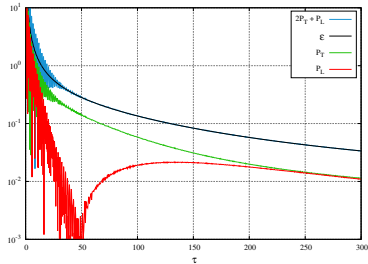
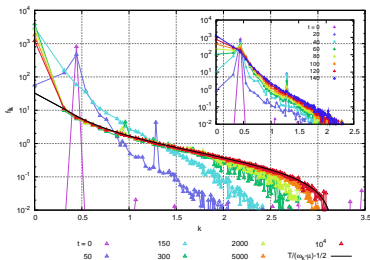


The early stages of heavy ion collisions



Trento, 18th June 2013

Thomas EPELBAUM
IPhT

OUTLINE

- ① INTRODUCTION
- ② THEORETICAL FRAMEWORK
- ③ FIXED VOLUME
- ④ EXPANDING VOLUME
- ⑤ QCD CASE
- ⑥ CONCLUSION

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Isotropization

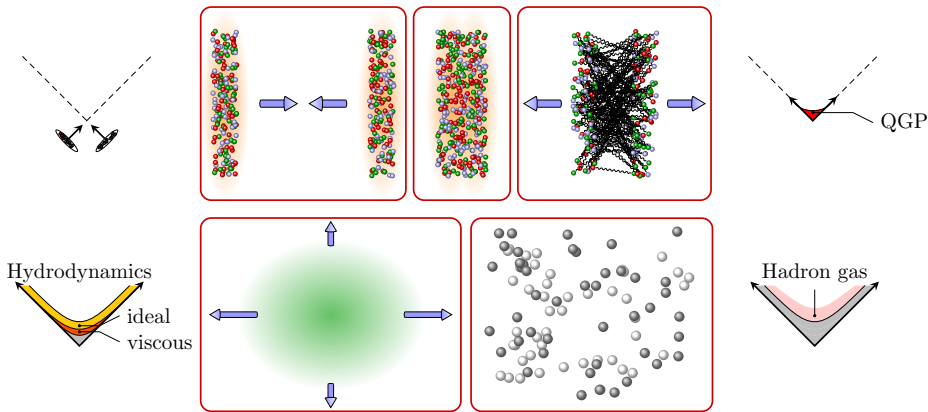
Comparison with Hydrodynamics

⑤ QCD CASE

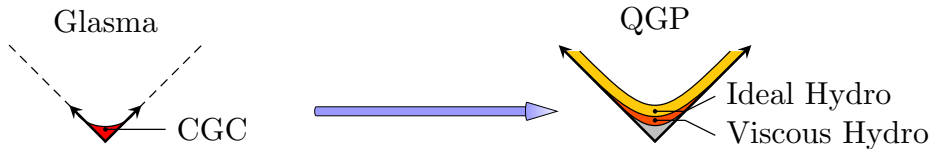
Spectrum of fluctuations

⑥ CONCLUSION

THE GENERAL PICTURE



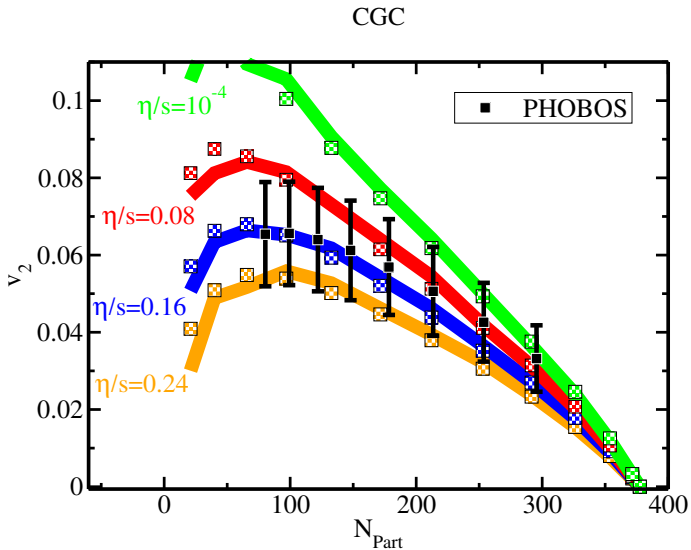
THE GENERAL PICTURE



Out of thermal equilibrium:
No Equation of state (EOS)
Huge anisotropy
 f_k far from thermal

Close to thermal equilibrium:
EOS
Small anisotropy
 f_k close to thermal

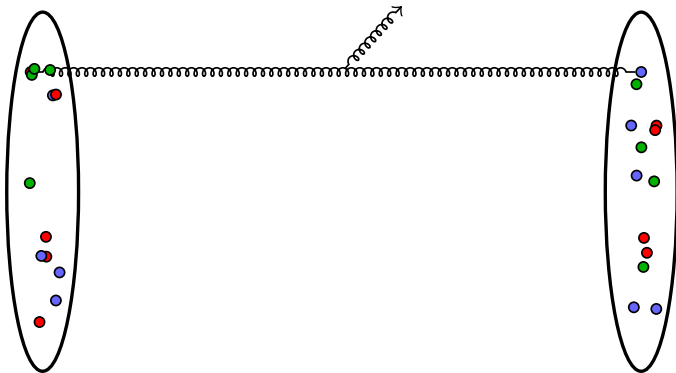
WHY DO WE TRUST HYDRODYNAMICS IN THE FIRST PLACE?



