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# Selected, recent results in high energy factorization

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Supported by grant: LIDER/02/35/L-2/10/NCBiR/2011

763/N-COPIN-ECT*/2010/0

Based on work with:
Krzysztof Bożek, Andreas van Hameren, Piotr Kotko
Wieslaw Placzek, Dawid Toton, Magda Slawinska

# QCD at high energies – high energy factorization

$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \sum_{a,b,c,d} \int \frac{d^2k_{1t}}{\pi}\frac{d^2k_{2t}}{\pi}\frac{1}{16\pi^2(x_1x_2S)^2}\overline{|\mathcal{M}_{ab\to cd}|}^2 \delta^2(\vec{k}_{1t}+\vec{k}_{2t}-\vec{p}_{1t}-\vec{p}_{2t})$$

$$\times\, \mathcal{F}_{a/A}(x_1, k_{1t}^2, \mu^2)\, \mathcal{F}_{b/B}(x_2, k_{2t}^2, \mu^2)\, \frac{1}{1+\delta_{cd}}$$

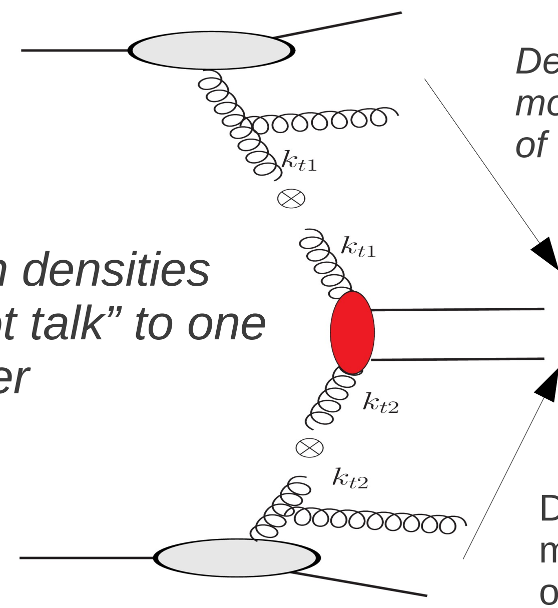
Unintegrated gluon density

$$k_1^\mu = x_1 P_1^\mu + \bar{x}_1 P_2^\mu + k_{1t}^\mu \qquad k_2^\mu = x_2 P_2^\mu + \bar{x}_2 P_1^\mu + k_{2t}^\mu$$

$$\bar{x}_1 = \frac{k_1^2 + \mathbf{k}^2}{S x_1} \qquad \bar{x}_2 = \frac{k_2^2 + \mathbf{k}^2}{S x_2}$$

$$|\mathcal{M}_{ab\to cd}|^2 = \frac{2x_1 k_1^{\mu_1} k_1^{\nu_1}}{k_1^2}\frac{2x_2 k_2^{\mu_2} k_2^{\nu_2}}{k_2^2}\mathcal{M}_{ab\to cd\,\mu_1\nu_1}\mathcal{M}^*_{ab\to cd\,\mu_2\nu_2}$$

*Parton densities "do not talk" to one another*




*Decreasing longitudianl momentum fractions of off-shell partons*

Decreasing longitudianl momentum fractions of off-shell partons

Gribov, Levin, Ryskin '81
Ciafaloni, Catani, Hautman '93

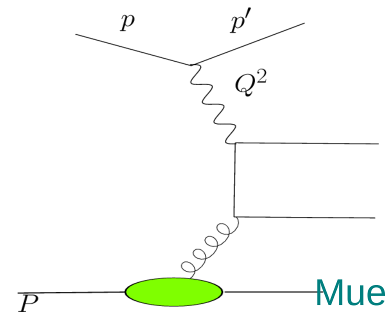
Originally derived for quarks in final state.
Lipatov provided general framework.

Recently new approach consistent with Lipatov's action allowed for formulation and numerical calculation of any tree level amplitude with off-shell gluons in initial state
Van Hameren, Kotko, KK '12

Attempts to generalize to p-A.
Dominguez, Huan, Marquet,Xiao '10

# Gluon density – practical definition

$$F_2(x,Q^2)=\frac{Q^2}{4\pi^2}\alpha_s\sum_q e_q^2\int d^2k\,\mathcal{F}(x,k^2)\left(S_L(k^2,Q^2,m_q^2)+S_T(k^2,Q^2,m_q^2)\right)$$

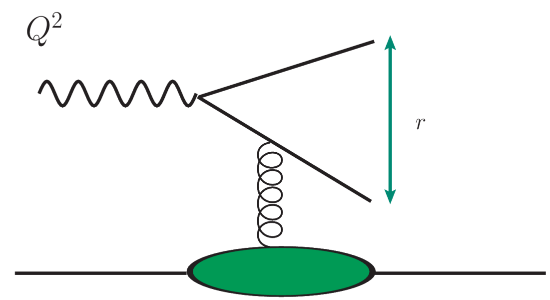



Mueller, Patel '95

Enters also into inclusive gluon production in adjoint representation recently called dipole gluon density

Impact factors from Feynman diagrams in momentum space

$$F_2(x,Q^2)=\frac{Q^2}{4\pi^2\alpha_{em}}\int d^2b\int_0^1 dz\int d^2r\left(|\psi_L(z,r)|^2+|\psi_T(z,r)|^2\right)N(x,r,b)$$

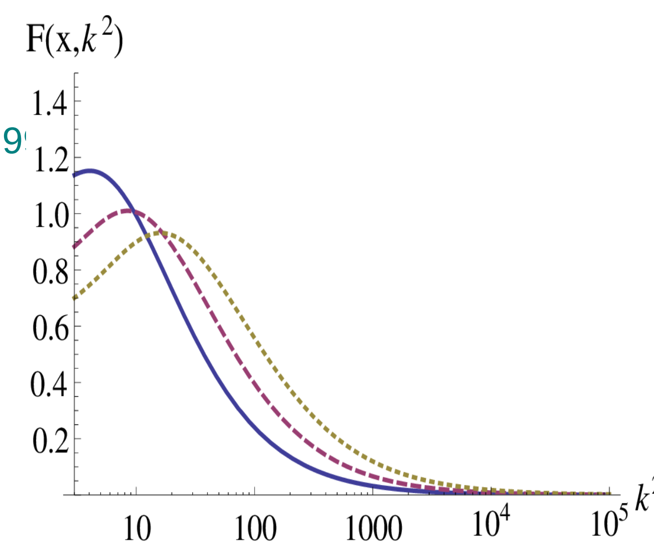



In the dipole formalism

$$\mathcal{F}(x,k^2)=\frac{N_c}{\alpha_s(2\pi)^3}\int d^2b\int d^2r\,e^{ik\cdot r}\nabla_r^2\,N(r,b,x)$$

