Jet broadening at NNLL in perturbation theory

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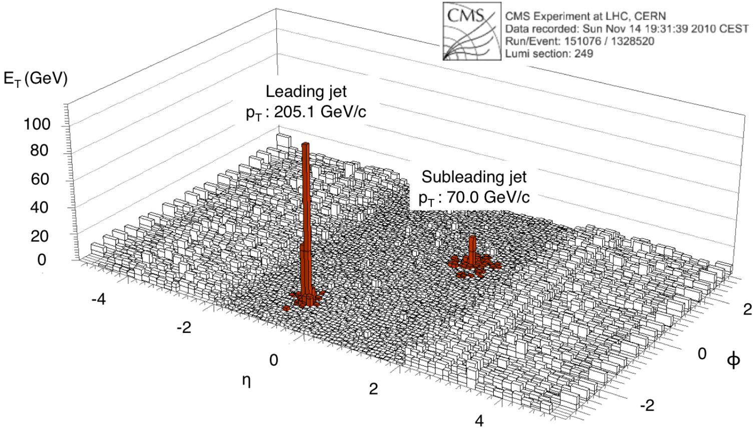
Outline

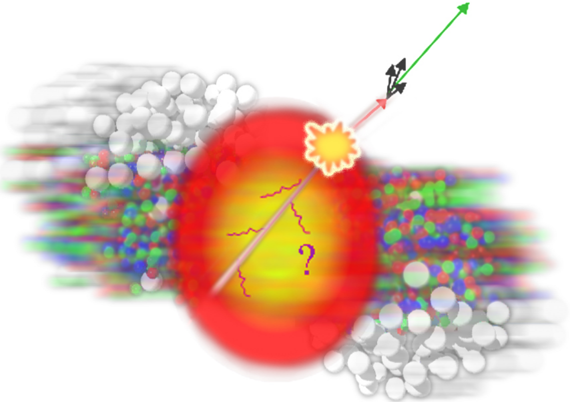
1. Jet quenching and jet broadening in perturbation theory and the $\hat{q}$ parameter
2. EFTs for the computation of jet broadening
3. The computation of $\hat{q}$ in MQCD
4. Conclusions

Jet quenching and jet broadening in perturbation theory and the $\hat{q}$ parameter

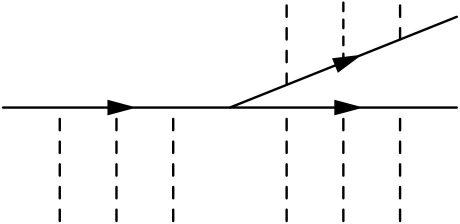
Jet quenching in heavy-ion collisions



Jet quenching



Processes for jet quenching



- Plain lines are partons with energy of order Q.
- Dashed lines are interaction with the medium with energy of order T or lower.
- The interactions with the medium do not make the high energy parton lose energy in a significant amount $Q \gg T$ but they enhanced the probability that the splitting happens.

Processes for jet quenching

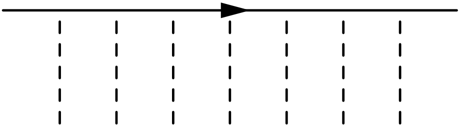
If the jet is a light quark

- Bremsstrahlung

If the jet is a gluon

- Bremsstrahlung
- Pair production of $q\bar{q}$

Jet broadening



In light-cone coordinates $(p^+, p^-, p_\perp)$ the initial parton has $(0, Q, 0)$, after the interaction with the medium it changes to $(\frac{k_\perp^2}{2Q}, Q, k_\perp)$. This is called jet broadening.
It is different from jet quenching but in order to compute jet quenching one needs information on jet broadening.

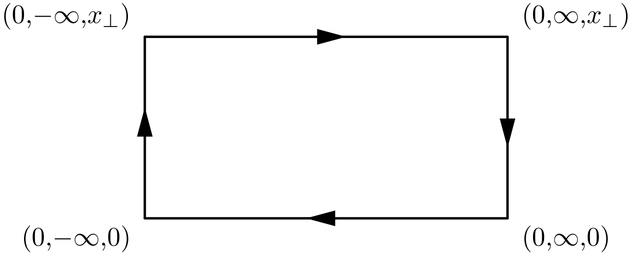
What is $\hat{q}$?