[LUZUM, ROMATSCHKE (2008)]

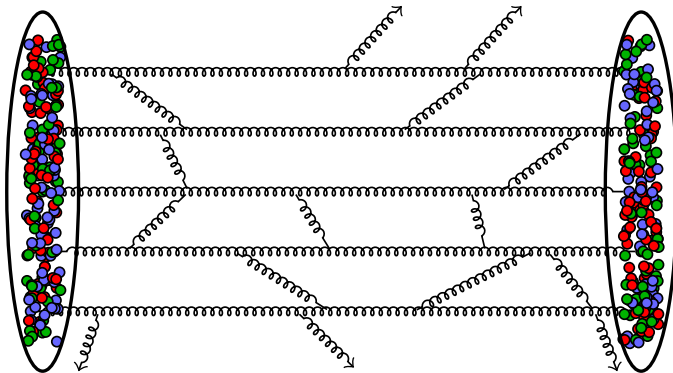
HOW TO STUDY THE TRANSITION THEN?

Dilute Regime



HOW TO STUDY THE TRANSITION THEN?

Dense regime:
 $\alpha_s \ll 1$ but $f_{\text{gluon}} \sim \frac{1}{\alpha_s} \dots$



HOW TO STUDY THE TRANSITION THEN?

THEORETICAL FRAMEWORK

- Color Glass Condensate (CGC)
- Resummation Scheme

Objectives:

TRANSITIONS?

- No EOS, big anisotropy \Rightarrow EOS, small anisotropy ?
- f_k far from Bose-Einstein $\Rightarrow f_k \sim \frac{1}{e^{\beta \omega_k} - 1}$?
- Thermalization time?

HYDRO PREREQUISITES?

- Actual value of ϵ , P_T , P_L ?
- Actual value of the transport coefficients? $\frac{\eta}{s}$...

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THE COLOR GLASS CONDENSATE (CGC) [MCLERRAN, VENUGOPALAN (1994)]

Fast and slow partons are not treated in the same way

THE MAIN ASSUMPTIONS

- Fast partons ($k > \Lambda$) \Rightarrow static color sources.

$$J^\mu(x^+, x^-, \mathbf{x}_\perp) = \delta^{\mu+} \delta(x^-) \rho^1(\mathbf{x}_\perp) + \delta^{\mu-} \delta(x^+) \rho^2(\mathbf{x}_\perp)$$

- Small $x \Rightarrow$ Gluon saturation $\Rightarrow J \propto g^{-1}$.
- Probabilistic knowledge of the $\rho \Rightarrow W_\Lambda[\rho]$.
- Slow partons $\Rightarrow \mathcal{A}^\mu$.
- System invariant under longitudinal boosts.

THE COLOR GLASS CONDENSATE (CGC) [MCLERRAN, VENUGOPALAN (1994)]

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Issues:

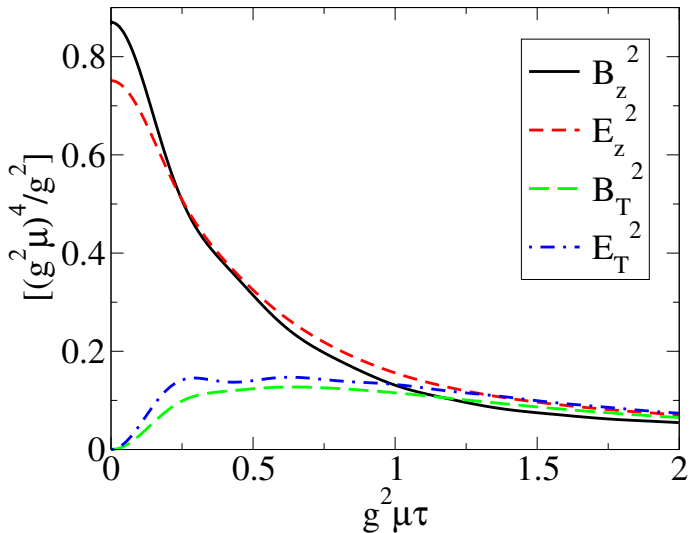
- Very Anisotropic system at $\tau = 0^+$
- Secular divergences.

ANISOTROPY ($\tau = 0^+$)

$$\epsilon = E_T^2 + B_T^2 + E_L^2 + B_L^2$$

$$P_T = E_L^2 + B_L^2$$

$$P_L = E_T^2 + B_T^2 - E_L^2 - B_L^2$$

ANISOTROPY ($\tau = 0^+$)

[LAPPI, MCLERRAN (2006)]

ANISOTROPY ($\tau = 0^+$)

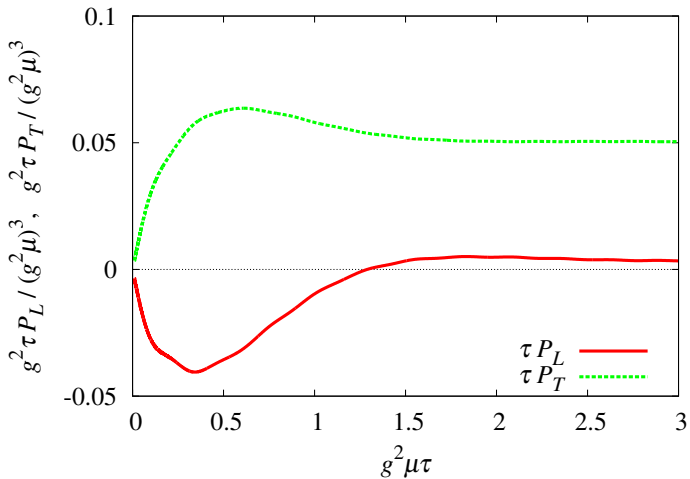
$$\epsilon = \underbrace{E_T^2}_0 + \underbrace{B_T^2}_0 + E_L^2 + B_L^2$$

$$P_T = E_L^2 + B_L^2$$

$$P_L = \underbrace{E_T^2}_0 + \underbrace{B_T^2}_0 - E_L^2 - B_L^2$$

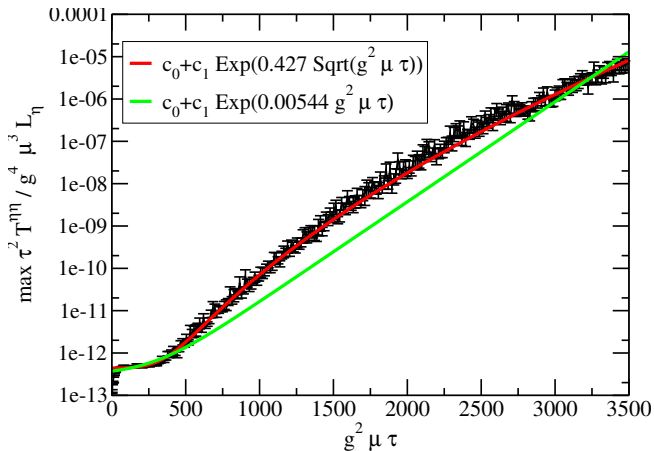
Initial $T^{\mu\nu}$ is $(\epsilon, \epsilon, \epsilon, -\epsilon)!$