Balitsky 96, Kovchegov 9

$$\mathcal{F}(x,k^2)=\mathcal{F}_0(x,k^2)+\overline{\alpha}_s\int_{x/x_0}^1\frac{dz}{z}\int_0^\infty\frac{dl^2}{l^2}\left[\frac{l^2\mathcal{F}(x/z,l^2)-k^2\mathcal{F}(x/z,k^2)}{|k^2-l^2|}+\frac{k^2\mathcal{F}(x/z,k^2)}{\sqrt{(4l^4+k^4)}}\right]$$
$$-\frac{2\alpha_s^2\pi}{N_cR^2}\int_{x/x_0}^1\frac{dz}{z}\left\{\left[\int_{k^2}^\infty\frac{dl^2}{l^2}\mathcal{F}(x/z,l^2)\right]^2+\mathcal{F}(x/z,k^2)\int_{k^2}^\infty\frac{dl^2}{l^2}\ln\left(\frac{l^2}{k^2}\right)\mathcal{F}(x/z,l^2)\right\}$$




K.K, Kwiecinski '2003
Nikolaev, Schaffer '04

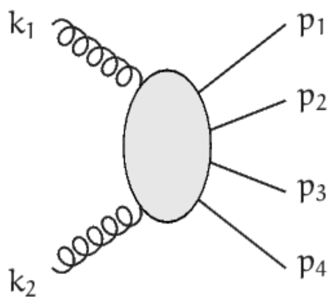
# High Energy Factorization - matrix elements

- General theory given by Lipatov effective action (Lipatov' 95; Antonov, Cherednikov, Kuraev,,Lipatov '05).
- Mainly analitical results. (Braun; Chachamis, Hentschinski, Madrigal, Sabio-Vera,)
- So far there is no numerical tool which generates matrix elements directly from the effective action.
- For collinear factorization there are: HELAC, Amagic++,AlpGen, MadGraph,...
- New framework which is equivalent to Lipatov effective action and which makes use of existing tools for evaluation of matrix elements

Van Hameren, Kotko, KK JHEP 1301 (2013) 078, Van Hameren, Kotko, KK JHEP 1212 (2012) 029 .

# Kinematics of High Energy Factorization

*Van Hameren, Kotko, KK JHEP 1301 (2013) 078*



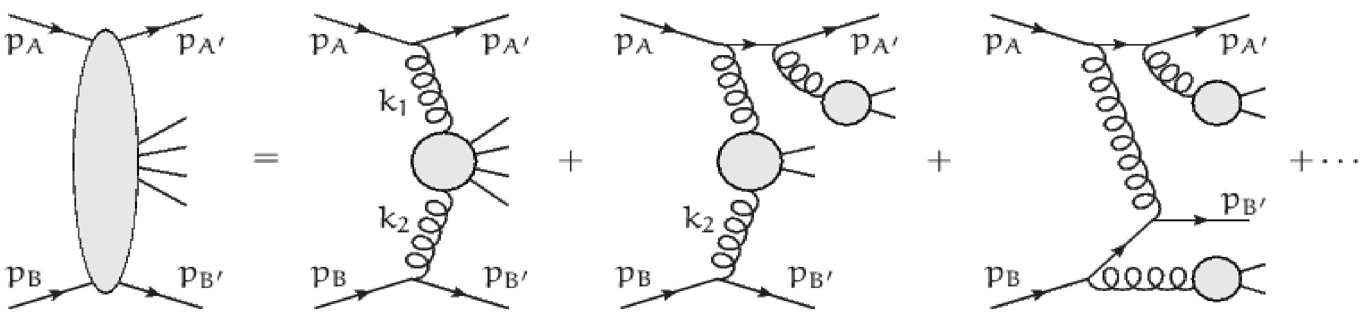

$$k_1 + k_2 = p_1 + p_2 + p_3 + p_4$$

$$k_1 = x_1 P_A + k_{\perp 1} \qquad k_2 = x_2 P_B + k_{\perp 2}$$

$$P_A \cdot k_{\perp 1} = P_A \cdot k_{\perp 2} = P_B \cdot k_{\perp 1} = P_B \cdot k_{\perp 2} = 0$$

$$P_A^2 = P_B^2 = 0$$

$$k_1^2 = k_{\perp 1}^2 \qquad k_2^2 = k_{\perp 2}^2$$




$$\ell_1 = (E, 0, 0, E) \qquad \ell_2 = (E, 0, 0, -E)$$

$$p_A - p_{A'} = k_1 = x_1 \ell_1 + k_{1\perp} + y_2 \ell_2 \qquad p_B - p_{B'} = k_2 = x_2 \ell_2 + k_{2\perp} + y_1 \ell_1$$

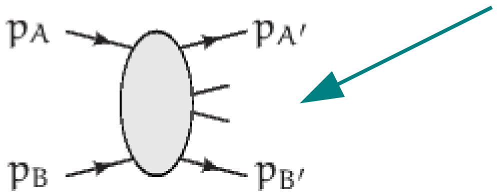
Needed to keep quarks on shell. Usually neglected

# Towards automation of High Energy Factorization

*Van Hameren, Kotko, KK*
*JHEP 1301 (2013) 078*

Let us consider $\mathcal{A}(g^* g^* \to X)$

Must be gauge invariant




$$p_A = (\Lambda + x_1)\ell_1 + \kappa_{13}\ell_3$$
$$p_{A'} = \Lambda\ell_1 - \kappa_{14}\ell_4$$
$$p_A - p_{A'} = x_1\ell_1 + k_{1\perp}$$

$$p_B = (\Lambda + x_2)\ell_2 + \kappa_{24}\ell_4$$
$$p_{B'} = \Lambda\ell_2 - \kappa_{23}\ell_3$$
$$p_B - p_{B'} = x_2\ell_2 + k_{2\perp}$$

