FROM JIMWLK EVOLUTION TO QCD REGGEON FIELD THEORY

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REGGEON FIELD THEORY

EVOLUTION OF SCATTERING OBSERVABLES WITH ENERGY AT HIGH ENERGY.
GRIBOV: POMERON - SCATTERING AMPLITUDE OF A PHYSICAL HADRON.
ITS RAPIDITY EVOLUTION IS GIVEN BY THE LAGRANGIAN

$$L = P^\dagger(x)\frac{dP(x)}{dY} - \kappa P^\dagger P + v[P^\dagger P^2 + PP^{\dagger 2}]$$

P - THE POMERON, $P^\dagger$ - THE CONJUGATE POMERON, κ - POMERON PROPAGATOR, v - THREE POMERON VERTEX.

RAPIDITY EVOLUTION:

$$\frac{dP}{dY} = \kappa P + v[P^2 + 2P^\dagger P]$$

EQUIVALENTLY - POMERON HAMILTONIAN

$$H[P, P^\dagger] = P^\dagger \kappa P - v[P^\dagger P^2 + PP^{\dagger 2}]$$

POMERON HAS ”VACUUM QUANTUM NUMBERS” - CHARGE CONJUGATION, TIME REVERSAL etc .EVEN, ISOSPIN SINGLET etc...

THERE MUST ALSO BE SIMILAR OBJECTS - ”REGGEONS” - WHICH CARRY NONTRIVIAL QUANTUM NUMBERS. THEY GIVE LEADING CONTRIBUTIONS TO SCATTERING WITH EXCHANGE OF CHARGE CONJUGATION (ODDERON) AND OTHER CHARGES.

THEY CAN IN PRINCIPLE COUPLE TO EACH OTHER IN THE EVOLUTION AND THESE COUPLINGS DEFINE THE MORE COMPLETE REGGEON FIELD THEORY.

SINCE EVERYTHING FOLLOWS FROM QCD, CAN ONE EXPLICITLY CONSTRUCT SUCH A QCD REGGEON FIELD THEORY?

BFKL AND REGGEONS

HIGH ENERGY QCD STARTS WITH THE BFKL EQUATION.

PERTURBATIVELY THE SCATTERING AMPLITUDE IS PROPORTIONAL TO GLUON DENSITY:

$$P(x,y) = \langle A_i^a(x) A_i^a(y) \rangle_{hadron}$$

BFKL EQUATION

$$\frac{dP(x,y)}{dY} = K(x,y;u,v)P(u,v)$$

FAMOUS LEADING SOLUTION

$$P \to_{Y\to\infty} \exp\{4\ln 2\bar{\alpha}_s Y\} P_0$$

P IS BILOCAL: SIMPLEST COLOR SINGLET OBJECT IN QCD IS A "COLOR DIPOLE", HAS TWO LEGS.

ANALOG OF REGGEONS: MULTIGLUON CORRELATION FUNCTIONS, SATISFY Bartels-Kwiecinski-Prazcalowicz EQUATION:

$$G^n(1,2...n) = \langle T^{a_1...a_n} A^{a_1}(1)...A^{a_n}(n)\rangle_{hadron}$$

EVOLUTION:

$$\frac{dG(1...n)}{dY} = \Sigma_{all\ pairs} K(ij;u,v) G(1..u,v...n)$$

EQUATIONS ARE LINEAR: LINEARITY BREAKS DOWN WHEN REGGEONS (AMPLITUDES) BECOME OF ORDER UNITY.

WHAT ARE THE VERTICES IN THE NONLINEAR REGIME?

BARTELS CONJECTURED THAT THERE IS A UNIVERSAL "THREE-POMERON VERTEX"

$$\delta H = \int V(xy,uv,wz)[P(xy)P(uv)P^\dagger(wz) + h.c.]$$

A LITTLE CONFUSING: BARTELS INTRODUCED DIFFERENT (BUT RELATED) QUANTITIES

$$D^n(x_1...x_n) = \langle \hat{\rho}(x_1)...\hat{\rho}(x_n)\rangle_{photon=dipole}$$

$\hat{\rho}(x)$ - COLOR CHARGE DENSITY OPERATOR

$$\frac{dD^2(xy)}{dY} = \bar{K}(xy, uv)D^2(uv); \qquad \bar{\mathrm{K}} \ - \ \text{BFKL up to kinematical factor}$$

THEN

$$\frac{dD^4_{IRREDUCIBLE}}{dY} = \bar{K}D^4_{IRREDUCIBLE} + VD^2$$

POSSIBLE CONJECTURE: V IS UNVERSAL

$$\frac{dD^6_{IRREDUCIBLE}}{dY} = \bar{K}D^6_{IRREDUCIBLE} + VD^4(?) + (\text{NO } D^2\text{TERM})(?)$$

REGGEIZATION : $D^4_{IRREDUCIBLE} = D^4 - \#D^2$ - WHAT IS IT?

