Di-gluon production at the LHC.

Julien Laidet

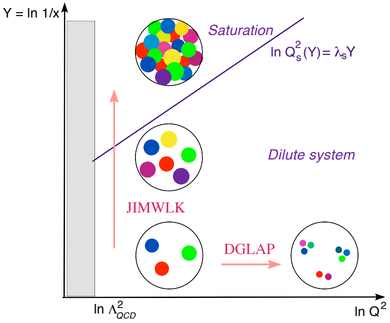
June 17th, 2013

Paper in collaboration with Edmond Iancu : arXiv:1305.5926 [hep-ph].

Plan

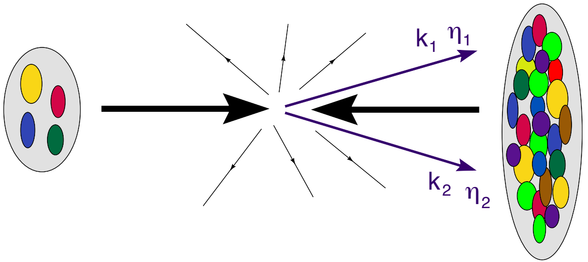
1. Why double gluon production ?
   - Why p-A ?
   - Why di-hadron correlations ?
   - Saturation and experiments : knowledge and expectation
2. General results for di-gluon production.
3. Hard transverse gluons
4. $q\bar{q}$ production.
5. Summary and outlook

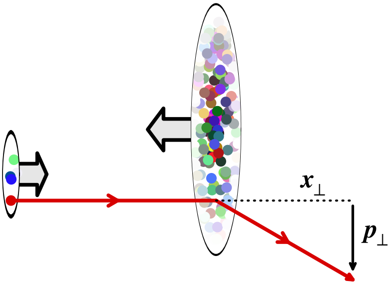
LHC $\longrightarrow$ saturated regime.



Good description provided by the Color Glass Condensate effective field theory.

Aim : clear probe of a saturated medium, the target, with a dilute projectile.



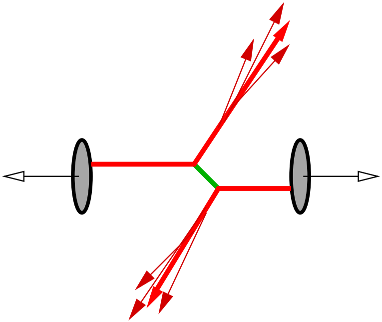


Single scattering :

$$x_1 = \frac{p_\perp}{\sqrt{s}} e^{Y} \sim 1 \qquad x_2 = \frac{p_\perp}{\sqrt{s}} e^{-Y} \ll 1$$

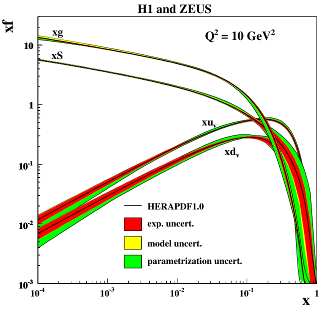
One has to look at forward rapidities ($Y > 0$).

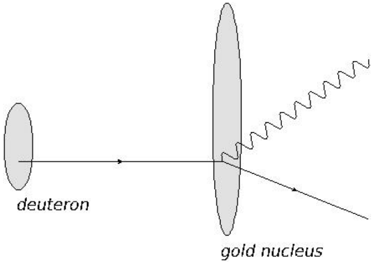
dilute-dilute collision :



dilute-dense collision : we expect that multiple scattering broaden the final state distribution in the transverse plane.

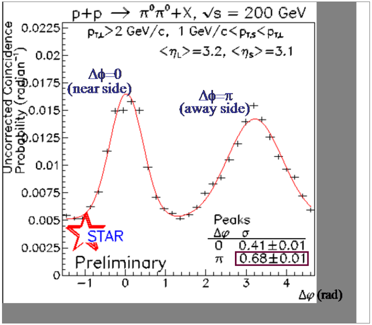
At RHIC, saturation has been marginally reached in d-Au collisions. Deuteron probed at $x_1 \sim 10^{-1}$.

