Numerical studies of JIMWLK evolution

T. Lappi

University of Jyväskylä, Finland

Trento 2013

Outline

- CGC, Glasma, JIMWLK evolution
- JIMWLK equation in Langevin form
- Running coupling in JIMWLK T.L., H. Mäntysaari EPJC 2013
- JIMWLK as initial condition for CYM T.L., PLB 2011
- Unequal rapidity correlations in progress

JIMWLK [“gym-walk”] Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

Gluon saturation, Glass and Glasma

Small x: the hadron/nucleus wavefunction is characterized by **saturation scale** $Q_s \gg \Lambda_{QCD}$.

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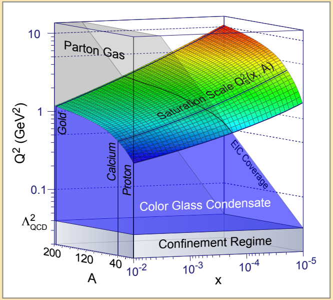
$\Downarrow$

p $\sim Q_s$: strong fields $A_\mu \sim 1/g$

- occupation numbers $\sim 1/\alpha_s$
- classical field approximation.
- small α_s, but nonperturbative



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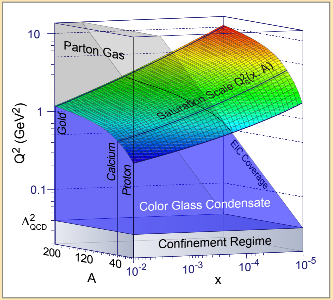

CGC: Effective theory for wavefunction of nucleus

- Large x = source ρ, **probability** distribution $W_y[\rho]$
- Small x = classical gluon field A_μ + quantum flucts.

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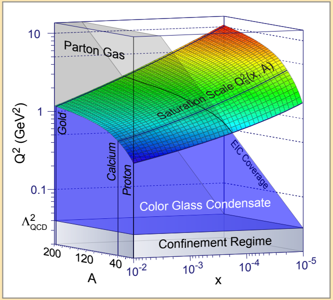

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Glasma field configuration of two colliding sheets of CGC.

Wilson line

Classical color field described as Wilson line

$$U(\mathbf{x}) = P \exp\left\{ ig \int \mathrm{d}x^- A^+_{\mathrm{cov}}(\mathbf{x}, x^-) \right\} \quad \in \quad \mathrm{SU}(3)$$

Relation to color charge

$$\nabla^2 A^+_{\mathrm{cov}}(\mathbf{x}, x^-) = -g\rho(\mathbf{x}, x^-)$$

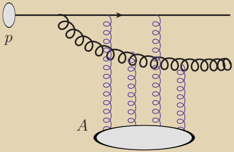
($x^\pm = \frac{1}{\sqrt{2}}(t \pm z)$; $A^\pm = \frac{1}{\sqrt{2}}(A^0 \pm A^z)$; $\mathbf{x}$ 2d transverse)

Example of usage: forward pA

- Quark from p (large x pdf) , radiate gluon
- Eikonal propagation $\Longrightarrow$ Wilson lines $U(\mathbf{x})$

Need target expectation values of operators:

$$\mathrm{Tr}\, U(\mathbf{x})U^\dagger(\mathbf{y}) \quad \mathrm{Tr}\, U(\mathbf{x})U^\dagger(\mathbf{y})U(\mathbf{u})U^\dagger(\mathbf{v}) \quad \ldots$$



JIMWLK evolution

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- Energy dependent **probability** distribution $W_y[U]$ $(y \sim \ln \sqrt{s})$
- Energy/rapidity dependence of $W_y[U]$ from JIMWLK renormalization group equation

$$\partial_y W_y[U(\mathbf{x})] = \mathcal{H} W_y[U(\mathbf{x})]$$

JIMWLK Hamiltonian: (fixed coupling)

$$\mathcal{H} \equiv \frac{1}{2}\alpha_s \int_{\mathbf{xyz}} \frac{\delta}{\delta A_c^+(\mathbf{y})} \mathbf{e}^{ba}(\mathbf{x},\mathbf{z}) \cdot \mathbf{e}^{ca}(\mathbf{y},\mathbf{z}) \frac{\delta}{\delta A_b^+(\mathbf{x})},$$

$$\mathbf{e}^{ba}(\mathbf{x},\mathbf{z}) = \frac{1}{\sqrt{4\pi^3}} \frac{\mathbf{x}-\mathbf{z}}{(\mathbf{x}-\mathbf{z})^2} \left(1 - U^\dagger(\mathbf{x}) U(\mathbf{z})\right)^{ba}$$

Fokker-Planck and Langevin

Textbook example: two descriptions of Brownian motion

- 1-d diffusion eq. ($\supset$ F.-P. eq.)

