Classical YM Dynamics and Turbulence Diffusion

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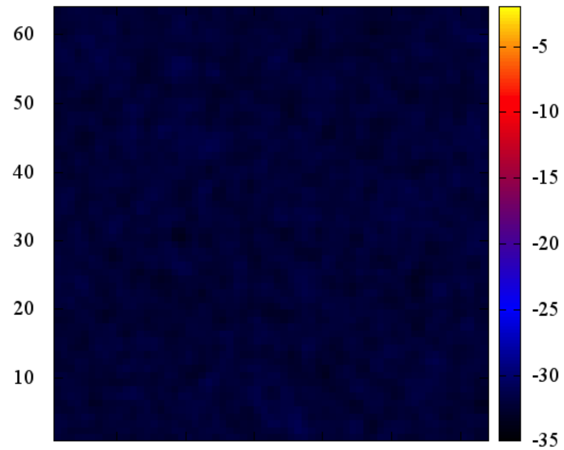
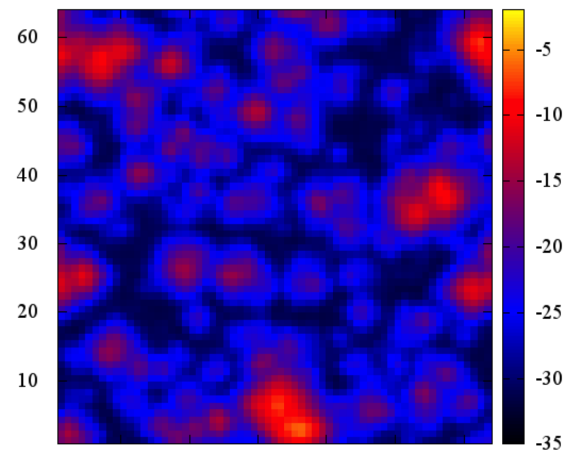
Transverse Pattern Formation

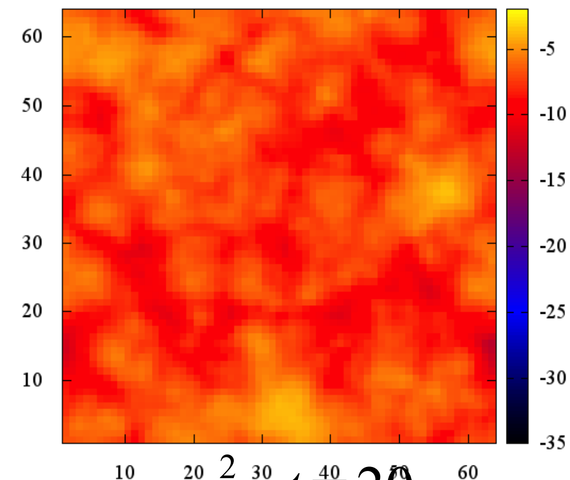
Central Results



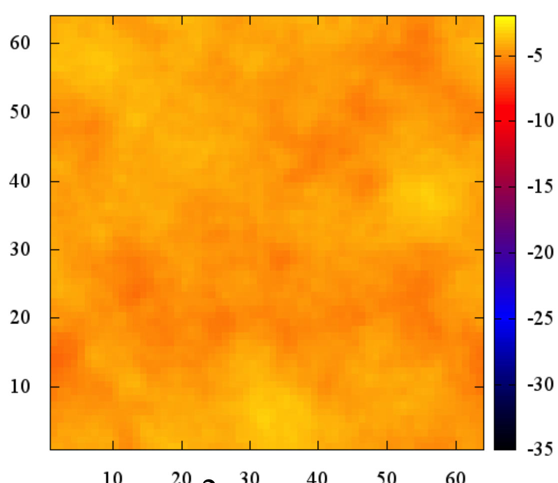
$g^2 \mu t = 0.1$



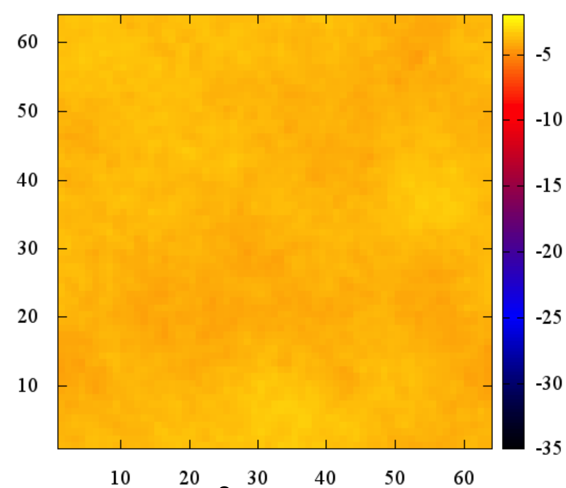
$g^2 \mu t = 10$



$g^2 \mu t = 20$

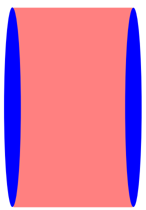


$g^2 \mu t = 30$



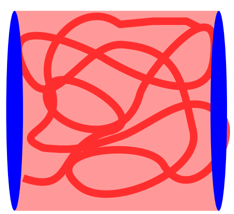
$g^2 \mu t = 40$

Schematic View of Four Regimes



Soft and coherent gluons
Color Glass Condensate (CGC)
Initial (quantum) fluctuations

$\tau < Q_s$
~ 0.1 fm/c

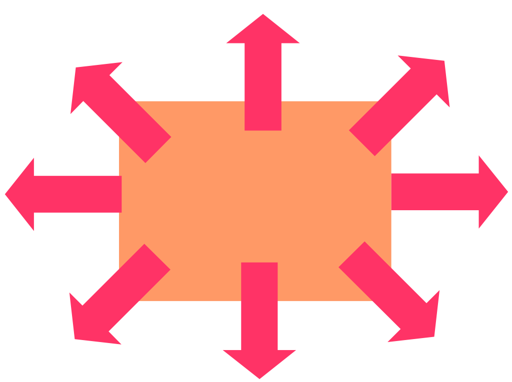


Instabilities $\rightarrow$ (toward) Isotropization
Glass + Plasma = Glasma
Quantum fluctuations
Particle (entropy) production
$\rightarrow$ Thermalization

0.1 fm/c
~ 1 fm/c

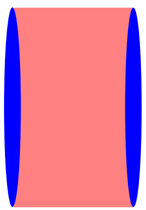
Hydrodynamic evolution + cascade
Relativistic Hydrodynamics

1 fm/c
~ 10 fm/c



Hadronization $\rightarrow$ Observation
Particle yields, distributions

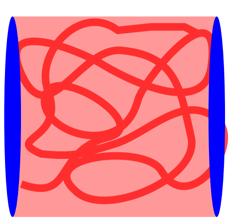
Missing Link



Soft and coherent gluons
Color Glass Condensate (CGC)
Initial (quantum) fluctuations

$\tau < Q_s$
~ 0.1 fm/c

If starting with the CGC what the "theory" predicts?



Instabilities → Isotropization
Glass + Plasma = Glasma
Quantum fluctuations
Particle (entropy) production
→ Thermalization