NO RULE HOW TO DEFINE $D_{IRREDUCIBLE}$ GIVEN, BUT IT FEELS THAT SOMETHING HAS TO BE UNIVERSAL...

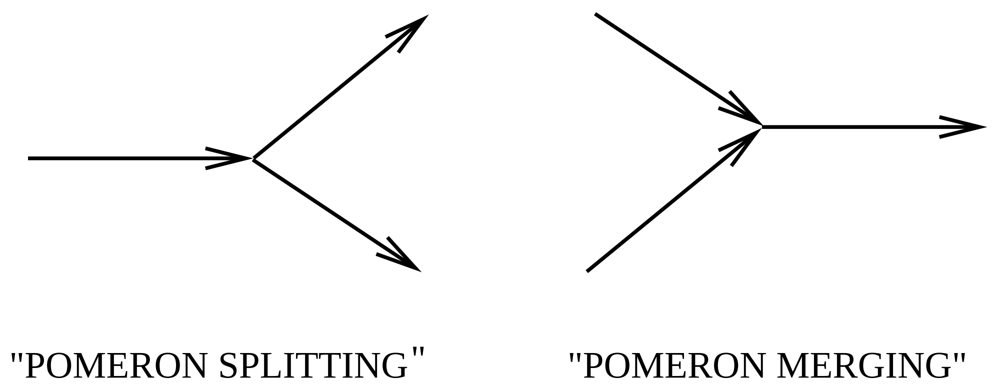
JIMWLK AND FRIENDS

PARALLEL DEVELOPEMENT FOCUSED ON THE IDEA OF PERTURBATIVE SATURATION: GRIBOV-LEVIN RYSKIN; MUELLER'S DIPOLE MODEL; MCLERRAN-VENUGOPALAN MODEL

BALITSKY-JIMWLK-KOVCHEGOV

KLWMIJ EVOLUTION- dilute projectile on dense target; "'Pomeron splittings"' but no "'Pomeron mergings"' - those are separate story.

$$PPP^{\dagger} \text{ vs } P^{\dagger}P^{\dagger}P$$



KLWMIJ IS A FIELD THEORY FOR RAPIDITY EVOLUTION.

THE RULES OF THE GAME:

S-MATRIX:

$$<< S >>= \langle \int d\rho\delta(\rho)W[\delta/\delta\rho]\exp\left\{\int_x i\rho^a(x)\alpha^a(x)\right\}\rangle_\alpha$$

IN FACT, ANY OBSERVABLE:

$$<< O >>= \langle \int d\rho\delta(\rho)W[\delta/\delta\rho]O[\rho,\alpha]\rangle_\alpha$$

W - "'WAVE FUNCTION OF THE PROJECTILE"'; α - COLOR FIELD OF THE TARGET (NOT IMPORTANT HERE - DOES NOT EVOLVE).

E.G. DIPOLE PROJECTILE:

$$W = \frac{1}{N_c}tr[R^\dagger(x)R(y)]; \qquad R(x) = \exp\{T^a\delta/\delta\rho^a(x)\}$$

$R_{\alpha\beta}(x)$ - SCATTERING MATRIX OF A PROJECTILE QUARK -

BASIC FIELD THEORETICAL DEGREE OF FREEDOM

EVOLUTION:

$$<< O >>_Y = \langle \int d\rho \delta(\rho) W_Y[R] O[\rho, \alpha] \rangle_\alpha$$

WHERE

$$\frac{dW}{dY} = -H_{KLWMIJ}[R, J] W[R]$$

WITH

$$H_{KLWMIJ} = \frac{\alpha_s}{2\pi^2} \int_{x,y,z} \frac{(x-z)_i (y-z)_i}{(x-z)^2 (y-z)^2} \left\{ J_L^a(x) J_L^a(y) + J_R^a(x) J_R^a(y) - 2 J_L^a(x) R_z^{ab} J_R^b(y) \right\}$$

THE LEFT AND RIGHT SU(N) GENERATORS:

$$J_L^a(x) = tr\left[\frac{\delta}{\delta R_x^T} T^a R_x\right] - tr\left[\frac{\delta}{\delta R_x^*} R_x^\dagger T^a\right]$$

$$J_R^a(x) = tr\left[\frac{\delta}{\delta R_x^T} R_x T^a\right] - tr\left[\frac{\delta}{\delta R_x^*} T^a R_x^\dagger\right]$$

A BONA FIDE QUANTUM FIELD THEORY OF A UNITARY MATRIX $R(x)$.

