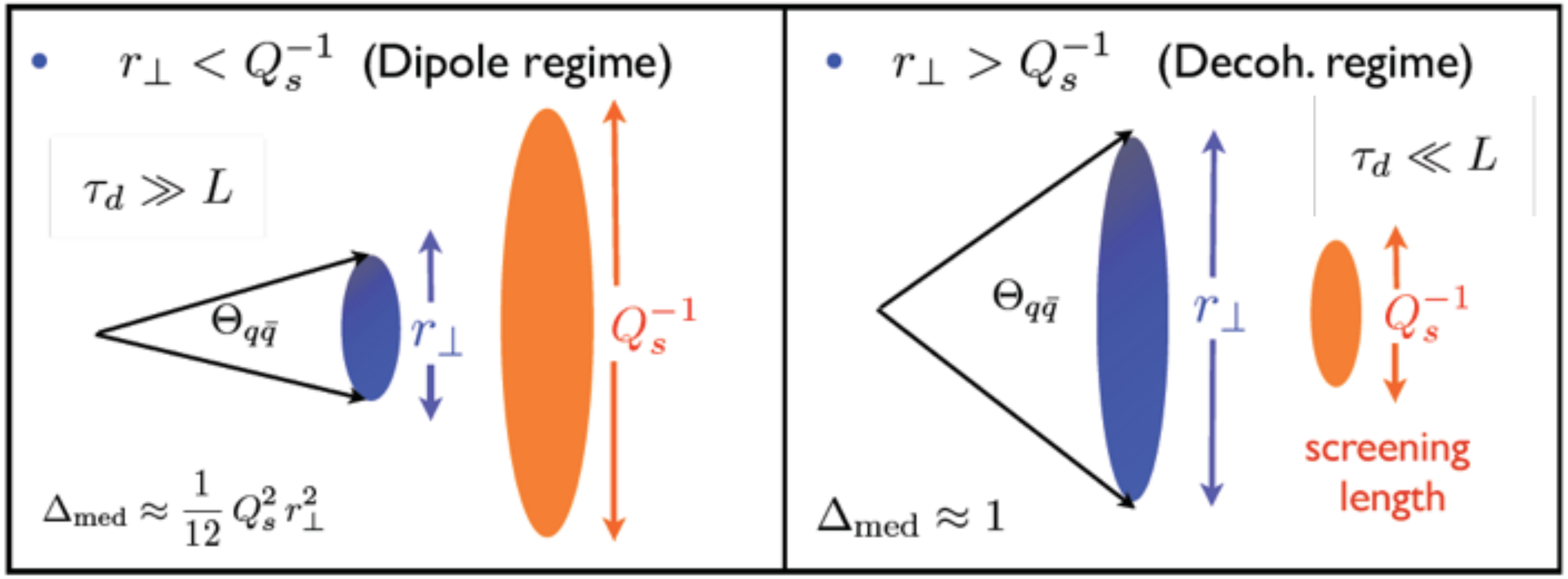


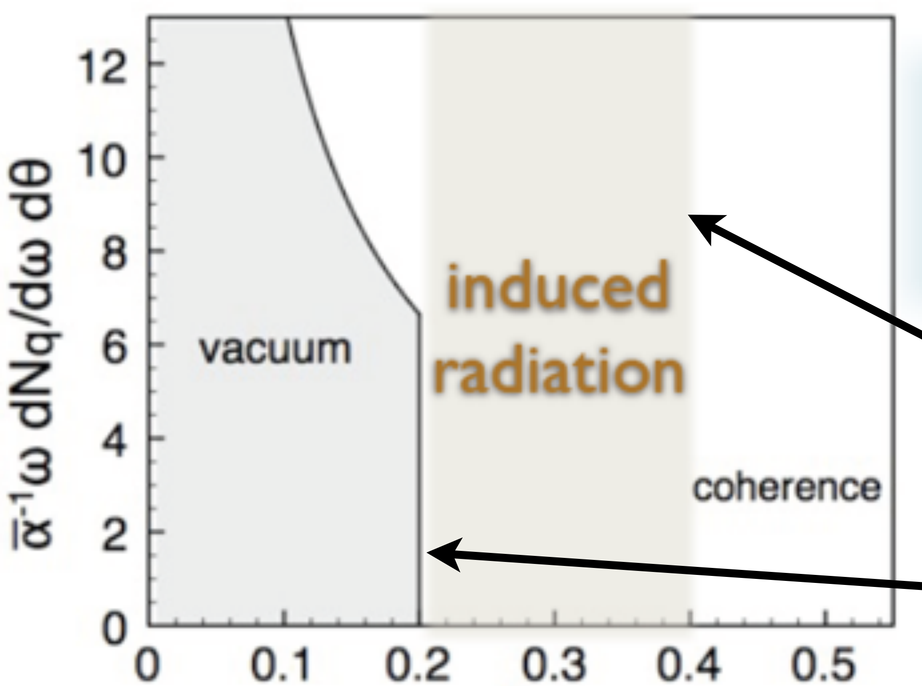
(iii) Significant dijet asymmetry



(iv) Vacuum-like fragmentation function

First steps: Antenna in a QCD medium





Meht

$$1 - \Delta_{\rm med}(t,0) \simeq \exp\left[-\frac{1}{12}\hat{q}\theta_{q\bar{q}}^2 t^3\right] \Rightarrow \tau_d = \left(\hat{q}\theta_{q\bar{q}}^2\right)^{-1/3}$$

decoherence parameter

characteristic decoherence time

Q_{hard}/ω

$\theta_{q\bar{q}}$

$$Q_{hard} = \max(r_\perp^{-1}, Q_s)$$

$$Q_s^2 = \hat{q}L, \qquad r_\perp = \theta_{q\bar{q}}L$$