$P(k_\perp)$ is the probability that an initial parton with momentum $(0, Q, 0)$ transforms into a parton with momentum $(\frac{k_\perp^2}{2Q}, Q, k_\perp)$ after going through a distance L in the medium.

$$\hat{q} = \frac{1}{L} \int \frac{d^2 k_\perp}{(2\pi)^2} k_\perp^2 P(k_\perp)$$

It is only related with jet broadening but it is needed as an input parameter in jet quenching computations.

$P(x_\perp)$ from Wilson lines



In covariant gauge. Baier et al. (1997), Zakharov (1996), Casalderrey-Solana and Salgado(2007).
In a general gauge. Benzke, Brambilla, M. A. E and Vairo (2012).

$\hat{q}$ in perturbation theory

$$\hat{q} = \frac{1}{L}\int \frac{d^2k_\perp}{(2\pi)^2} k_\perp^2 P(k_\perp)$$

$P(k_\perp)$ is related with the Fourier transform of a gluonic operator. The scales that appear are

- $k_\perp$ itself.
- The energy scales that characterize the medium.

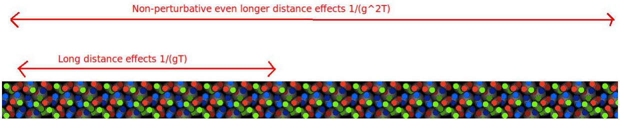
Energy scales in a thermal bath

In the weak-coupling regime

- We have an almost free gas of quarks and gluons with typical energy πT.
- At long distances (order $\frac{1}{gT}$), non-trivial collective phenomena arise, as for example chromoelectric static fields screening.
- At even longer distances (order $\frac{1}{g^2 T}$), non-perturbative phenomena arise, as for example chromomagnetic static fields screening.

Energy scales in a thermal bath

In the weak-coupling regime



What is know about $P(k_\perp)$?

$$\hat{q} = \frac{1}{L}\int \frac{d^2k_\perp}{(2\pi)^2} k_\perp^2 P(k_\perp)$$

$k_\perp$	$\alpha_s^2 T^3$	$\alpha_s^2 g_s T^3$	$\alpha_s^3 T^3$
πT	computed	no contribution	not known
gT	computed	computed	not known, $\alpha_s^3 \log(g) T^3$
$g^2 T$	no contribution	no contribution	computed

- $\hat{q}$ at LO, Arnold and Xiao (2008).
- NLO, Caron-Huot (2008).
- NNLO contribution from $g^2 T$, Benzke, Brambilla, M. A. E and Vairo (2012), Laine (2012).

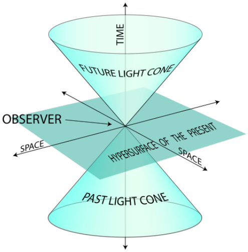
Why logarithmic enhancement? Answer later.

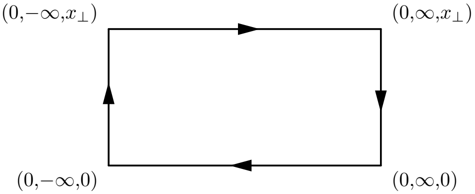
Summary

- Jet quenching provides information about the medium formed in heavy-ion collisions.
- To predict jet quenching we need and understanding of jet broadening and $\hat{q}$.
- What are the relevant degrees of freedom at a given temperature? To answer this we also need precise perturbative computations.
- We are going to compute $P(k_\perp)$ for $k_\perp \sim gT$ which is going to give us the NNLL correction to $\hat{q}$.

EFTs for the computation of jet broadening.

Jet broadening as a static phenomena





- All the operators are separated by a space-like distance or a light-cone distance.
- Caron-Huot (2008) argued that one can deform infinitesimally the light-cone Wilson lines to make them slightly space-like.
- All the separations are space-like.

Imaginary-time formalism

Observables at finite temperature that do not depend on time.
Euclidean field theory.