$$\partial_t P(x,t) = D\partial_x^2 P(x,t)$$

- $P(x,t)$=probability for particle to be at location x at time t.
- For particle starting at $x=0$ at $t=0$ solution is

$$P(x,t) = \frac{1}{\sqrt{4\pi Dt}} \exp\left\{-\frac{x^2}{4Dt}\right\}$$

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- Langevin equation:

$$\begin{aligned} x(t) &= \sqrt{2D}\eta(t) \\ \langle \eta(t)\eta(t')\rangle &= \delta(t-t') \end{aligned}$$

- $$\begin{aligned} \langle x(t)\rangle &= 0 \\ \langle x^2(t)\rangle &= 2Dt \end{aligned}$$

  $\Longrightarrow$ same as F.-P.
- Langevin also gives $t \neq t'$

$$\langle x(t)x(t')\rangle = 2D\min(t,t')$$

- Now $x \Longrightarrow U(\mathbf{x})$ and $t \Longrightarrow y$.
- $({N_c}^2-1)N_\perp^2$-dimensional nonlinear diffusion equation.
  ($N_\perp^2$ = number of lattice points in transverse plane.)

Langevin formulation

Fokker-Planck $\Longrightarrow$ Langevin in JIMWLK Blaizot, Iancu, Weigert 2002

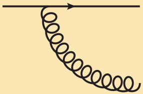
Original Langevin form: only right derivative ($\xi_{\mathbf{z}}^{b,i}$ is noise)

$$U_{\mathbf{x}}(y+\mathrm{d}y) = U_{\mathbf{x}}(y) \exp\left\{ i t^a \int_{\mathbf{z}} \varepsilon_{\mathbf{x},\mathbf{z}}^{ab,i} \xi_{\mathbf{z}}^{b,i} \sqrt{\mathrm{d}y} + \sigma_{\mathbf{x}}^a \mathrm{d}y \right\}.$$

Simpler, equivalent (for $\mathrm{d}y \to 0$) form T.L., H.M.

$$U_{\mathbf{x}}(y+\mathrm{d}y) = \exp\left\{ -i \frac{\sqrt{\alpha_s \mathrm{d}y}}{\pi} \int_{\mathbf{z}} \mathbf{K}_{\mathbf{x}-\mathbf{z}} \cdot (U_{\mathbf{z}} \xi_{\mathbf{z}} U_{\mathbf{z}}^\dagger) \right\} \times U_{\mathbf{x}}(y) \exp\left\{ i \frac{\sqrt{\alpha_s \mathrm{d}y}}{\pi} \int_{\mathbf{z}} \mathbf{K}_{\mathbf{x}-\mathbf{z}} \cdot \xi_{\mathbf{z}} \right\},$$

$$K_{\mathbf{x}-\mathbf{z}}^i = \frac{(\mathbf{x}-\mathbf{z})^i}{(\mathbf{x}-\mathbf{z})^2}$$



$i = x, y$

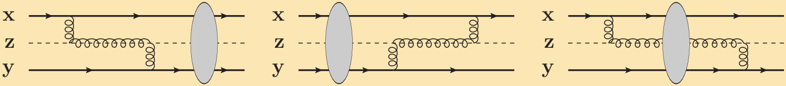
Fixed α_s noise: $\langle \xi_{\mathbf{x}}(y_m)_i^a \xi_{\mathbf{y}}(y_n)_j^b \rangle = \alpha_s \delta^{ab} \delta^{ij} \delta_{\mathbf{xy}}^{(2)} \delta_{mn}, \quad \xi = \xi^a t^a$

Multiply from left **and** right $\Longrightarrow$ remove deterministic term

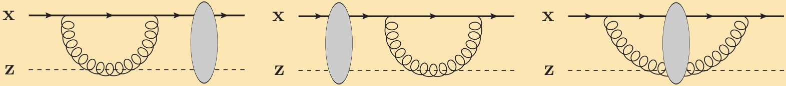
Interpreting JIMWLK: derive BK

$$U_{\mathbf{x}}(y + \mathrm{d}y) = e^{-i\frac{\sqrt{\alpha_s \mathrm{d}y}}{\pi}\int_{\mathbf{z}} \mathbf{K}_{\mathbf{x}-\mathbf{z}}\cdot(U_{\mathbf{z}}\xi_{\mathbf{z}}U^\dagger_{\mathbf{z}})} U_{\mathbf{x}} e^{i\frac{\sqrt{\alpha_s \mathrm{d}y}}{\pi}\int_{\mathbf{z}} \mathbf{K}_{\mathbf{x}-\mathbf{z}}\cdot\xi_{\mathbf{z}}},$$

- At $\mathrm{d}y \to 0$ develop to $\mathcal{O}(\xi^2)$ and take expectation values.
- BK Balitsky-Kovchegov is equation for **dipole** $\hat{D}_{\mathbf{x},\mathbf{y}} = \mathrm{Tr}\, U^\dagger(\mathbf{x})U(\mathbf{y})/N_c$
- Contract ξ's from timestep of $U^\dagger(\mathbf{x})$ with one from $U(\mathbf{y})$: real terms



