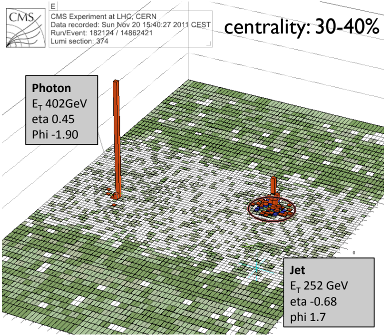
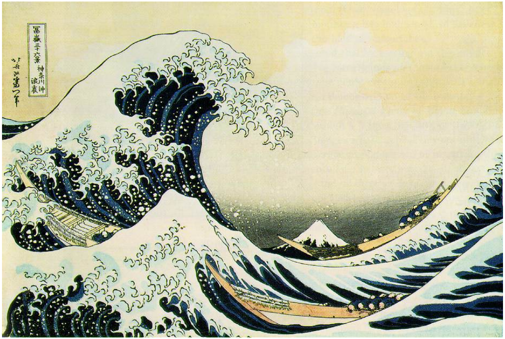
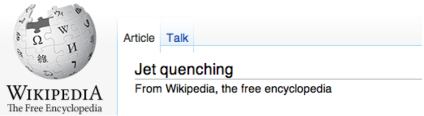
From Jet Quenching to Turbulence

Edmond Iancu
IPhT Saclay & CNRS





With due respect to Wikipedia ...



- *Collisions of ultra-relativistic heavy-ion beams create a hot and dense medium comparable to the conditions in the early universe, and then these jets interact strongly with the medium, leading to a marked reduction of their energy. This energy reduction is called jet quenching.*

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Article Talk

Jet quenching

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- *Collisions of ultra-relativistic heavy-ion beams create a hot and dense medium comparable to the conditions in the early universe, and then these jets interact strongly with the medium, leading to a marked reduction of their energy. This energy reduction is called jet quenching.*

- A familiar concept, hiding a complex reality

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Wave turbulence

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- *Wave turbulence is a set of waves deviated far from thermal equilibrium. Such state is accompanied by dissipation. It is either decaying turbulence or requires external source of energy to sustain it. Examples are waves on a fluid surface excited by winds or ships, and waves in plasma excited by electromagnetic waves etc. The wave system can be described by kinetic equations and their stationary solutions called Kolmogorov-Zakharov (KZ) energy spectra. They have the form* $1/k^{\nu}$*, with* k *the wavenumber and* ν *a positive constant depending on the specific wave system. The form of KZ-spectra does not depend on the initial magnitude of the total energy or on the details of initial energy distribution over the modes. Only the fact the energy is conserved at some inertial interval is important.*

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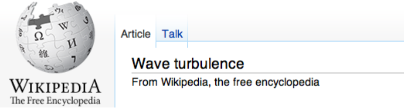

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- A rather elusive concept, with an unambiguous mathematical signature

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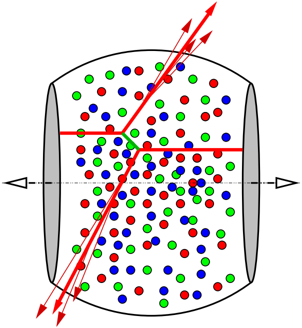
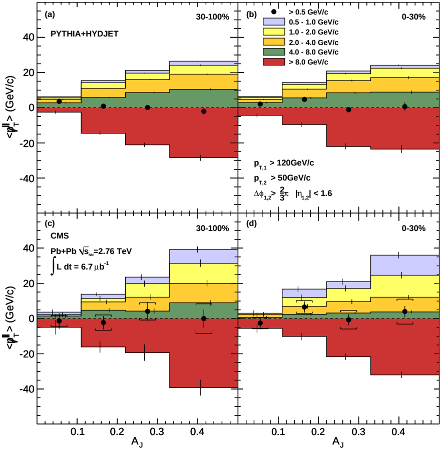
- *V. Zakharov, V. L´vov, and G. Falkovich, Kolmogorov Spectra of Turbulence, Wave Turbulence (Springer-Verlag, 1992)*