Introduce complex $p_A$, $p_B$, $p_{A'}$ $p_{B'}$

$$\ell_3^\mu = \tfrac{1}{2}\langle \ell_2 - | \gamma^\mu | \ell_1 - \rangle \qquad \ell_4^\mu = \tfrac{1}{2}\langle \ell_1 - | \gamma^\mu | \ell_2 - \rangle$$

$$p_A^\mu = (\Lambda + x_1)\ell_1^\mu - \frac{\ell_4 \cdot k_{1\perp}}{\ell_1 \cdot \ell_2}\ell_3^\mu \qquad p_{A'}^\mu = \Lambda\ell_1^\mu + \frac{\ell_3 \cdot k_{1\perp}}{\ell_1 \cdot \ell_2}\ell_4^\mu$$

$$p_B^\mu = (\Lambda + x_2)\ell_2^\mu - \frac{\ell_3 \cdot k_{2\perp}}{\ell_1 \cdot \ell_2}\ell_4^\mu \qquad p_{B'}^\mu = \Lambda\ell_2^\mu + \frac{\ell_4 \cdot k_{2\perp}}{\ell_1 \cdot \ell_2}\ell_3^\mu$$

$$p_A^\mu - p_{A'}^\mu = x_1\ell_1^\mu + k_{1\perp}^\mu \qquad p_B^\mu - p_{B'}^\mu = x_2\ell_2^\mu + k_{2\perp}^\mu$$

$$p_A^2 = p_{A'}^2 = p_B^2 = p_{B'}^2 = 0$$

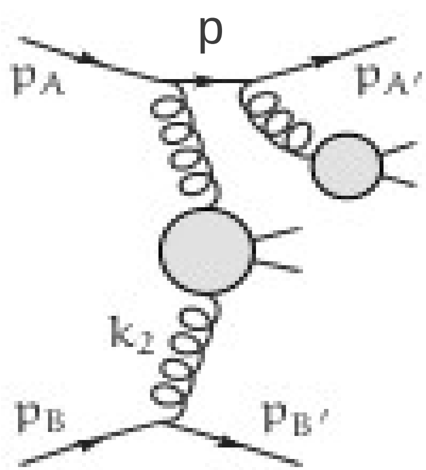
For a given process amplitude is evaluated numerically.

# Extraction of physical amplitude

External spinors

$$|p_A] = \frac{\sqrt{\Lambda + x_1} + \kappa_{13}}{\sqrt{|\sqrt{\Lambda + x_1} + \kappa_{13}|}} |\ell_1] \qquad \langle p_{B'}| = \sqrt{|\sqrt{\Lambda} - \kappa_{23}|}\, \langle \ell_2|$$

$$|p_B] = \frac{\sqrt{\Lambda + x_2} + \kappa_{24}}{\sqrt{|\sqrt{\Lambda + x_2} + \kappa_{24}|}} |\ell_2] \qquad \langle p_{A'}| = \sqrt{|\sqrt{\Lambda} - \kappa_{14}|}\, \langle \ell_1|$$




For A- quark line propagator

$$\frac{\not p}{p^2} = \frac{(\Lambda + x)\not\ell_1 + y\not\ell_2 + \not p_\perp}{2(\Lambda + x) y\, \ell_1 \cdot \ell_2 + p_\perp^2} \xrightarrow{\Lambda \to \infty} \frac{\not\ell_1}{2y\, \ell_1 \cdot \ell_2} = \frac{\not\ell_1}{2\, \ell_1 \cdot p}$$

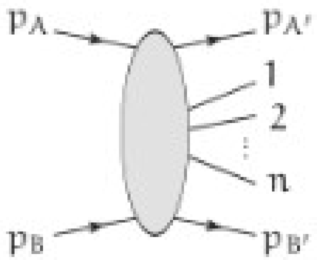
Gluons attach via eikonal coupling

$$\langle \ell_1 | \gamma^{\mu_1} \not\ell_1 \gamma^{\mu_2} \not\ell_1 \cdots |\ell_1] = \langle \ell_1 | \gamma^{\mu_1} |\ell_1] \langle \ell_1 | \gamma^{\mu_2} |\ell_1] \langle \ell_1 | \cdots |\ell_1]$$
$$= (2\ell_1^{\mu_1})(2\ell_1^{\mu_2}) \cdots \, .$$

g*g* → g, g*g* →gg, g*g →gg,.. agree with RR → g, RR →gg, Rg →gg from Lipatov action

# Prescription to get amplitude with off-shell gluons

1. Consider the process $q_A\, q_B \to q_A\, q_B\, X$, where $q_A, q_B$ are distinguishable massless quarks not occurring in X, and with momentum flow as if the momenta $p_A, p_B$ of the initial-state quarks and $p_{A'}, p_{B'}$ of the final-state quarks are given by




$$p_A^\mu = k_1^\mu \quad , \quad p_B^\mu = k_2^\mu \quad , \quad p_{A'}^\mu = p_{B'}^\mu = 0$$

2. Associate the number 1 instead of spinors with the end points of the A-quark line, interpret every vertex on the A-quark line as $g_s T_{ij}^a \ell_1^\mu$ instead of $-ig_s T_{ij}^a \gamma^\mu$, interpret every propagator on the A-quark line as $\delta_{ij}/\ell_1 \cdot p$ instead of $i\delta_{ij}/\not{p}$.

3. Associate the number 1 instead of spinors with the end points of the B-quark line, interpret every vertex on the B-quark line as $g_s T_{ij}^a \ell_2^\mu$ instead of $-ig_s T_{ij}^a \gamma^\mu$, interpret every propagator on the B-quark line as $\delta_{ij}/\ell_2 \cdot p$ instead of $i\delta_{ij}/\not{p}$.

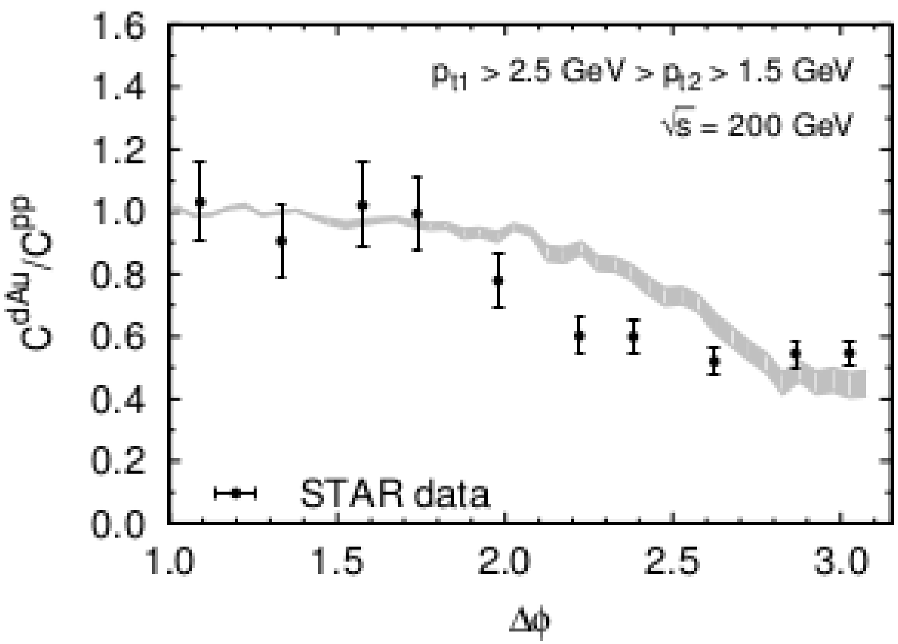
4. Multiply the amplitude with $F = \dfrac{i\, x_1 \sqrt{-2k_{1\perp}^2}}{g_s} \times \dfrac{i\, x_2 \sqrt{-2k_{2\perp}^2}}{g_s}$ .