- Contract two ξ's from timestep of $U^\dagger(\mathbf{x})$ or $U(\mathbf{y})$: virtual terms



- Result

$$\partial_y \hat{D}_{\mathbf{x},\mathbf{y}}(y) = \frac{\alpha_s N_c}{2\pi^2}\int_{\mathbf{z}} \left(\mathbf{K}^2_{\mathbf{x}-\mathbf{z}} + \mathbf{K}^2_{\mathbf{y}-\mathbf{z}} - 2\mathbf{K}_{\mathbf{x}-\mathbf{z}}\cdot\mathbf{K}_{\mathbf{y}-\mathbf{z}}\right)\left[\hat{D}_{\mathbf{x},\mathbf{z}}\hat{D}_{\mathbf{z},\mathbf{y}} - \hat{D}_{\mathbf{x},\mathbf{y}}\right].$$

Scale of running α_s in JIMWLK

BK for $\hat{D}_{\mathbf{x},\mathbf{y}}(y)$ describes dipole splitting $\mathbf{x}-\mathbf{y} \quad \longrightarrow \quad \mathbf{x}-\mathbf{z}\ ;\ \mathbf{z}-\mathbf{y}$

- α_s given by parent $\mathbf{x}-\mathbf{y}$: easy in BK, but funny in JIMWLK: Langevin is only for one Wilson line
- Daughter (scale in $\mathbf{K}$): easy to implement as $\sqrt{\alpha_s}$, but why?

$$\sqrt{\alpha_s}\mathbf{K}_{\mathbf{x}-\mathbf{z}} \to \sqrt{\alpha_s(\mathbf{x}-\mathbf{z})}\mathbf{K}_{\mathbf{x}-\mathbf{z}}$$

- Used in BK: combinations of these two.
- Suggestion T.L., H.Mäntysaari 2012 : natural scale is momentum of radiated gluon.
- Implemented by modifying momentum space noise correlator

$$\langle \xi_{\mathbf{x}}(m)_i^a \xi_{\mathbf{y}}(n)_j^b \rangle \sim \alpha_s \delta^{(2)}_{\mathbf{xy}} = \alpha_s \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}$$

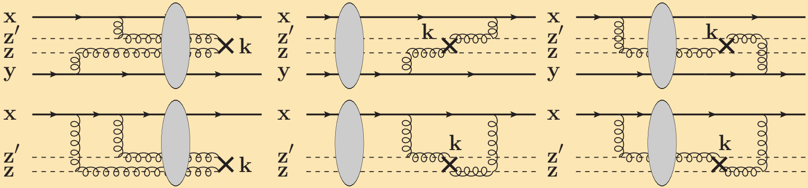
$$\Longrightarrow \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \alpha_s(\mathbf{k})$$

Reinterpreting JIMWLK

$$U_{\mathbf{x}}(y+\mathrm{d}y) = \exp\left\{-i\frac{\sqrt{\mathrm{d}y}}{\pi}\int_{\mathbf{z}} \mathbf{K}_{\mathbf{x}-\mathbf{z}}\cdot(U_{\mathbf{z}}\xi_{\mathbf{z}}U_{\mathbf{z}}^{\dagger})\right\}$$

$$\times\, U_{\mathbf{x}}(y)\ \exp\left\{i\frac{\sqrt{\mathrm{d}y}}{\pi}\int_{\mathbf{z}'} \mathbf{K}_{\mathbf{x}-\mathbf{z}'}\cdot\xi_{\mathbf{z}'}\right\},$$

$$\langle \xi_{\mathbf{x}}(m)_i^a \xi_{\mathbf{y}}(n)_j^b\rangle \sim \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\alpha_{\mathrm{s}}(\mathbf{k}) \equiv \widetilde{\alpha}_{\mathbf{x}-\mathbf{y}}$$



- Breaks time-reversal-symmetry: choose scale as momentum of gluon either before or after the target
- Two gluon coordinates instead of one

Recovering BK

- Equation for dipole now involves higher point functions:

$$\partial_y \hat{D} = \frac{N_c}{2\pi^2} \int_{\mathbf{u},\mathbf{v}} \widetilde{\alpha}_{\mathbf{u}-\mathbf{v}} \Big(\mathbf{K}_{\mathbf{x}-\mathbf{u}} \cdot \mathbf{K}_{\mathbf{x}-\mathbf{v}} + \mathbf{K}_{\mathbf{y}-\mathbf{u}} \cdot \mathbf{K}_{\mathbf{y}-\mathbf{v}} - 2\mathbf{K}_{\mathbf{x}-\mathbf{u}} \cdot \mathbf{K}_{\mathbf{y}-\mathbf{v}} \Big) \times \frac{1}{2} \left[\hat{D}_{\mathbf{x},\mathbf{u}} \hat{D}_{\mathbf{u},\mathbf{y}} + \hat{D}_{\mathbf{x},\mathbf{v}} \hat{D}_{\mathbf{v},\mathbf{y}} - \hat{D}_{\mathbf{x},\mathbf{y}} - \hat{D}_{\mathbf{v},\mathbf{u}} \hat{Q}_{\mathbf{x},\mathbf{v},\mathbf{u},\mathbf{y}} \right],$$