0.1 fm/c
~ 1 fm/c

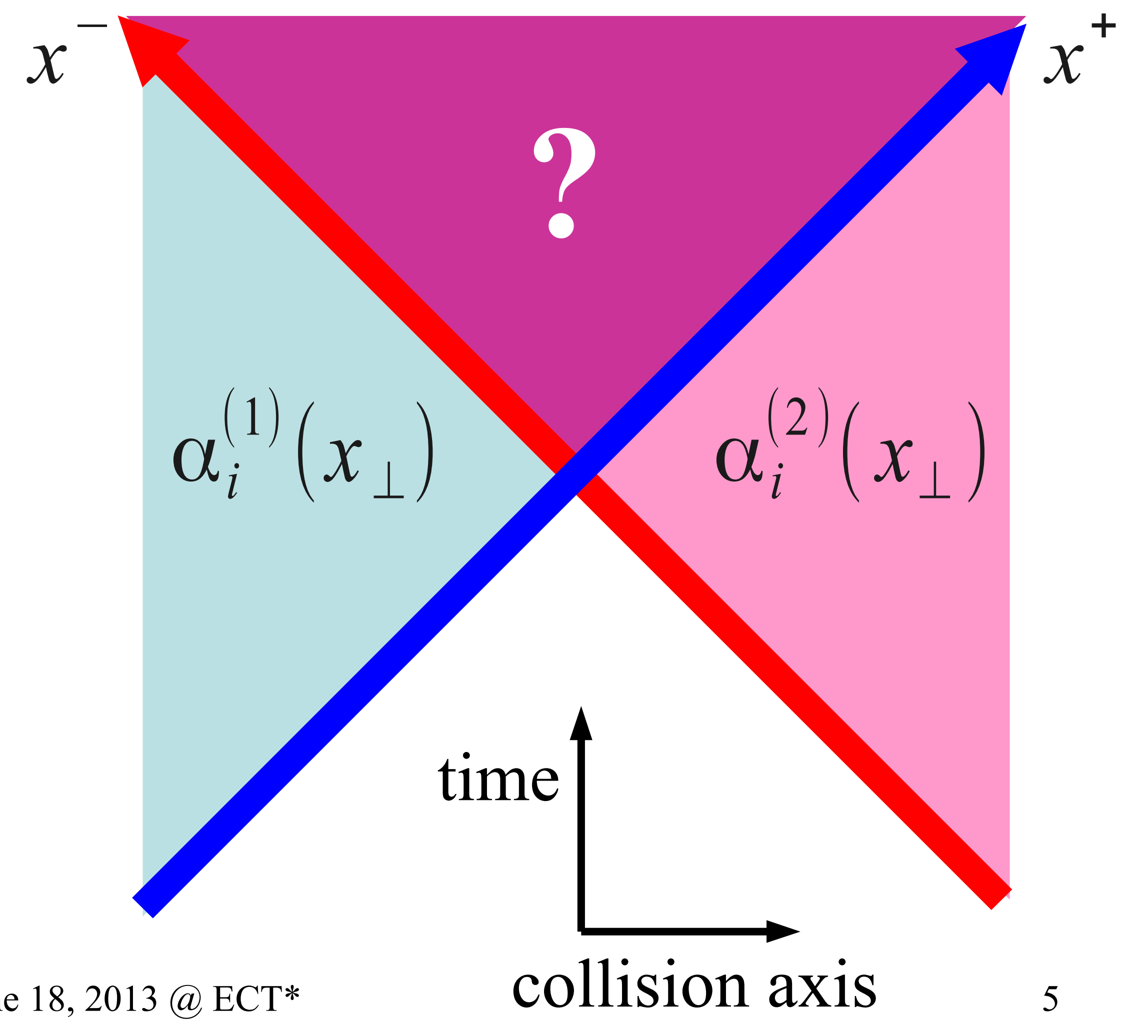
Initial Condition

Fields made by colliding two sources

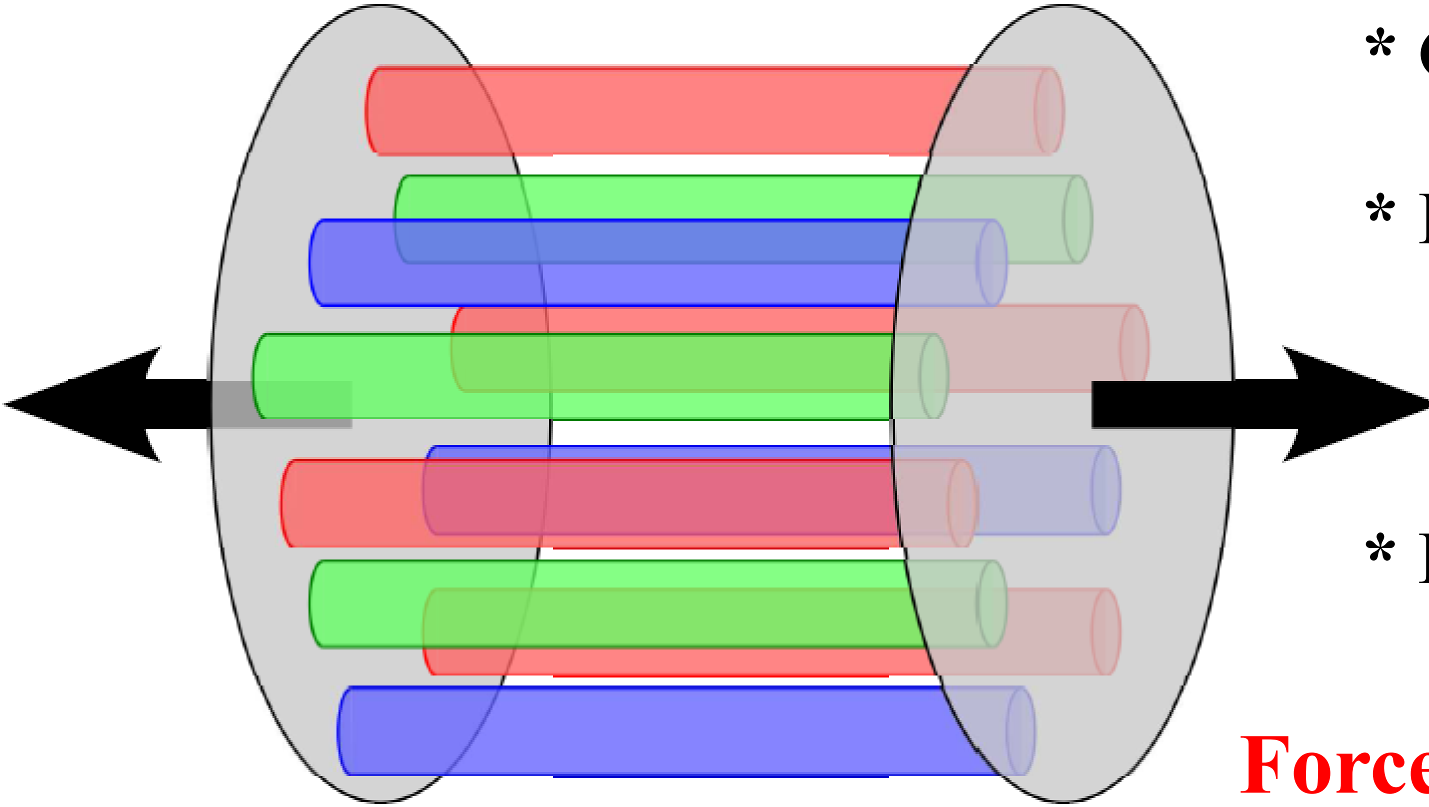
Initial condition is known on the light-cone

$$\mathcal{A}_i = \alpha_i^{(1)} + \alpha_i^{(2)}$$
$$\mathcal{A}_\eta = 0$$
$$\mathcal{E}^i = 0$$
$$\mathcal{E}^\eta = ig\left[\alpha_i^{(1)}, \alpha_i^{(2)}\right]$$

Kovner-McLerran-Weigert (1995)



Intuitive Picture of "Glasma"



*** Boost Invariant**
*** Coherent Fields**
(amp. ~ $1/g$)
*** Flux Tube**
(size ~ $1/Q_s$)

*** Expanding**

Longitudinal $E \sim B$

Force from the tube should be overcome

McLerran-Lappi (2006)

Formulation

Time Evolution

$$E^i = \tau \partial_\tau A_i , \qquad E^\eta = \tau^{-1} \partial_\tau A_\eta$$

$$\partial_\tau E^i = \tau^{-1} D_\eta F_{\eta i} + \tau D_j F_{ji}$$

$$\partial_\tau E^\eta = \tau^{-1} D_j F_{j\eta}$$

Classical Equations of Motion in the Expanding System

Ensemble Average

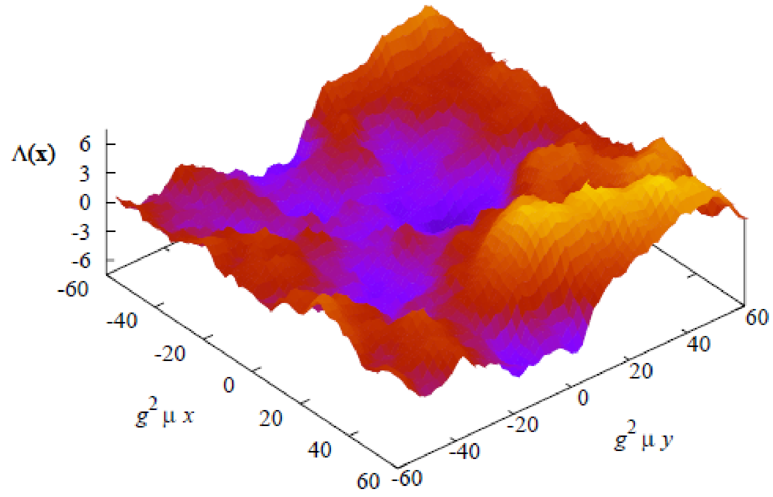
$$\langle\langle \mathcal{O}[A] \rangle\rangle_{\rho_t,\rho_p} \sim \int D\rho_t \, D\rho_p W_x[\rho_t] W_{x'}[\rho_p] \, \mathcal{O}[\mathcal{A}[\rho_t,\rho_p]]$$

Quantum fluctuations partially included in the initial state

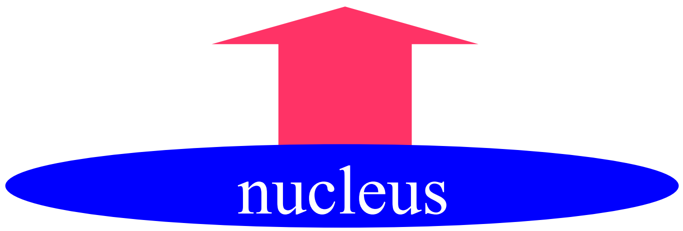
Initial Configurations

Solve the Poisson Eq

$$\partial_\perp^2 \Lambda^{(m)}(\boldsymbol{x}_\perp) = -\rho^{(m)}(\boldsymbol{x}_\perp)$$

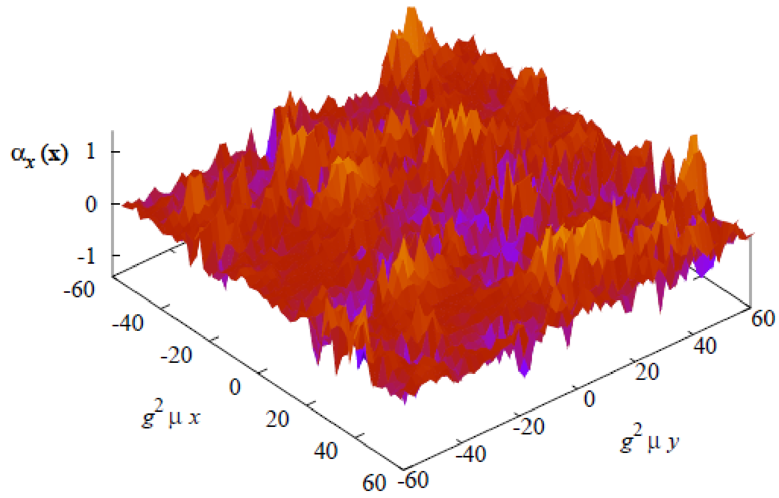


Transverse Distribution



Gauge Configuration

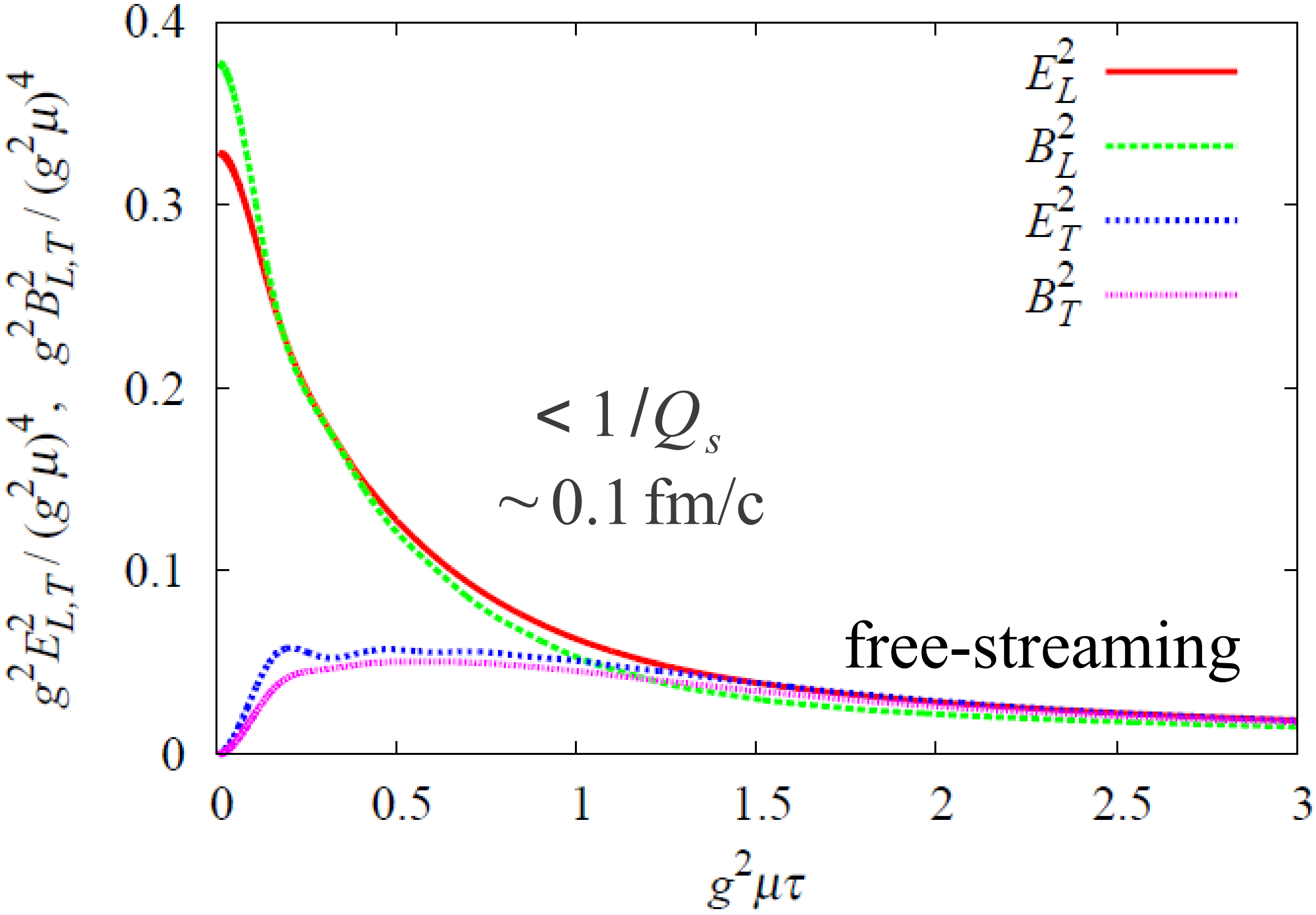
$$e^{-ig\Lambda(\boldsymbol{x}_\perp)} e^{ig\Lambda(\boldsymbol{x}_\perp + \hat{i})} = \exp[-ig\alpha_i(\boldsymbol{x}_\perp)]$$