5. For the rest normal Feynman rules apply.

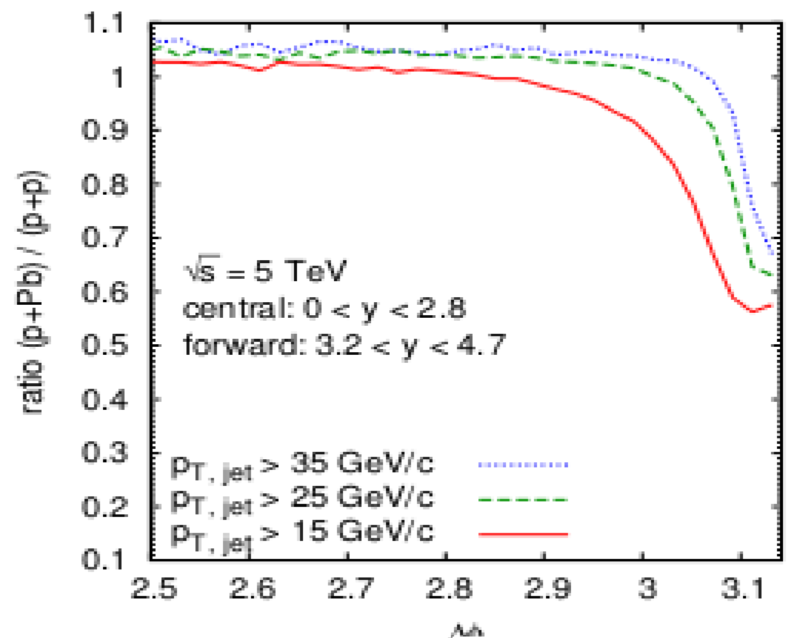
In agreement with Lipatov's effective action.

# Application to some hadronic observables

*Sapeta, KK '12*

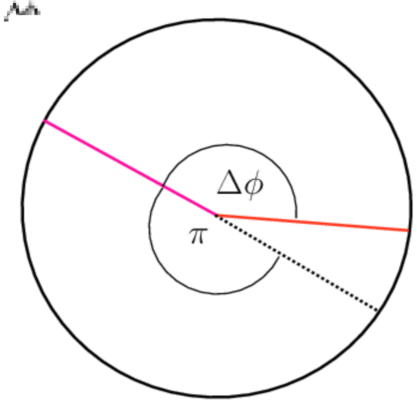



*Sapeta, KK '13*




Framework works well for di-hadron production at RHIC

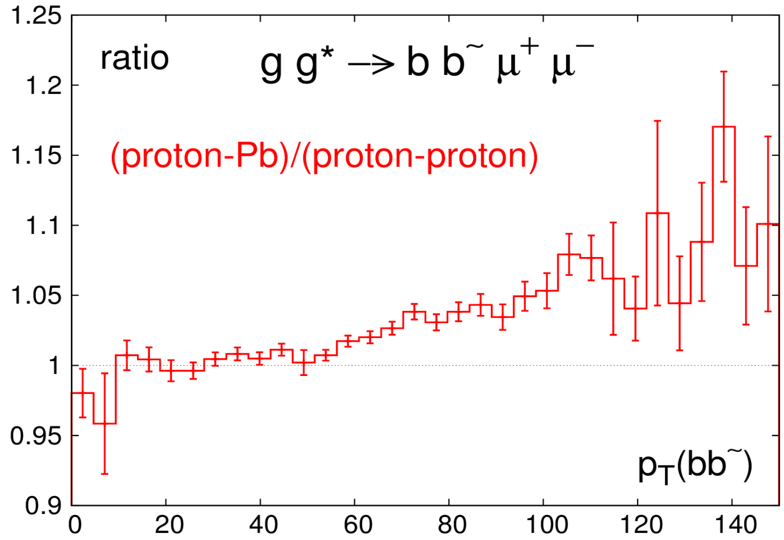
also *Albacete, Marquet '10, Juan,Stasto,Xiao '11 ,...*