Jet quenching & Wave turbulence

- Jet quenching in pQCD is wave turbulence with KZ power $\nu = 1/2$

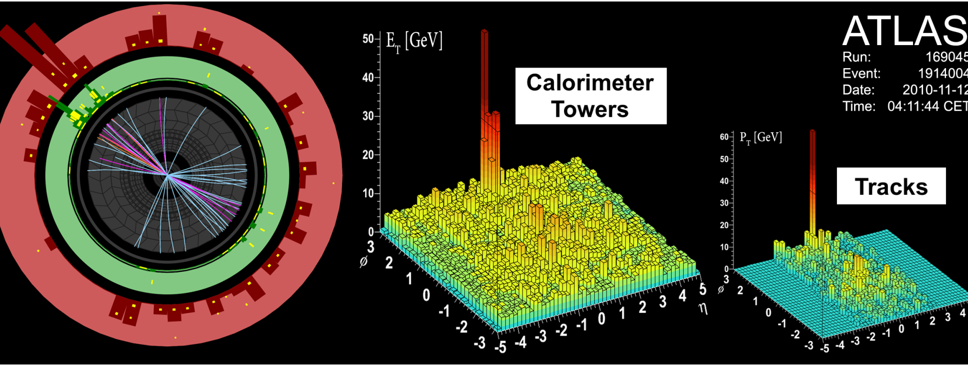




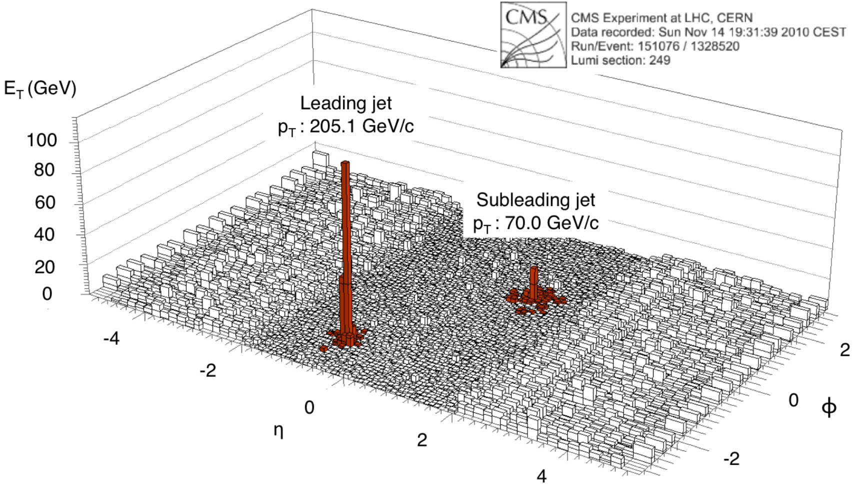
- Conceptually interesting & useful for the phenomenology

  new insight into some remarkable features of the di–jet asymmetry

Di–jet asymmetry *(ATLAS)*



- Central Pb+Pb: 'mono–jet' events
- The secondary jet cannot be distinguished from the background: $E_{T1} \geq 100$ GeV, $E_{T2} > 25$ GeV
- Additional energy imbalance as compared to p+p : 20 to 30 GeV

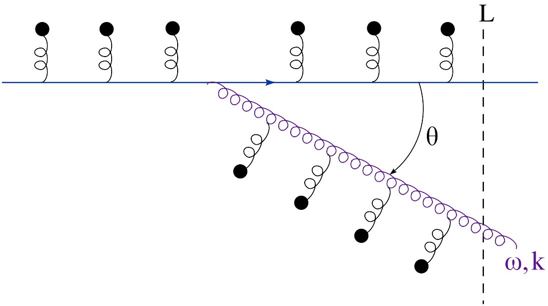


- Detailed studies show that the 'missing energy' is associated with the additional radiation of many soft quanta at large angles *(cf. the talks by Guilherme Milhano, Brian Cole and Yen–Jie Lee)*

pQCD : the BDMPSZ mechanism

- Gluon radiation triggered by interactions in the medium

  Baier, Dokshitzer, Mueller, Peigné, Schiff, Zakharov ∼ 1996



- Gluon emission is linked to transverse momentum broadening

$$\Delta k_\perp^2 \simeq \hat{q}\,\Delta t \quad \text{with} \quad \hat{q} \simeq \frac{m_D^2}{\lambda} = \frac{(\text{Debye mass})^2}{\text{mean free path}}$$

  - destroys the coherence between the gluon and its parent parton
  - increases the emission angle

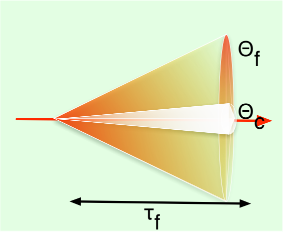
Formation time (τ_f) & angle (θ_f)

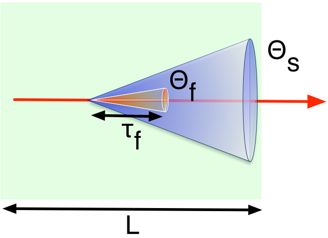
$$\tau_f \simeq \sqrt{\frac{\omega}{\hat{q}}} \qquad \theta_f \equiv \frac{\Delta k_\perp}{\omega} \simeq \left(\frac{\hat{q}}{\omega^3}\right)^{1/4}$$

- Maximal ω for this mechanism : $\tau_f \simeq L \;\Rightarrow\; \omega_c = \hat{q}L^2$
- The BDMPSZ gluon spectrum (probability for one gluon emission)

$$\omega\,\frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq \alpha_s\,\frac{L}{\tau_f(\omega)} \simeq \alpha_s\sqrt{\frac{\omega_c}{\omega}}$$

