NLO photons and soft physics

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1302.5970

- Soft gT physics
- Sensitivity to soft physics in γ-production
- Light-cone condensates
- Structure on the NLO rate

How to deal with soft physics:

- Scales of hot QCD: $D_\mu \sim p_\mu + g\langle A \rangle \sim p_\mu + gT$
  - "Hard scale": $P \sim T$, narrow quasi-particles, kinetic theory
    - Pert. expansion in $\frac{g^2}{(4\pi)^2}$
  - "Soft scale": $P \sim gT$, Interaction with medium non-perturbative
    - Can be resummed, exp. in g
  - "Ultrasoft scale": $P \sim g^2 T$, interaction among ultrasoft modes non-perturbative
    - No diagrammatic expansion
- Observables become sensitive to soft physics at some order in g

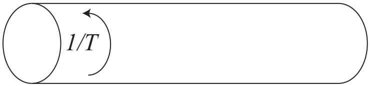
$$P(T) \sim \int d^3p E(p) n_B(p) \overset{\text{for } p\sim gT}{\sim} \int_{gT} dp p^3 \frac{T}{p} \sim g^3 T^4$$

How does one deal with the soft contributions?

How to deal with soft physics: The static case

A familiar story Braaten & Nieto 80's

- Separate the hard and soft contributions: $P(T) = P_{\rm hard} + P_{\rm soft}$
  - $P_{\rm hard}$ computable in PT
  - $P_{\rm soft}$ depends only on integrals of equal time correlators $\equiv$ *condensates*. E.g. $\langle F_{ij}^2 \rangle$, $\langle {\rm Tr}\, A_0^2 \rangle$
- Soft condensates computable in 3D effective field theory, EQCD
  - Pert. theory in eff. theory: expansion in $(g_3^2)/m_E \sim g$
  - Simulate eff. theory on lattice
    - Lat. theory super-renormalizable $\rightarrow$ simple continuum limit
- EQCD result of dimensional reduction of imaginary time in Euclidean formalism.



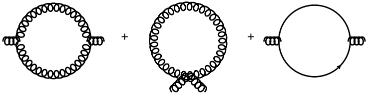
What to do when interested in dynamical quantities $\Delta t \neq 0$, $\omega \neq 0$?

How to deal with soft physics: The dynamical case Braaten, Pisarski; Frenkel, Taylor; Blaizot-Iancu

- gT-scale can be dealt with perturbatively within HTL eff. field theory.
- Interaction generates a $\mathcal{O}(gT)$ correction dispersion

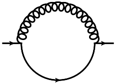
- Dominated by scattering with "Hard" particles at the scale T

$$\Pi_{gluon}(p \sim gT) =$$



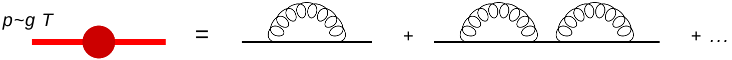
$$\sim g^2 T^2$$

$$\Sigma_{quark}(p \sim gT) =$$



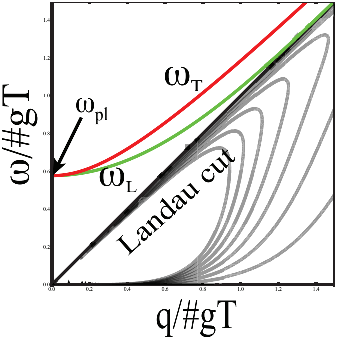
$$\sim gT$$

- Correction not small for "soft" $\mathcal{O}(gT)$ modes: need to resum
  $\Rightarrow$ HTL resummed perturbation theory



In-medium dispersion relations

For momenta $\sim gT$, qualitatively different disp. rel.:



- Transverse and Longitudinal polarizations: ω_T, ω_L
- Minimum frequency: Plasma frequency: $\omega_{\rm pl}$
- Asymptotic mass: $m_{\infty,g}$
- Non-zero spectral weight in spacelike region: Landau cut

- Similarly for quarks:
  - Thermal asymptotic masses (m_∞), plasma frequencies and Landau cut
  - Positive and negative helicity/chirality modes: Plasminos

How to deal with soft physics: The dynamical case

But what about non-pert.? Lattice? g^2T?

