# NEW RESULTS WITH LIPATOV'S EFFECTIVE ACTION

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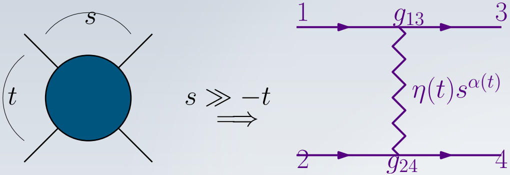
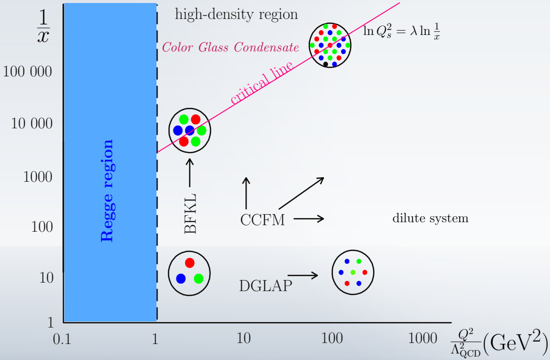
# High-Energy QCD

## High-Energy (Regge) Limit



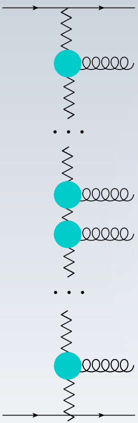




Wide spectrum of applications:

- From perturbative forward/ diffractive scattering...
- ...over TMDs at low-$x$...
- ...to complicated phenomena in heavy ion collisions

And many interesting properties:

- Similarity to $\mathcal{N} = 4$ SYM
- Appearance of conformal invariance and integrability



Despite the presence of specific physical environments (e.g. collinear/soft logarithms, high-parton density) *there is a common basis*

- High-Energy Factorization
- Resummation of Leading Logarithms ($\ln s$)

$\Longrightarrow$ BFKL Equation

★ Based on **Reggeization** Ansatz

★ Consistency with **Bootstrap**

$$A^{\rm MRK}_{2\to 2+n} = A^{\rm tree}_{2\to 2+n} \prod_{i=1}^{n+1} s_i^{\omega(t_i)},$$

$$A^{\rm tree}_{2\to 2+n} = 2gsT^{c_1}_{A'A}\Gamma_1 \frac{1}{t_1} gT^{d_1}_{c_2 c_1}\Gamma^1_{2,1}\frac{1}{t_2}\cdots gT^{d_n}_{c_{n+1}c_n}\Gamma^n_{n+1,n}\frac{1}{t_{n+1}} gT^{c_{n+1}}_{B'B}\Gamma_2,$$

# The Need for an Effective Action in High-Energy QCD

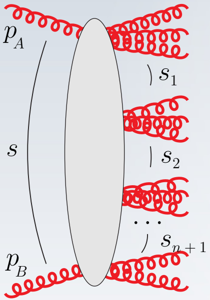
The Effective Action for High-Energy QCD Aims at

- Providing a unified formalism for different phenomena in high-energy QCD
- Incorporating all requirements of unitarity
- Setting a connection with Gribov's reggeon field theory
- Simplifying the computation of scattering amplitudes in the Regge limit

★ It is expected to be an integrable theory in the leading log approximation [Lipatov'91; Kirschner, Lipatov & Szymanowski'93,'94]

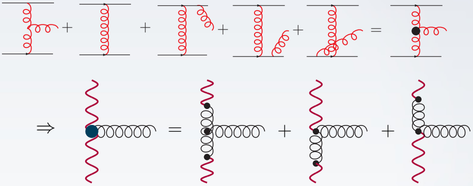
# Quasi-Multi-Regge Kinematics & High-Energy Factorization

Generalized **Quasi-Multi-Regge Kinematics** (QMRK)
[Fadin&Lipatov'89]




Clusters strongly ordered in rapidity:
$y_0 \gg y_1 \gg \cdots \gg y_{n+1}$,
$y_k = \frac{1}{2} \ln \frac{k^+}{k^-}$