- Soft gluons (small ω) : short formation times & large emission angles





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- Relatively hard emissions with $\omega \sim \omega_c$:
  - rare events : probability of $\mathcal{O}(\alpha_s)$
  - control energy loss by leading particle: $E = \int_\omega \omega\, (\mathrm{d}N/\mathrm{d}\omega) \sim \alpha_s \omega_c$
  - small angles though ($\theta_f \sim \theta_c$) $\Longrightarrow$ the energy remains inside the jet
  - arguably, not so important for the di–jet asymmetry

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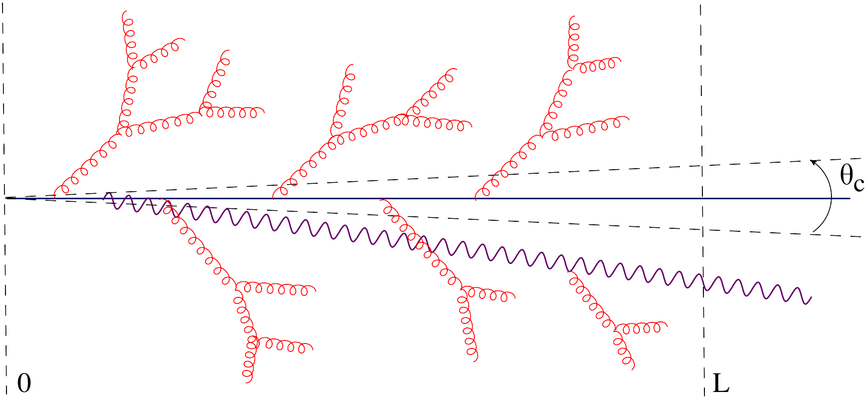
- Soft emissions with $\omega \ll \omega_c$:
  - quasi–deterministic : probability of $\mathcal{O}(1)$ for $\omega \lesssim \alpha_s^2\, \omega_c \sim 5\,$GeV
  - less energy is lost in this way : $E_{\rm soft} \sim \alpha_s^2 \omega_c$
  - ... but this can be lost at arbitrarily large angles
  - potentially relevant for the di–jet asymmetry

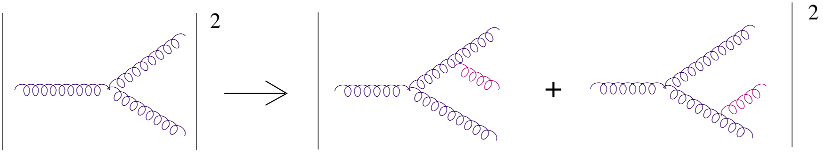
A typical cascade



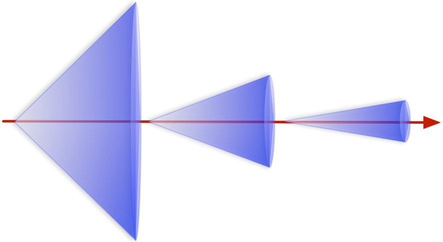
- One needs to understand multiple medium–induced branchings
- A rain of soft gluons ($\omega \ll \omega_c$) plus (sometimes) a harder one ($\omega \sim \omega_c$)

Multiple emissions : vacuum

- Successive medium–induced branchings are independent
- Non–trivial ! Not true for jet evolution in the vacuum !

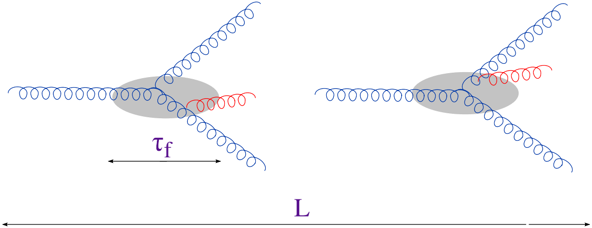


- In vacuum, interference effects lead to angular ordering



Multiple emissions : medium

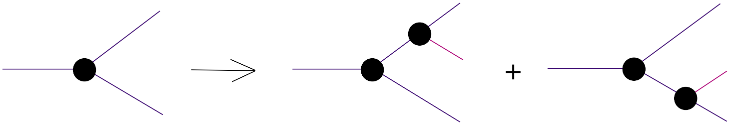
- In the medium, color coherence is rapidly lost via rescattering
  Mehtar-Tani, Salgado, Tywoniuk (arXiv: 1009.2965; 1102.4317); Casalderrey-Solana, E. I. (arXiv: 1106.3864)



- The interference effects are suppressed by a factor $\tau_f/L \ll 1$
  Blaizot, Dominguez, E.I., Mehtar-Tani (arXiv: 1209.4585)
- A correlator of 4 adjoint Wilson lines ('quadrupole') describing 2 correlated gluons factorizes into 2 'dipoles' (= independent gluons)

A classical branching process

- Medium–induced jet evolution $\approx$ a classical branching process
  - a Markovian process in $D = 3 + 1$: ω, $k_\perp$, time t (or medium size L)



- Successive branchings are independent and quasi–local ($\tau_f \ll L$)
  - the $g \to gg$ splitting vertex (the 'blob') : the BDMPSZ spectrum
  - the propagator (the 'line') : transverse momentum broadening
- Here, I will restrict myself to the 1+1 process involving (ω, t)