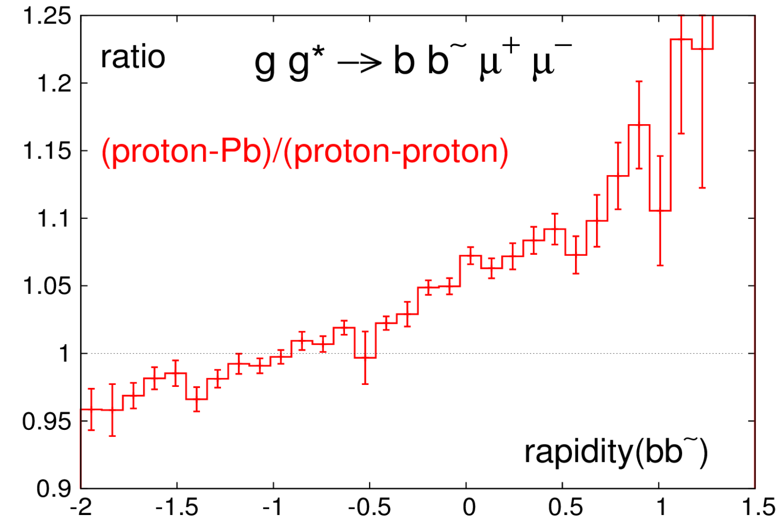



Obtained with gluon from BK
with collinear imrovements - model

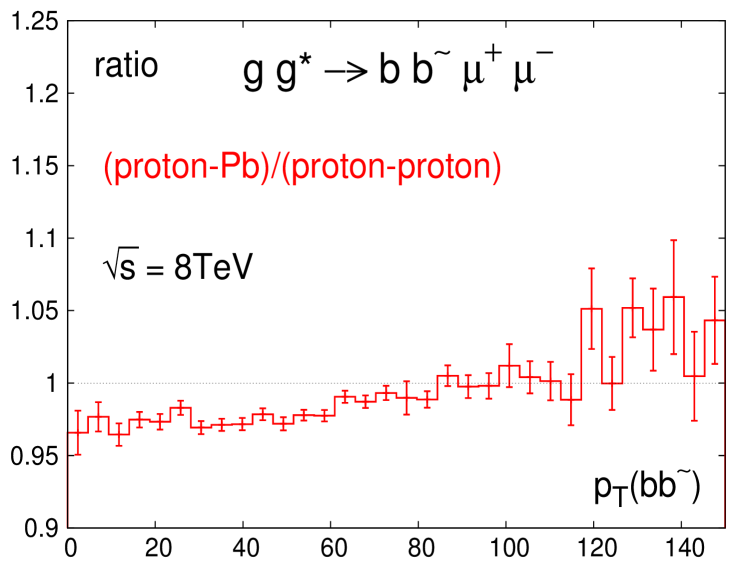
# Z production in p-p vs. p-Pb

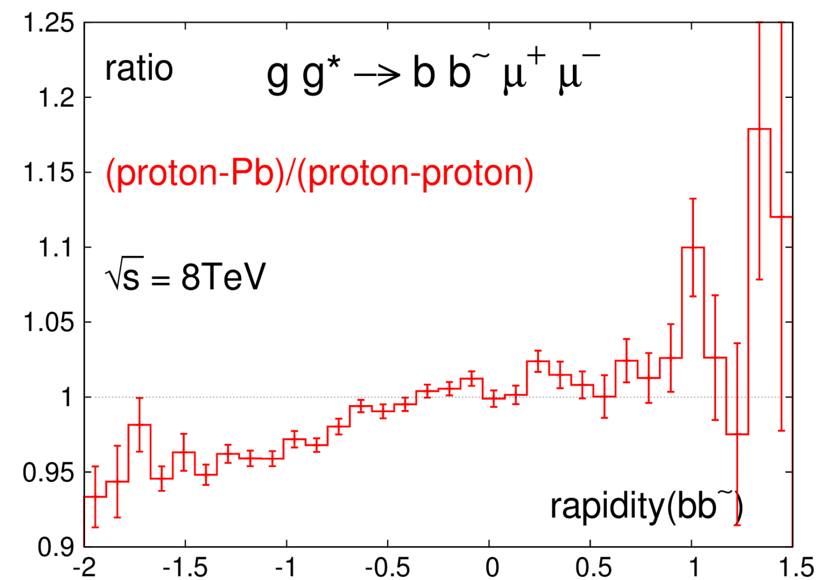
5 TeV




8 TeV



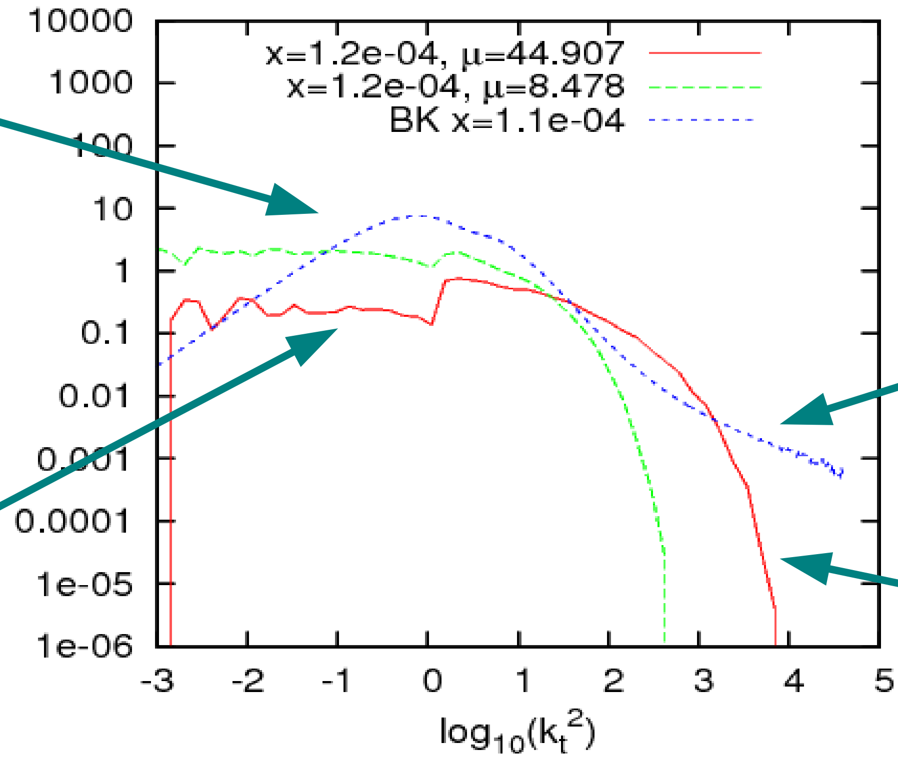

*pt of b barb > 20GeV, y of b, barb < 2.5*
*Pt of lepton >20GeV, y of lepton < 2.1*
*Et > 20 GeV*

*Work in progress with Andreas van Hameren and Piotr Kotko*

# Needed unification of recombination and coherence

universal

no universal




Too hard spectrum

ok

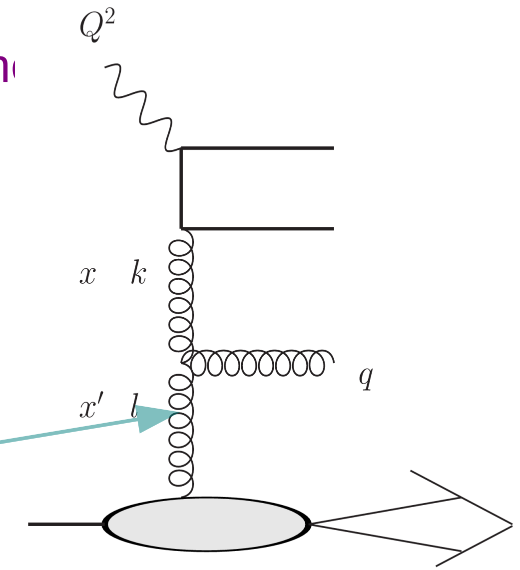
Use reformulation of BK to combine nonlinearities with coherence

talk by Dawid Toton

# Towards exclusive processes
# resummed form of the BK

The strategy:

- Use the BK equation for gluon number density. Simple nonlinear term
- Split linear kernel into resolved and unresolved parts
- Resumm the virtual contribution and unresolved ones in th
- Use analogy to postulate a nonlinear CCFM




The starting point:

Integration over l

$$\Phi(x,k^2) = \Phi_0(x,k^2) + \overline{\alpha}_s \int_{x/x_0}^{1} \frac{dz}{z} \int_0^{\infty} \frac{dl^2}{l^2} \left[ \frac{l^2\Phi(x/z,l^2) - k^2\Phi(x/z,k^2)}{|k^2 - l^2|} + \frac{k^2\Phi(x/z,k^2)}{\sqrt{(4l^4 + k^4)}} \right] - \frac{\overline{\alpha}_s}{\pi R^2} \int_{x/x_0}^{1} \frac{dz}{z} \Phi^2(x/z,k^2)$$

let $\pi R^2 = 1$

$$\mathcal{F}(x,k^2) = \frac{N_c}{4\alpha_s \pi^2} k^2 \nabla_k^2 \Phi(x,k^2)$$

# Resummed form of the BK

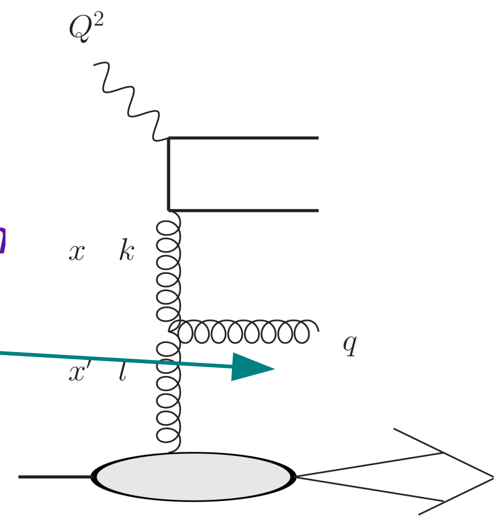
*K. K, Jadach, Golec, Skrzypek '11*

$$\Phi(x,k^2) = \Phi^0(x,k^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \int \frac{d^2\mathbf{q}}{\pi q^2} \left[\Phi(x/z,|\mathbf{k}+\mathbf{q}|^2) - \theta(k^2-q^2)\Phi(x/z,k)\right] - \overline{\alpha}_s \int_x^1 \frac{dz}{z} \Phi^2(x/z,k)$$

*Write in exclusive*
*Form. Combine virtual con*

$$\Phi(x,k^2) = \Phi^0(x,k^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \int \frac{d^2\mathbf{q}}{\pi q^2} \Phi(x/z,|\mathbf{k}+\mathbf{q}|^2)\theta(q^2-\mu^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \int \frac{d^2\mathbf{q}}{\pi q^2} \left[\Phi(x/z,|\mathbf{k}+\mathbf{q}|^2)\theta(\mu^2-q^2) - \theta(k^2-q^2)\Phi(x/z,k)\right] - \overline{\alpha}_s \int_x^1 \frac{dz}{z} \Phi^2(x/z,k)\,.$$

Integration over q




Resolution scale
introduced

Perform Mellin transform w.r.t x to get rid of
"z" integral

$$\overline{\Phi}(\omega,k^2) = \int_0^1 dx x^{\omega-1}\Phi(x,k^2)$$

$$\Phi(x,k^2) = \int_{c-i\infty}^{c+i\infty} d\omega\, x^{-\omega}\overline{\Phi}(\omega,k^2)$$

# Towards exclusive and resummed form of BK

Using in unresolved real part $|\mathbf{k}+\mathbf{q}|^2 \approx \mathbf{k}^2 \longleftarrow q^2 < \mu^2$

$$\overline{\Phi}(\omega,k^2) = \overline{\Phi}^0(\omega,k^2) + \frac{\overline{\alpha}_s}{\omega}\int \frac{d^2\mathbf{q}}{q^2}[\overline{\Phi}(\omega,|\mathbf{k}+\mathbf{q}|^2)\theta(q^2-\mu^2)] + \frac{\overline{\alpha}_s}{\omega}\int \frac{d^2\mathbf{q}}{q^2}\overline{\Phi}(\omega,k^2)[\theta(\mu^2-q^2)-\theta(k^2-q^2)] - \frac{\overline{\alpha}_s}{\omega}\int_0^1 dy y^{\omega-1}\Phi^2(y,k^2) \tag{8}$$

$$\overline{\Phi}(\omega,k^2) = \overline{\Phi}^0(\omega,k^2) + \frac{\overline{\alpha}_s}{\omega}\int \frac{d^2\mathbf{q}}{\pi q^2}\overline{\Phi}(\omega,|\mathbf{k}+\mathbf{q}|^2)\theta(q^2-\mu^2) - \frac{\overline{\alpha}_s}{\omega}\overline{\Phi}(\omega,k^2)\ln\frac{k^2}{\mu^2} - \frac{\overline{\alpha}_s}{\omega}\int_0^1 dy y^{\omega-1}\Phi^2(y,k^2)\,.$$

move to left hand side

$$\overline{\Phi}(\omega,k^2) = \hat{\Phi}^0(\omega,k^2) + \frac{\overline{\alpha}_s}{\overline{\omega}+\omega}\int \frac{d^2\mathbf{q}}{\pi q^2}\overline{\Phi}(\omega,|\mathbf{k}+\mathbf{q}|^2)]\theta(q^2-\mu^2) - \frac{\overline{\alpha}_s}{\omega+\overline{\omega}}\int_0^1 dy y^{\omega-1}\Phi^2(y,k^2)$$

Inverting the transform we obtain:

# BK equation in the resummed exclusive form

$$\Phi(x,k^2) = \tilde{\Phi}^0(x,k^2) + \overline{\alpha}_s \int_x^1 d\,z \int \frac{d^2\mathbf{q}}{\pi q^2}\,\theta(q^2-\mu^2)\,\frac{\Delta_R(z,k,\mu)}{z}\left[\Phi(\frac{x}{z},|\mathbf{k}+\mathbf{q}|^2) - q^2\delta(q^2-k^2)\,\Phi^2(\frac{x}{z},q^2)\right]$$

$$\Delta_R(z,k,\mu) \equiv \exp\left(-\overline{\alpha}_s \ln\frac{1}{z}\ln\frac{k^2}{\mu^2}\right)$$