- But recall that α_s is a slowly varying function of the scale:

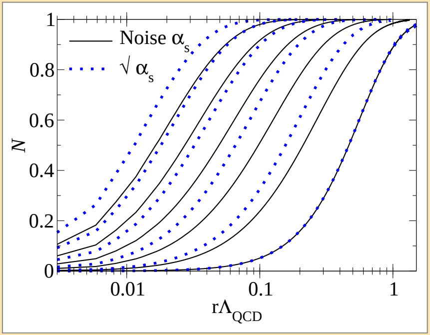
$$\widetilde{\alpha}_{\mathbf{x}-\mathbf{y}} \equiv \int \frac{d^2\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \alpha_s(\mathbf{k}) \sim \alpha_s \delta^2(\mathbf{x}-\mathbf{y})$$

$\Longrightarrow$ $\mathbf{u} \approx \mathbf{v}$ and structure simplifies to BK:

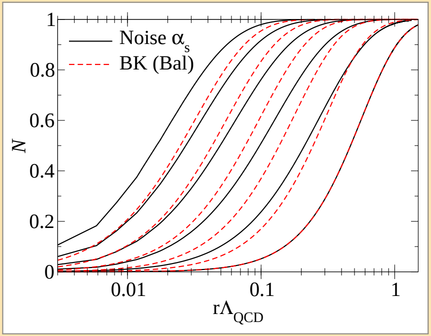
$$\frac{1}{2} \left[\hat{D}_{\mathbf{x},\mathbf{u}} \hat{D}_{\mathbf{u},\mathbf{y}} + \hat{D}_{\mathbf{x},\mathbf{v}} \hat{D}_{\mathbf{v},\mathbf{y}} - \hat{D}_{\mathbf{x},\mathbf{y}} - \hat{D}_{\mathbf{v},\mathbf{u}} \hat{Q}_{\mathbf{x},\mathbf{v},\mathbf{u},\mathbf{y}} \right] \approx \hat{D}_{\mathbf{x},\mathbf{u}} \hat{D}_{\mathbf{u},\mathbf{y}} - \hat{D}_{\mathbf{x},\mathbf{y}}$$

- Parametrically dominant length scale in coupling is “smallest dipole”, just like in Balitsky prescription for BK.

Comparison BK/JIMWLK



Evolution with our prescription is slower than with $\sqrt{\alpha_s}$.
This is good, data favors slower evolution

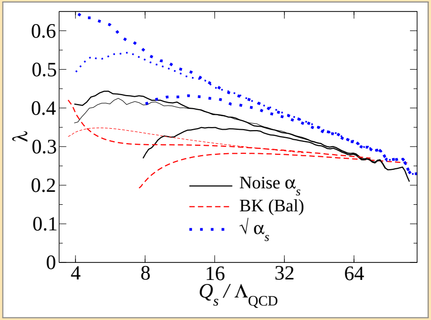


But this is still faster than with Balitsky prescription in BK
(Although parametricallly dominant scales are the same.)

Note: rcBK fits to HERA data need to take $\Lambda_{QCD} \approx$ 50MeV to make evolution slow enough.

Evolution speed

$$\lambda \equiv \frac{\mathrm{d}\ln Q_s^2}{\mathrm{d}y}$$

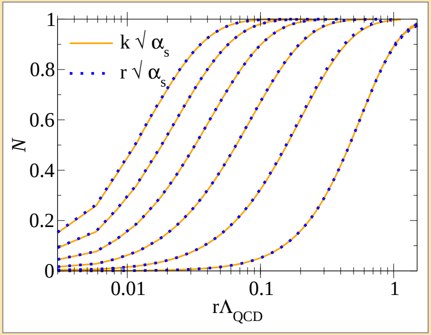


At very IR scales also dependence on how the Landau pole is regulated (different line shapes)

Side note: scale in coordinate vs momentum space

If running coupling depends only on scale in **K** ($\sqrt{\alpha_s}$-prescription), can use either coordinate or momentum space:

$$\sqrt{\alpha_s(\mathbf{x})}\frac{\mathbf{x}}{\mathbf{x}^2} \quad \text{vs.} \quad \sqrt{\alpha_s(\mathbf{k})}\frac{\mathbf{k}}{\mathbf{k}^2}$$