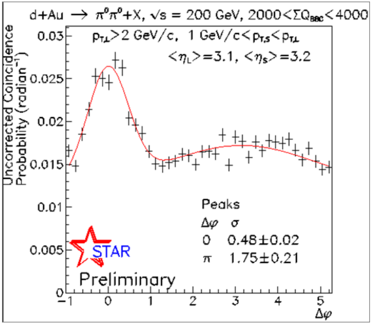




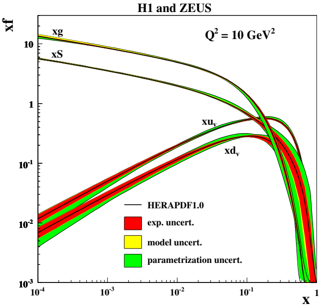
CGC-based predictions provide a good description for the $Aq \longrightarrow qgX$ inclusive cross-section (Albacete, Marquet, 2010 - Stasto, Xiao, Yuan, 2012).

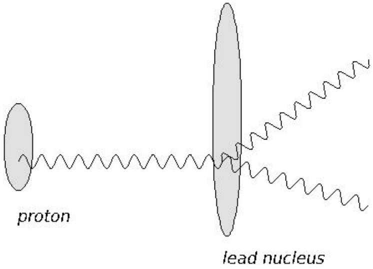
Evidences of saturation : pp vs. pA





At the LHC, the first p-Pb runs just occurred.





Proton probed at $x_1 \sim 10^{-2} - 10^{-3}$. Dominant processes : $Ag \to ggX$ and $Ag \to q\bar{q}X$.

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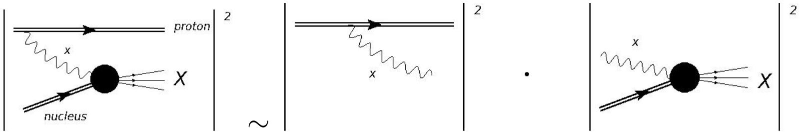
Proton = dilute medium → collinear factorization ∼ the gluon is an in-state.



$$\sigma(pA \to pX) \quad = \quad \int \mathrm{d}x G(x,\mu^2)\,\sigma(gA \to X)$$

Collinear factorization at the projectile (proton) level :

$$\frac{\mathrm{d}\sigma(pA \to ggX)}{\mathrm{d}y_1\,\mathrm{d}y_2\,\mathrm{d}^2k_{1,\perp}\,\mathrm{d}^2k_{2,\perp}} = \int \mathrm{d}x_1\, G(x_1,\mu^2)\frac{\mathrm{d}\sigma(gA \to ggX)}{\mathrm{d}y_1\,\mathrm{d}y_2\,\mathrm{d}^2k_{1,\perp}\,\mathrm{d}^2k_{2,\perp}}$$
$$= \frac{1}{256\pi^5(p^+)^2} x_1\, G(x_1,\mu^2)\left\langle \overline{|\mathcal{M}(g(p)A \to g(k_1)g(k_2))|^2}\right\rangle_Y .$$

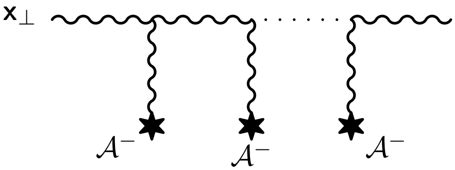
$\left\langle \overline{|\mathcal{M}|^2}\right\rangle_Y$ will be computed thank to the CGC effective theory.

Nucleus = dense medium = CGC.

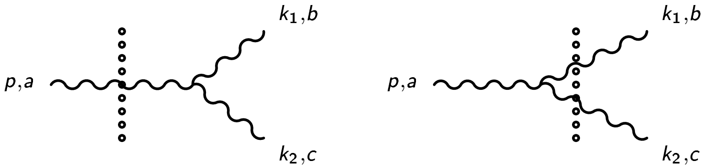
- Left-moving classical source $\mathcal{J}^-$.
- It is flat by Lorentz length contraction in the lab frame, i.e. $\mathcal{J}^-(x) \sim \delta(x^+)$. Referred to as a shockwave.
- The hadronic fluctuations are frozen during collision duration : $\mathcal{J}^-(x)$ does not depend on x^-.
- The classical source generates a single component classical field $\mathcal{A}^-$ that shares the two previous properties of $\mathcal{J}^-$ as well.