PROJECTING KLWMIJ ONTO RFT

THIS IS A FIELD THEORY, BUT NOT QUITE A FIELD THEORY OF REGGEONS. REGGEONS ARE PHYSICAL AMPLITUDES - COLOR SINGLETS.

Q: CAN WE PROJECT H_{KLWMIJ} ONTO COLOR SINGLETS AND DELIVER THE QCD REGGEON FIELD THEORY?

FIRST - CHOOSE EFFECTIVE DEGREES OF FREEDOM. AS IN ANY EFFECTIVE THEORY IS INFORMED BY THE SYMMETRIES.

$SU_L(N) \times SU_R(N)$ - ALL MUST BE SCALARS.

CHARGE CONJUGATION Z_2: $R(x) \rightarrow R^*(x)$

TIME REVERSAL (SIGNATURE) Z_2: $R(x) \rightarrow R^\dagger(x)$

NATURAL CONDITION: IN THE LINEAR REGIME $(R = 1 - \alpha_s + \ldots)$ SHOULD REDUCE TO THE BKP REGGEONS.

THERE IS INFINITE NUMBER OF INDEPENDENT COLOR SINGLET OBSERVABLES :(

BUT THERE IS A NATURAL HIERARCHY :)

DIPOLE: $d(x,y) = \frac{1}{N_c}\mathrm{Tr}[R(x)R^\dagger(y)]$

QUADRUPOLE: $Q(x,y,u,v) = \frac{1}{N_c}\mathrm{Tr}[R(x)R^\dagger(y)R(u)R^\dagger(v)]$

NATURALLY DECOMPOSE INTO

POMERON: - C, T EVEN $P(1,2) = \frac{1}{2}[2 - d(1,2) - d(2,1)]$

ODDERON: - C, T ODD $O(1,2) = \frac{1}{2}[d(1,2) - d(2,1)]$

B-REGGEON: C,T EVEN, PERTURBATIVELY ORTHOGONAL TO P

$B_{1,2,3,4} = \frac{1}{4}\left[4 - Q_{1,2,3,4} - Q_{4,1,2,3} - Q_{3,2,1,4} - Q_{2,1,4,3}\right] - [P_{12} + P_{14} + P_{23} + P_{34} - P_{13} - P_{24}$

OTHER 'ONS

C ODD, T EVEN: $C_{1,2,3,4} = \frac{1}{4}\left[Q_{1,2,3,4} + Q_{4,1,2,3} - Q_{3,2,1,4} - Q_{2,1,4,3}\right]$

T ODDS: $D^{\pm}_{1,2,3,4} = \frac{1}{4}\left[Q_{1,2,3,4} - Q_{4,1,2,3}\right] \pm \frac{1}{4}\left[Q_{3,2,1,4} - Q_{2,1,4,3}\right]$

CAN CONTINUE TO HIGHER MULTIPOLES, BUT WILL STOP HERE

THE RFT HAMILTONIAN

FROM HERE ON WE WORK AT LARGE N_C.

HOW DO WE FIND H_{RFT}?

ASSUME

$$W[R] = \tilde{W}[P, O\ldots]$$

NOW ACT ON IT WITH H_{KLWMIJ} AND DISCARD SUBLEADING TERMS IN N_C.

THE EVOLUTION EQUATIONS FOR P AND O HAVE BEEN KNOWN FOR A WHILE - THIS IS JUST THE BK EQUATION (OR KOVCHEGOV EQUATION, OR THE FIRST EQUATION IN BALITSKY'S HIERARCHY)

$$\frac{d}{dY}P_{x,y} = \frac{\bar{\alpha}_s}{2\,\pi}\int_z \frac{(x\,-\,y)^2}{(x\,-\,z)^2\,(y\,-\,z)^2}\,[P_{x,z} + P_{z,y}\,-\,P_{x,y} - P_{x,z}P_{z,y} - O_{x,z}O_{z,y}];$$

$$\frac{d}{dY}O_{x,y} = \frac{\bar{\alpha}_s}{2\,\pi}\int_z \frac{(x\,-\,y)^2}{(x\,-\,z)^2\,(y\,-\,z)^2}\,[O_{x,z} + O_{z,y}\,-\,O_{x,y} - O_{x,z}P_{z,y} - P_{x,z}O_{z,y}]$$