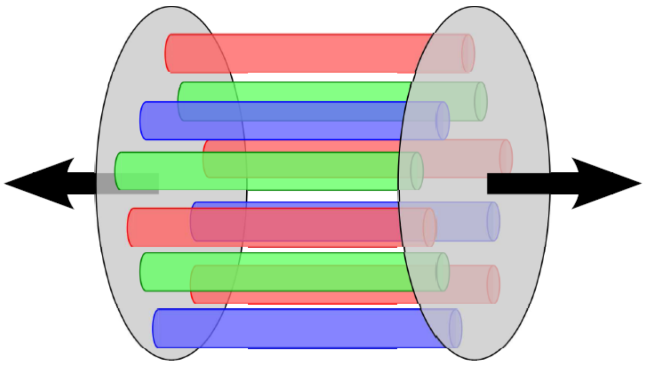


No structure because of the Gaussian wave-function

Chromo-Electric and Magnetic Fields

Longitudinal and Transverse Fields

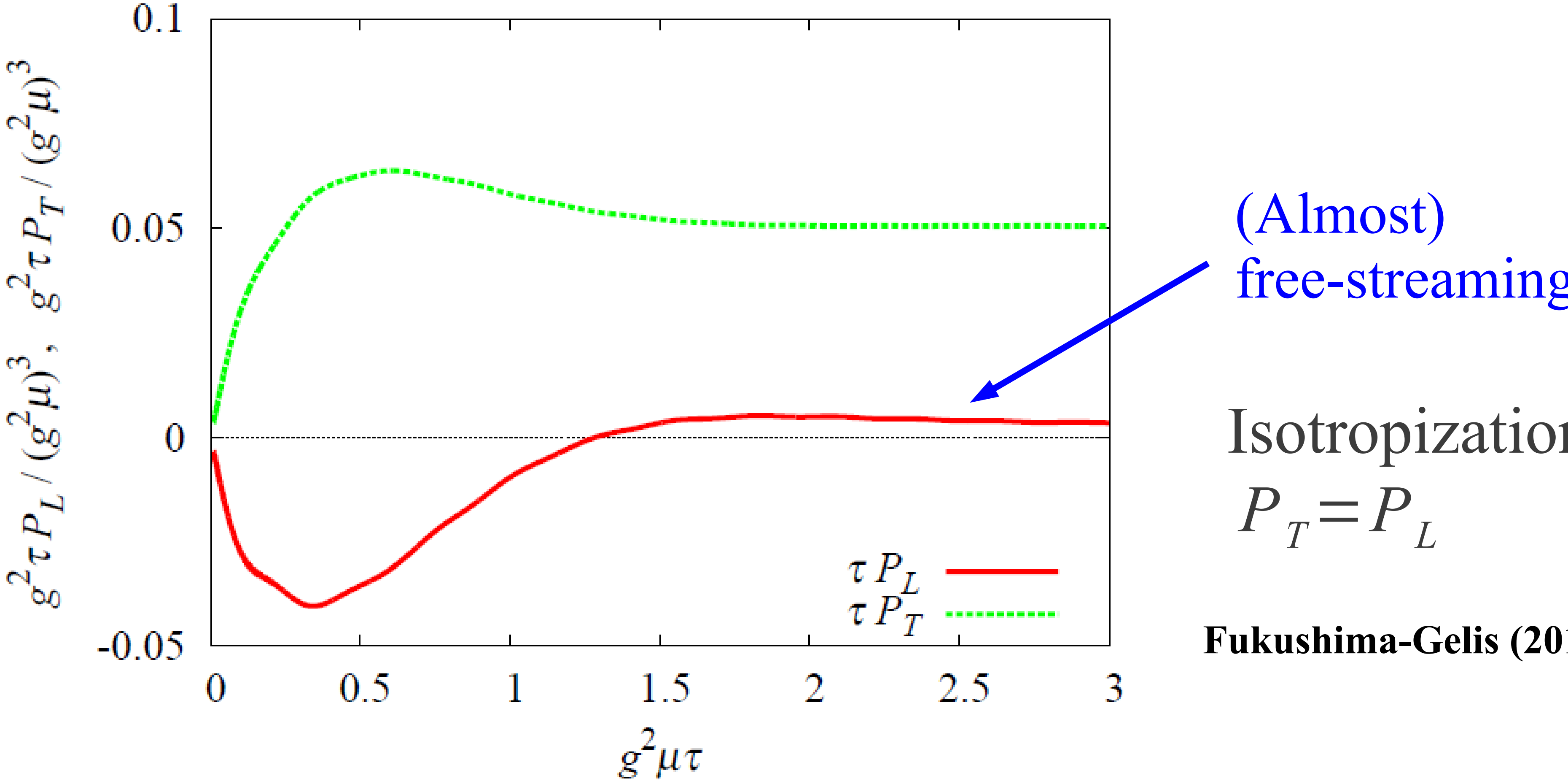




Lappi-McLerran (2006)
Fukushima-Gelis (2011)

Longitudinal and Transverse Pressure

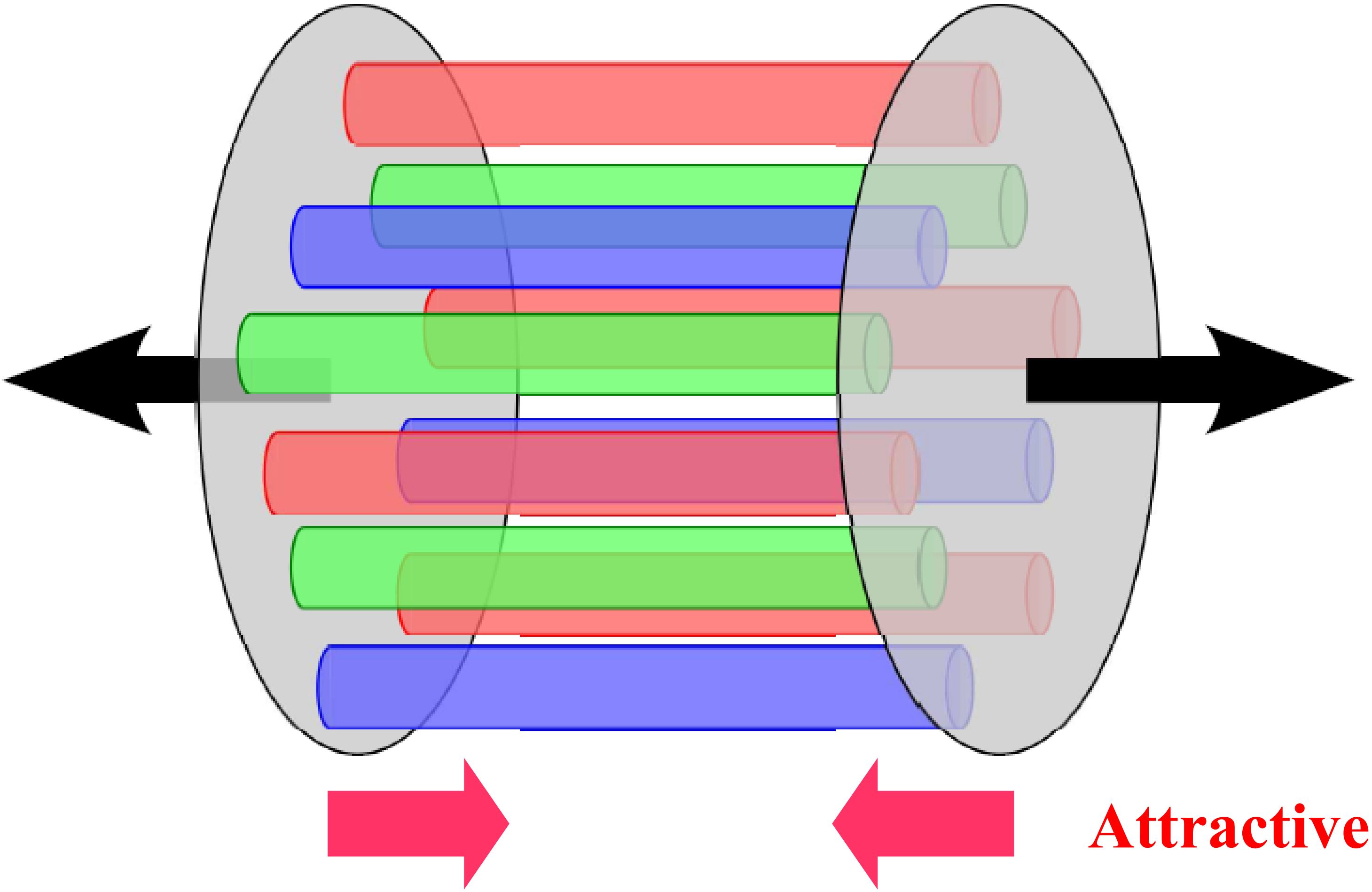
$$P_T = \frac{1}{2}\left\langle T^{xx} + T^{yy} \right\rangle = \left\langle \mathrm{tr}\left[E_L^2 + B_L^2\right]\right\rangle ,$$
$$P_L = \left\langle \tau^2 T^{\eta\eta} \right\rangle = \left\langle \mathrm{tr}\left[E_T^2 + B_T^2 - E_L^2 - B_L^2\right]\right\rangle$$



Isotropization
$P_T = P_L$

Fukushima-Gelis (2011)