No general answer here, but we learned something from computing the NLO thermal photon rate.

Why did we look at NLO photons?

Phenomenologically interesting:

- Photons created in HIC:
  - in the collision: primary photons
  - in the QGP: Thermal and Jet-termal photons
  - in hadroninc interactions: Hardon gas and decay photons
- Photons created in the QGP escape plasma without rescattering
  - May provide direct info about the QGP

LO result used in hyrdo: improve, or at least provide with error bars

Why did we look at NLO photons?

- Theoretically clean:
  - To first order in $\alpha_{EM} \ll 1$ (no rescat.)

$$\frac{d\Gamma_\gamma}{d^3k} = \frac{e^2}{(2\pi)^3 2k^0} \int d^4Y \, e^{-iK\cdot Y} \langle J^\mu(Y) J_\mu(0) \rangle$$

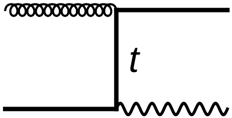
$$J^\mu = \sum_{q=uds} e_q \bar{q} \gamma^\mu q :$$

- Perturbation theory maybe not that bad:
  - additional hard scale $K \gg T$
- The second dynamical NLO transport coefficient computed
  - Heavy quark diffusion Caron-Huot, Moore

- Soft gT physics
- Sensitivity to soft physics in γ-production
- Light-cone condensates
- Structure on the NLO rate

Where does sensitivity to soft physics show up for Γ_γ?

Consider Compton scattering in kinetic theory:



$$\frac{d\Gamma_\gamma}{d^3k} \sim \int_{p_1,p_2,p_3} n_F(p_1)n_B(p_2)(1-n_F(p_3))\delta(p_1+p_2-p_3-k)|\mathcal{M}_{\text{compt.}}|^2$$

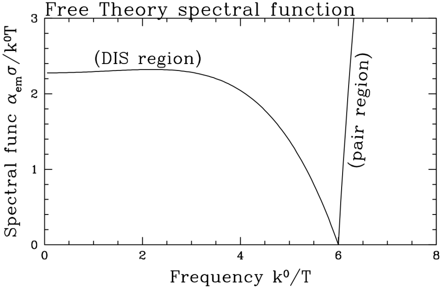
$$\underset{\text{for } q_\perp\to 0}{\sim} \int d^2q_\perp \frac{d\sigma}{dq_\perp^2} \sim \alpha_s\alpha_{EM}\int \frac{d^2q_\perp}{q_\perp^2}$$

- Total cross section of a transfer of massless particle IR divergent
- $q_\perp \to 0$ corresponds to a *conversion* process
- Equally large contribution from all logarithmic momentum scales, including gT, where kinetic theory fails
  - LO sensitivity to conversion rate from soft collisions
  - Similarity from pair annihilation

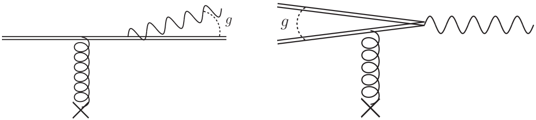
Where does sensitivity to soft physics arise for Γ_γ?

Also:

- On-shell quark cannot emit a photon, but a near on-shell one can



- Even a soft scattering can bring enough off-shell: Brem/Pair annihl.

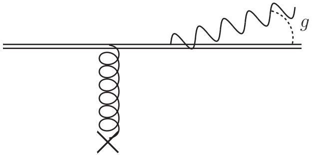


$$\Gamma_\gamma \sim \alpha_{EM}\Gamma_{\text{elastic}} \sim \alpha_{EM}\alpha_s^2 \int d^2q_\perp \frac{T^3}{(q_\perp^2)^2} \overset{\text{for } q_\perp \sim gT^3}{\sim} \alpha_{EM}\alpha_s^2 T^3/(gT)^2$$

- LO sensitivity to a rate of acquiring virtuality from soft collisions

Where does sensitivity to soft physics come up for Γ_γ?