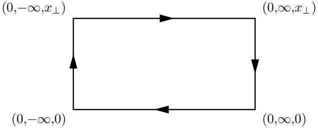
$$\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2+m^2} \to T\sum_{n=-\infty}^{\infty}\int \frac{d^3k}{(2\pi)^3}\frac{1}{4\pi^2n^2T^2+k^2+m^2}$$

Lorenz boost with $v = 1+\epsilon$.

$$p_z \to p_z + 2\pi T n$$

Dimensional reduction

$P(k_\perp)$ for $k_\perp \sim gT$.



- Only modes with Euclidean virtuality smaller than πT are important.
- The starting point can be an Effective Field Theory where high energy mode has been integrated out.
- $n = 0$, a zero temperature three dimension EFT with temperature dependent Wilson coefficients.

Electrostatic QCD

Braaten (1995), integrate out scale πT.

$$\mathcal{L}_{EQCD} = \frac{1}{4}F^a_{ij}F^a_{ij} + \frac{1}{2}(D_i A^a_0)(D_i A^a_0) + \frac{1}{2}m_D^2 A^a_0 A^a_0$$

- 3D QCD + additional scalar field with mass $m_D \sim gT$.
- Coupling constant $g_E = g\sqrt{T}$.
- Infrared behaviour is dominated by the magnetic part.

Consequences of EQCD for light-cone Wilson lines

$$A_+ = \frac{1}{\sqrt{2}}(A_0 + A_3) \rightarrow_{EQCD,\, Infrared} \frac{A_3}{\sqrt{2}}$$

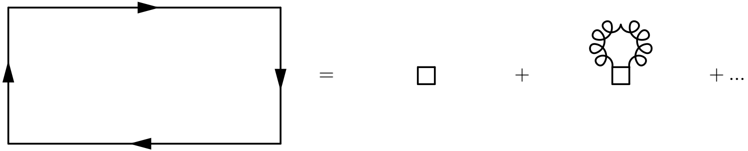
- Identify A_3 as a temporal component in Euclidean time.
- Infrared behaviour of $P(x_\perp)$ for $x_\perp \sim 1/gT$ is the same as static potential $V(x_\perp)$ in 3D QCD.
- Naive computation of the static potential in 3D QCD has infrared divergence at three loops. Schroeder (1999).
- This divergence was found to cancel with an ultraviolet divergence coming from the energy scale of the potential (ladder resummation). Pineda and Stahlhofen (2010).

Magnetostatic QCD

Braaten (1995), integrate out also scale gT.

$$\mathcal{L}_{MQCD} = \frac{1}{4} F^a_{ij} F^a_{ij}$$

At leading order 3D QCD with $g_E = g\sqrt{T}$



Match operator in EQCD to set of operators in MQCD

Operators in MQCD

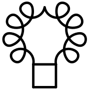
$$\Box \qquad = Z(x_\perp)e^{-\frac{V_s(x_\perp,\mu)L}{\sqrt{2}}}$$

At leading order

$$V_s(x_\perp,\mu) = -\frac{g^2 T C_F}{2\pi}\left(\frac{1}{2} - \gamma - \log(2\pi) - \log(m_D x_\perp) - k_0(m_D x_\perp)\right)$$

- In the limit $m_D x_\perp \to \infty$ coincides with static potential.
- $gT \gg \mu \gg g^2 T$. Factorization scale.
- V_s itself is a new energy scale that appears in the problem.

Operators in MQCD



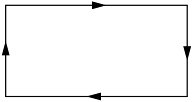
$$= -\frac{\delta h_s(x_\perp,\mu)L}{\sqrt{2}} Z(x_\perp) e^{-\frac{V_s(x_\perp,\mu)L}{\sqrt{2}}}$$

$$\delta h_s(x_\perp,\mu) =$$

$$\frac{g_E^2 T_F}{C_A} x_\perp^i x_\perp^j \int_0^\infty d\tau e^{-(V_o(x_\perp,\mu)-V_s(x_\perp,\mu))\tau} \langle F_\perp^{3i,a}(\tau,0)\phi_{adj}^{ab}(\tau)F_\perp^{3j,b}(0,0)\rangle$$

Same operator as in pNRQCD in 3D. Pineda and Stahlhofen (2010). But with different values of V_s and V_o.