AT LARGE N_C THIS IS EQUIVALENT TO

$$H_{KLWMIJ}W[P,O] = -\frac{d}{dY}W[P,O] = -\int_{x,y}\frac{d}{dY}P_{x,y}\frac{\delta}{\delta P_{x,y}}W + \frac{d}{dY}O_{x,y}\frac{\delta}{\delta O_{x,y}}W$$

AND SO

$$H_{KLWMIJ} = H_P + H_O$$

$$H_P = -\frac{\bar{\alpha}_s}{2\,\pi}\int_{x,y,z}\frac{(x\;-\;y)^2}{(x\;-\;z)^2\,(y\;-\;z)^2}\Big\{\left[P_{x,z}+P_{z,y}-P_{x,y}-P_{x,z}P_{z,y}-O_{x,z}O_{z,y}\right]P^{\dagger}_{x,y}\Big\}$$

$$H_O = -\frac{\bar{\alpha}_s}{2\,\pi}\int_{x,y,z}\frac{(x\;-\;y)^2}{(x\;-\;z)^2\,(y\;-\;z)^2}\Big\{\left[O_{x,z}+O_{z,y}-O_{x,y}-O_{x,z}P_{z,y}-P_{x,z}O_{z,y}\right]O^{\dagger}_{x,y}\Big\}$$

ALLOWING DEPENDENCE ON B AND C (AS WE SHOULD) GETS US

$$\delta H = H_B + H_C$$

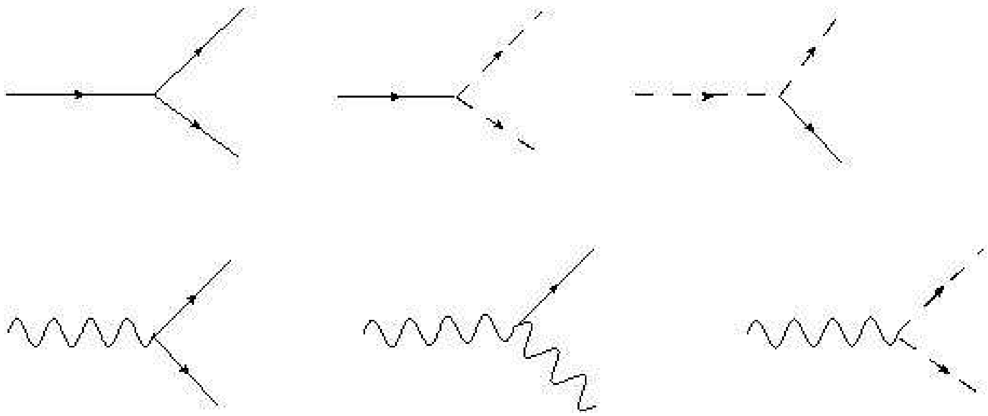
$$H_C = -\frac{\bar{\alpha}_s}{2\,\pi}\int_{x,y,u,v,z}\Bigg\{\Big[-\left[M_{x,y;z}+M_{u,v;z}-L_{x,u,v,y;z}\right]C_{xyuv}+4L_{x,v,u,v;z}C_{xyuz}\Big]C^{\dagger}_{xyuv}$$
$$-4L_{x,v,u,v;z}C_{xyuz}P_{zv}C^{\dagger}_{xyuv}-4L_{x,v,u,v;z}D^{-}_{xyuz}O_{zv}C^{\dagger}_{xyuv}\Bigg\}$$

$$H_B = -\frac{\bar{\alpha}_s}{2\,\pi}\int_{xyuvz}\Bigg\{\Big[-\left[M_{x,y;z}+M_{u,v;z}-L_{x,u,v,y;z}\right]B_{xyuv}+4L_{x,v,u,v;z}B_{xyuz}\Big]B^{\dagger}_{xyuv}$$
$$-2L_{x,y,u,v;z}\Big[P_{xv}P_{uy}+O_{xv}O_{uy}\Big]B^{\dagger}_{xyuv}-2P_{xz}P_{yz}\Big[2L_{x,y,u,v;z}B^{\dagger}_{xyuv}-\Big(L_{x,u,y,v;z}+L_{x,v,y,u;z}\Big)B^{\dagger}_{xuyv}\Big]$$
$$-4P_{xz}P_{yu}\Big[2L_{x,y,x,v;z}B^{\dagger}_{xyuv}-L_{x,y,x,u;z}B^{\dagger}_{xyvu}\Big]-4B_{xyuz}P_{zv}L_{x,v,u,v;z}B^{\dagger}_{xyuv}$$
$$-4D^{+}_{xyuz}O_{zv}L_{x,v,u,v;z}B^{\dagger}_{xyuv}\Bigg\}$$