Negative Longitudinal Pressure



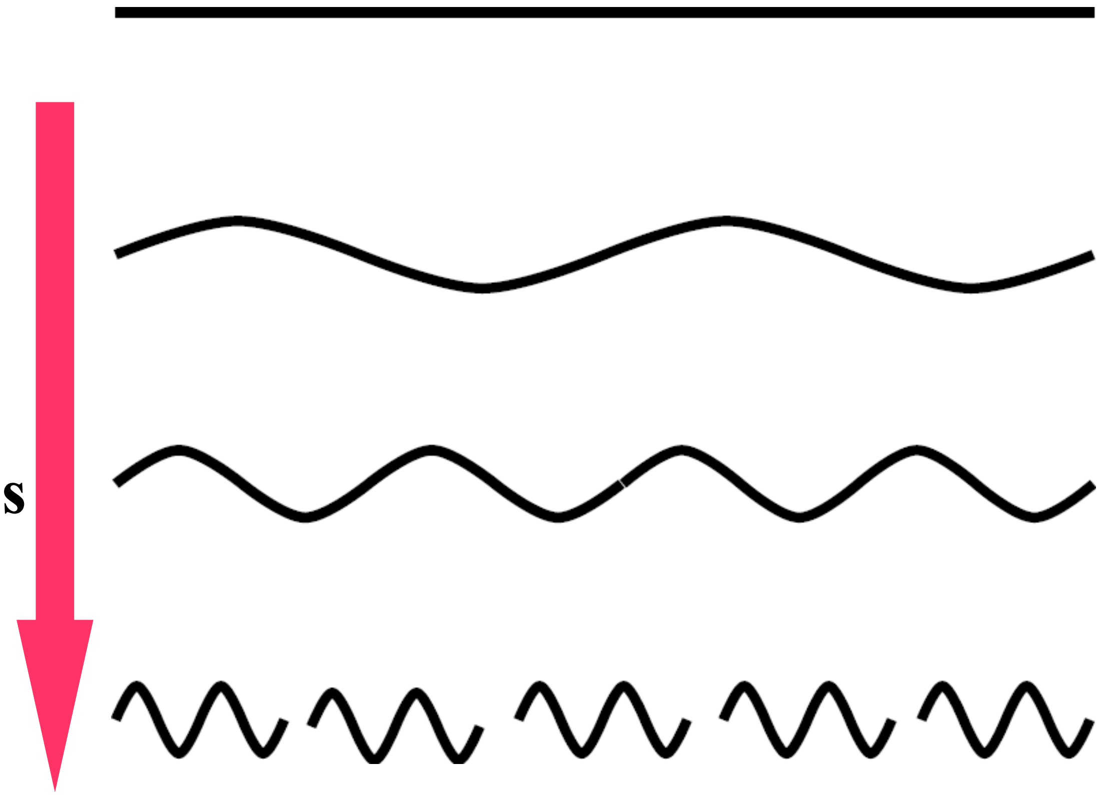
Attractive Force

Flux tubes have a positive energy

Missing Dynamics

Boost Invariant *E* and *B*

Flux tube **~ QCD string**



Instability

c.f. Plasma instability

Talk by Attems

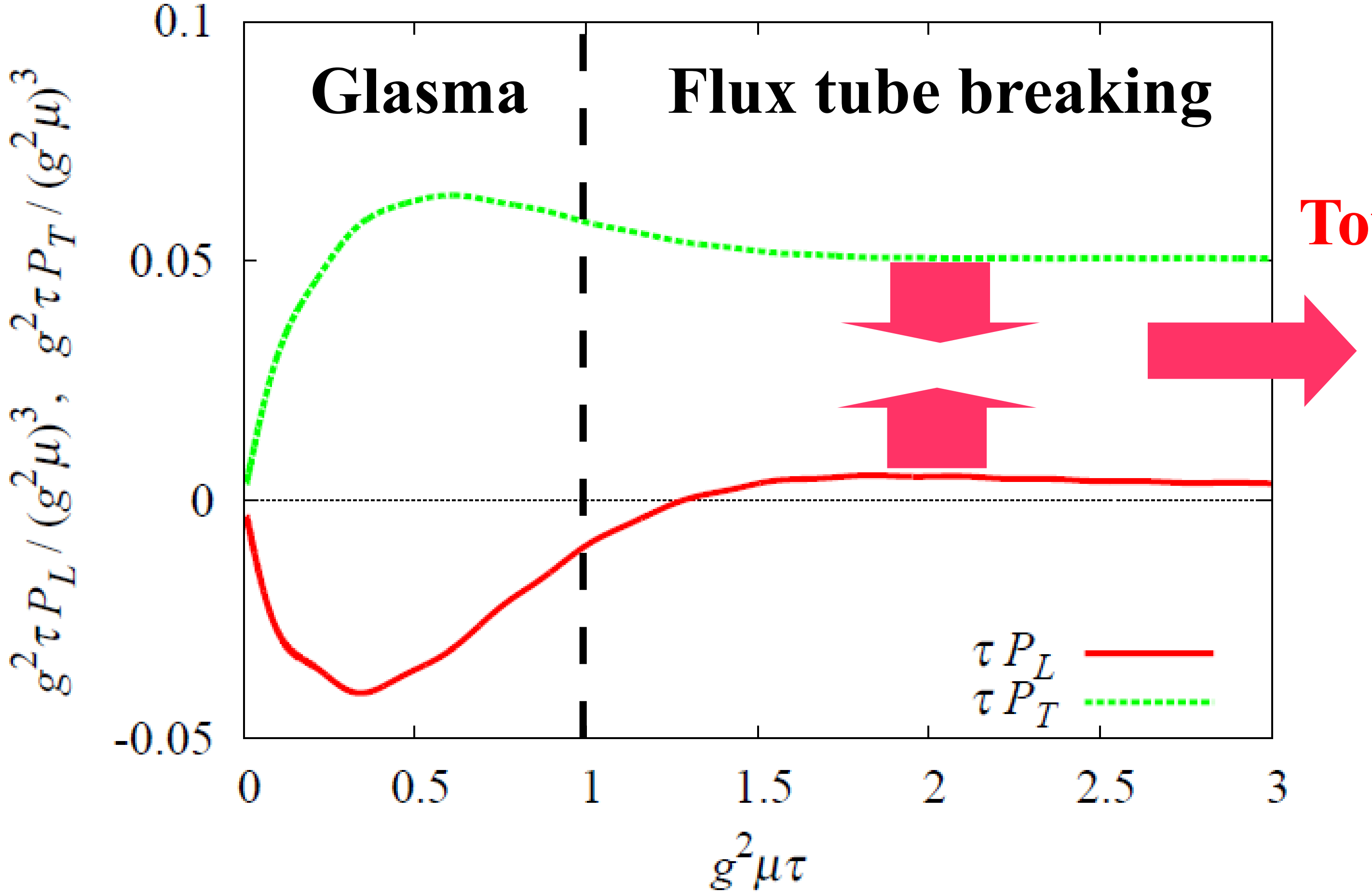
c.f. Deconfinement at high T (entropy wins)

String breaking → Particle production (Schwinger mechanism)

Expectation

$$P_T = \frac{1}{2}\left\langle T^{xx} + T^{yy}\right\rangle = \left\langle \mathrm{tr}\left[E_L^2 + B_L^2\right]\right\rangle ,$$

$$P_L = \left\langle \tau^2 T^{\eta\eta}\right\rangle = \left\langle \mathrm{tr}\left[E_T^2 + B_T^2 - E_L^2 - B_L^2\right]\right\rangle$$



Toward thermalization

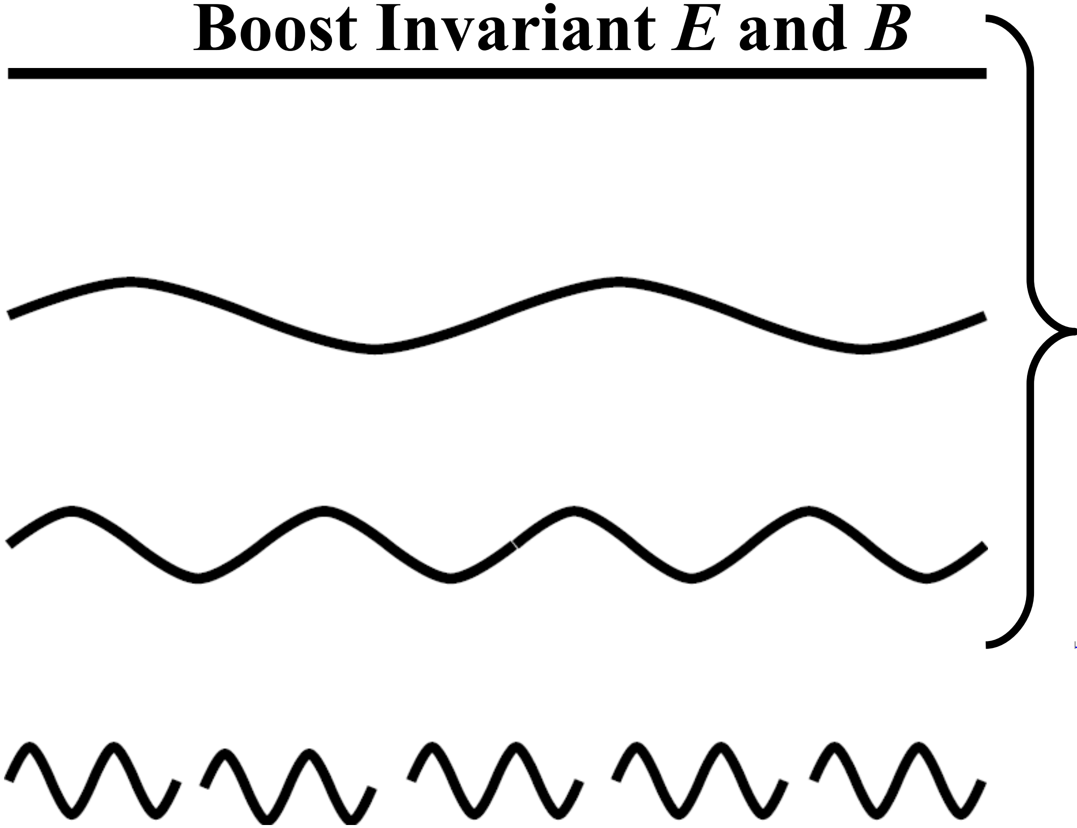
E_L^2

B_L^2

Classical Statistical Simulation

Talk by Tanji

Boost Invariant *E* and *B*



Instability

Classical Dynamics

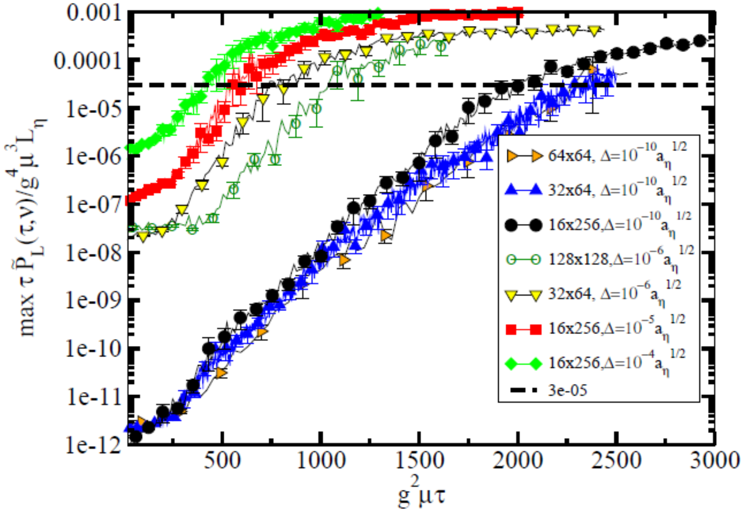
+

Small Fluctuations

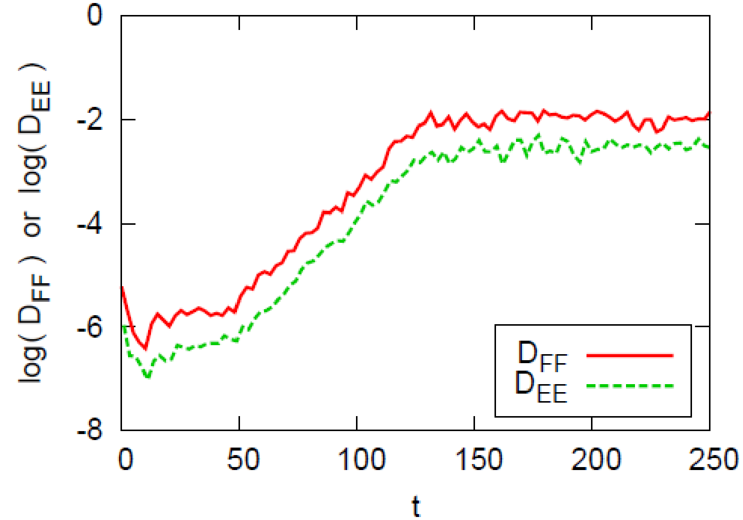
What is the dynamics of the background *E* and *B*?