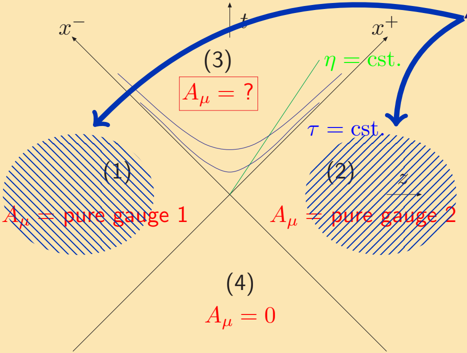


Numerically verified identification (for this kernel)

$$\ln\frac{\mathbf{k}^2}{\Lambda^2_{\text{QCD}}} \sim \ln\frac{4e^{-2\gamma_E}}{r^2\Lambda^2_{\text{QCD}}}$$

Gluon fields in AA collision

Classical **Y**ang-**M**ills



2 pure gauges

$$A^i_{(1,2)} = \frac{i}{g} U_{(1,2)}(\mathbf{x}) \partial_i U^\dagger_{(1,2)}(\mathbf{x})$$

$$U_{(1,2)}(\mathbf{x}) = P e^{ig \int dx^- \frac{\rho(\mathbf{x}, x^-)}{\nabla^2}}$$

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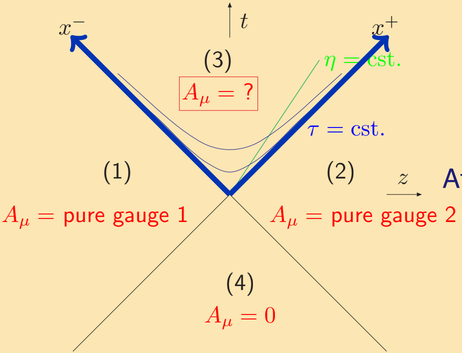

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At $\tau = 0$:

$$A^i\Big|_{\tau=0} = A^i_{(1)} + A^i_{(2)}$$

$$A^\eta\Big|_{\tau=0} = \frac{ig}{2}[A^i_{(1)}, A^i_{(2)}]$$

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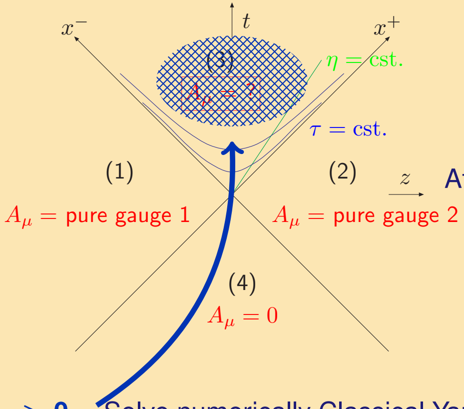

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$\tau > 0$ Solve numerically Classical Yang-Mills **CYM** equations.
This is the **glasma** field $\Longrightarrow$ Then average over ρ.

Gluons with $p \sim Q_s$ — strings of size $R \sim 1/Q_s$

Gluon spectrum in the glasma

T.L., *Phys.Lett.* **B703** (2011) 325 ; 1st calculation to actually use JIMWLK in CYM calculation

Q_s is only dominant scale

Parametrically gluon spectrum $$\frac{dN_g}{dy\, d^2\mathbf{x}\, d^2\mathbf{p}} = \frac{1}{\alpha_s} f\left(\frac{p}{Q_s}\right)$$

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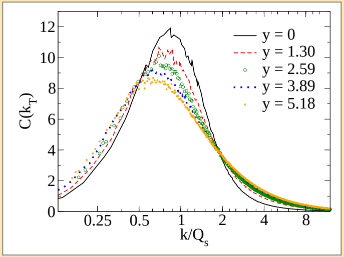

Unintegrated gluon distribution
$$C(\mathbf{k}) = \frac{k^2}{N_c} \mathrm{Tr} \left\langle U(\mathbf{k}) U^\dagger(\mathbf{k}) \right\rangle$$
becomes **harder** with evolution.

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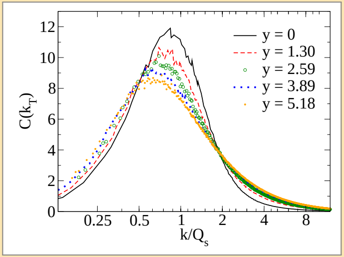

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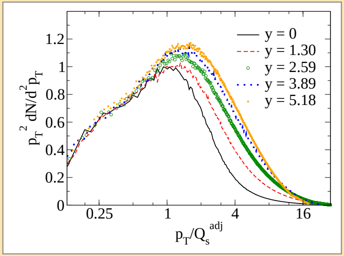

Produced gluon spectrum: harder at higher $\sqrt{s}$ (Here: midrapidity, $y \equiv \ln \sqrt{s/s_0}$)

Gluon multiplicity and mean p_T

Q_s is only dominant scale

Parametrically $$\frac{\mathrm{d}N_g}{\mathrm{d}y\,\mathrm{d}^2\mathbf{x}} = c_N \frac{C_F}{2\pi^2\alpha_s} Q_s^2 \qquad \langle p \rangle \sim Q_s$$

Note: in full CYM total gluon multiplicity is IR finite, no cutoff.