The interaction with the background field is encoded into Wilson lines (eikonal approximation) :

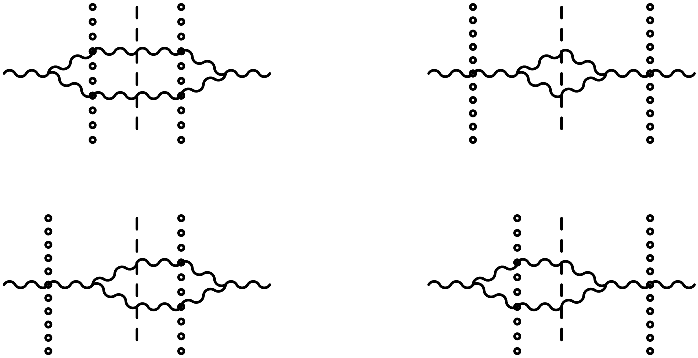
$$U(\mathbf{x}_\perp) = \mathcal{P} \exp \left[ig \int \mathrm{d}x^+ \mathcal{A}_a^-(x^+, \mathbf{x}_\perp) T^a \right].$$

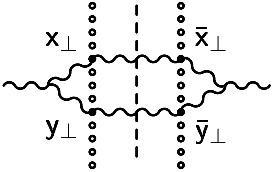


Two contributions to the amplitude :



$\overline{|\mathcal{M}|^2}$ receives 4 contributions :

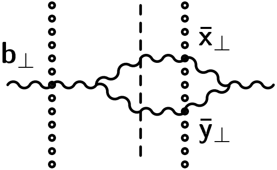




$$\sim \mathrm{Tr}\left[\tilde{U}(\mathbf{x}_\perp)\tilde{T}^a\tilde{U}^\dagger(\mathbf{y}_\perp)\tilde{U}(\bar{\mathbf{y}}_\perp)\tilde{T}^a\tilde{U}^\dagger(\bar{\mathbf{x}}_\perp)\right]$$

4 points intersecting the shockwave $\rightarrow$ four adjoint Wilson lines $\rightarrow$ quadrupole operator

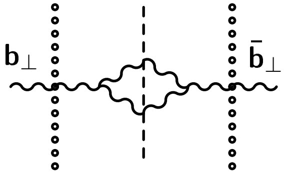
$$\tilde{S}^{(4)}(\mathbf{x}_\perp,\mathbf{y}_\perp,\bar{\mathbf{x}}_\perp,\bar{\mathbf{y}}_\perp) = \frac{1}{N_c(N_c^2-1)} f^{aef} f^{ae'f'}\,\tilde{U}_{be}(\mathbf{x}_\perp)\tilde{U}_{cf}(\mathbf{y}_\perp) \times \tilde{U}_{be'}(\bar{\mathbf{x}}_\perp)\tilde{U}_{cf'}(\bar{\mathbf{y}}_\perp).$$



$$\sim \tilde{U}_{ab}(\mathbf{b}_\perp)\mathrm{Tr}\left[\tilde{T}^a \tilde{U}(\bar{\mathbf{y}}_\perp)\tilde{T}^b \tilde{U}^\dagger(\bar{\mathbf{x}}_\perp)\right]$$

with $\mathbf{b}_\perp = z\mathbf{x}_\perp + (1-z)\mathbf{y}_\perp$.
3 points intersecting the shockwave $\rightarrow$ three adjoint Wilson lines

$$\tilde{S}^{(3)}(\mathbf{b}_\perp, \bar{\mathbf{x}}_\perp, \bar{\mathbf{y}}_\perp) = \frac{1}{N_c(N_c^2-1)} f^{dbc} f^{aef}\, \tilde{U}_{da}(\mathbf{b}_\perp)\tilde{U}_{be}(\bar{\mathbf{x}}_\perp)\tilde{U}_{cf}(\bar{\mathbf{y}}_\perp).$$



$$\sim \tilde{U}_{ac}\tilde{U}^\dagger_{cb}(\bar{\mathbf{b}}_\perp)(\mathbf{b}_\perp)\mathrm{Tr}\left[\tilde{T}^a\tilde{T}^b\right]$$