Summary



$$= Z(x_\perp) e^{-\frac{h_s(x_\perp)L}{\sqrt{2}}}$$

$$h_s(x_\perp) = V_s(x_\perp, \mu) + \delta h_s(x_\perp, \mu)$$

- Compute $\frac{\partial \delta h_s}{\partial \mu}$.
- Get NNLL corrections to $P(k_\perp)$.
- NNLL corrections to $\hat{q}$.
- Most of the computations already done in the static potential case. Pineda and Stahlhofen (2010).

The computation of $\hat{q}$ in MQCD

The renormalization group equation

$$\frac{\partial h_s(x_\perp)}{\partial \mu} = 0 \to \frac{\partial V_s(x_\perp)}{\partial \mu} = -\frac{\partial \delta h_s(x_\perp)}{\partial \mu}$$

but

$$\delta h_s(x_\perp) = \delta h_s(x_\perp, \Delta V(x_\perp, \mu), g_E(\mu), \mu)$$

In 3D $\frac{\partial g_E}{\partial \mu} = 0$

$$\Delta V(x_\perp) = V_o(x_\perp) - V_s(x_\perp) =$$
$$\frac{g^2 T C_A}{4\pi}\left(\frac{1}{D-4} - \frac{\gamma}{2} - \frac{1}{2}\log\pi - \log(x_\perp \mu) - k_0(m_D x_\perp)\right)$$

Result for $V_s(x_\perp,\mu)$

$$\mu\frac{\partial V_s}{\partial\mu} = \frac{C_F}{2}x_\perp^2\alpha T(\Delta V_{MS})^2 + C_F C_A\alpha^2 T^2 x_\perp^2\Delta V_{MS} + \frac{x_\perp^2\alpha^3 T^3 C_F(13\pi^2-2208)C_A^2}{384}$$

$$\begin{aligned}\delta V_s^{NNLL}(x_\perp) = C_F C_A^2\alpha^3 T^3 x_\perp^2 \Big[&\frac{1}{6}(\log(x_\perp\mu))^3 \\ &+\frac{1}{2}\left(\frac{\gamma}{2}+\frac{1}{2}\log\pi + k_0(m_D x_\perp) - 1\right)(\log(x_\perp\mu))^2 \\ &+\Big(\frac{\gamma^2}{8}+\frac{\gamma}{4}\log\pi+\frac{\gamma}{2}k_0(m_D x_\perp)+\frac{1}{8}(\log\pi)^2+\frac{1}{2}\log\pi\, k_0(m_D x_\perp) \\ &+(k_0(m_D x_\perp))^2-\frac{\gamma}{2}-\frac{1}{2}\log\pi - k_0(m_D x_\perp) \\ &+\frac{13\pi^2-2208}{384}\Big)\log(x_\perp\mu)\Big] + \mathcal{O}(\alpha^3 T^3 x_\perp^2)\end{aligned}$$

Improving the perturbative expansion

$$h_s(x_\perp) = V_s(x_\perp, \log(x_\perp \mu)) + \delta h_s(x_\perp, \Delta V(\mu), \log\left(\frac{\Delta V_{MS}}{\mu}\right))$$

All the logs are included in $V_s(x_\perp)$ if we choose $\mu = \Delta V_{MS}(x_\perp, \mu)$.

$$\mu = \alpha T C_A W\left(\frac{1}{\alpha C_A e^{\frac{\gamma}{2}+k_0(m_D x_\perp)}\sqrt{\pi} x_\perp T}\right)$$

W is the Lambert function. $W(x)$ is logarithmically big for $x \to 0$.