ANISOTROPY ($\tau = 0^+$)



[FUKUSHIMA, GELIS (2012)]

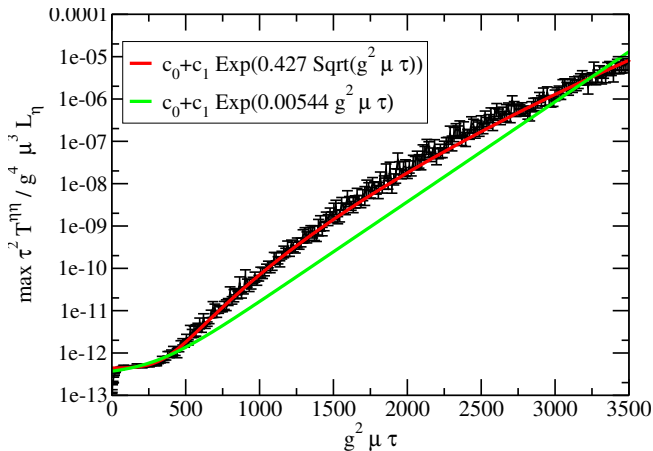
SECULAR DIVERGENCES



[ROMATSCHKE, VENUGOPALAN (2006)]

Fluctuations grows like $e^{\sqrt{\mu\tau}}$!

SECULAR DIVERGENCES



[ROMATSCHKE, VENUGOPALAN (2006)]

when $ge^{\sqrt{\mu\tau}} \approx 1$, perturbative expansion breaks down!

② THEORETICAL FRAMEWORK

CGC

Resummation Scheme

RESUMMATION

$$T_{\text{NLO}}^{\mu\nu}[\varphi_0] = \hat{\mathcal{O}} T_{\text{LO}}^{\mu\nu}[\varphi_0]$$

$$T_{\text{resum}}^{\mu\nu} = e^{\hat{\mathcal{O}}} T_{\text{LO}}^{\mu\nu}[\varphi_0] = T_{\text{LO}}^{\mu\nu}[\varphi_0] + T_{\text{NLO}}^{\mu\nu}[\varphi_0] + \dots$$

RESUMMATION FORMULA

RESUMMATION

$$T_{\text{NLO}}^{\mu\nu}[\varphi_0] = \hat{\mathcal{O}} T_{\text{LO}}^{\mu\nu}[\varphi_0]$$

$$T_{\text{resum}}^{\mu\nu} = e^{\hat{\mathcal{O}}} T_{\text{LO}}^{\mu\nu}[\varphi_0] = T_{\text{LO}}^{\mu\nu}[\varphi_0] + T_{\text{NLO}}^{\mu\nu}[\varphi_0] + \dots$$

INITIAL VALUE PROBLEM

$$\phi(t_0, \mathbf{x}) = \varphi_0(\mathbf{x}) + \text{Re} \int_k c_k a_k(t_0, \mathbf{x})$$

$$0 = \square \phi(t, \mathbf{x}) + V'[\phi(t, \mathbf{x})]$$

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LAGRANGIAN OF THE THEORY

$$\mathcal{L}(\phi) = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \underbrace{\frac{g^2}{4!}\phi^4}_{V(\phi)} + J\phi$$

WHY DO WE USE THIS MODEL?

- Scale invariance in 3 + 1 dimensions
- Parametric resonance
- Much simpler than QCD

INITIAL CONDITION

INITIAL CONDITION OF THE EOM

$$\phi(t_0, \mathbf{x}) = \varphi_0(\mathbf{x}) + \sum_k \text{Re} [c_k a_k]$$

with a_k given by

$$[\square + V''(\varphi_0)] a_k(t, \mathbf{x}) = 0 \quad \lim_{t \rightarrow -\infty} a_k(t, \mathbf{x}) = e^{ikx}$$