2 points intersecting the shockwave → two adjoint Wilson lines → dipole operator

$$\tilde{S}^{(2)}(\mathbf{b}_\perp,\bar{\mathbf{b}}_\perp) = \frac{1}{N_c(N_c^2-1)} f^{dbc} f^{d'bc}\tilde{U}_{da}(\mathbf{b}_\perp)\tilde{U}_{d'a}(\bar{\mathbf{b}}_\perp)$$
$$= \frac{1}{N_c^2-1}\mathrm{Tr}\left[\tilde{U}(\mathbf{b}_\perp)\tilde{U}^\dagger(\bar{\mathbf{b}}_\perp)\right].$$

The squared averaged $\mathcal{M}$-matrix for the $gA \to ggX$ process reads :

$$\left\langle \overline{|\mathcal{M}(g(p)A \to g(k_1)g(k_2))|^2} \right\rangle_Y = \frac{4g^2 N_c}{\pi^2} (p^+)^2 z(1-z) P_{g\leftarrow g}(z)$$
$$\times \int \mathrm{d}^2 x_\perp \mathrm{d}^2 y_\perp \mathrm{d}^2 \bar{x}_\perp \mathrm{d}^2 \bar{y}_\perp \frac{(\mathbf{x}_\perp - \mathbf{y}_\perp)\cdot(\bar{\mathbf{x}}_\perp - \bar{\mathbf{y}}_\perp)}{(\mathbf{x}_\perp - \mathbf{y}_\perp)^2(\bar{\mathbf{x}}_\perp - \bar{\mathbf{y}}_\perp)^2}$$
$$\times e^{-i\mathbf{k}_{1,\perp}\cdot(\mathbf{x}_\perp - \bar{\mathbf{x}}_\perp) - i\mathbf{k}_{2,\perp}\cdot(\mathbf{y}_\perp - \bar{\mathbf{y}}_\perp)} \left\langle \tilde{S}^{(2)}(\mathbf{b}_\perp, \bar{\mathbf{b}}_\perp) \right.$$
$$\left. - \tilde{S}^{(3)}(\mathbf{b}_\perp, \bar{\mathbf{x}}_\perp, \bar{\mathbf{y}}_\perp) - \tilde{S}^{(3)}(\bar{\mathbf{b}}_\perp, \mathbf{x}_\perp, \mathbf{y}_\perp) + \tilde{S}^{(4)}(\mathbf{x}_\perp, \mathbf{y}_\perp, \bar{\mathbf{x}}_\perp, \bar{\mathbf{y}}_\perp) \right\rangle_Y .$$

with

$$P_{g\leftarrow g}(z) = \frac{z}{1-z} + \frac{1-z}{z} + z(1-z).$$

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CGC predicts that partons within a dense hadron typically carry transverse momenta of order Q_s.

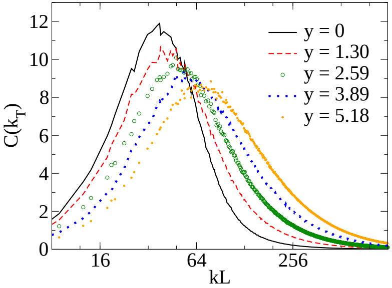


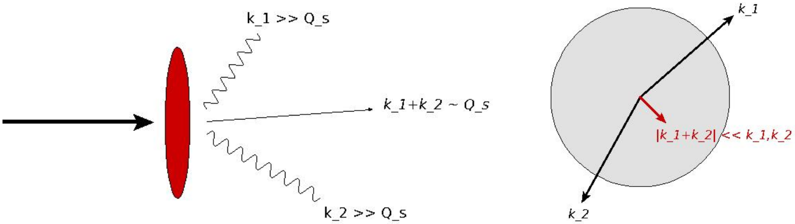
Figure: T. Lappi (numerical solution to JIMWLK)

Transverse momenta of the two final gluons :

- both very large w.r.t. Q_s $\longrightarrow$ hard process
- both smaller or of order Q_s $\longrightarrow$ semi-hard process

cannot get one $\gg Q_s$ and one $\lesssim Q_s$.