How fluctuations grow?

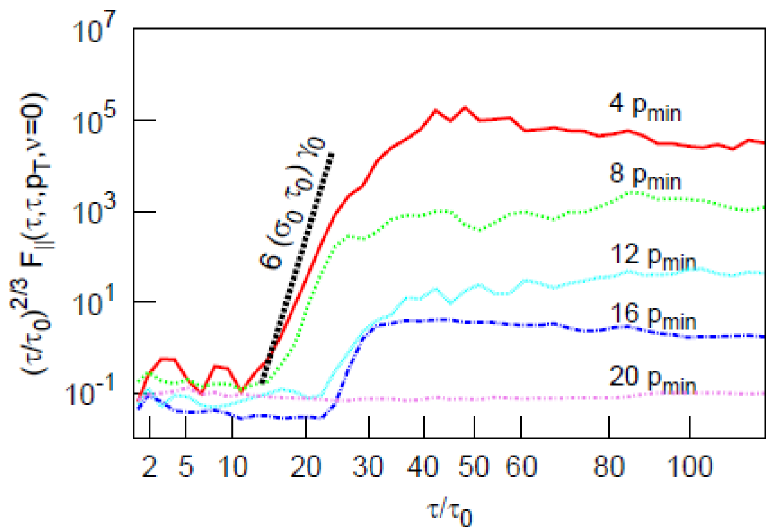
Instabilities in the Classical YM



Romatschke-Venugopalan (2005)



Kunihiro et al. (2010)



Berges-Boguslavski-Schlichting (2012)

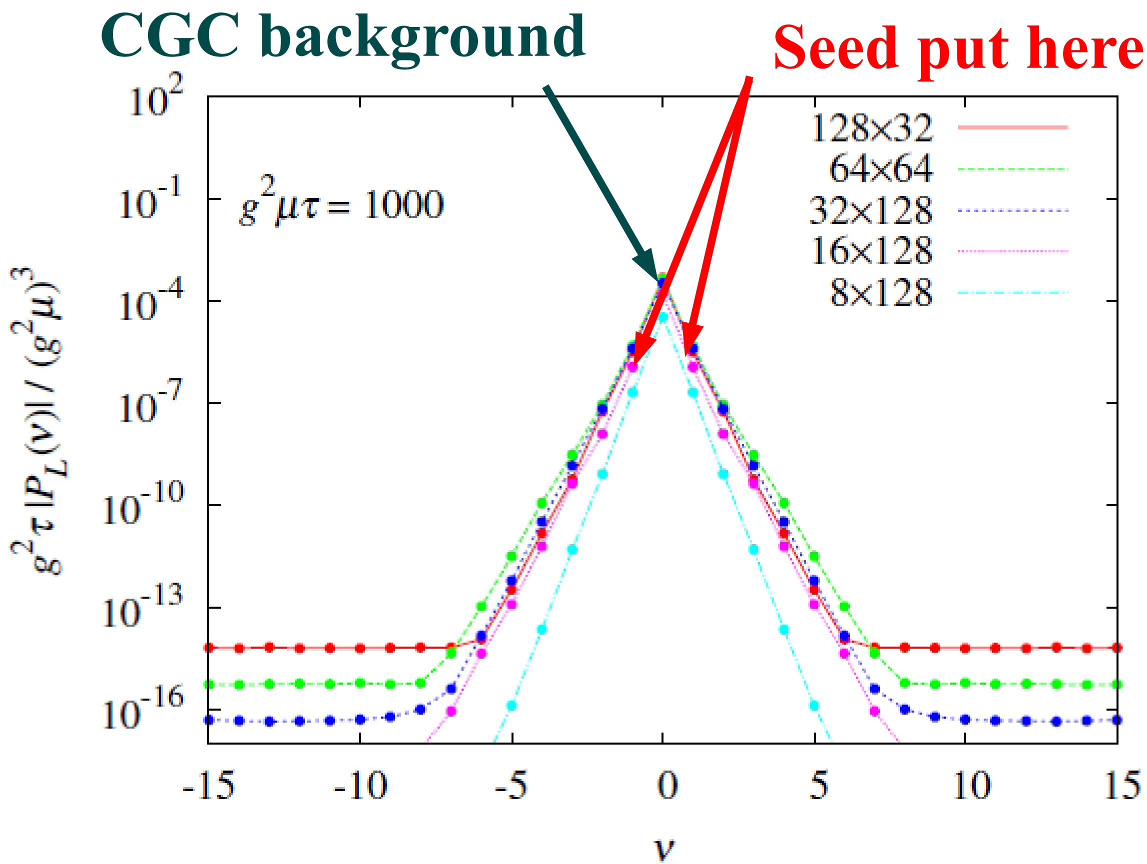
Weibel instability
Nielsen-Olesen instability
Parametric resonance

Talk by Schlichting

Minimal Perturbation

$\delta E^{ia} = \bar{E}^{ia} \cos(2\pi\eta/L_\eta)$ $\delta E^{\eta a}$ from the Gauss law

$\bar{E}^{ia} \sim \Delta \ll 1$



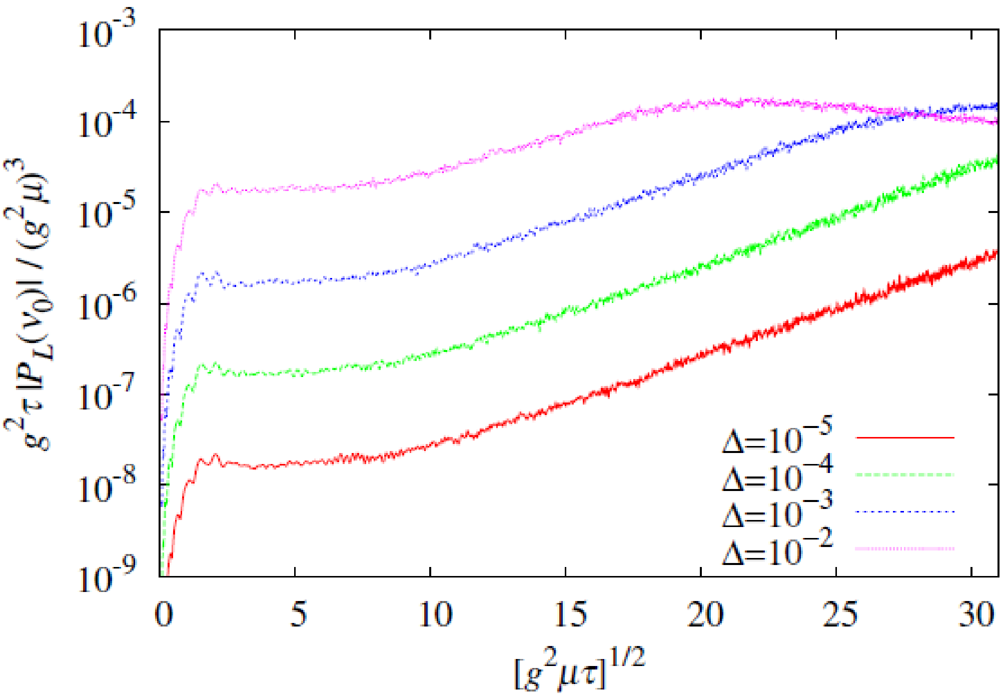
Because the zero-mode background is so huge, it keeps supplying the energy (or particle) injection.

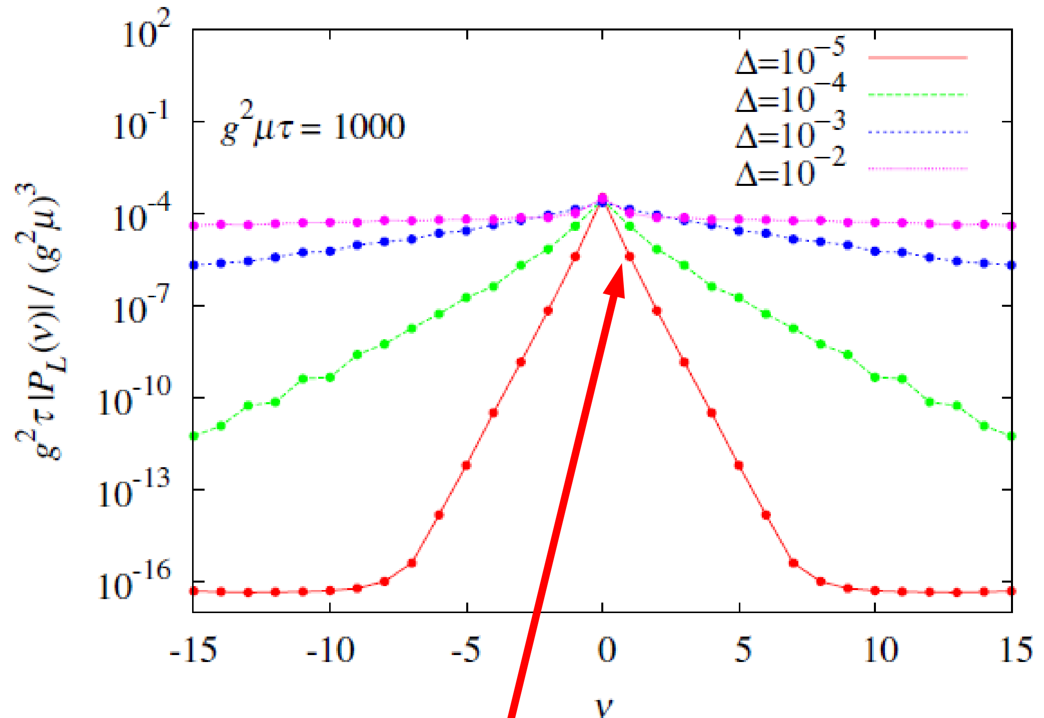
Wave-number along the longitudinal direction

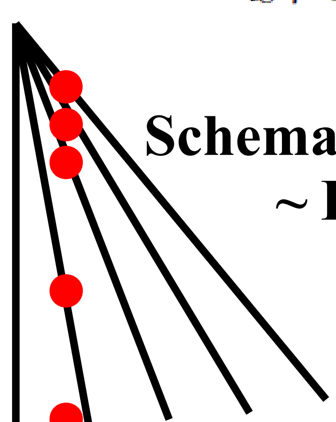
Initial spectrum:: Dusling-Gelis-Venugopalan (2011), Dusling-Epelbaum-Gelis-Venugopalan

Amplitude Decay from Zero-Mode

Fukushima-Gelis (2011)







Schematic Behavior
~ Diffusion

How this mode grows

Instability... but too weak and the bulk thermodynamics unaffected

Turn expansion off

- Although the expanding system is more realistic, numerical simulations in a fixed-volume box would be useful to make underlying physics clear.