AT LEADING N_C WE HAVE A VARIETY OF VERTICES - BUT ALL OF THEM HAVE THE NATURE OF SPLITTING: ONE REGGEON TURNS INTO TWO

$$PPP^\dagger, \quad OOP^\dagger; \quad POO^\dagger; \quad PPB^\dagger; \quad BPB^\dagger; \quad CPC^\dagger \ \ldots$$

AND ANYTHING ALLOWED BY THE SYMMETRIES



INTERESTINGLY AT SUBLEADING N_C ONE GETS ALSO MERGING VERTICES

$$\frac{1}{N_c^2} BP^\dagger P^\dagger \ \ etc.$$

REGGEONS AND BARTELS' D'S

THE D-FUNCTIONS ARE NOT EXACTLY REGGEONS. THEY ARE CORRELATIONS OF COLOR CHARGE DENSITIES - SO CLOSELY RELATED TO CONJUGATE REGGEONS.

WHAT IS $P^\dagger$ EXCEPT FOR FORMALLY DEFINED FUNCTIONAL DERIVATIVE?

NOT TOO DIFFICULT TO IDENTIFY, SINCE COLOR CHARGE DENSITIES ARE $SU(N)$ ROTATION GENERATORS

$$\frac{1}{2N_c}\left[J^a_L(1)J^a_L(2)+J^a_R(1)J^a_R(2)\right]=\left[1-P_{12}\right]P^\dagger_{12}-O_{12}O^\dagger_{12}-\delta_{12}\int_x\left[\left[1-P_{1x}\right]P^\dagger_{1x}-O_{1x}O^\dagger_{1x}\right]$$

FROM HERE FOR BARTELS' FUNCTIONS WE HAVE:

$$D^2(1,2)\approx\left[1-P_{12}\right]P^\dagger_{12}$$

$$D^4(1,2,3,4)\approx D^2(1,2)D^2(3,4)=P^\dagger_{12}P^\dagger_{34}-\delta_{12,34}P^\dagger(1,2)$$

THE LAST TERM IS PRECISELY THE REGGEIZATION OF BARTELS ET.AL.

"'REGGEIZED"' TERMS ARE EXACTLY EXTRA TERMS IN THE RELATION OF D^{2n} AND $(P^\dagger)^n$

THE BARTELS VERTEX V IS THE SAME AS APPEARS IN THE KLWMIJ-RFT HAMILTONIAN $PPP^\dagger$ TERM.

EQUATIONS FOR $D^n_{IRREDUCIBLE}$ ARE EXACTLY EQUATIONS FOR $P^\dagger P^\dagger$, $B^\dagger$, ETC...

CONSEQUENCE: THERE WILL BE OTHER VERTICES (ALTHOUGH SIMILAR IN STRUCTURE) IN THE EVOLUTION OF D^6, D^8 ETC - THEY CAN ALL BE READ OFF THE HAMILTONIAN.

(THIS IS SIMPLIFIED SOMEWHAT, BUT BASICALLY TRUE)

IT IS NICE THAT ONCE WE HAVE CHOSEN OUR BASIC VARIABLES, THERE IS NO AMBIGUITY ANYWHERE. ALL REGGEIZATION CORRECTIONS AND VERTICES ARE UNIQUELY DETERMINED.

CONCLUSIONS

KLWMIJ IS INDEED THE REGGEON FIELD THEORY, OR AT LEAST HALF OF IT. IT ONLY CONTAINS REGGEON SPLITTINGS. WE KNEW THAT - WE NEED TO INTEGRATE IT WITH JIMWLK INTO A SINGLE HAMILTONIAN IN ORDER TO SEE ALL VERTICES CONSISTENTLY.

ACTUALLY AT SUBLEADING ORDER REGGEON MERGINGS ARE PRESENT EVEN IN KLWMIJ!

EVEN IN LEADING ORDER IN N_C IT CONTAINS MANY REGGEONS - NOT ONLY THE POMERON.

ARE OTHERS IMPORTANT?

CERTAINLY SOME OF THEM GIVE LEADING CONTRIBUTIONS TO SOME OBSERVABLES: ODDERON TO C-ODD AMPLITUDES, B-REGGEON TO PARTICLE PRODUCTION.

MAYBE THIS SET CAN BE TRUNCATED, BUT NOT ON THE BASIS OF $1/N_c$ EXPANSION.