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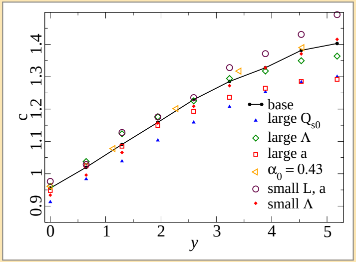

Scaled multiplicity increases with energy (Midrapidity, $y \equiv \ln \sqrt{s/s_0}$)

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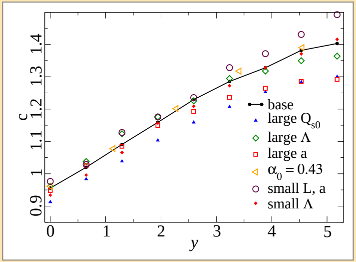
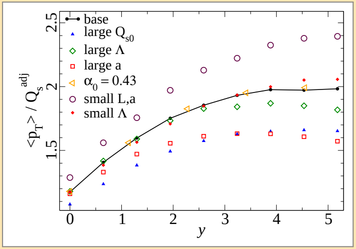




Scaled multiplicity increases with energy (Midrapidity, $y \equiv \ln\sqrt{s/s_0}$)

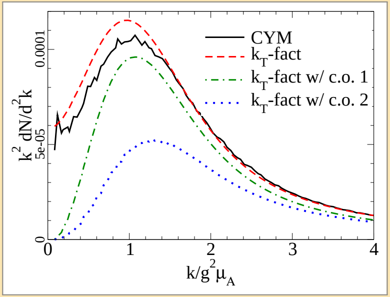
Harder gluon spectrum $\Longrightarrow$ higher $\langle p \rangle / Q_s$ as scaling regime sets in.

(Still very large lattice cutoff effects.)

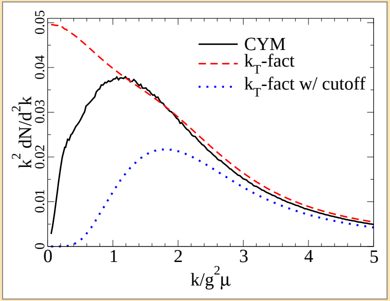
Side note: CYM vs. k_T-factorization

Blaizot, T.L., Mehtar-Tani 2010

$$\frac{dN}{d^2\mathbf{p}\,dy} = \frac{\#}{\alpha_s}\frac{1}{\mathbf{p}^2}\int_{\mathbf{k}}\left[\theta(p-k)\right]\phi_y(\mathbf{k})\phi_y(\mathbf{p}-\mathbf{k})$$



pA: k_T-factorization works



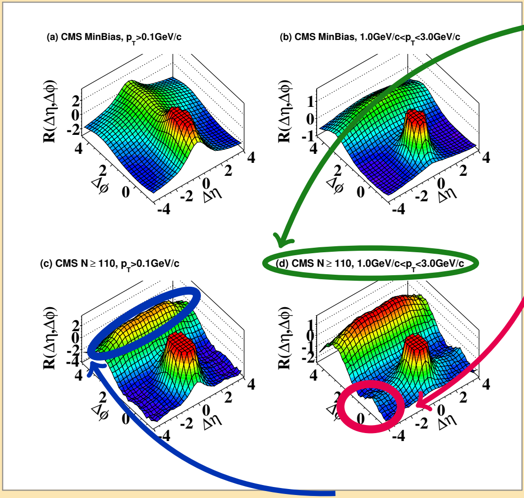
AA: no k_T-factorization

Not directly observable

Do not measure gluon spectrum with $\mathbf{p} \lesssim 1\text{GeV}$!
Centrality, rapidity, energy dependence from $N \sim S_\perp Q_s^2$

Suggested interpretation Levin, 2010 : Sudakov suppression factor.

Unequal rapidity: context



High N_{ch} trigger
$\Longrightarrow$ central events.