  (see the talk by Yacine for the full 3+1 process and its surprises)
- Previously conjectured and used for phenomenological studies

  (Baier, Mueller, Schiff, Son '01; Jeon, Moore '05; MARTINI ; Q-PYTHIA)

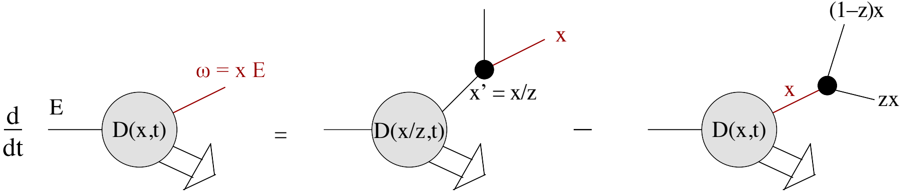
The spectrum from multiple branchings

(J.-P. Blaizot, E. I., Y. Mehtar-Tani, arXiv: 1301.6102)

- Evolution equation for the gluon spectrum (integrated over $k_\perp$)

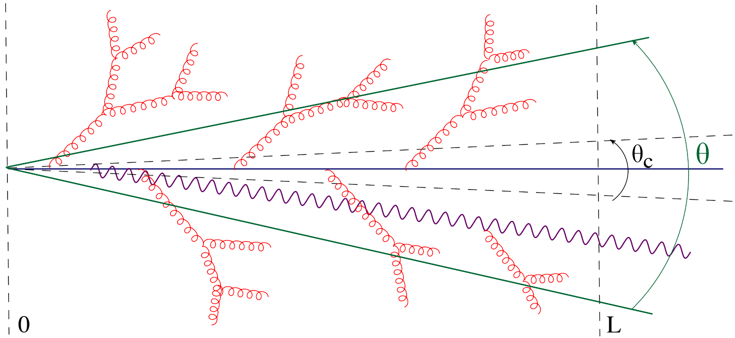
$$D(x,t) \equiv x\frac{\mathrm{d}N}{\mathrm{d}x} \quad \text{where} \quad x = \frac{\omega}{E} \quad \text{(energy fraction)}$$

- t : the time/distance traveled by the jet inside the medium



- $t \to t + \mathrm{d}t$: one additional branching with splitting fraction z
- Rate for change = 'Gain' - 'Loss'
- Formally similar to DGLAP ... but different kernel & physics !

Some interesting questions



- How much of the jet energy goes outside a given angle θ ?

  (the actual energy lost by the 'jet', according to Guilherme's 'jet' definition)

- How is this energy distributed among the emitted quanta ?
- Exploit the correlation between energy (x) and the emission angle (θ_f)

BDMPSZ spectrum revisited

- First iteration $\Longrightarrow$ BDMPSZ spectrum by the leading particle

$$D^{(1)}(x,t) \simeq \alpha_s \frac{L}{\tau_f(\omega)} = \frac{t}{\sqrt{x}} \qquad (t = L \text{ in appropriate units})$$

- Energy fraction transported at large angles $\theta_f > \theta_0$ via BDMPSZ

$$\mathcal{E}^{(1)}(\theta_f > \theta_0, t) = \int_0^{x_0} \mathrm{d}x D^{(1)}(x,t) \simeq 2t\sqrt{x_0}\,, \qquad \theta_0 \propto \frac{1}{x_0^{3/4}}$$

- The larger $\theta_0 \Rightarrow$ the smaller $x_0 \Rightarrow$ the smaller the energy loss
- Direct radiation by the leading particle is not very efficient in transporting the jet energy towards large angles
- Will the situation improve after including multiple branching ?

A fake scenario inspired by DGLAP

- Via successive branchings, gluons fall at smaller and smaller values of x
- Naively, energy conservation seems to imply the sum-rule

$$\int_0^1 \mathrm{d}x\, D(x,t) \,=\, 1 \quad \text{for any } t$$

- In particular, this is what happens for the DGLAP evolution !
- If this was true, then with increasing t the spectrum should become steeper and steeper at small x, but in such a way to remain integrable
- The energy fraction contained in the modes with $x < x_0$, that is,

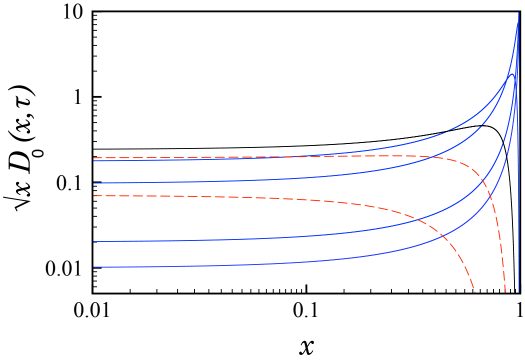
$$\mathcal{E}(x < x_0, t) \,\equiv\, \int_0^{x_0} \mathrm{d}x\, D(x,t)$$

  - would be dominated by the upper limit x_0
  - would rapidly vanish when $x_0 \to 0$ (say, as a power of x_0)
- This would not be very effective in transporting energy at large angles !