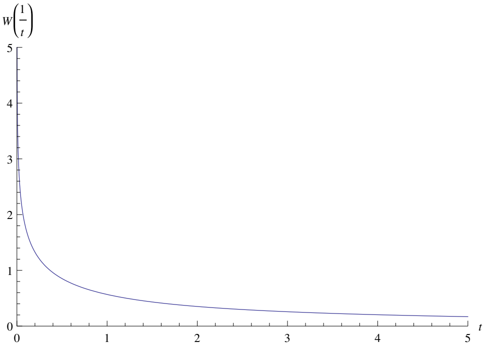
Improving the perturbative expansion

$$V_s^{NNLL}(x_\perp) = -C_F C_A^2 \alpha^3 T^3 x_\perp^2 \left[\frac{1}{6} W^3\left(\frac{1}{\alpha C_A e^{\frac{\gamma}{2}+k_0(m_D x_\perp)}\sqrt{\pi} x_\perp T}\right)\right.$$

$$+\frac{1}{2} W^2\left(\frac{1}{\alpha C_A e^{\frac{\gamma}{2}+k_0(m_D x_\perp)}\sqrt{\pi} x_\perp T}\right)$$

$$\left.+\frac{13\pi^2-2208}{384} W\left(\frac{1}{\alpha C_A e^{\frac{\gamma}{2}+k_0(m_D x_\perp)}\sqrt{\pi} x_\perp T}\right)\right]$$

Properties of the Lambert function



Properties of the Lambert function

$m_D x_\perp \gg 1$ implies $t \sim 1$. No logarithmic enhancement.
$m_D x_\perp \sim 1$ implies $t \ll 1$.

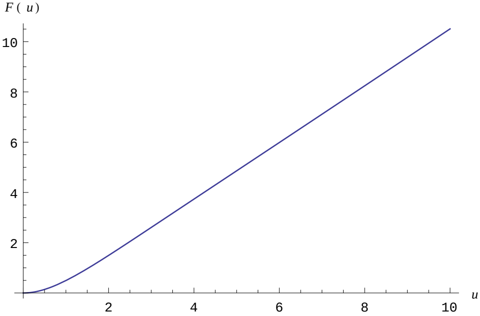
$$W(1/t) \sim -\log t - \log(-\log t) + \ldots$$

$m_D x_\perp \ll 1$ implies

$$t = \frac{\alpha T C_A 2\sqrt{\pi}}{m_D e^{\gamma/2}}$$

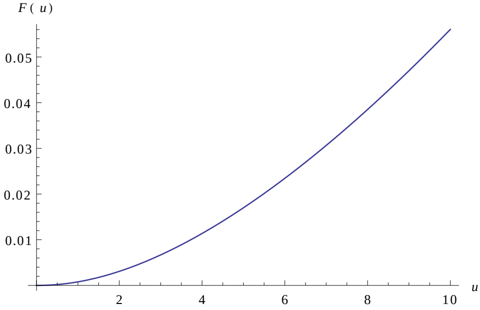
Properties of V_s^{NNLL}

Define $u = m_D x_\perp$, $\lambda = \frac{\alpha C_A T}{m_D}$ and $F[u] = \frac{V_s^{NNLL}(u)}{C_F \alpha T}$
For $\lambda = 0.5$



Properties of V_s^{NNLL}

Define $u = m_D x_\perp$, $\lambda = \frac{\alpha C_A T}{m_D}$ and $F[u] = \frac{V_s^{NNLL}(u)}{C_F \alpha T}$
For $\lambda = 0.01$



Properties of $P(x_\perp)$

$$P(x_\perp) = e^{-h_s(x_\perp)L}$$

- It happens that $Z(x_\perp) = 1$ at our order of accuracy.
- To be able to compute $\hat{q}$ analytically one assumes $h_s(x_\perp)L \ll 1$.
- Starting from here we use this approximation, however what we learn until now about $h_s(x_\perp)$ is still valid if this does not happen.