- The same resumed piece for linear and nonlinear
- Initial distribution also gets multiplied by the Regge form factor
- New scale introduced to equation. One has to check dependence of the solution on it
- Suggestive form to promote the CCFM equation to nonlinear equation which is more suitable for description of final states

talk of Dawid Toton

# *Extension of CCFM to non linear equation for gluon number density*

The same procedure of resummation can be applied to the high energy factorizable gluon density. The stucture of nonlinearity does not introduce complications:

*K.K. '12*

$$\mathcal{F}(x,k^2)=\tilde{\mathcal{F}}_0(x,k^2)+\overline{\alpha}_s\int_{x/x_0}^1\frac{d\,z}{z}\Delta_R(z,k,\mu)\left\{\int\frac{d^2\mathbf{q}}{\pi q^2}\theta(q^2-\mu^2)\mathcal{F}(\frac{x}{z},|\mathbf{k}+\mathbf{q}|^2)\right.$$
$$\left.-\frac{\pi\alpha_s^2}{4N_cR^2}k^2\nabla_k^2\left[\int_{k^2}^{\infty}\frac{dl^2}{l^2}\ln\frac{l^2}{k^2}\mathcal{F}(x,l^2)\right]^2\right\}$$

# High Energy Factorization –
# towards parton shower Monte Carlo

General purpose Monte Carlos

- Collinear factorization PYTHIA, HERWIG, SHERPA
- High energy factorization CASCADE (CCFM based), HEJ (BFKL + DGLAP base) , ARIADNE (dipole picture based but no saturation), DIPSY(dipole picture based with saturation)

The goal is to develop forward MC based on Markov chain approach.

- No need to pretabulate gluon density. Probablilistic inerpretation.
- Well developed algorithms for linear equations.
- Easy to generalize to higher dimensions
- First step towards this goal is to solve BK using Markov chain approach

# Equivalent linear system for the BK equation

$$\Phi(y,\kappa) = \Phi^0(y,\kappa) + \int_0^y dt \int_0^\infty d\lambda\, K\left(y,t,\kappa,\lambda,\Phi(t,\lambda)\right)$$

$y = -\ln x$

$\kappa = \ln(k^2/\mu^2)$

$$K\left(y,t,\kappa,\lambda,\Phi(t,\lambda)\right) = \bar{\alpha}_s \left[\frac{e^\lambda \Phi(t,\lambda) - e^\kappa \Phi(t,\kappa)}{|e^\kappa - e^\lambda|} + \frac{e^\kappa \Phi(t,\kappa)}{\sqrt{4e^{2\lambda} + e^{2\kappa}}}\right] - \bar{\alpha}_s\, \delta(\lambda-\kappa)\, \Phi^2(t,\lambda)$$

$\lambda = \ln(l^2/\mu^2)$

$$K\left(y,t,\kappa,\lambda,\Phi(t,\lambda)\right) = K\left(y,t,\kappa,\lambda,\bar{\Phi}(t,\lambda)\right) + K'_\Phi\left(y,t,\kappa,\lambda,\bar{\Phi}(t,\lambda)\right)\left[\Phi(t,\lambda) - \bar{\Phi}(t,\lambda)\right] \mathcal{O}\left(\left[\Phi(t,\lambda) - \bar{\Phi}(t,\lambda)\right]^2\right)$$

$$\Psi(t,\lambda) = \Phi(t,\lambda) - \bar{\Phi}(t,\lambda)$$

Newton-Kantorovich algorithm

$$K'_\Phi\left(y,t,\kappa,\lambda,\bar{\Phi}(t,\lambda)\right) = \bar{\alpha}_s \left[\frac{e^\lambda \delta(\lambda-\kappa) - e^\kappa}{|e^\kappa - e^\lambda|} + \frac{e^\kappa}{\sqrt{4e^{2\lambda} + e^{2\kappa}}}\right] - 2\bar{\alpha}_s\, \delta(\lambda-\kappa)\, \bar{\Phi}(t,\lambda)$$

$$\Phi(y,\kappa) = \bar{\Phi}(y,\kappa) + \Psi(y,\kappa)$$

$$\Psi(y,\kappa) = \Lambda(y,\kappa) + \int_{y_0}^y dt \int_0^{+\infty} d\lambda\, K'_\Phi\left(y,t,\kappa,\lambda,\bar{\Phi}(t,\lambda)\right) \Psi(t,\lambda)$$

linear

$$\Lambda(y,\kappa) = \Phi^0(y,\kappa) + \int_{y_0}^y dt \int_0^{+\infty} d\lambda\, K\left(y,t,\kappa,\lambda,\bar{\Phi}(t,\lambda)\right) - \bar{\Phi}(y,\kappa)$$

solution

$$\Psi_{n-1}(y,\kappa) = \Lambda_{n-1}(y,\kappa)$$
$$+ \sum_{m=1}^{\infty} \prod_{i=1}^{m} \left[\int_{y_0}^y dt_i \int_0^{+\infty} d\lambda_i\, \theta(t_{i-1} - t_i)\, K'_\Phi\left(t_{i-1}, t_i, \lambda_{i-1}, \lambda_i, \Phi_{n-1}(t_i,\lambda_i)\right)\right] \Psi_{n-1}(t_m, \lambda_m)$$

# Markov chain algorithm

K.Bozek, W. Placzek, KK
arxiv:1305.4154

Start a random walk (Markov chain) from the point $(t_0, \lambda_0) = (y, \kappa)$

Being at the point $(t_i, \lambda_i)$:

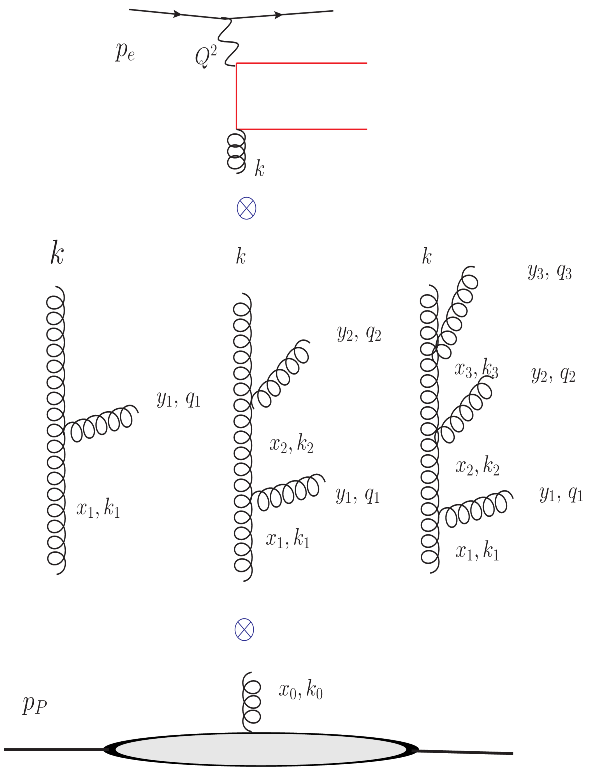
(i) generate a random step in the $t$-direction $\tau_{i+1} = t_{i+1} - t_i < 0$ according to some probability density function (pdf) $\rho(\tau)$, with the normalisation contidion