c_k random gaussian numbers

$$\langle c_k c_l^* \rangle = \delta(k - l)$$

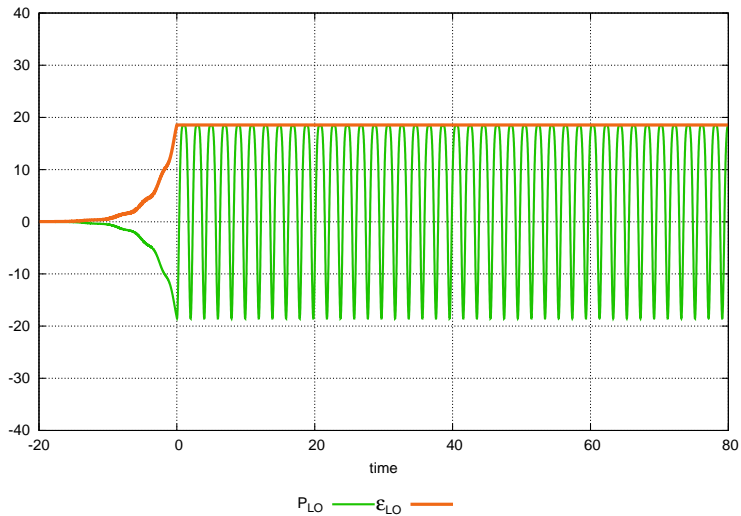
③ FIXED VOLUME

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Energy-Momentum Tensor

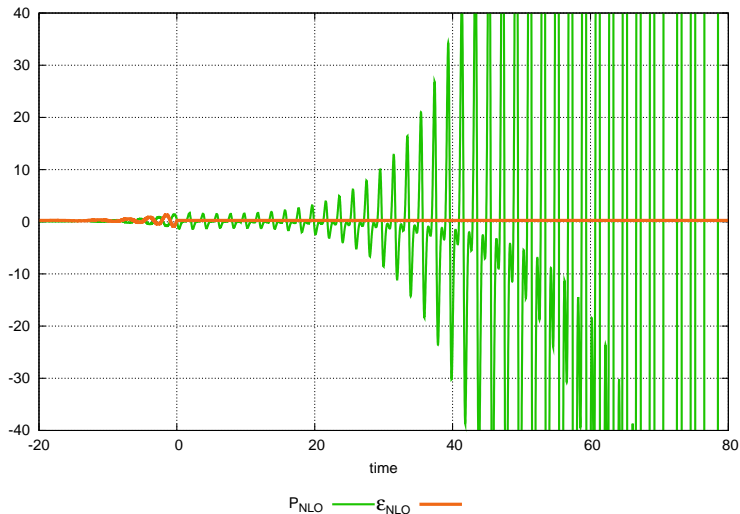
Distribution Function

Bose-Einstein Condensation

$T_{LO}^{\mu\nu}$ 

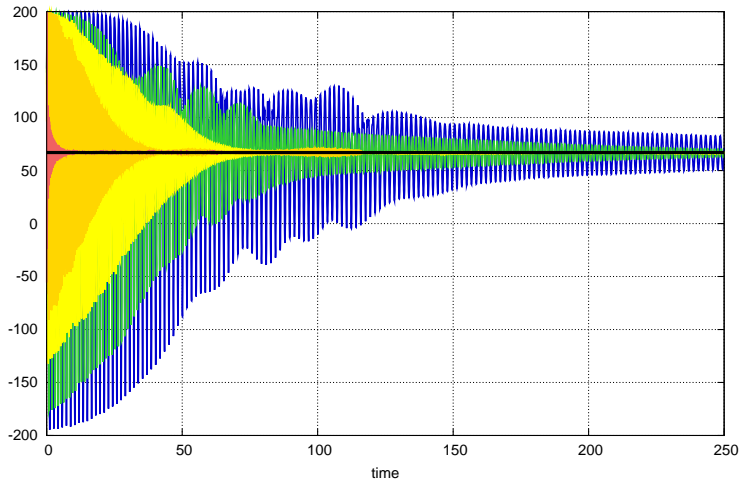
$T_{\text{NLO}}^{\mu\nu}$: SECULAR DIVERGENCES

$T_{\text{NLO}}^{\mu\nu}$



$T_{\text{RESUM}}^{\mu\nu}$: PRESSURE EQUILIBRATION

$T_{\text{resum}}^{\mu\nu}$



③ FIXED VOLUME

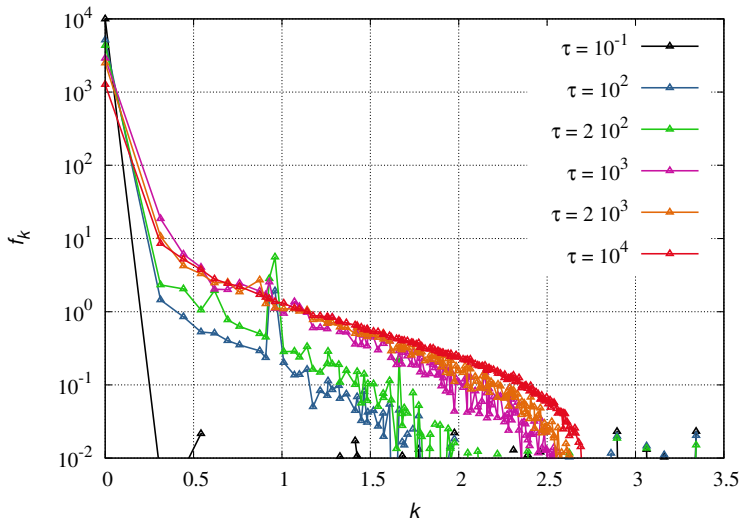
The Model

Energy-Momentum Tensor

Distribution Function

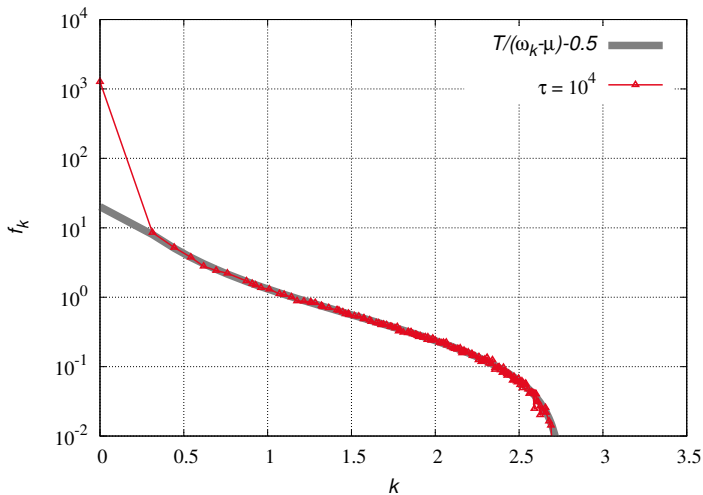
Bose-Einstein Condensation

TIME EVOLUTION OF THE OCCUPATION NUMBER [TE, GELIS (2011)]



"CLASSICAL" EQUILIBRIUM DISTRIBUTION

- [BERGES, SIXTY(2012)]
- [MICHA, TKACHEV(2004)]