The scaling spectrum

- However, this is not what actually happens ! Rather, one finds

$$D(x \ll 1, t) \simeq \frac{t}{\sqrt{x}}\, \mathrm{e}^{-\pi t^2} \qquad \& \qquad \int_0^1 \mathrm{d}x\, D(x,t) = \mathrm{e}^{-\pi t^2}$$



- The spectrum does not get steeper at small x !
- The energy sum–rule is not obeyed !

Where does the energy flow ?

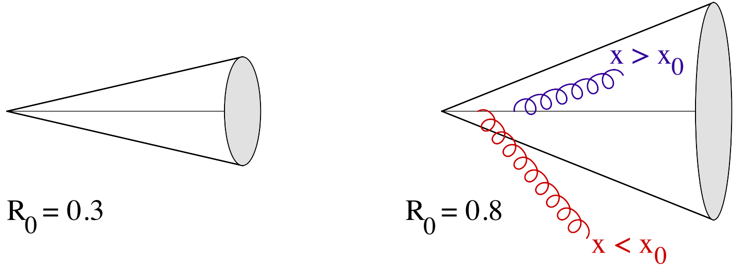
- The scaling spectrum in $\frac{1}{\sqrt{x}}$ is the KZ spectrum for jet quenching
  - a 'fixed point' : fine cancellations between 'gain' and 'loss'
- Via successive branchings, the energy flows from large x to small x without accumulating in any bin $x > 0$
  - formally, it accumulates into a condensate at $x = 0$
  - physically, it goes below $x_{\rm th} = T/E \ll 1$, meaning it thermalizes
- In principle, an arbitrary large fraction of the jet energy can flow out to arbitrarily large angles

$$E_{\rm flow}(t) \equiv 1 - \int_0^1 {\rm d}x\, D(x,t) = 1 - {\rm e}^{-\pi t^2}$$

- In practice, $t = \alpha_s \sqrt{\omega_c/E} \sim 0.3$ for $E = 100$ GeV is not that large
  - $1 - {\rm e}^{-\pi t^2} \sim 0.2 \Rightarrow$ about 20% of the energy is lost at large angles.
  - ... independently of the jet angle & and the medium temperature

Energy flow at large angles

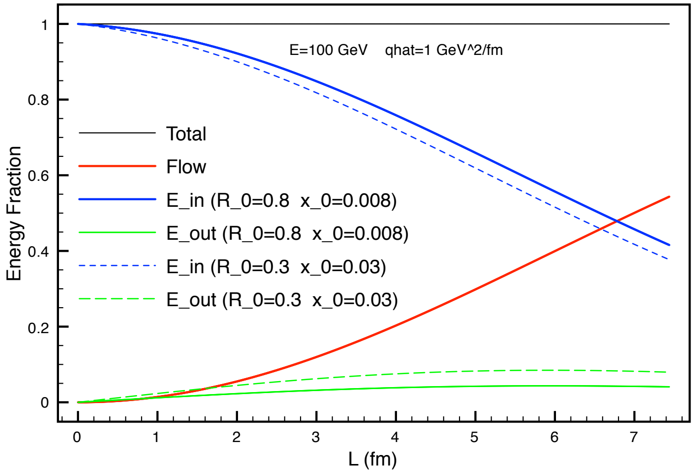
- The energy inside the jet is only weakly dependent upon the jet angular opening R_0, within a wide range of values for R_0



- The energy inside the jet $E_{\rm in}$: the energy in the spectrum at $x > x_0$
- The energy outside the jet : $E_{\rm out}\,(x_{\rm th} < x < x_0) \; + \; E_{\rm flow}$
- The flow component: independent of R_0 and the original energy E

$$E_{\rm flow} = v\,\alpha_s^2\,\hat{q}L^2 \qquad (\sim 20\,\text{GeV for } L = 5\text{ fm})$$

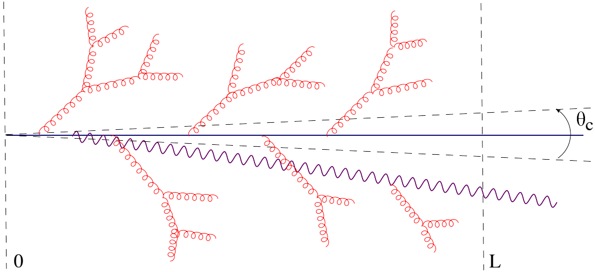
Energy flow at large angles



- Good agreement with the analysis by CMS (arXiv:1102.1957)

Conclusions

- 3+1 space-time picture for medium–induced jet evolution in pQCD
  - hard emissions at small angles (energy loss by leading particle, R_{AA})
  - multiple soft branchings leading to turbulent flow (energy loss at large angles, di–jet asymmetry)