- If we cannot account for isotropization in a fixed-volume box, we have no chance to realize it in an expanding case.

- Because there is no coordinate singularity, we can take as large initial fluctuations as we like.
  (In the expanding system the transverse energy becomes singular if not renormalized properly.)

Formulation

Equations of Motion

$$D_\mu F^{\mu\nu} = j^\nu = 0$$ in the Cartesian coordinates

"Glasma" Initial Conditions

Background Fields

$$\mathcal{A}_i = \alpha_i^{(1)} + \alpha_i^{(2)}$$

$$\mathcal{A}_z = 0$$

$$\mathcal{E}^i = 0$$

$$\mathcal{E}^z = ig[\alpha_i^{(1)}, \alpha_i^{(2)}]$$

+

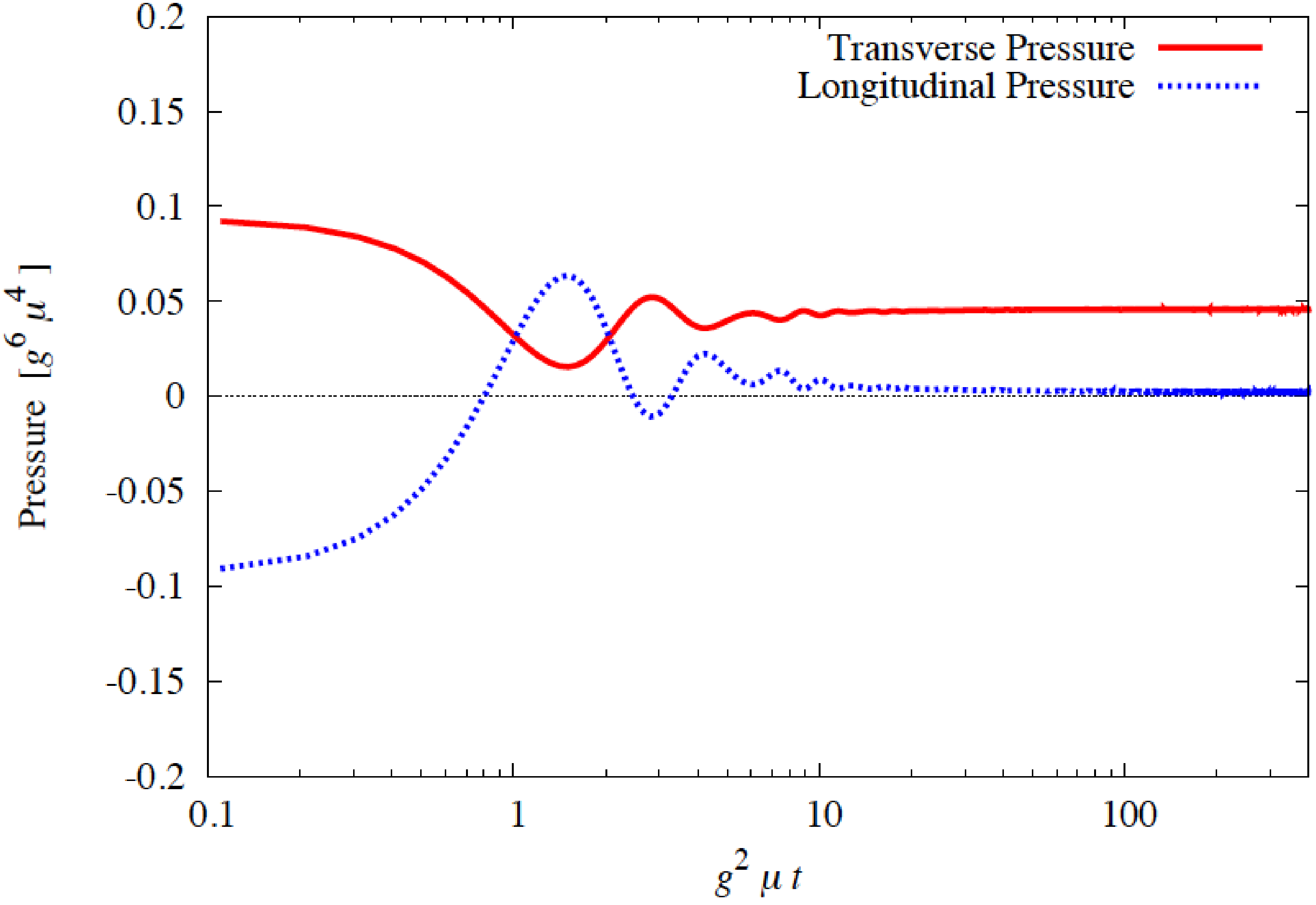
Fluctuation Fields

$$\delta E^i(x,y,z) \quad \delta E^z(x,y,z)$$

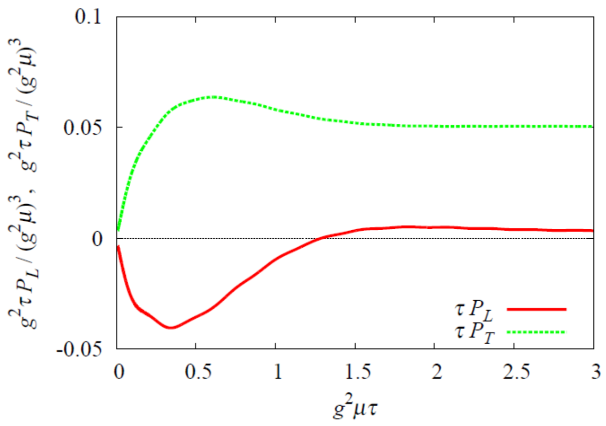
$$\delta A^i(x,y,z) \quad \delta A^z(x,y,z)$$

Results without Fluctuations

Vanishing longitudinal pressure P_z not because of the expansion but because of the invariance.



c.f. expanding

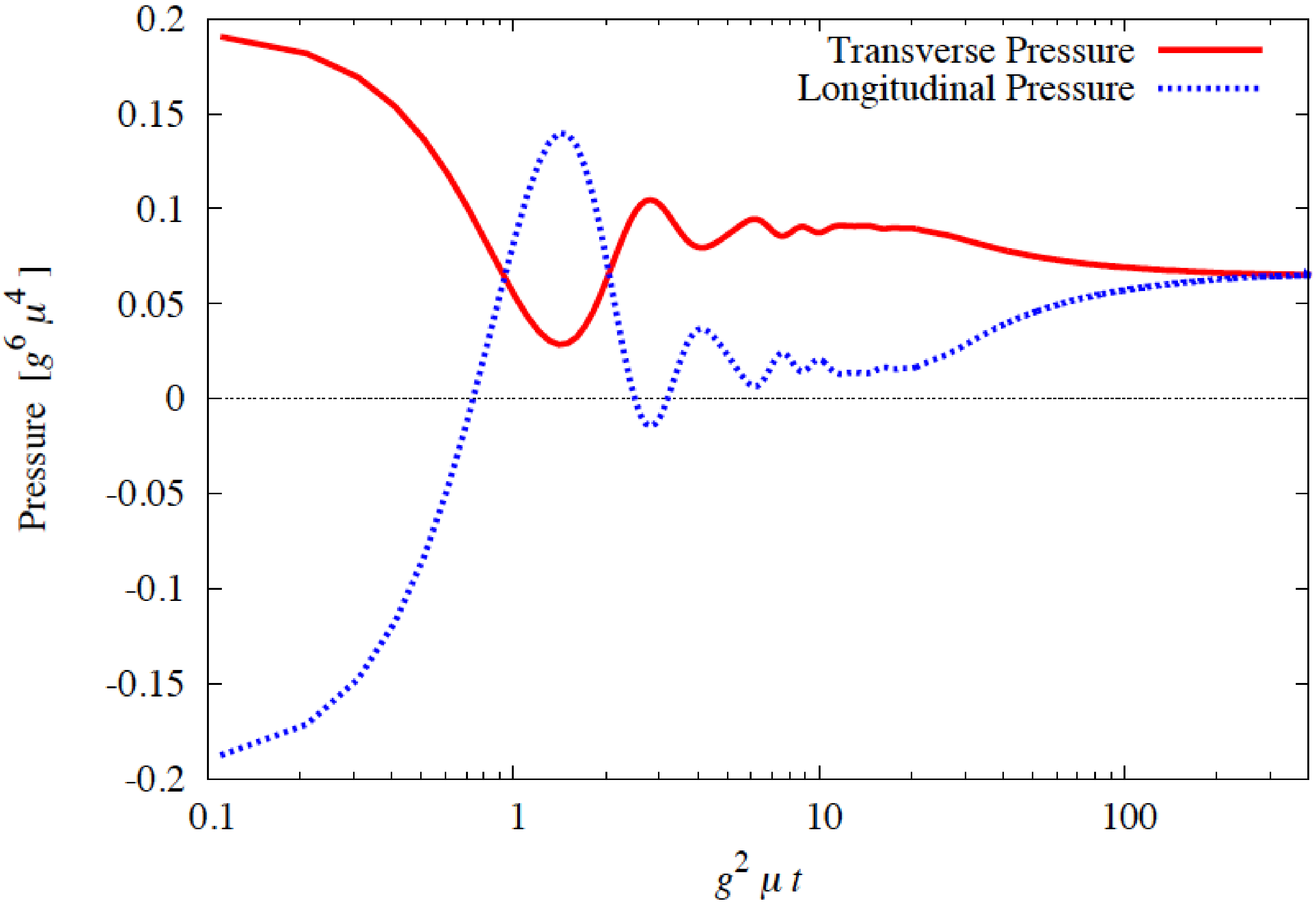


$64 \times 64 \times 128$

30 configurations

Results with Fluctuations

Isotropization is certainly reached if we wait for a sufficiently long time (but too long yet...)