- Strong rapidity ordering simplifies polarization tensor of $t$-channel reggeons:
  $g_{\mu\nu} \to \frac{1}{2}(n^+)_\mu (n^-)^\nu + \mathcal{O}(1/s)$
- Reggeized gluons couple to quarks and gluons through effective vertices local in rapidity:
  ***Effective vertex*** $=$ ***Kinematic Projection*** $+$ ***Induced Contributions***



- Reggeon propagators are essentially transverse: $q_i^2 = -\boldsymbol{q}_i^2$

$p_a + p_b \to p_1 + p_2; \quad n^{+,-} = 2p_{a,b}/\sqrt{s},$
$k = k^+ \frac{n^-}{2} + k^- \frac{n^+}{2} + \boldsymbol{k}$

# The Pillars of Lipatov's Action

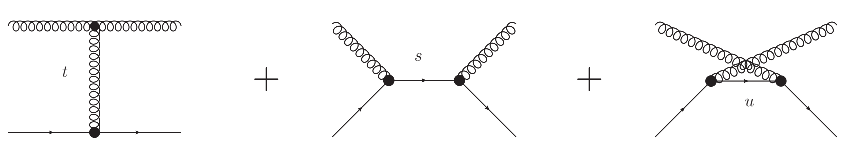
[Lipatov'95,'97]

**HIGH-ENERGY (QMRK) FACTORIZATION**

**+**

**GAUGE INVARIANCE**

**= Lipatov's Action**

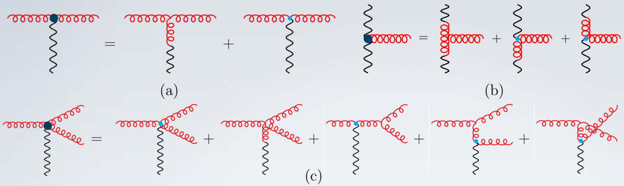



- All three diagrams contribute in covariant gauge

# Gauge Invariance & Lipatov's Action

[Lipatov'95,'97]

Consider local-in-rapidity production scattering amplitudes



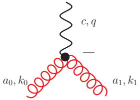

*In order for these amplitudes to be* **gauge invariant**, *a new induced vertex has to be introduced at each order in perturbation theory*
[Ward Identities⟹Recurrence Relations]

$$\Delta^{\nu_0\nu_1\cdots\nu_r +}_{d_0 d_1\cdots d_n c}(k_0^+,\cdots,k_r^+) = \frac{(n^+)^{\nu_r}}{k_r^+}\sum_{i=0}^{r-1} t^a_{a_r a_i}\Delta^{\nu_0\nu_1\cdots\nu_{r-1}+}_{d_0 d_1\cdots d_n c}(k_0^+,\cdots k_{r-1}^+), \quad (n>2)$$

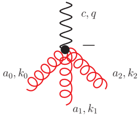
- For an action linear in reggeon fields, these identities build a current coupling to the reggeon of the form

$$W_\pm[v(x)] = -g^{-1}\partial_\pm \mathcal{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{x^\mp} dz^\pm v_\pm(z)\right)$$


$$= \Delta^{\nu_0 -}_{a_0 c} = -i\boldsymbol{q}^2 \delta^{a_0 c} (n^-)^{\nu_0},$$




$$= g\boldsymbol{q}^2 f^{a_0 a_1 c} \frac{1}{k_0^-} (n^-)^{\nu_0} (n^-)^{\nu_1},$$




$$= \Delta^{\nu_0 \nu_1 \nu_2 -}_{a_0 a_1 a_2 c} = i g^2 \boldsymbol{q}^2 \left( \frac{f^{a_2 a_1 a} f^{a_0 a c}}{k_2^- k_0^-} + \frac{f^{a_2 a_0 a} f^{a_1 a c}}{k_2^- k_1^-} \right) (n^-)^{\nu_0} (n^-)^{\nu_1} (n^-)^{\nu_2},$$


$$= \frac{i}{2\boldsymbol{q}^2}.$$

[Antonov, Cherednikov, Kuraev & Lipatov'05]

$$S_{\text{eff}} = S_{\text{QCD}} + S_{\text{ind}};$$

$$S_{\text{ind}} = \int d^4x \, \text{Tr} \left[ \left( W_+[v(x)] - \mathscr{A}_+(x) \right) \partial_\perp^2 \mathscr{A}_-(x) \right] + \int d^4x \, \text{Tr} \left[ \left( W_-[v(x)] - \mathscr{A}_-(x) \right) \partial_\perp^2 \mathscr{A}_+(x) \right];$$

$$W_\pm[v] = v_\pm \frac{1}{D_\pm} \partial_\pm = v_\pm - g v_\pm \frac{1}{\partial_\pm} v_\pm + \cdots$$