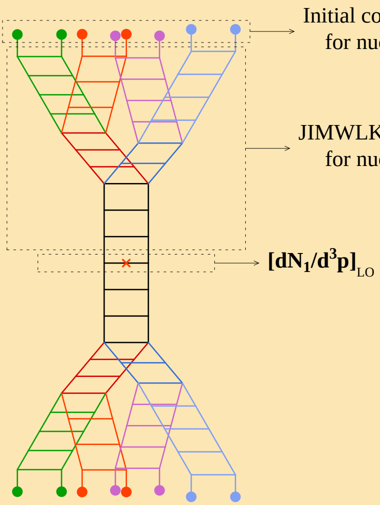
"Ridge" structure:

- $\Delta\phi \sim 0$, large $\Delta\eta$
- New at high $\sqrt{s}$
- Stronger in AA

Normal away-side ($\Delta\phi = \pi$) jet peak;

Unequal rapidity correlations in glasma

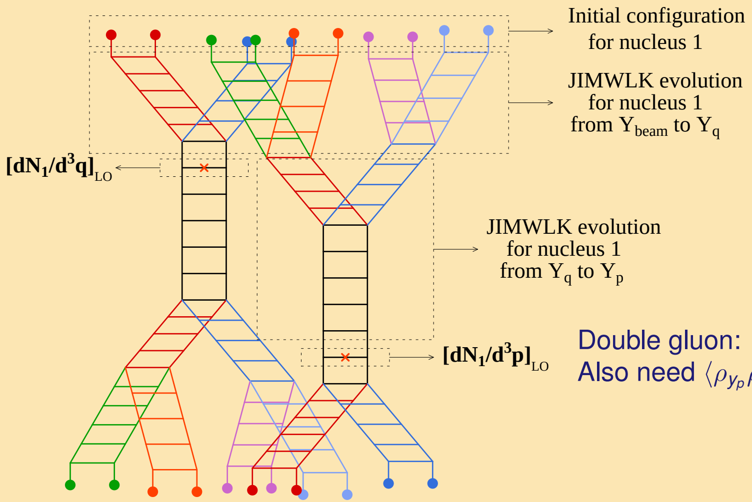
Single and double inclusive gluon production



Single gluon:
Need only $\langle \rho_y \rho_y \rangle$

Unequal rapidity correlations in glasma

Single and double inclusive gluon production



Double gluon:
Also need $\langle \rho_{y_p} \rho_{y_q} \rangle$

Unequal rapidity: ridge in AA or pp

CGC ridge phenomenology so far (Dumitru, Dusling, Gelis, Jalilian-Marian, T.L., Venugopalan, several papers) based on k_T-factorized approximation

$$C(\mathbf{p},\mathbf{q}) \sim \int_{\mathbf{k}} \Big\{ \overbrace{\Phi^2_{A_1}(y_p,\mathbf{k})\Phi_{A_2}(y_p,\mathbf{p}-\mathbf{k})}^{3 \text{ at } y_p} \overbrace{\Phi_{A_2}(y_q,\mathbf{q}+\mathbf{k})}^{1 \text{ at } y_q} + (\mathbf{k} \leftrightarrow -\mathbf{k}) + (A_1 \leftrightarrow A_2) \Big\}$$

Based on leading α_s, (i.e. no decorrelation in rapidity) , MV/Gaussian approximation:

$$\left\langle \hat{D}(\mathbf{k})_{y_p} \hat{D}(\mathbf{k})_{y_q} \right\rangle - \left\langle \hat{D}(\mathbf{k})_{y_p} \right\rangle \left\langle \hat{D}(\mathbf{k})_{y_q} \right\rangle \sim \frac{1}{{N_c}^2 - 1} \left\langle \hat{D}(\mathbf{k})_{y_{\min}} \right\rangle^2$$

Is this valid? Real evolution causes decorrelation; how fast?

Where did this come from?

Produce two gluons in two 2→1 processes; subtract uncorrelated

$$\Big\langle \rho_A(\mathbf{p}-\mathbf{k}, y_p)\rho_A(\mathbf{k}-\mathbf{p}, y_p)\rho_B(\mathbf{k}, y_p)\rho_B(-\mathbf{k}, y_p)$$

$$\times\ \rho_A(\mathbf{q}-\mathbf{k}', y_q)\rho_A(\mathbf{k}'-\mathbf{q}, y_q)\rho_B(\mathbf{k}', y_q)\rho_B(-\mathbf{k}', y_q)\Big\rangle$$

$$-\Big\langle \rho_A(\mathbf{p}-\mathbf{k}, y_p)\rho_A(\mathbf{k}-\mathbf{p}, y_p)\rho_B(\mathbf{k}, y_p)\rho_B(-\mathbf{k}, y_p)\Big\rangle$$

$$\times\Big\langle \rho_A(\mathbf{q}-\mathbf{k}', y_q)\rho_A(\mathbf{k}'-\mathbf{q}, y_q)\rho_B(\mathbf{k}', y_q)\rho_B(-\mathbf{k}', y_q)\Big\rangle$$

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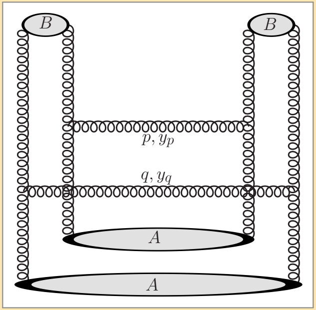
$$\times\Big\langle \rho_A(\mathbf{q}-\mathbf{k}',y_q)\rho_A(\mathbf{k}'-\mathbf{q},y_q)\rho_B(\mathbf{k}',y_q)\rho_B(-\mathbf{k}',y_q)\Big\rangle$$