Hard regime :



- The transverse momentum distribution of the final gluons is centered around a relative angle $\Delta\phi = \pi$.
- One nevertheless has to consider multiple scatterings, i.e. non linear effects, since $|\mathbf{k}_{1,\perp} + \mathbf{k}_{2,\perp}| \sim Q_s$.

This is the back-to-back regime.

- The fine detail of the $\Delta\phi$ distribution is still sensitive to saturation.
- The back-to-back limit allows generalizations of unintegrated distribution functions in presence of non-linear effects $\rightarrow$ effective gluon distributions that take saturation into account.

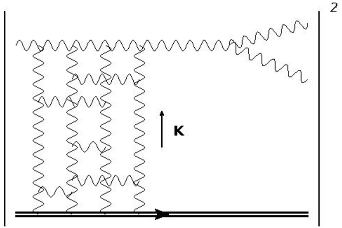
In the back-to-back limit, the squared averaged $\mathcal{M}$-matrix reads :

$$\left\langle |\mathcal{M}(g(p)A \to g(k_1)g(k_2))|^2 \right\rangle_Y = 16g^4 N_c^2 S_\perp \frac{(p^+)^2 z(1-z)}{((1-z)\mathbf{k}_{1,\perp} - z\mathbf{k}_{2,\perp})^4}$$
$$\times P_{g\leftarrow g}(z) \left[f_Y^{\mathrm{quad}}(\mathbf{k}_{1,\perp} + \mathbf{k}_{2,\perp}) - z(1-z) f_Y^{\mathrm{dip}}(\mathbf{k}_{1,\perp} + \mathbf{k}_{2,\perp}) \right].$$

f_Y^{dip} is the ordinary unintegrated distribution function associated to the dipole but including non linear effects :

$$S_\perp f_Y^{dip}(\mathbf{K}_\perp) \equiv \frac{\mathbf{K}_\perp^2}{g^2 N_c} \int \mathrm{d}^2 b_\perp \mathrm{d}^2 \bar{b}_\perp e^{-i\mathbf{K}_\perp \cdot (\mathbf{b}_\perp - \bar{\mathbf{b}}_\perp)} \times \left\langle \tilde{S}^{(2)}(\mathbf{b}_\perp, \bar{\mathbf{b}}_\perp) \right\rangle_Y .$$

This already appears in the single gluon production.



$f_Y^{\rm quad}$ is a new distribution function associated with the quadrupole :

$$S_\perp f_Y^{\rm quad}(\mathbf{K}_\perp) \equiv \frac{1}{g^2 N_c} \int {\rm d}^2 b_\perp {\rm d}^2 \bar{b}_\perp e^{-i\mathbf{K}_\perp \cdot (\mathbf{b}_\perp - \bar{\mathbf{b}}_\perp)} \times \left\langle \partial_x^i \partial_u^i \tilde{S}^{(4)}(\mathbf{x}_\perp, \mathbf{b}_\perp, \mathbf{u}_\perp, \bar{\mathbf{b}}_\perp) \Big|_{\mathbf{b}_\perp \mathbf{b}_\perp \bar{\mathbf{b}}_\perp \bar{\mathbf{b}}_\perp} \right\rangle_Y .$$

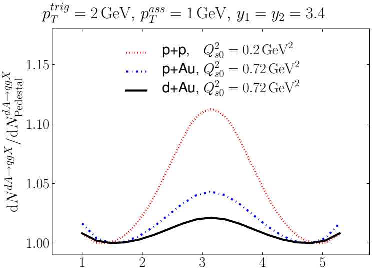
In the dilute limit = single gluon exchanged with the target = second order expansion of the Wilson lines in the background field. f_Y^{dip} and f_Y^{quad} both reduce to the unintegrated gluon distribution f_Y :

$$S_\perp f_Y(\mathbf{K}_\perp) = \mathbf{K}_\perp^2 \int \mathrm{d}b^+ \mathrm{d}\bar{b}^+ \mathrm{d}^2 b_\perp \mathrm{d}^2 \bar{b}_\perp e^{-i\mathbf{K}_\perp \cdot (\mathbf{b}_\perp - \bar{\mathbf{b}}_\perp)} \times \left\langle \mathcal{A}_a^-(\vec{b}) \mathcal{A}_a^-(\vec{\bar{b}}) \right\rangle_Y .$$