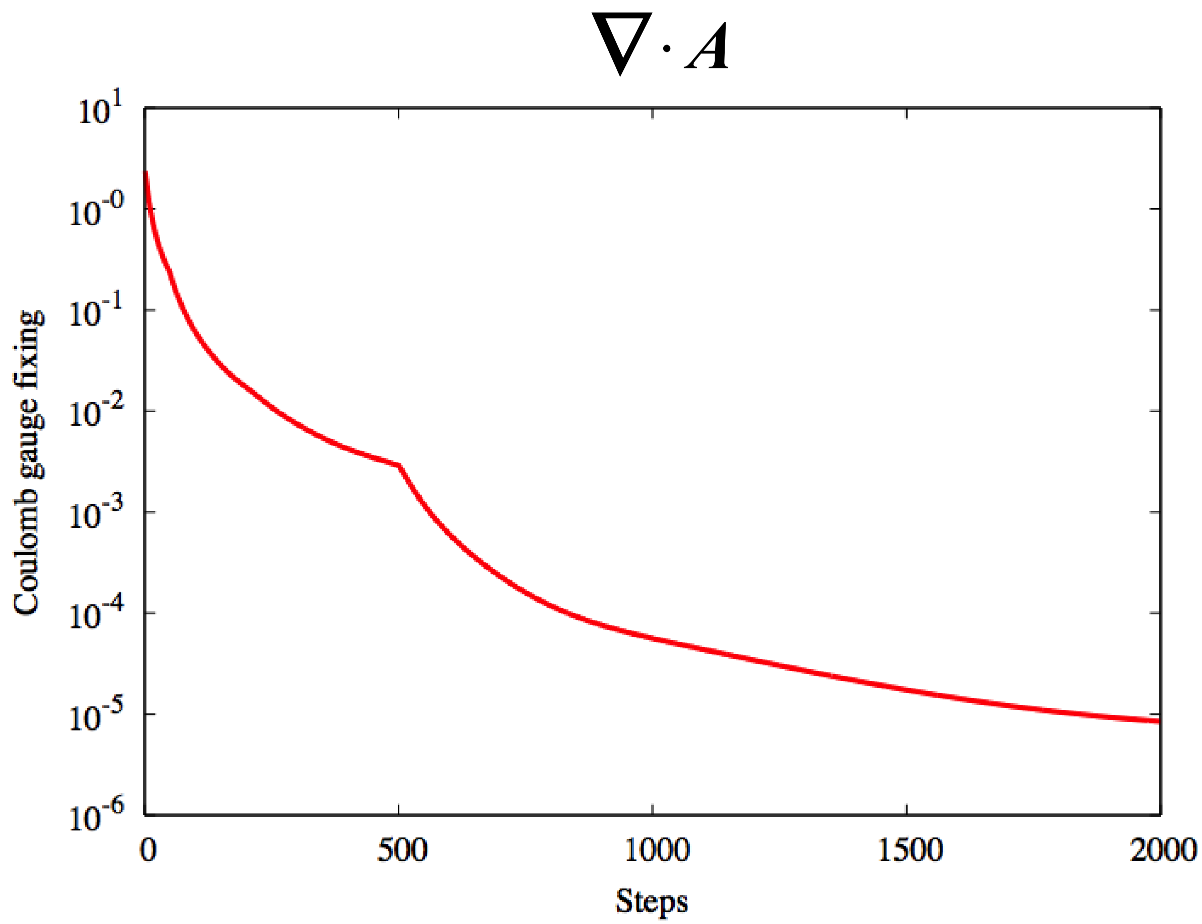
$\Delta = 0.1\, g^2 \mu\, a$

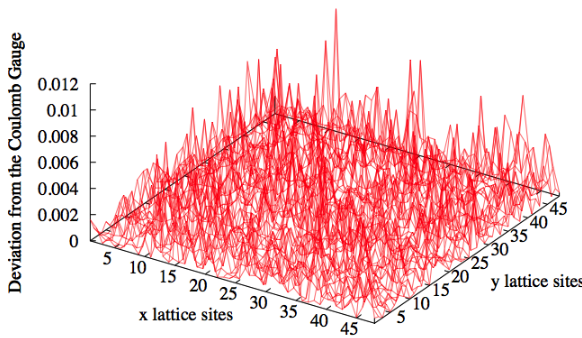
$64 \times 64 \times 128$

30 configurations

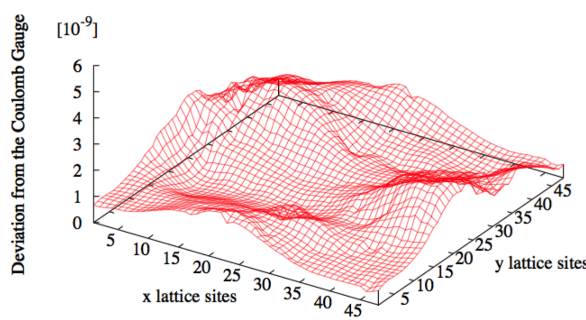
Coulomb Gauge Fixing

2000 steps using the overrelaxation method



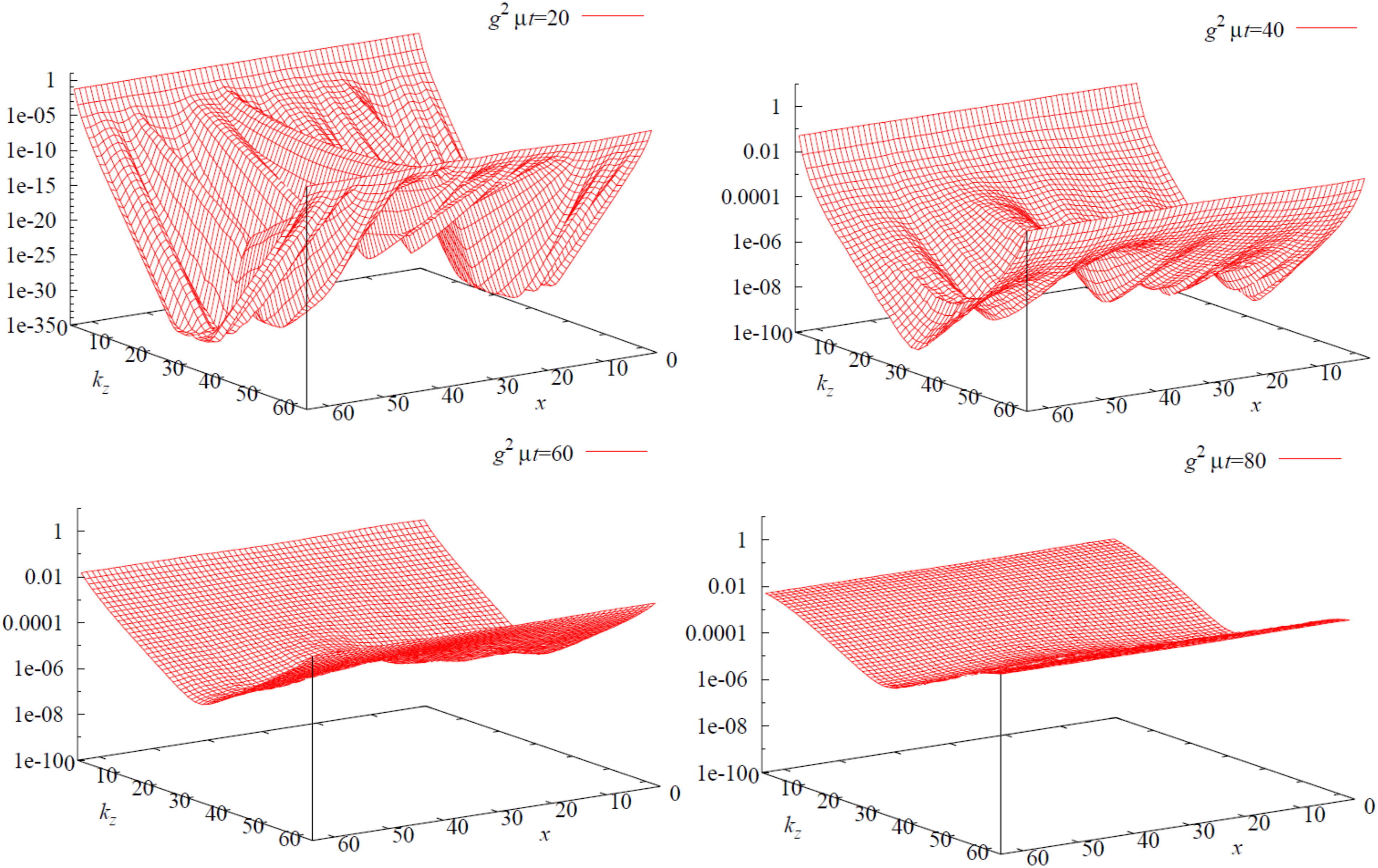


Before gauge fixing



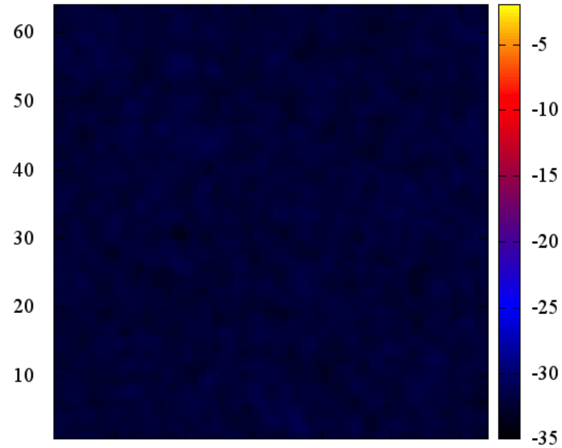
After gauge fixing

Diffusion in x and k_z with k_y=0



Transverse Pattern Formation

Central Results at maximum k_z



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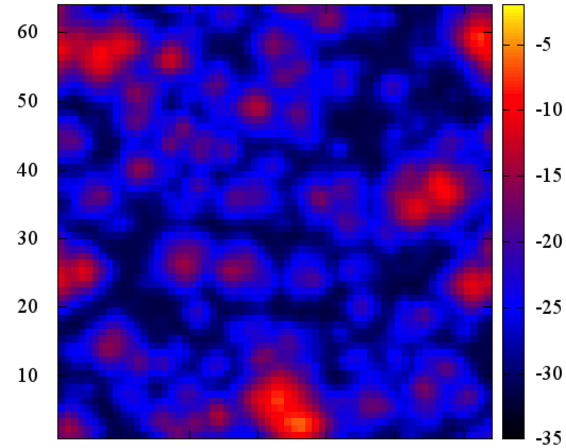
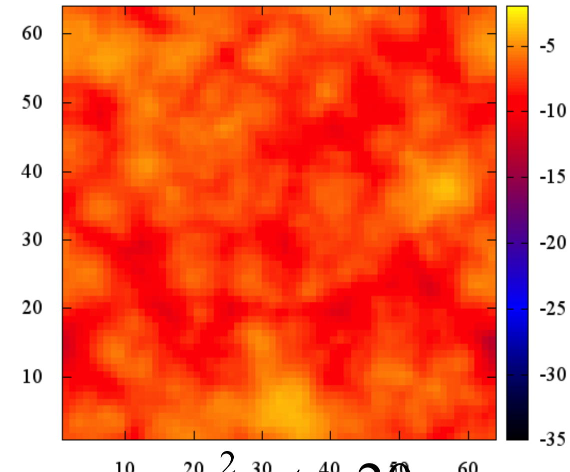
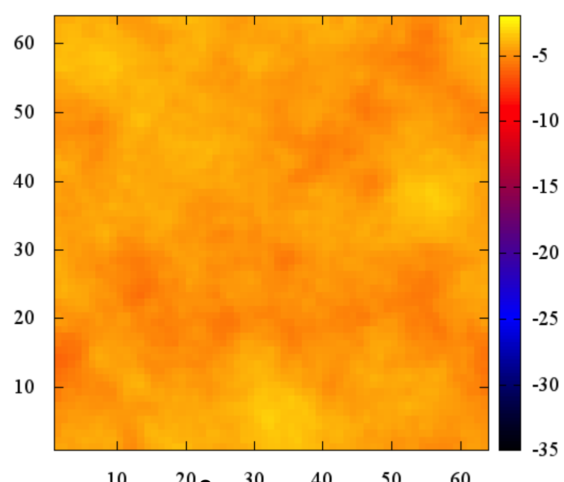