Assumption: $\rho(y+\Delta y)=\rho(y)+\Delta\rho$ with $\langle(\Delta\rho)(\rho(y))\rangle = 0 \Longrightarrow$

$$\sim \quad \frac{1}{{N_c}^2-1}\langle\rho_A^2\rangle_{y_p}\langle\rho_A^2\rangle_{y_q}\left(\langle\rho_B^2\rangle_{y_p}\right)^2$$

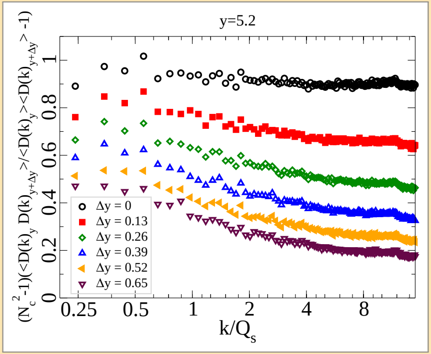
(When y_p is earlier in evolution of B, i.e. $y_p = y_{\rm min}$)

Then replace $\rho\rho$-correlator in linearized expression by dipole from BK.

Decorrelation in rapidity: preliminary results

Is this MV result valid in JIMWLK? Study numerically

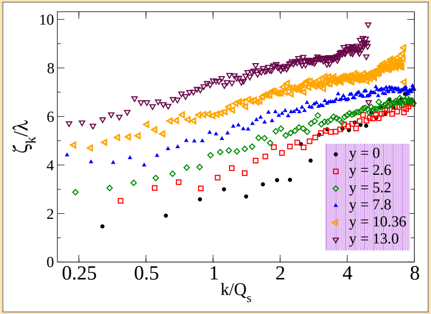
$$(N_c{}^2 - 1)\left[\frac{\left\langle \hat{D}(\mathbf{k})_y \hat{D}(\mathbf{k})_{y+\Delta y}\right\rangle}{\left\langle \hat{D}(\mathbf{k})_y\right\rangle \left\langle \hat{D}(\mathbf{k})_{y+\Delta y}\right\rangle} - 1\right]$$



Decorrelation speed vs. $\mathbf{k}$

$$\frac{\left\langle \hat{D}(\mathbf{k})_y \hat{D}(\mathbf{k})_{y+\Delta y} \right\rangle - \left\langle \hat{D}(\mathbf{k})_y \right\rangle \left\langle \hat{D}(\mathbf{k})_{y+\Delta y} \right\rangle}{\left\langle \hat{D}(\mathbf{k})_y \right\rangle \left\langle \hat{D}(\mathbf{k})_{y+\Delta y} \right\rangle} \sim \exp\{-\zeta_{\mathbf{k}} \Delta y\}$$

Compare decorrelation speed ζ to evolution speed $Q_s^2 \sim e^{-\lambda y}$:

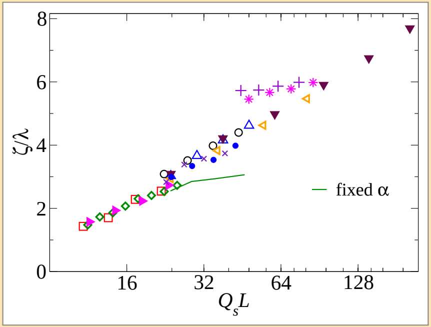


Decorrelation fast, and gets faster with evolution.

Decorrelation speed weighted with k

Weighted with same power of **k** as in double inclusive spectum
($\Phi(\mathbf{k}) \sim \mathbf{k}^2 D(\mathbf{k})$ is the correct unintegrated distribution)

$$\frac{\int d^2\mathbf{k}\mathbf{k}^4 \left[\left\langle \hat{D}(\mathbf{k})_y \hat{D}(\mathbf{k})_{y+\Delta y}\right\rangle - \left\langle \hat{D}(\mathbf{k})_y\right\rangle \left\langle \hat{D}(\mathbf{k})_{y+\Delta y}\right\rangle\right]}{\int d^2\mathbf{k}\mathbf{k}^4 \left\langle \hat{D}(\mathbf{k})_y\right\rangle \left\langle \hat{D}(\mathbf{k})_{y+\Delta y}\right\rangle} \sim \exp\{-\zeta \Delta y\}$$



Different symbols: different lattice size, rapidity step, initial Q_s, coupling freeze . . .
$\Longrightarrow$ Fast, and seems to be IR-unsafe (depends on system size L)

Conclusions

- JIMWLK equation is beginning to be actually applied
- Provides initial state for AA collision in CYM
- Running coupling, going towards NLO ... necessary for phenomenology
- Unequal rapidity correlations? See Dionysis' talk.