$k_\perp$-factorization recovered on the target side
$(k_{1,\perp} \simeq k_{2,\perp} \gg |\mathbf{k}_{1,\perp} + \mathbf{k}_{2,\perp}|)$:

$$\left\langle |\mathcal{M}(g(p)A \to g(k_1)g(k_2))|^2 \right\rangle_Y \to 16 g^4 N_c^2 S_\perp (p^+)^2 \left[1 - z(1-z)\right]^3 \frac{f_Y(\mathbf{k}_{1,\perp} + \mathbf{k}_{2,\perp})}{\mathbf{k}_{1,\perp}^4}.$$

The semi-hard regime : we expect a broadening of the $\Delta\phi = \pi$ peak as the momenta of final gluons approach Q_s. This has been shown by T. Lappi for qg production.



We have the master formula, the gg case is right now a matter of programming.

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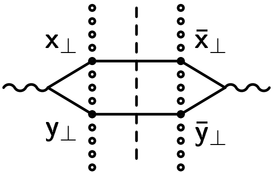
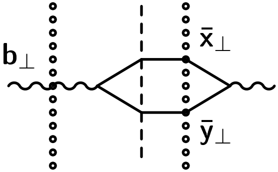


4 fermions intersecting the shockwave → four fundamental Wilson lines → quadrupole operator

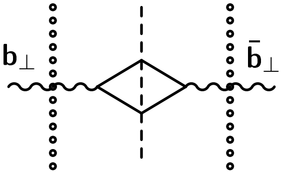
$$S^{(4)}(\mathbf{x}_\perp, \mathbf{y}_\perp, \bar{\mathbf{y}}_\perp, \bar{\mathbf{x}}_\perp) = \frac{2}{N_c^2 - 1} \mathrm{tr}\left[U(\mathbf{x}_\perp) T^a U^\dagger(\mathbf{y}_\perp) U(\bar{\mathbf{y}}_\perp) T^a U^\dagger(\bar{\mathbf{x}}_\perp)\right].$$



2 fermions and 1 gluon intersecting the shockwave → two fundamental and one adjoint Wilson lines

$$S^{(3)}(\mathbf{b}_\perp, \bar{\mathbf{y}}_\perp, \bar{\mathbf{x}}_\perp) = \frac{2}{N_c^2 - 1} \tilde{U}_{ab}(\mathbf{b}_\perp) \mathrm{tr}\left[T^a U(\bar{\mathbf{y}}_\perp) T^b U^\dagger(\bar{\mathbf{x}}_\perp) \right].$$

with $\mathbf{b}_\perp = z\mathbf{x}_\perp + (1-z)\mathbf{y}_\perp$.



2 gluons intersecting the shockwave → two adjoint Wilson lines → dipole operator

$$S^{(2)}(\mathbf{b}_\perp, \bar{\mathbf{b}}_\perp) = \frac{2}{N_c^2 - 1} \tilde{U}_{ac}(\mathbf{b}_\perp) \tilde{U}_{bc}(\bar{\mathbf{b}}_\perp) \left[T^a T^b \right]$$
$$= \frac{1}{N_c^2 - 1} \mathrm{Tr}\left[\tilde{U}(\mathbf{b}_\perp) \tilde{U}^\dagger(\bar{\mathbf{b}}_\perp) \right].$$

The squared averaged $\mathcal{M}$-matrix for the $gA \to q\bar{q}X$ process reads :