Also:



- For a soft (space-like) momentum transfer, virtuality of the hard intermediate quark is $P^2 \sim g^2T^2$
  - Long lifetime $t_{\text{emit}} \sim 1/(g^2T)$, the quark has a long time to "feel" the medium
    - Modification to dispersion relation $m_\infty \sim gT$
    - and damping rate $\Gamma \sim g^2T$
- LO sensitivity also from lines with small virtuality

- Soft gT physics
- Sensitivity to soft physics in γ-production
- Light-cone condensates
- Structure on the NLO rate

Eikonalization

- For photons, soft sector enters though modifying propagation of a hard nearly on-shell quark
  - Soft conversion rate
  - Soft rate to acquire virtuality
  - Soft modifications to dispersion
- Sensitive only to near- light-cone correlation functions!

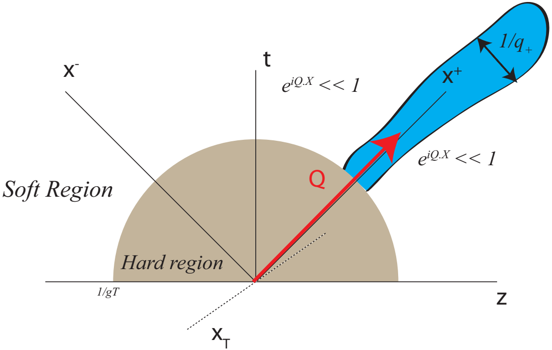
Eikonalization

Consider a quark with $Q = (q_+, q_-, q_\perp)$, with $q^+ \gg (q_-, q_\perp)$

$q_+ = \frac{1}{2}(q_0 + q_z)$, $q_- = (q_0 - q_z)$

- Correlation function in coordinate space:

$$G(Q) = \int d^4X e^{i(q_+x_- + q_-x_+ + q_\perp \cdot x_\perp)} G(X)$$



- Consider only soft contributions: large X
- q_+ large, x_- large: Phase oscillates rapidly

$\Rightarrow$ Expect soft physics to appear only through correlators near light-cone $\equiv$ *light-cone condensates*, e.g., $\int dx^+ \langle B(0)U(0,x_+)B(x_+)\rangle$, $\int dx^+ \langle \bar\psi U(0,x_+)\psi(x_+)\rangle$

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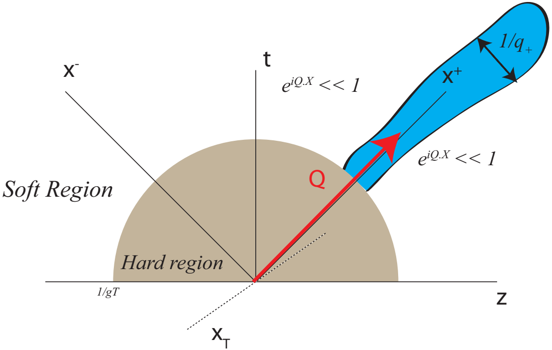

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- Magic: These can be evaluated in EQCD! (Guy's talk) Caron-Huot 2009

Eikonalization

The NLO photon production is sensitive to soft physics only though:

- Condensates related to the asymptotic mass $m_\infty^2 = g^2(Z_g + Z_f)$

$$\begin{aligned} Z_g &\propto \int_0^\infty dx^+\, x^+ \langle v_{k\,\mu} F_a^{\mu\nu}(x^+) U_A^{ab}(x^+,0) v_{k\,\rho} F_{b\ \nu}^{\rho}(0)\rangle, \\ Z_f &\propto \int_0^\infty dx^+ \langle \overline{\psi}(x^+)\, \not{v}_k\, U_R(x^+,0)\psi(0)\rangle \end{aligned}$$

- Condensates related to momentum broadening
  - A generalized momentum diffusion coefficient $\hat{q}$

$$\hat{q}(\delta E) = \int_{-\infty}^{\infty} dx^+\, e^{ix^+\delta E}\, \langle v_k^\mu F_\mu{}^\nu(x^+) U_A(x^+,0) v_k^\rho F_{\rho\nu}(0)\rangle,$$