③ FIXED VOLUME

The Model

Energy-Momentum Tensor

Distribution Function

Bose-Einstein Condensation

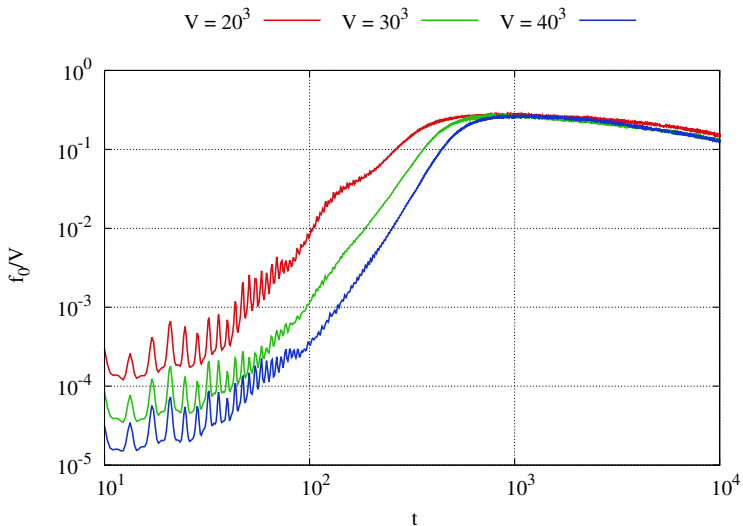
EVOLUTION OF THE CONDENSATE

$$f_{\mathbf{k}} = \frac{T}{\omega_{\mathbf{k}} - \mu} - \frac{1}{2} + n_0 \delta(\mathbf{k})$$

implies

$$\frac{f_0}{V} = \text{cte}$$

EVOLUTION OF THE CONDENSATE



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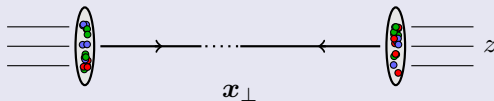
④ EXPANDING VOLUME

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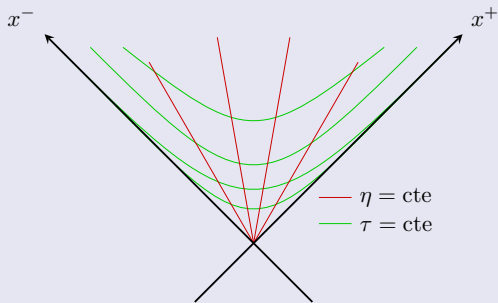
ADAPTED COORDINATE SYSTEM TO DESCRIBE A HEAVY ION COLLISION?



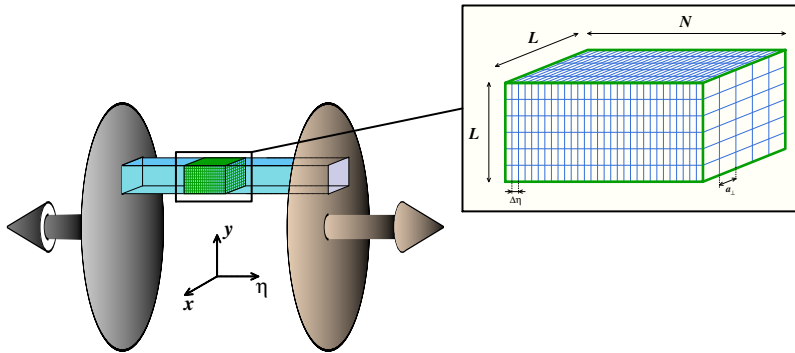
System boost invariant in z direction

SCALAR FIELD THEORY IN AN EXPANDING VOLUME

PROPER TIME/RAPIDITY COORDINATE SYSTEM



SCALAR FIELD THEORY IN AN EXPANDING VOLUME



INITIAL CONDITION

INITIAL CONDITION OF THE EOM

$$\phi_i(\tau_0, \mathbf{x}_\perp, \eta) = \varphi_0(\mathbf{x}_\perp) + \sum_{k_\perp, \nu} \text{Re} [c_{\nu k_\perp} a_{\nu k_\perp}(\tau, \mathbf{x}_\perp, \eta)]$$

INITIAL CONDITION

INITIAL CONDITION OF THE EOM

$$\phi_i(\tau_0, \mathbf{x}_\perp, \eta) = \varphi_0(\mathbf{x}_\perp) + \sum_{\mathbf{k}_\perp, \nu} \text{Re} [c_{\nu \mathbf{k}_\perp} e^{i\nu\eta} b_{\nu \mathbf{k}_\perp}(\tau, \mathbf{x}_\perp)]$$

with a_k given by

$$[\square + V''(\varphi_0)] a_k(\tau, \eta, \mathbf{x}_\perp) = 0 \quad \lim_{t \rightarrow -\infty} a_k(t, \mathbf{x}) = e^{ikx}$$

$c_{\nu \mathbf{k}_\perp}$ random gaussian numbers

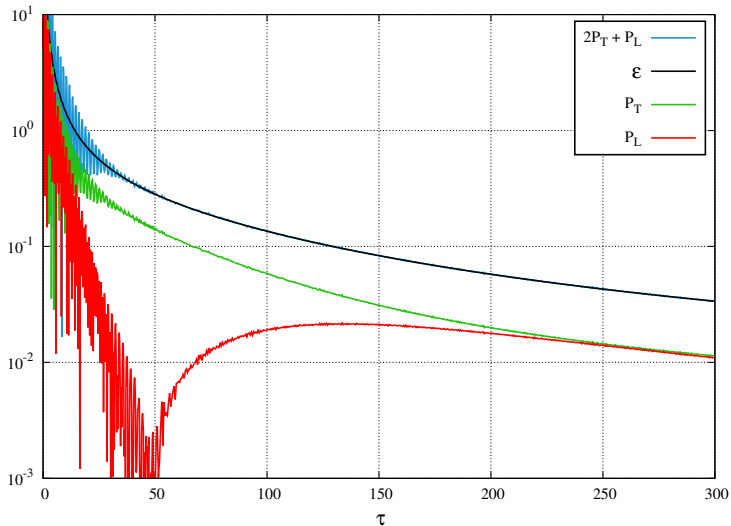
$$\langle c_{\nu \mathbf{k}_\perp} c_{\mu \mathbf{l}_\perp}^* \rangle = \delta(\mathbf{k}_\perp - \mathbf{l}_\perp) \delta(\nu - \mu)$$