$\mathscr{A}_\pm$: reggeons, $v_\mu$: gluons

**Kinematical Constraints**

$$\partial_\pm \mathscr{A}_\mp(x) = 0, \quad \sum_{i=0}^{r} k_i^\pm = 0$$

Reggeon fields invariant under *local* gauge transformations

# Connection to Other Formalisms

☐ On quite general grounds, effective theory of high-energy QCD is 2-dimensional and written in terms of Wilson lines [Nachtmann'91, Verlinde & Verlinde'93]

☐ Reggeon Field Theory obtained after integrating out quark and gluon fields

☐ Apparent similarities with other alternative formulations

- Balitsky's High-Energy OPE [Balitsky'96,'99]
- Color Glass Condensate / JIMWLK equation [Jalilian-Marian, Kovner, Leonidov, McLerran, Weigert'96,'97,'99; Iancu, Leonidov, McLerran '00,'01]

(...) but formal similarities usually obscured by different physical interpretations [Jalilian-Marian et al.'97, Hatta'06]

# Lipatov's Action Beyond Tree Level

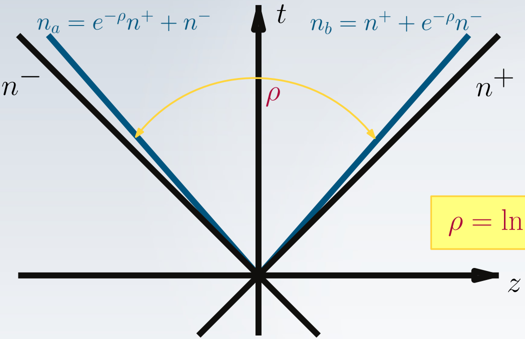
The Problems...

- Appearance of new rapidity divergences in longitudinal momentum integrations
- Apparent overcounting of degrees of freedom

★ Enforcing locality in rapidity with a cutoff manifestly breaks gauge invariance and makes computations unwieldy [Bartels, Hentschinski, Lipatov'08]

The Proposed Solutions...

# Regularization & Scaling



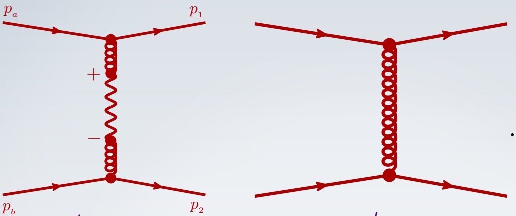

$\rho = \ln s$

Tilting the light-cone vectors appearing in the induced vertices

[Collins & Soper'81,'82]
[Korchemsky & Radyushkin'87]
[Balitsky'96]
[Hentschinski & Sabio Vera'11]

- Regularization needed to make sense of non-local $\frac{1}{\partial_\pm}$
- Rest of divergences $\Longrightarrow$ dimensional regularization
- $\rho \to \infty$ in high-energy limit
- Scaling arguments select small number of $\rho$-enhanced diagrams
- Pole prescription consistent with Hermiticity [Hentschinski'11]