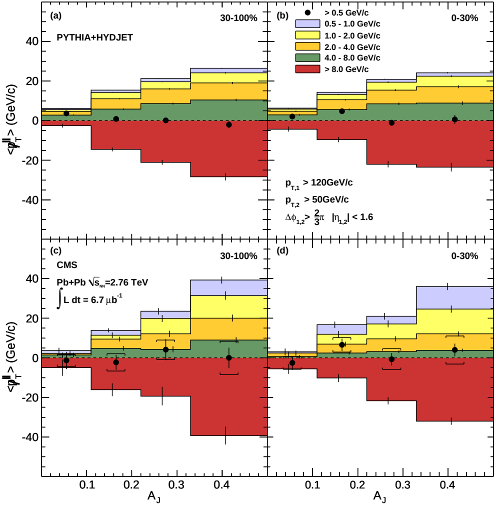
- Anomalously large value for $\hat{q}$ due to pQCD evolution

  (see the next talks by Yacine and by Bin)

No missing energy ! (CMS, arXiv:1102.1957)

- ... but a pronounced difference in the distribution of the total energy in bins of $\omega \equiv p_T$ and in the angle w.r.t. the jet axis

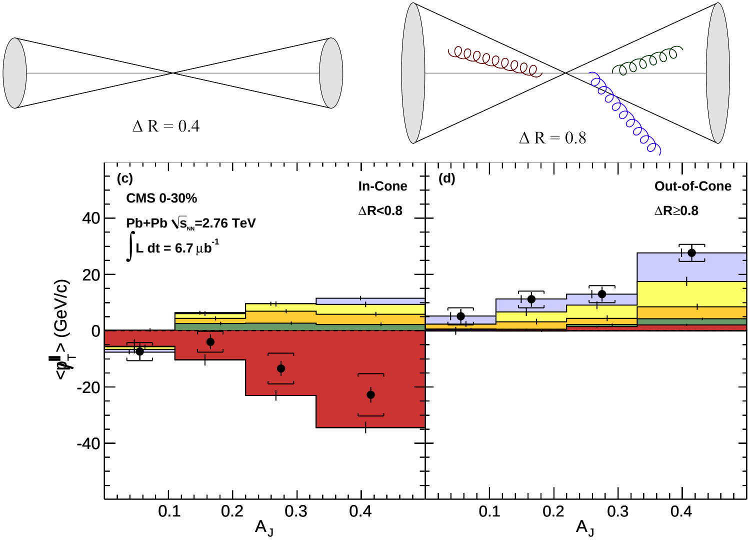
- $p_T^{\parallel}$: projection of the (transverse) energy along the jet axis
- $p_T^{\parallel} < 0$: same hemisphere as the trigger jet
- $p_T^{\parallel} > 0$: same hemisphere as the secondary jet
- all hadrons with $p_T > 0.5$ GeV are measured



- Excess of soft quanta (≤ 4 GeV) in the hemisphere of secondary jet

In–out asymmetry

- Increase the angular opening ΔR of the jet

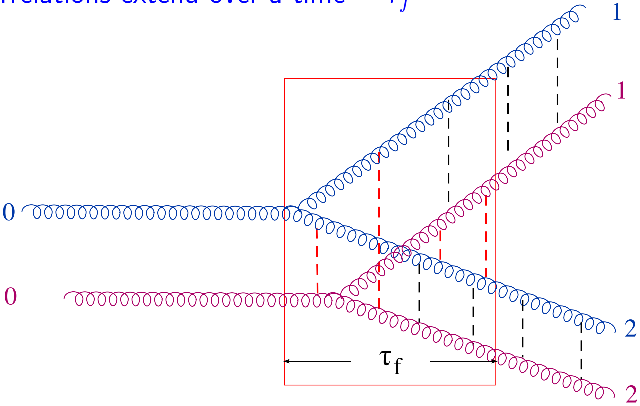


- The soft energy in excess is found at very large angles

Medium–induced decoherence

- The mechanism for decorrelation is the same as in the process of gluon formation: via medium rescattering

  $\Longrightarrow$ correlations extend over a time $\sim \tau_f$

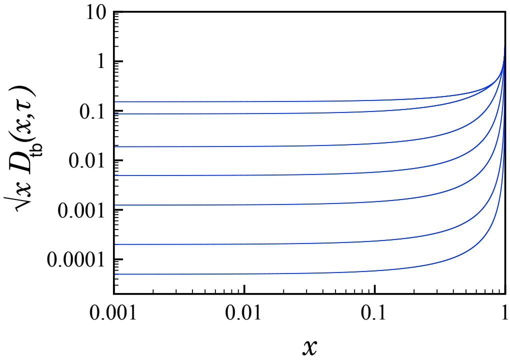


- When $\tau_f \ll L$, their effects are parametrically suppressed ✓
- By the same token, the branching process looks quasi–local