$$\int_{-\infty}^{0} d\tau\, \rho(\tau) = 1;$$

(ii) for a given value $\tau_{i+1}$, generate a random step in the $\lambda$ direction: $\xi_{i+1} = \lambda_{i+1} - \lambda_i$ according to some pdf $\eta_{\tau_{i+1}}(\xi)$, with the normalisation condition

$$\int_{0}^{+\infty} d\xi\, \eta_{\tau_{i+1}}(\xi) = 1,$$

Stop the random walk when some $t_{m+1}$ jumps beyond the lower $t$-integral limit, i.e. $t_{m+1} \leq y_0$ following the sequence $t_0 > t_1 > t_2 > \ldots > t_m > y_0$.




To each trajectory

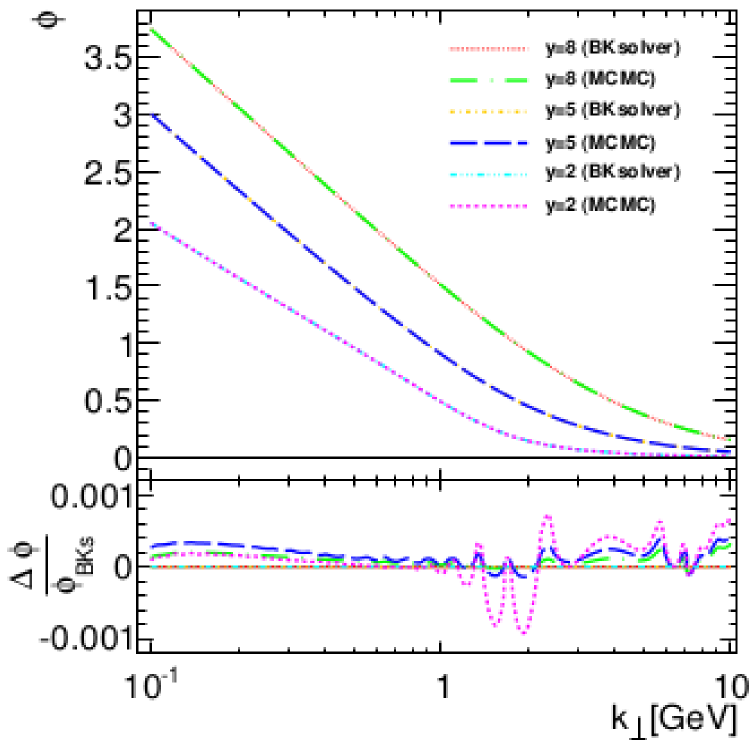
$$\gamma_m = \{(t_0, \lambda_0), (t_1, \lambda_1), \ldots, (t_m, \lambda_m) : y = t_0 > t_1 > t_2 > \ldots > t_m > y_0 \geq t_{m+1}\}$$

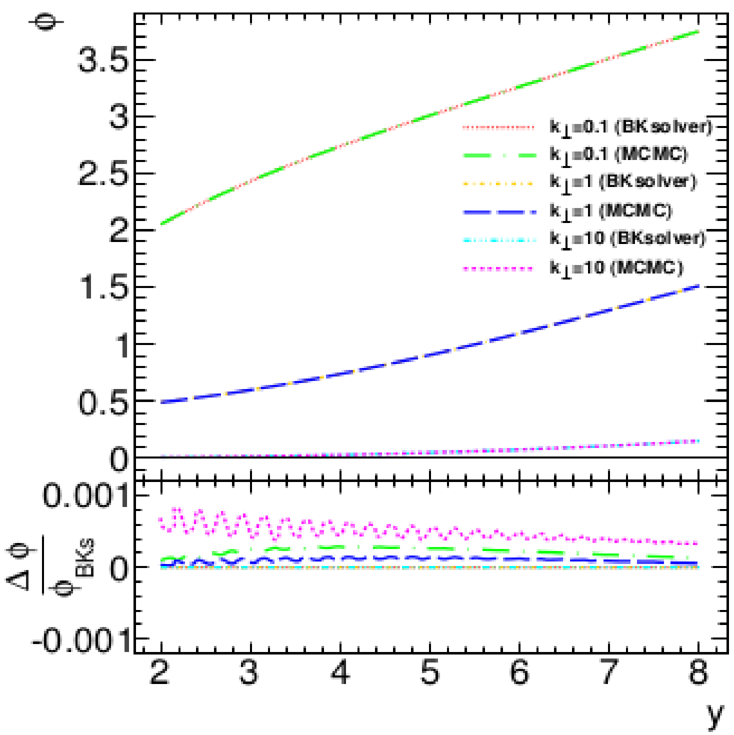
$$w(y, \kappa) = \frac{v(\gamma_n)\Lambda_{n-1}(t_m, \lambda_m)}{R(t_m)}$$

Repeat the above steps $N$ times and compute the MCMC estimate of $\Psi_{n-1}(y, \kappa)) = \frac{1}{N}\sum_{k=1}^{N} w_k(y, \kappa)$

implemented by master student Krzysztof Bozek

# Monte Carlo vs. BK-Solver




- Grid 100 (y) X 128 (k)
- 1000 trajectories for linear equation
- 15 iterations of the set
- 20 minutes CPU 2.2 GHz Pentium Dual Core
- Agreement within 1 per mil

$$\Phi^0(y,\kappa) = \exp\left(-\frac{\mu^2 e^{\kappa}}{\mathrm{GeV}^2}\right)$$

$$\rho(\tau_i) = e^{\tau_i}, \quad \eta_{\lambda_{i-1}}(\xi_i) \pm e^{-\lambda_i}$$

$$\tau_i = \ln U_i, \quad \lambda_i = -\ln V_i$$

random (0,1)

# Conclusions and outlook

•Efficient framework for evaluation of Matrix Elements with high energy projectors

•The new representation of the  BK equation has been found

•Well motivated anzatz for the equations  which incorporate both saturation effects and coherence.

•Prospects for  Monte Carlo simulation of exclusive processes in physics of saturation