Computation of $\hat{q}^{NNLL}$

$$\hat{q}^{NNLL} = -\int_{k_2}^{k_1} \frac{d^2k_\perp}{(2\pi)^2} k_\perp^2 \int d^2x_\perp e^{-i\mathbf{x}_\perp \mathbf{k}_\perp} V_s^{NNLL}(x_\perp)$$

- $\pi T \gg k_1 \gg m_D$
- $m_D \gg k_2 \gg \alpha T$

Computation of $\hat{q}^{NNLL}$

$$\hat{q}^{NNLL} = \hat{q}^{NNLL,r} + \hat{q}^{NNLL,UV} + \hat{q}^{NNLL,IR}$$

$$\hat{q}^{NNLL,r} = -\int \frac{d^2k_\perp}{(2\pi)^2} k_\perp^2 \int d^2x_\perp e^{-i\mathbf{x}_\perp \mathbf{k}_\perp} V_s^{NNLL}(x_\perp)$$

$$\hat{q}^{NNLL,UV} = \int_{k_1}^{\infty} \frac{d^2k_\perp}{(2\pi)^2} k_\perp^2 \int d^2x_\perp e^{-i\mathbf{x}_\perp \mathbf{k}_\perp} V_s^{NNLL}(x_\perp)$$

$$\hat{q}^{NNLL,IR} = \int_0^{k_2} \frac{d^2k_\perp}{(2\pi)^2} k_\perp^2 \int d^2x_\perp e^{-i\mathbf{x}_\perp \mathbf{k}_\perp} V_s^{NNLL}(x_\perp)$$

Computation of $\hat{q}^{NNLL,r}$

$$\hat{q}^{NNLL,r} = \nabla_\perp^2 V_s^{NNLL}(x_\perp)|_{x_\perp=0}$$

$$\hat{q}^{NNLL,r} = -4C_F C_A^2 \alpha^3 T^3 \left[\frac{1}{6} W^3\left(\frac{m_D e^{\frac{\gamma}{2}}}{\alpha T C_A 2\sqrt{\pi}}\right) + \frac{1}{2} W^2\left(\frac{m_D e^{\frac{\gamma}{2}}}{\alpha T C_A 2\sqrt{\pi}}\right) + \frac{13\pi^2-2208}{384} W\left(\frac{m_D e^{\frac{\gamma}{2}}}{\alpha T C_A 2\sqrt{\pi}}\right)\right]$$

Computation of $\hat{q}^{NNLL,UV}$

For $m_D x_\perp \ll 1$

$$V_s^{NNLL}(x_\perp) = A x_\perp^2 + \dots$$

which implies

$$\hat{q}^{NNLL,UV} = A \int_{k_1}^{\infty} \frac{d^2 k_\perp}{(2\pi)^2} \delta^2(k_\perp) = 0$$

In fact it is a contribution to N^3LL and not to $NNLL$.

Computation of $\hat{q}^{NNLL,IR}$

For $m_D x_\perp \gg 1$ the Lambert function is not big.
No logarithmic enhancement. It is a *NNLO* contribution and not a *NNLL* one.

Final result

$$\hat{q} = \hat{q}^{LO} + \hat{q}^{NLO} + \hat{q}^{NNLL} + \dots$$

$$\hat{q}^{LO}(\Lambda) = \frac{2g^4T^3}{3\pi}\left[\frac{3}{2}\log\left(\frac{T}{m_D}\right) + \frac{7\zeta(3)}{4\zeta(2)}\log\left(\frac{\Lambda}{T}\right) - 0.105283\right]$$

Arnold and Xiao (2008)

$$\hat{q}^{NLO} = \frac{g^4T^2m_D}{8\pi^2}(3\pi^2 + 10 - 4\log(2))$$

Caron-Huot (2008)

$$\hat{q}^{NNLL} = -48\alpha^3T^3\left[\frac{1}{6}W^3\left(\frac{m_De^{\frac{\gamma}{2}}}{6\sqrt{\pi}\alpha T}\right) + \frac{1}{2}W^2\left(\frac{m_De^{\frac{\gamma}{2}}}{6\sqrt{\pi}\alpha T}\right) + \frac{13\pi^2-2208}{384}W\left(\frac{m_De^{\frac{\gamma}{2}}}{6\sqrt{\pi}\alpha T}\right)\right]$$

Conclusions

Conclusions

- The understanding of jet broadening is important for the interpretation of jet observables in heavy-ion collisions.
- It is important to improve perturbative computations.
- We obtained NNLL contributions to $P(k_\perp)$ and $\hat{q}$.
- In order to get the correct result one needs to consider the ladder resummation. This is easily done using EFTs.