The usual turbulence set-up

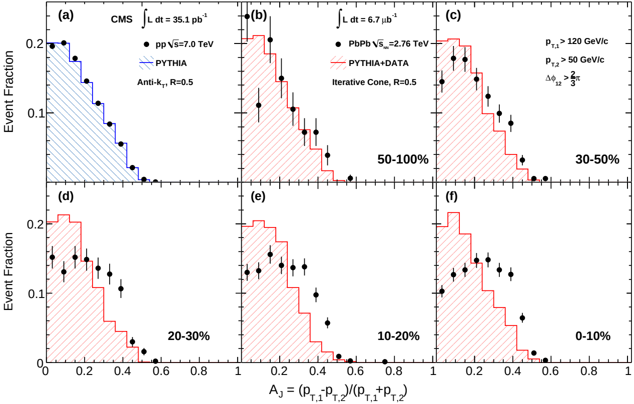
- Steady source at $x = 1$ and sink at $x = x_{\rm th}$ (here $x_{\rm th} = 0$)

$$D_{\rm tb}(x,t) \;=\; \frac{1}{2\pi\sqrt{x(1-x)}}\,\left(1 - {\rm e}^{-\pi\frac{t^2}{1-x}}\right)$$



- The spectrum approaches a steady shape when $\pi t^2 \gtrsim 1$

Di–jet asymmetry : A_J (CMS)

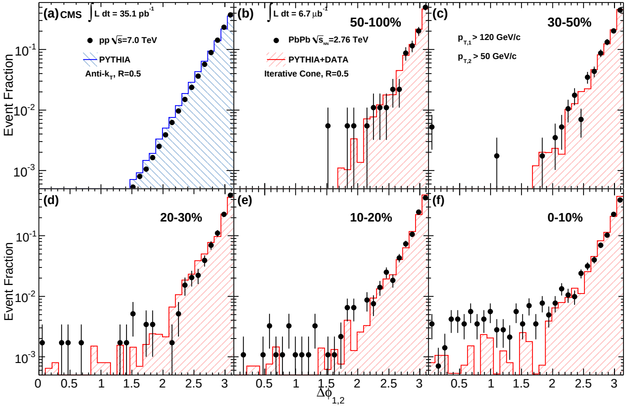


- Event fraction as a function of the di–jet energy imbalance in p+p (a) and Pb+Pb (b–f) collisions for different bins of centrality

$$A_J = \frac{E_1 - E_2}{E_1 + E_2} \qquad (E_i \equiv p_{T,i} = \text{ transverse energy})$$

- Additional energy loss of 20 to 30 GeV due to the medium

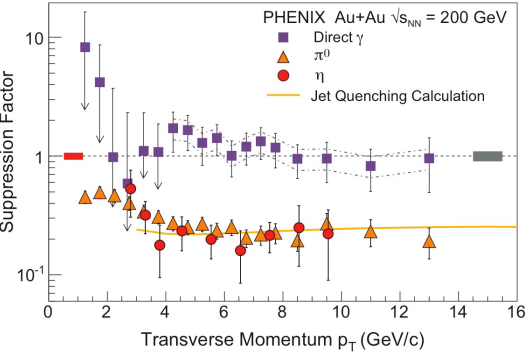
Di–jet asymmetry : $\Delta\phi$ (CMS)

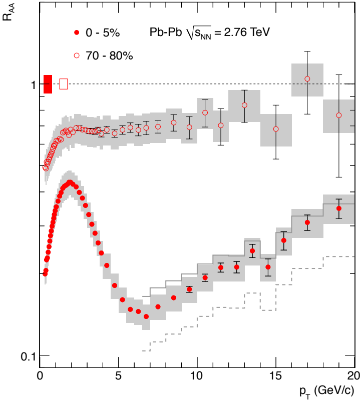


- Event fraction as a function of the azimuthal angle $\Delta\phi$
- Typical event topology: still a pair of back–to–back jets
- The secondary jet loses energy without being deflected
- The additional in–medium radiation is relatively soft

Nuclear modification factor at RHIC & the LHC

$$R_{A+A} \equiv \frac{1}{A^2}\frac{dN_{A+A}/d^2p_\perp d\eta}{dN_{p+p}/d^2p_\perp d\eta}$$



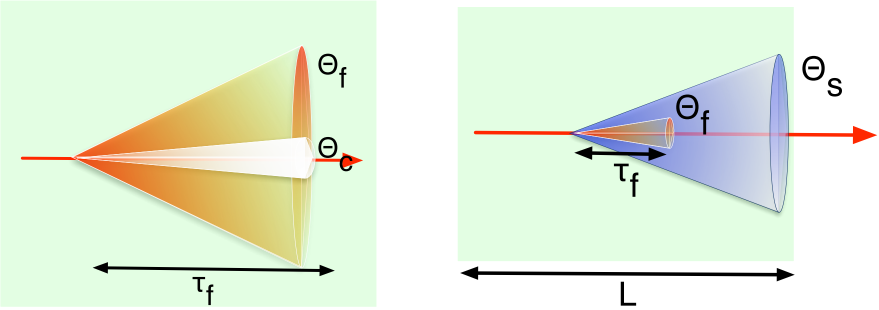


- Strong suppression $(R_{AA} \lesssim 0.2)$ at moderate $p_\perp$
- Probing the energy loss by the leading particle