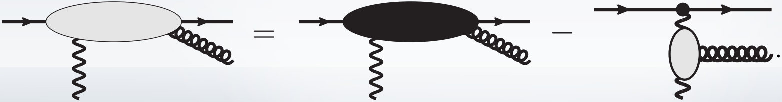
# The Subtraction Mechanism




$\frac{-t}{s} < \eta$ $\frac{-t}{s} > \eta$

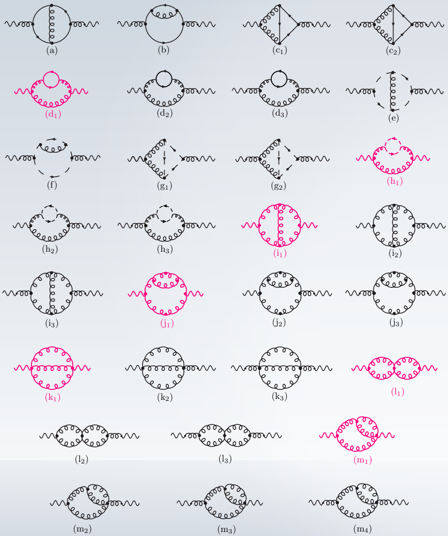
Subtraction Mechanism respects gauge (and Lorentz) invariance *and agrees by construction with QCD amplitude*

Subtract non-local contributions mediated by reggeon exchange

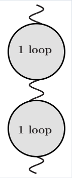


# The Two-Loop Gluon Regge Trajectory

[Chachamis, Hentschinski, JDM & Sabio Vera'12,'13]




Subtractions




- We get trajectory from subtracted self-energy requiring $\rho$-independence of renormalized gluon propagator
- Ambiguities from mixed divergences fixed [Chachamis, Hentschinski, JDM & Sabio Vera'13]

- Exact agreement with literature [Fadin, Fiore & Kotsky'95,'96; Fadin, Fiore & Quartarolo'96; Del Duca & Glover'01]

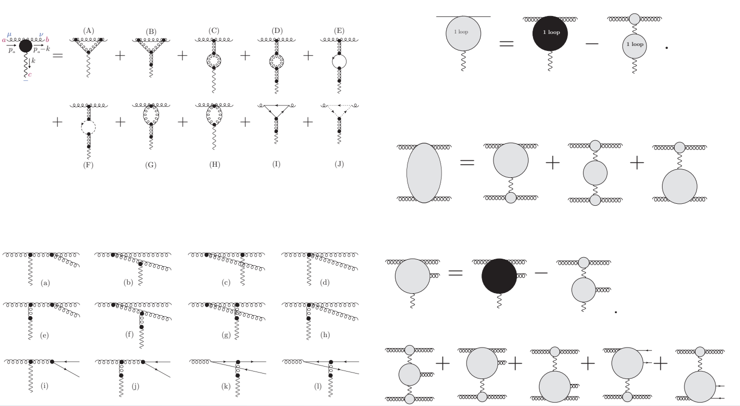
$$\omega^{(2)}(\boldsymbol{q}^2) = \frac{(\omega^{(1)}(\boldsymbol{q}^2))^2}{4}\left[\frac{11}{3} - \frac{2n_f}{3N_c} + \left(\frac{\pi^2}{3} - \frac{67}{9}\right)\epsilon + \left(\frac{404}{27} - 2\zeta(3)\right)\epsilon^2\right]$$

- Easy extraction of cusp anomalous dimension from this expression [Korchemskaya & Korchemsky'96]
- Universality explicitly revealed in our formalism
- Imaginary parts under control: important role of symmetric prescription

# Forward Jet Vertex

[Hentschinski & Sabio Vera'11; Chachamis, Hentshcinski, JDM & Sabio Vera'12]

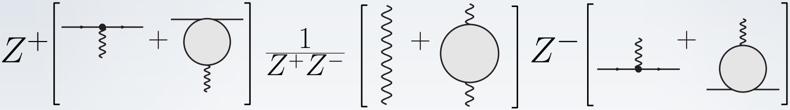
1-loop quark- and gluon initiated jet vertices found in agreement with previous results [Bartels, Colferai & Vacca'01, '02]



# Renormalization Interpretation

Exact cancellation of $\rho$-divergences between impact factors and reggeon self-energy explicitly checked

- Makes possible definition of reggeon vertex and wavefunction renormalization




- Reggeon initially taken as background field, renormalized by perturbative corrections $\sim$ MATCHING

# Technical Details: Loop Integrations

- [IMPORTANT] Our regularization/subtraction allows the usage of conventional techniques for Feynman integrals