$$\left\langle \overline{|\mathcal{M}(g(p)A \to q(k_1)\bar{q}(k_2))|^2} \right\rangle_Y = \frac{g^2}{\pi^2}(p^+)^2 z(1-z)P_{q\leftarrow g}(z)$$
$$\times \int \mathrm{d}^2x_\perp \mathrm{d}^2y_\perp \mathrm{d}^2\bar{x}_\perp \mathrm{d}^2\bar{y}_\perp \frac{(\mathbf{x}_\perp - \mathbf{y}_\perp)\cdot(\bar{\mathbf{x}}_\perp - \bar{\mathbf{y}}_\perp)}{(\mathbf{x}_\perp - \mathbf{y}_\perp)^2(\bar{\mathbf{x}}_\perp - \bar{\mathbf{y}}_\perp)^2}$$
$$\times e^{-i\mathbf{k}_{1,\perp}\cdot(\mathbf{x}_\perp - \bar{\mathbf{x}}_\perp) - i\mathbf{k}_{2,\perp}\cdot(\mathbf{y}_\perp - \bar{\mathbf{y}}_\perp)} \Big\langle S^{(2)}(\mathbf{b}_\perp, \bar{\mathbf{b}}_\perp)$$
$$-S^{(3)}(\mathbf{b}_\perp, \bar{\mathbf{y}}_\perp, \bar{\mathbf{x}}_\perp) - S^{(3)}(\bar{\mathbf{b}}_\perp, \mathbf{x}_\perp, \mathbf{y}_\perp) + S^{(4)}(\mathbf{x}_\perp, \mathbf{y}_\perp, \bar{\mathbf{y}}_\perp, \bar{\mathbf{x}}_\perp) \Big\rangle_Y .$$

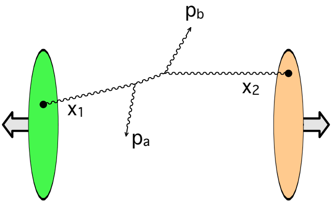
with

$$P_{q\leftarrow g}(z) = z^2 + (1-z)^2.$$

Plan

- We obtained a general formula for the di-gluon (and $q\bar{q}$) production cross-section that stands for a large range of kinematics encountered at the LHC for finite N_c.
- The back-to-back limit is interesting in the sense it is expected for hard processes : hard transverse gluons are strongly correlated up to $\sim Q_s$ momentum imbalance that is very small w.r.t their intrinsic transverse momentum. We now have a quantitative description of this regime.
- The semi-hard regime is the signature of saturation but its quantitative predictions requires numerical devices for computing the averages of color operators (mean field approximation to JIMWLK : Iancu, Triantafyllopoulos - 2011).

Thanks for paying attention.



$$x_1 = \frac{p_{a,\perp}}{\sqrt{s}} e^{y_1} + \frac{p_{b,\perp}}{\sqrt{s}} e^{y_2}$$

$$x_2 = \frac{p_{a,\perp}}{\sqrt{s}} e^{-y_1} + \frac{p_{b,\perp}}{\sqrt{s}} e^{-y_2}.$$

The Color Glass Condensate :

- Effective field theory
- Semi-hard gluons = gluons with $x' \gg x$ considered = classical field $\mathcal{A}$ radiated by classical sources ρ
- Source configuration ρ randomly frozen during the process
- CGC weight function $\mathcal{W}_Y[\rho]$ = probability of occurrence of a given source configuration
- Independance of physical observables on Y $\rightarrow$ renormalization group equation known as the JIMWLK equation (Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner; 1997-2000) :

$$\frac{\partial \mathcal{W}_Y}{\partial Y} = \mathcal{H}_{\mathrm{JIMWLK}} \mathcal{W}_Y$$

CGC requires to perform averages over the classical field weighted with the CGC weight function $W_Y[\rho]$ for observables :

$$\langle \mathcal{O} \rangle_Y = \int \mathcal{D}[\rho]\ \mathcal{W}_Y[\rho]\mathcal{O}[\rho].$$

- $G(x;\mu^2)$ is the integrated gluon distribution
- it represents the (average) number of gluons of momentum fraction x in the proton

$$G(x;\mu^2) = \frac{\mathrm{d}N_g}{\mathrm{d}x}$$

- it is the integral over transverse momenta of the number of gluons per phase space volume element up to the scale $|\mathbf{p}_\perp| < \mu \ll p^+$.

$$xG(x,\mu^2) = \int^{\mu^2} \mathrm{d}^2p_\perp \frac{\mathrm{d}N_g}{\mathrm{d}Y\mathrm{d}^2p_\perp} = \frac{2(N_c^2-1)S_\perp}{(2\pi)^3}\int^{\mu^2}\mathrm{d}^2p_\perp f_Y(\mathbf{p}_\perp).$$

$f_Y(\mathbf{p}_\perp)$ is the unintegrated gluon distribution