$g^2 \mu t = 10$



$g^2 \mu t = 20$



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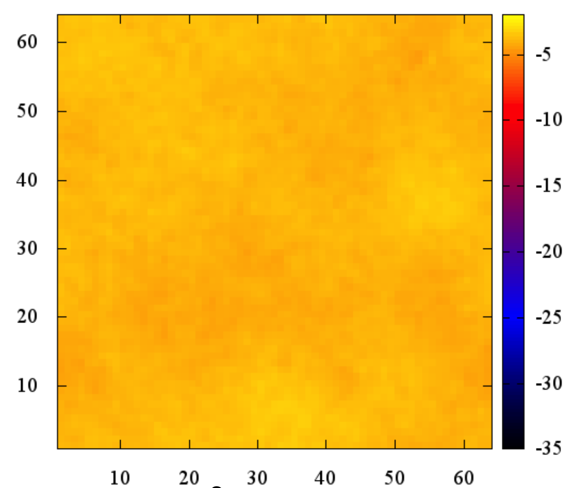

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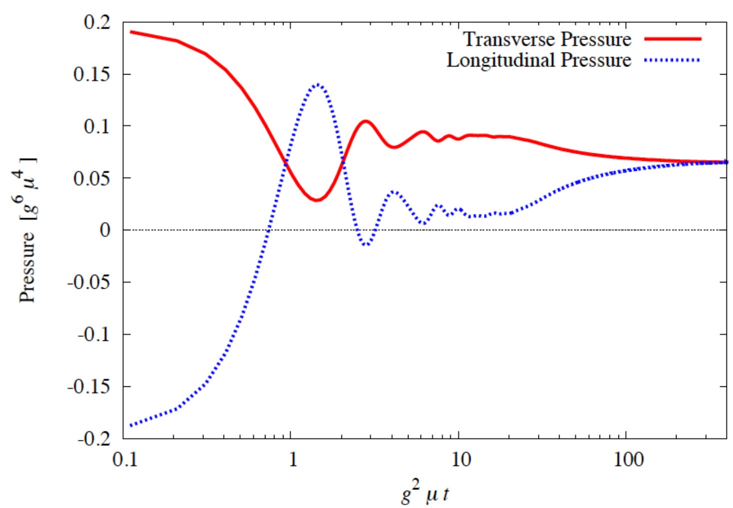


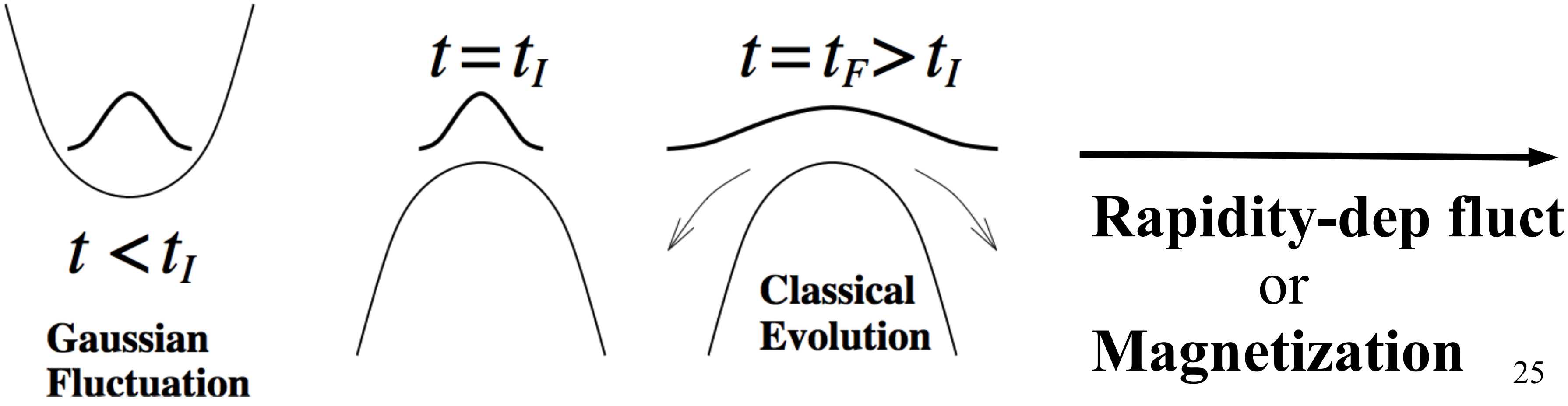
Similarity to Magnetization

Spontaneous pattern formation from "**uniform**" to "**non-uniform**" distribution in the Glasma

Movie of pattern formation in the Glasma

Spontaneous pattern formation from "**disordered**" to "**ordered**" state in spin systems

Movie of spin pattern formation (Kudo et al.)

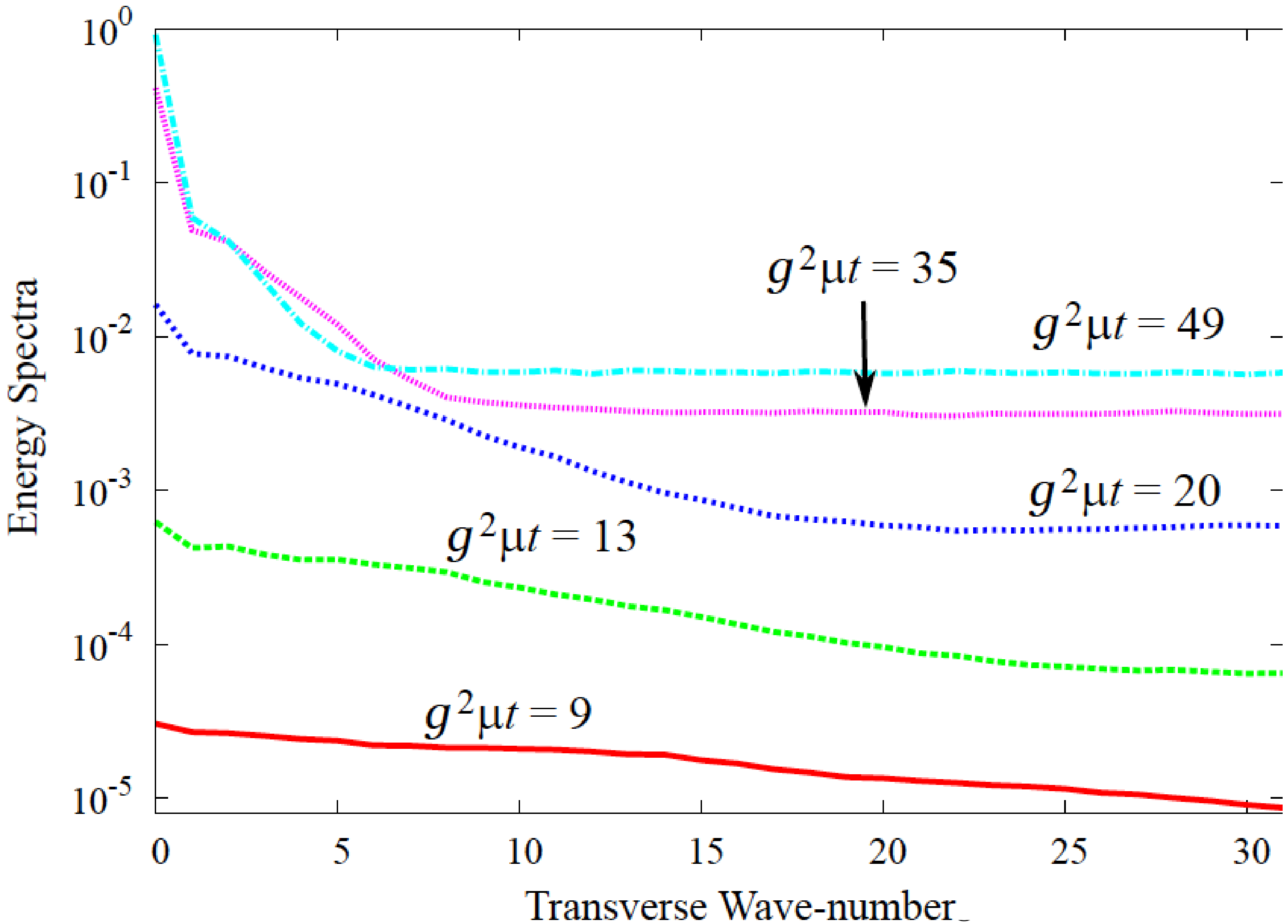


Rapidity-dep fluct
or
Magnetization

Time Evolution of Transverse Spectrum

Hint to the BEC?

See: Blaizot-Gelis-Liao-McLerran-Venugopalan (2011)



Still the box size is not large enough to be conclusive.

Yet, suggestive!

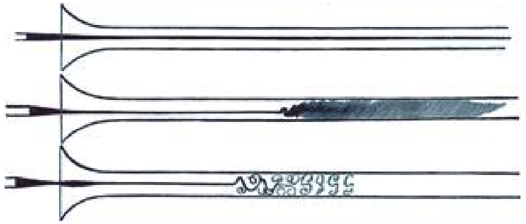
Depends on μ

Conceivable Scenario

- Spontaneous pattern formation: cores of more complex structures (probably related to the vortex dynamics; see **talk by Dumitru**)

- After development of cores, they spread in the transverse plane – looks like turbulence diffusion! (See **talk by Schlichting**)

  Reynolds' famous pipe experiment:
  Analogous to the Glasma situation!?
  Small viscosity → Turbulence



- Rapid turbulence diffusion could lead to a BEC formation in the transverse distribution

Works to be done

- Does it survive the expansion?
- Systematic study of the system size dependence
- Correct spectrum of quantum fluctuation
- Where is the shear as a source of turbulence?
- More idealized simulation with a single shear source

Summary

- Early-time evolution of the relativistic heavy-ion collision was investigated in the classical statistical simulation of the non-expanding Glasma.

- Isotropization was achieved.

- Spontaneous pattern formation in the transverse plane was observed in an analogous way to the magnetization formation.

- BEC-like distribution was found after the diffusion of cores of formed pattern.