  - and time-like Wilson loop with transverse size $x_\perp$

$$C(q_\perp) = \lim_{x^+\to\infty} -(x^+)^{-1}\log(W(x^+,x_\perp))$$

$$W(x^+,x_\perp) \equiv \mathrm{Tr}\left\langle U_R(0,0,x_\perp;x^+,0,x_\perp)\, U_R(0,0,0;0,0,x_\perp) \right.$$
$$\left. U_R(x^+,0,0;0,0,0)\, U_R(x^+,0,x_\perp;x^+,0,0)\right\rangle.$$

- Soft gT physics
- Sensitivity to soft physics in γ-production
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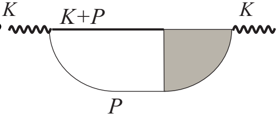
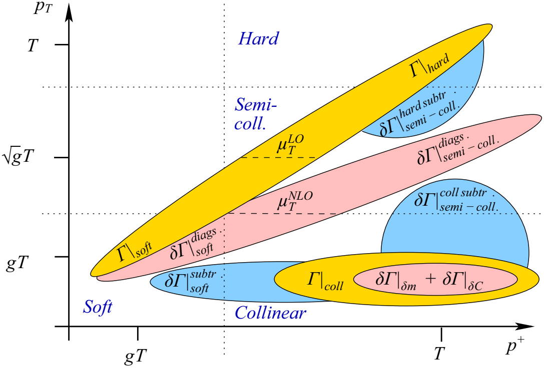
Organizing the calculation

$$\langle J_\mu J^\mu \rangle = \int d^4P$$

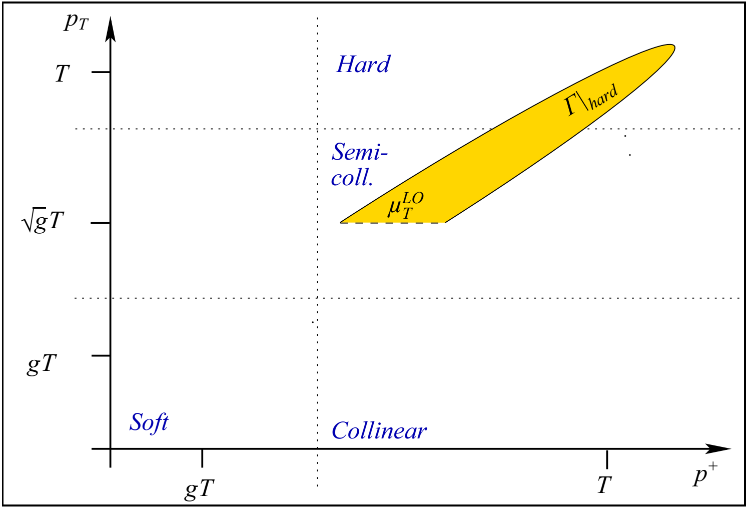


- LO and NLO given by a single quark-loop with gluon lines
- Line with $K + P$ can be arranged to be cut giving $\delta[(K+P)^2]$



- Yellow: LO
- Red: NLO
- Blue: LO-subraction

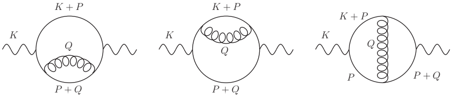
Hard sector



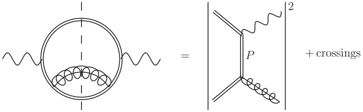
p_T
T
$\sqrt{g}T$
gT
Hard
Semi-
coll.
μ_T^{LO}
$\Gamma\backslash_{hard}$
Soft
Collinear
gT
T
p^+

Hard sector: Kapusta, Lichard, Seibert 1991

- Leading order diagrams:

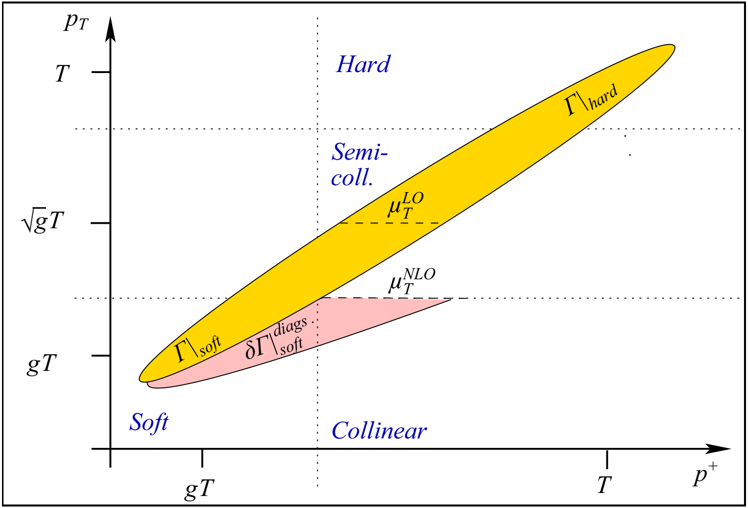


- Cuts correspond to kinetic theory: Compton and Pair-annih.



- Regulate at IR: $\int d^4P\delta[(K+P)^2] \to \int_{\mu_\perp^{LO}}^{\infty} d^2p_\perp \int_{\mu_+^{LO}}^{\infty} dp_+$, with $gT \ll \mu \ll T$
- Hard contribution: $\Gamma|_{\mathrm{hard}} \propto \alpha_s \alpha_{EM} \left[\log(\frac{T}{\mu_\perp^{\mathrm{LO}}}) + C_{\mathrm{hard}}(k/T)\right]$
  - $\mu_\perp^{LO}$ dependence due to logarithmic singularity at small $p_\perp$
- All lines hard and far off shell, no soft sensitivity, no NLO corrections

Soft

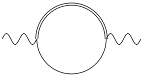


p_T
T
$\sqrt{g}T$
gT
Hard
Semi-
coll.
Soft
Collinear
$\Gamma\backslash_{hard}$
μ_T^{LO}
μ_T^{NLO}
$\Gamma\backslash_{soft}$
$\delta\Gamma\backslash_{soft}^{diags}$
gT
T
p^+

Soft sector

LO:

- In leading order diagrams:

$\Gamma|_{\text{soft}} =$ 

- Cuts correspond to conversion processes
- Can be calculated using novel fermionic sum rules (previously brute force numerics)
- Soft contribution: $\Gamma_{\text{soft}} \propto \alpha_{EM}\alpha_s \int^{\mu_\perp^{LO}} d^2p_\perp \frac{m_\infty^2}{p_\perp^2+m_\infty^2}$ Independ. by Bödeker & Besak
  - $\mu_\perp^{LO}$ dependence cancels against the hard sector
- The m_∞^2 is related to two light-cone condensates: $m_\infty^2 = g^2(Z_g + Z_f)$

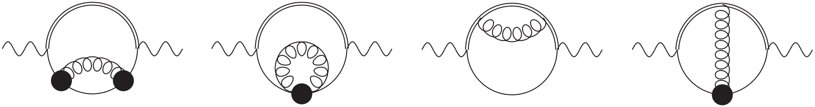
$$Z_g \propto \int_0^\infty dx^+ \, x^+ \langle v_{k\,\mu} F_a^{\mu\nu}(x^+) U_A^{ab}(x^+,0) v_{k\,\rho} F_{b\;\nu}^{\rho}(0)\rangle,$$

$$Z_f \propto \int_0^\infty dx^+ \langle \overline{\psi}(x^+)\, \not{v}_k \, U_R(x^+,0)\psi(0)\rangle$$

Soft sector:

In NLO:

- Soft expansion parameter g:



- Single lines HTL props., blobs HTL vertices.
- But, magic happens! Using sum rules:

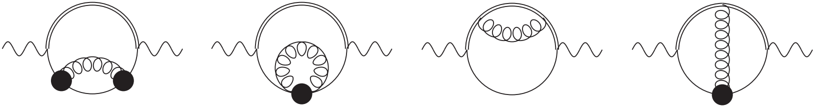
$$\Gamma_{\mathrm{soft}}^{\mathrm{NLO}} \propto \delta m_\infty^2 \int d^2p_\perp \frac{p_\perp^2}{(p_\perp^2 + m_\infty^2)^2} \propto \log(\mu_\perp^{\mathrm{NLO}}/T)$$

with $m_{\infty,\mathrm{NLO}}^2 = m_\infty^2 + \delta m_\infty^2$

Soft sector:

In NLO:

- Soft expansion parameter g:



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$$\Gamma_{\rm soft}^{\rm NLO} \propto \delta m_\infty^2 \int d^2 p_\perp \frac{p_\perp^2}{(p_\perp^2 + m_\infty^2)^2} \propto \log(\mu_\perp^{\rm NLO}/T)$$

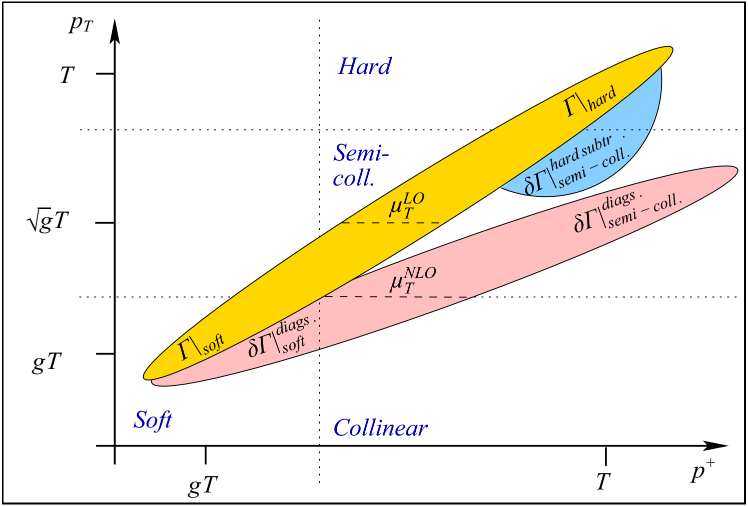
with $m_{\infty,{\rm NLO}}^2 = m_\infty^2 + \delta m_\infty^2$
...and actually

$$\Gamma_{\rm soft}^{\rm LO} + \Gamma_{\rm soft}^{\rm NLO} = \int d^2 p_\perp \frac{m_{\infty,{\rm NLO}}^2}{p_\perp^2 + m_{\infty,{\rm NLO}}^2}$$

Depends only on two NLO light-cone condensates, Z_g, Z_f

- Both Z_g and Z_f get their leading order from hard sector, NLO from soft.

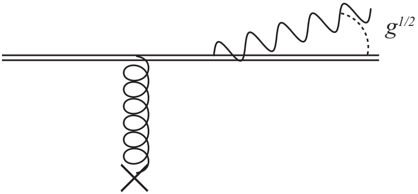
Semi-Collinear



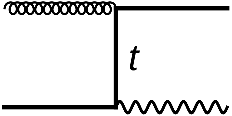
Semi-collinear

Same diagrams as hard, different kinematics. Cuts corresponds to:

- Space-like Q^2: Bremsstrahlung of photon with an angle $\mathcal{O}(\sqrt{g})$ WRT quark



- Time-like Q^2: Compton/pair-annih. with in/outgoing gluon at plasmon pole $E(q) \sim \sqrt{q^2 + m_\infty^2}$. Correction important only in NLO



  - We actually already included this in the LO computation of hard+soft sector
  - ...But we did it wrong by an extra $g \Rightarrow$ for NLO remove wrong and replace with correct

Semi-collinear

- Can be computed using same methods as the soft part:

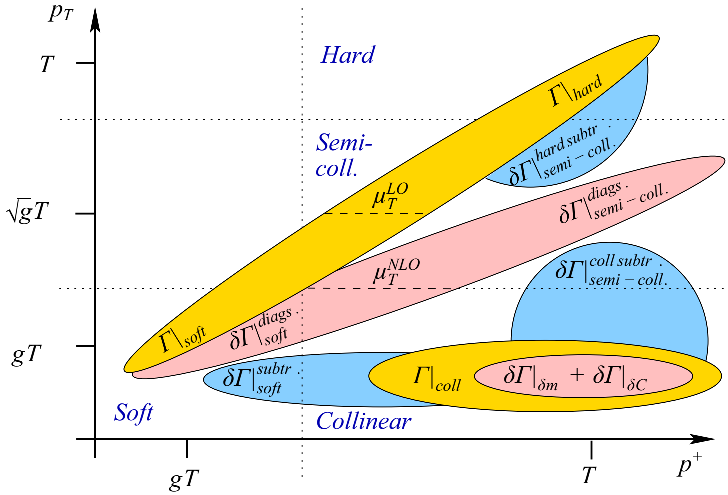
$$\Gamma_{\mathrm{semi-coll.}} \propto \int dp^+ \left[\frac{(p^+)^2 + (p^+ + k)^2}{(p^+)^2 (p^+ + k)^2} \right] \frac{n_f(k+p^+)[1-n_f(p^+)]}{n_f(k)}$$
$$\int \frac{d^2 p_\perp}{(2\pi)^2} \frac{4(p^+)^2(p^+ + k)^2}{k^2 p_\perp^4} \hat{q}(\delta E) \,.$$

with $\delta E = \frac{p_\perp^2 + m_\infty^2}{2p^+}$ and

$$\hat{q}(\delta E) = \int_{-\infty}^{\infty} dx^+ \, e^{ix^+ \delta E} \, \langle v_k^\mu F_\mu{}^\nu(x^+) U_A(x^+) v_k^\rho F_{\rho\nu}(0) \rangle,$$

Again, all information about soft sector in a new *light-cone condensate*.

Collinear



p_T
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Hard
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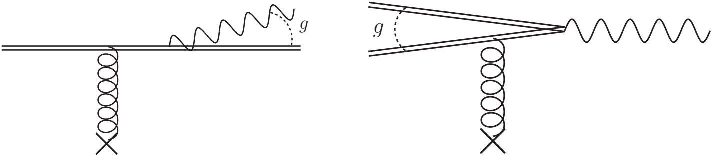
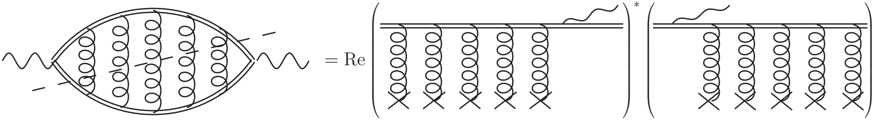
Soft
Collinear
$\Gamma\backslash_{hard}$
$\delta\Gamma\backslash^{hard\,subtr.}_{semi-coll.}$
$\delta\Gamma\backslash^{diags.}_{semi-coll.}$
μ_T^{LO}
μ_T^{NLO}
$\delta\Gamma|^{coll\,subtr.}_{semi-coll.}$
$\Gamma\backslash_{soft}$
$\delta\Gamma\backslash^{diags.}_{soft}$
$\delta\Gamma|^{subtr.}_{soft}$
$\Gamma|_{coll}$
$\delta\Gamma|_{\delta m} + \delta\Gamma|_{\delta C}$
gT
T
p^+

Collinear

- Contributes to LO, originally spotted by Aurenche Gelis Kobes Petitgirard Zaraket 1998-2000
- Cuts Correspond to collinear Brem./Pair. annihilation with angle g.



- The long formation time of emitting the photon leads to need to resum ladder diagrams (effect: $\mathcal{O}(1)$ suppression of the coll. rate).