④ EXPANDING VOLUME

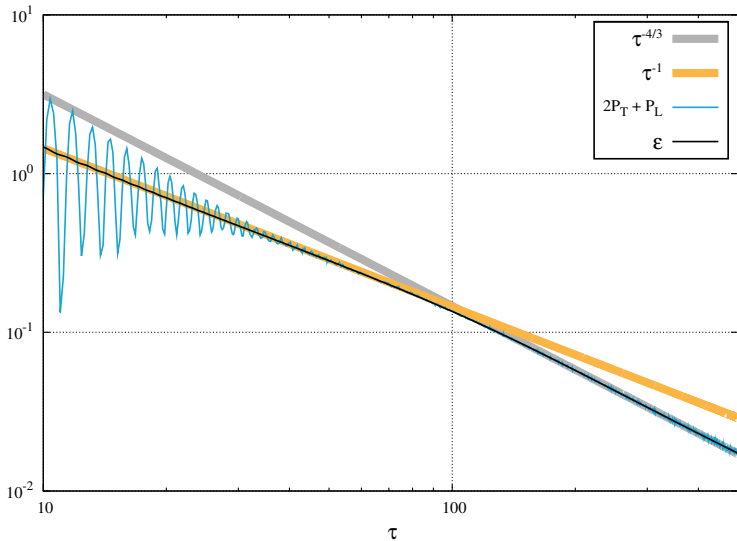
The model

Isotropization

Comparison with Hydrodynamics



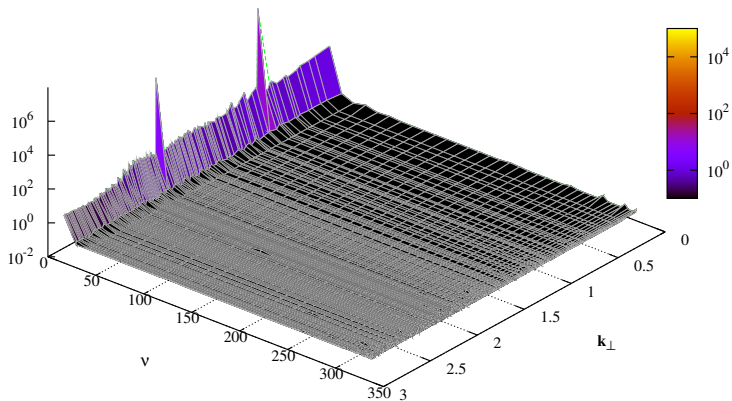
ϵ BEHAVIOUR



Bjorken Law: $\partial_\tau \epsilon + \frac{\epsilon + P_L}{\tau} = 0$

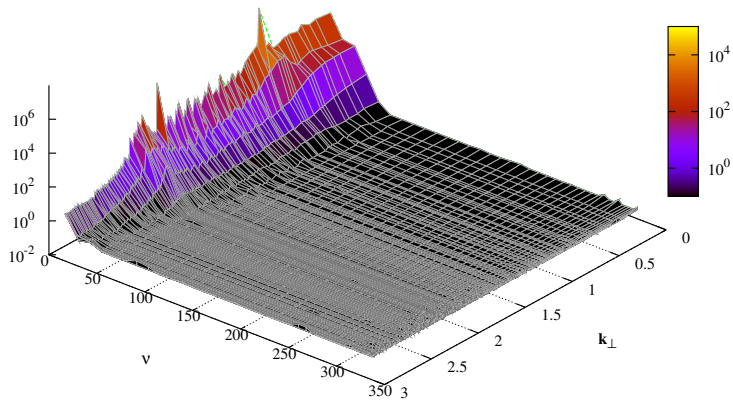
EVOLUTION OF $f_{\nu k_{\perp}}$

$\tau = 10$ [40 × 40 × 320]



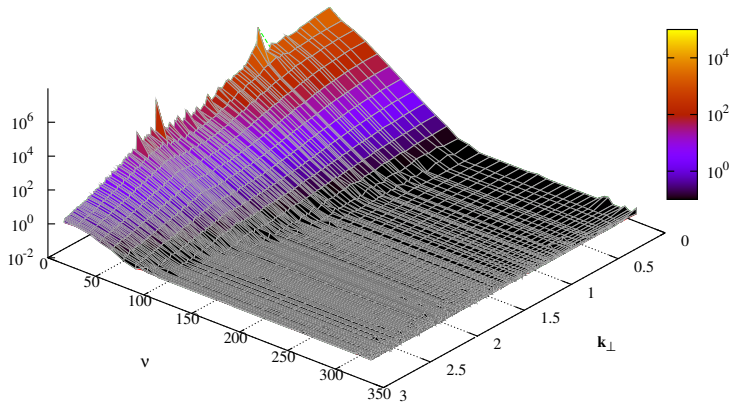
EVOLUTION OF $f_{\nu k_{\perp}}$

$\tau = 50$ [40 × 40 × 320]



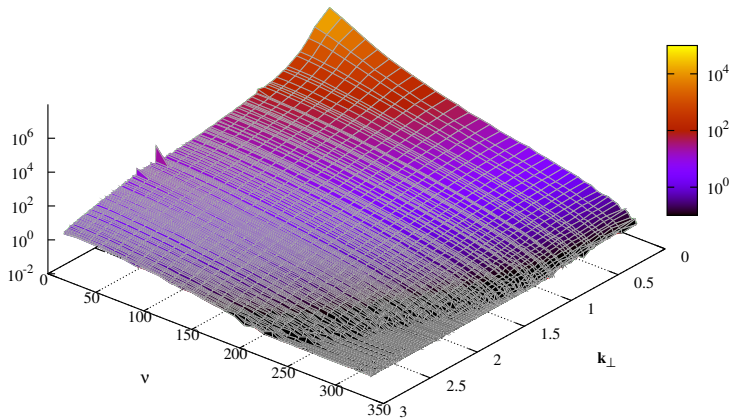
EVOLUTION OF $f_{\nu k_{\perp}}$

$\tau = 100$ [40 × 40 × 320]