Formation time (τ_f) & angle (θ_f)

$$\tau_f \simeq \sqrt{\frac{\omega}{\hat{q}}} \qquad \theta_f \equiv \frac{\Delta k_\perp}{\omega} \simeq \left(\frac{\hat{q}}{\omega^3}\right)^{1/4}$$

- Soft gluons (small ω) : short formation times & large emission angles
- Maximal ω for this mechanism : $\tau_f \simeq L \Rightarrow \omega_c = \hat{q}L^2$



- Soft gluons ($\omega \ll \omega_c$) have $\tau_f \ll L$ & $\theta_f \gg \theta_c$

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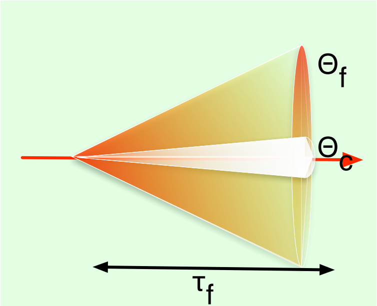

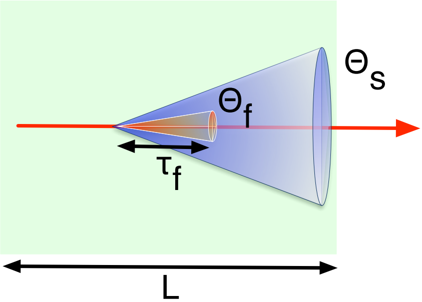

- After emission, the angle can further increase via medium rescattering

Hard vs. soft emissions

- The BDMPSZ gluon spectrum (probability for one gluon emission)

$$\omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq \alpha_s \frac{L}{\tau_f(\omega)} \simeq \alpha_s \sqrt{\frac{\omega_c}{\omega}}$$

- Typical range: $T \simeq 1\,\text{GeV} < \omega \leq \omega_c \simeq 50\,\text{GeV}$

  T ('temperature') : typical momentum scale of the medium ('QGP')

- Relatively hard emissions with $\omega \sim \omega_c$:
  - rare events : probability of $\mathcal{O}(\alpha_s)$
  - control energy loss by leading particle: $E = \int_\omega \omega\,(\mathrm{d}N/\mathrm{d}\omega) \sim \alpha_s \omega_c$
  - small angles though $(\theta_f \sim \theta_c) \Longrightarrow$ the energy remains inside the jet
  - arguably, not so important for the di–jet asymmetry

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- Soft emissions with $\omega \ll \omega_c$:
  - quasi–deterministic : probability of $\mathcal{O}(1)$ for $\omega \lesssim \alpha_s^2\, \omega_c \sim 5\,\text{GeV}$
  - less energy is lost in this way : $E_{\text{soft}} \sim \alpha_s^2 \omega_c$
  - ... but this can be lost at arbitrarily large angles
  - potentially relevant for the di–jet asymmetry

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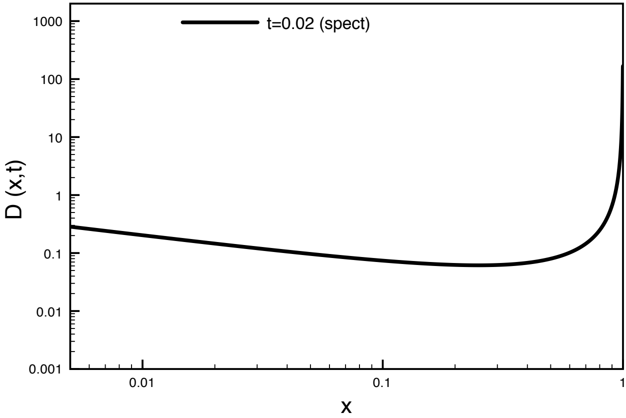
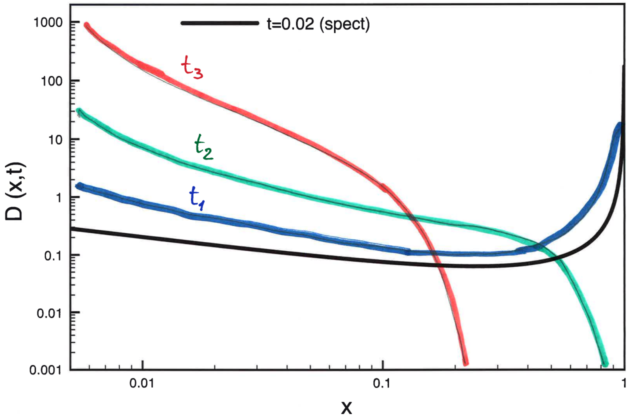
- One needs to understand multiple medium–induced branchings

A fake scenario based on DGLAP (cont.)



- BDMPSZ spectrum at small times ...

A fake scenario based on DGLAP (cont.)



- DGLAP–like spectrum at larger times.
- A DGLAP–like evolution would not be very effective in transporting energy at large angles neither !