Collinear

- AMY formalism for resumming diagrams by solving a diff. equation at LO. Arnold, Moore, Yaffe 2001
  - Requires information about collision kernel $C(q_\perp)$ and m_∞, both related to light-cone condensates

$$C(q_\perp) = \lim_{x^+ \to \infty} -(x^+)^{-1} \log(W(x^+, x_\perp))$$

$$W(x^+, x_\perp) \equiv \mathrm{Tr} \Big\langle U_R(0,0,x_\perp; x^+, 0, x_\perp)\, U_R(0,0,0;0,0,x_\perp) \\ U_R(x^+,0,0;0,0,0)\, U_R(x^+,0,x_\perp;x^+,0,0) \Big\rangle .$$

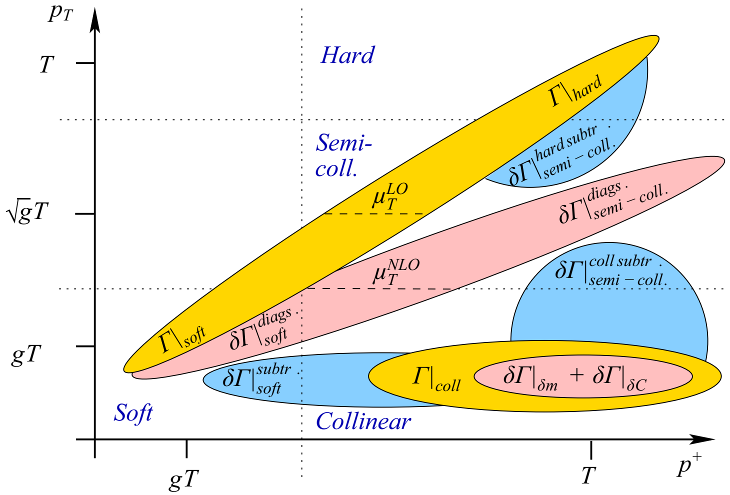
  - Works equally well at NLO: Replace $C_{LO}(q_\perp) \to C_{\mathrm{NLO}}(q_\perp) = C_{\mathrm{LO}}(q_\perp) + \delta C(q_\perp)$, and $m^2_{\infty,\mathrm{LO}} \to m^2_{\infty,\mathrm{LO}} + \delta m^2_\infty$
    - $C_{\mathrm{NLO}}(q_\perp)$ computed by Caron-Huot using EQCD

  - In leading order calculation extended integration outside region

$$\int_0^\infty dp_+ \int_0^\infty d^2p_\perp \frac{d\Gamma_{\mathrm{brem}}}{d^3p}$$

    - Both $p_z \sim gT$ (soft) and $p_\perp^2 \sim gT^2$ contribution to integral $\mathcal{O}(g)$. Need to be removed in NLO calculation.

Collinear



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$\sqrt{g}T$
gT
gT
T
p^+
Hard
Semi-coll.
Soft
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$\Gamma\backslash_{hard}$
$\delta\Gamma\backslash^{hard\,subtr.}_{semi-coll.}$
μ_T^{LO}
$\delta\Gamma\backslash^{diags.}_{semi-coll.}$
μ_T^{NLO}
$\delta\Gamma|^{coll\,subtr.}_{semi-coll.}$
$\Gamma\backslash_{soft}$
$\delta\Gamma\backslash^{diags.}_{soft}$
$\delta\Gamma|^{subtr.}_{soft}$
$\Gamma|_{coll}$
$\delta\Gamma|_{\delta m} + \delta\Gamma|_{\delta C}$

(Partial) conclusions

- We did an NLO calculation using HTL and found out that:
- To NLO, all the information of soft physics found to be contained in *light-cone condensates* $\{Z_g, Z_f, C(q_\perp), \hat{q}(\delta E)\}$.
- Soft contributions (and only soft) can be computed using EQCD.
- We needed and computed a new *light-cone condensates*

$$\hat{q}(\delta E) = \int_{-\infty}^{\infty} dx^+ \, e^{ix^+ \delta E} \, \langle v_k^\mu F_\mu{}^\nu(x^+) U_A(x^+) v_k^\rho F_{\rho\nu}(0) \rangle,$$

  - Related to collision kernel $\lim_{x_\perp \to 0} \partial_{x_\perp} C(x_\perp) \sim \lim_{\delta E \to 0} \hat{q}(\delta E)$
  - Not just a momentum diffusion coefficient: Includes contributions from the pole and cut
    $\Rightarrow$ Includes splitting
- Not a general solution to problem of soft sector:
  - At higher order (and different observables), new condensates.
  - Sensitivity to time-like soft correlators would prevent evaluation of the condensates non-perturbatively.