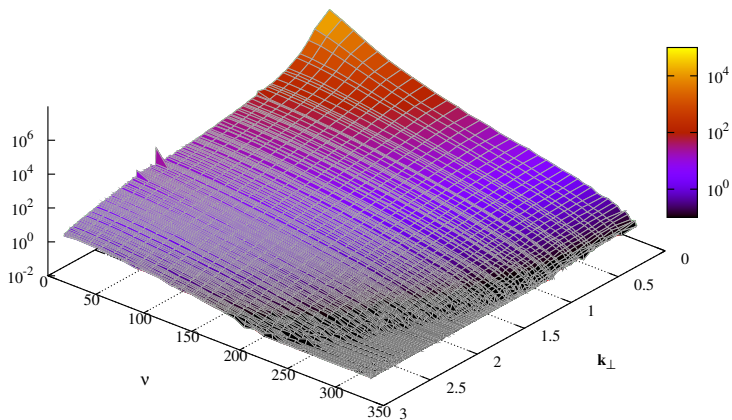
EVOLUTION OF $f_{\nu k_{\perp}}$

$\tau = 300$ [40 × 40 × 320]



EVOLUTION OF $f_{\nu k_{\perp}}$

$\tau = 300$ [40 × 40 × 320]



$$\epsilon \sim T^4 \sim \tau^{-4/3}$$

$$\nu \sim k_z \tau \sim \tau^{2/3}$$

④ EXPANDING VOLUME

The model

Isotropization

Comparison with Hydrodynamics

Equation of state: $\epsilon = 2P_L + P_T$

IDEAL HYDRO

Isotropic system

$$T_{\text{ideal}}^{\mu\nu} = \epsilon u^\mu u^\nu - P(g^{\mu\nu} - u^\mu u^\nu)$$

Equation of state: $\epsilon = 2P_L + P_T$

IDEAL HYDRO

Isotropic system

$$T_{\text{ideal}}^{\mu\nu} = \epsilon u^\mu u^\nu - P(g^{\mu\nu} - u^\mu u^\nu)$$

VISCOUS HYDRO

Anisotropic system

$$T^{\mu\nu} = T_{\text{ideal}}^{\mu\nu} + \Pi^{\mu\nu}$$

With first order corrections

$$P_T = \frac{\epsilon}{3} + \frac{2\eta}{3\tau} \qquad P_L = \frac{\epsilon}{3} - \frac{4\eta}{3\tau}$$

BJORKEN's Law (coming from $\partial_\mu T^{\mu\nu} = 0$):

$$\partial_\tau \epsilon + \frac{\epsilon + P_L}{\tau} = 0 \rightarrow \partial_\tau \epsilon + \frac{4}{3} \frac{\epsilon}{\tau} = \frac{4}{3} \frac{\eta}{\tau^2}$$

BJORKEN's Law (coming from $\partial_\mu T^{\mu\nu} = 0$):

$$\partial_\tau \epsilon + \frac{\epsilon + P_L}{\tau} = 0 \rightarrow \partial_\tau \epsilon + \frac{4}{3} \frac{\epsilon}{\tau} = \frac{4}{3} \frac{\eta}{\tau^2}$$

assuming $\eta = \frac{\eta_0}{\tau}$ and STEFAN-BOLTZMANN entropy $s \approx \epsilon^{\frac{3}{4}}$

$$\partial_\tau \epsilon + \frac{4}{3} \frac{\epsilon}{\tau} = \frac{4}{3} \underbrace{\frac{\eta}{s}}_{\text{cte}} \frac{\epsilon^{\frac{3}{4}}}{\tau^2}$$

BJORKEN's Law (coming from $\partial_\mu T^{\mu\nu} = 0$):

$$\partial_\tau \epsilon + \frac{\epsilon + P_L}{\tau} = 0 \rightarrow \partial_\tau \epsilon + \frac{4}{3} \frac{\epsilon}{\tau} = \frac{4}{3} \frac{\eta}{\tau^2}$$

assuming $\eta = \frac{\eta_0}{\tau}$ and STEFAN-BOLTZMANN entropy $s \approx \epsilon^{\frac{3}{4}}$

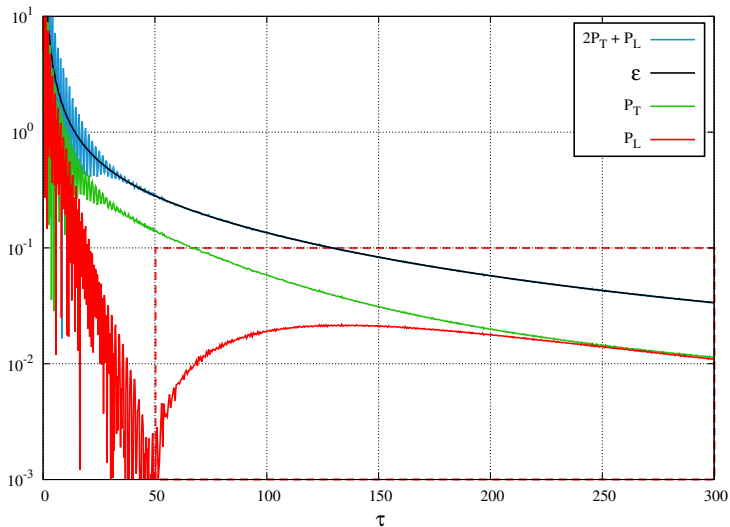
$$\partial_\tau \epsilon + \frac{4}{3} \frac{\epsilon}{\tau} = \frac{4}{3} \underbrace{\frac{\eta}{s}}_{\text{cte}} \frac{\epsilon^{\frac{3}{4}}}{\tau^2}$$