## Mellin-Barnes Representations

$$\mathcal{S} = \iint \frac{d^dk}{(2\pi)^d}\frac{d^dl}{(2\pi)^d}\frac{1}{[-k^2-i0]^{\alpha_1}[-(k-q)^2-i0]^{\alpha_2}[-l^2-i0]^{\alpha_3}[-(l-q)^2-i0]^{\alpha_4}}$$
$$\times\frac{1}{[-(k-l)^2-i0]^{\alpha_5}(-n_a\cdot k-i0)^{\alpha_6}(-n_b\cdot k-i0)^{\alpha_7}(n_a\cdot l-i0)^{\alpha_8}(-n_b\cdot l-i0)^{\alpha_9}}$$
$$=\frac{-\left(\boldsymbol{q}^2\right)^{d-\alpha_{12345}-\frac{\alpha_{6789}}{2}}}{2(4\pi)^d}\prod_{i=1}^{6}\int\frac{dz_i}{2\pi i}\Gamma(-z_i)\frac{\Gamma\left(z_{1234}+\alpha_{345}+\frac{\alpha_{6789}}{2}-\frac{d}{2}\right)\Gamma(-z_4+\alpha_2)}{\prod_{j=1}^{9}\Gamma(\alpha_j)\Gamma(-2z_1)\Gamma(-2z_6)}$$
$$\frac{\Gamma\left(-z_{12345}-\alpha_{345}+\frac{\alpha_{6789}}{2}+\frac{d}{2}\right)\Gamma\left(-z_1+z_{2345}+\alpha_{345}-\frac{\alpha_{6789}}{2}-\frac{d}{2}\right)\Gamma(-z_3+\alpha_1)}{\Gamma(-2z_2-z_{34}-\alpha_{126789}-2\alpha_{345}+2d)}$$
$$\frac{\Gamma\left(-z_{23}-\alpha_{2345}-\frac{\alpha_{6789}}{2}+d\right)\Gamma\left(-z_{24}-\alpha_{1345}-\frac{\alpha_{6789}}{2}+d\right)\Gamma(z_{2345}-z_6+\alpha_{3458}-\frac{d}{2})}{\Gamma(2z_{234}+z_5+2\alpha_{345}+\alpha_{789}-d)}$$
$$\frac{\Gamma(2z_{234}+z_5+2\alpha_{345}+\alpha_{89}-d)\Gamma(-z_{234}+z_6-\alpha_{345}+\frac{d}{2})\Gamma\left(z_2+\alpha_{12345}+\frac{\alpha_{6789}}{2}-d\right)}{\Gamma(-z_5+\alpha_6)}$$
$$\frac{\Gamma(-z_{23456}-\alpha_{3458}+\frac{d}{2})\Gamma(-2z_2-z_{34}-2\alpha_{34}-\alpha_{589}+d)\Gamma(z_{23}+\alpha_3)\Gamma(z_{24}+\alpha_4)}{\Gamma(-\alpha_{34589}+d)}e^{-z_{16}\rho}$$

- Extraction of leading $\rho$ and $\epsilon$ terms with `MB Tools` [Czakon, Smirnov]

# Outlook

**Many Issues Still To Be Understood...**

- Relation with Other Formalisms [JIMWLK Hamiltonian, Balitsky OPE]; Relation of QMRK Factorization with $k_T$ and Dipole Factorizations
- Formalization of the Procedure
- Renormalization Group Equation for Evolution with Respect to *Factorization Scale* $\rho$ (Possibly Hindered by Different Regularization of Real Corrections)
- Extension of Lipatov's Action to Dense Regime; Exploitation of Symmetries etc.

**Application to Phenomenology....**

- e.g. NLO Impact Factors for Jet-Gap-Jet Production [Hentschinski, Madrigal, Murdaca, Sabio Vera & Salas, to appear soon]

# Conclusions

- Consistency of Lipatov's action checked at loop level via explicit non-trivial computations (2-loop gluon Regge trajectory + 1-loop impact factors)
- General mechanism that allows for application of usual loop integration techniques
- Very useful tool for computation of effective vertices needed in phenomenology