At a given time, knowing ϵ, P_T, P_L and assuming

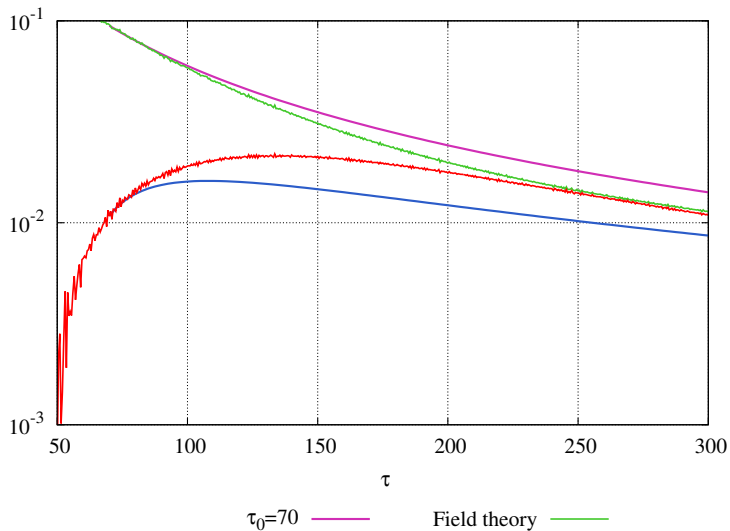
- an EOS
- STEFAN-BOLTZMANN entropy
- $\eta = \frac{\eta_0}{\tau}$
- $\frac{\eta}{s} = \text{cte}$

gives a very simple hydrodynamical model.

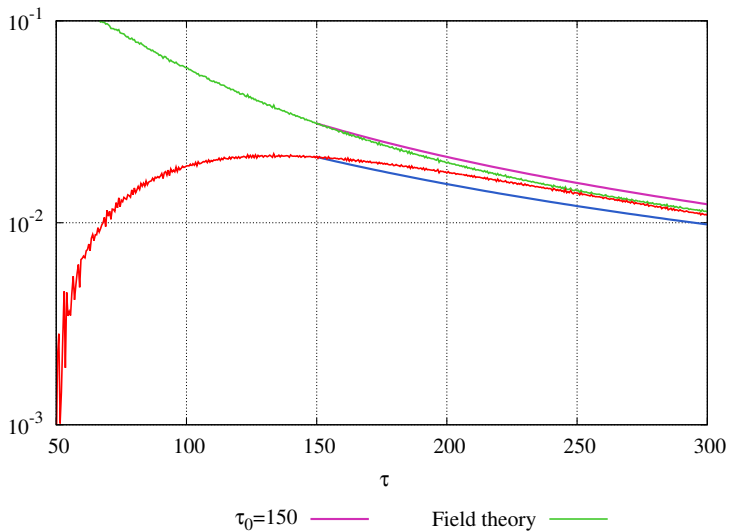
COMPARISON WITH HYDRODYNAMICS: ISOTROPIZATION



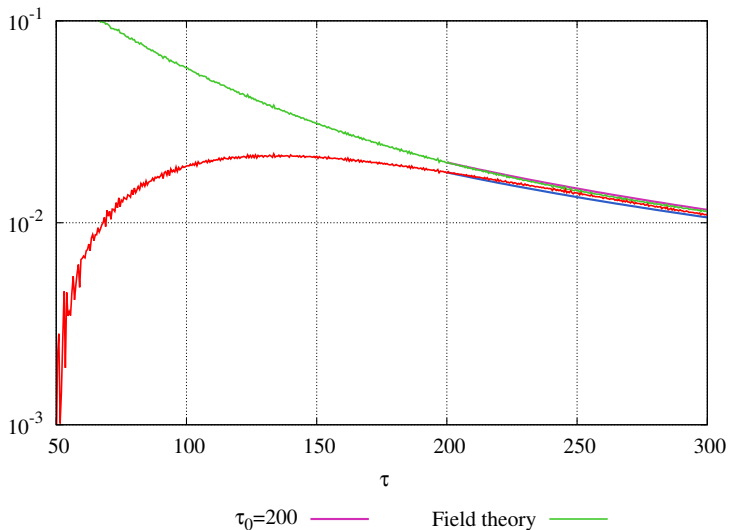
COMPARISON WITH HYDRODYNAMICS: ISOTROPIZATION



COMPARISON WITH HYDRODYNAMICS: ISOTROPIZATION

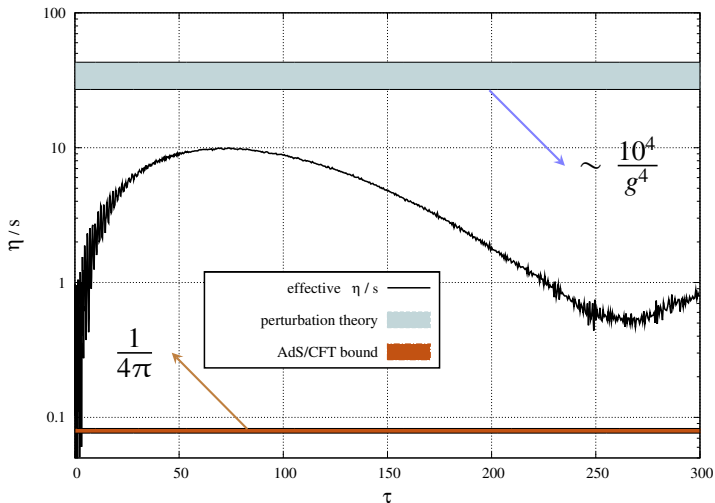


COMPARISON WITH HYDRODYNAMICS: ISOTROPIZATION



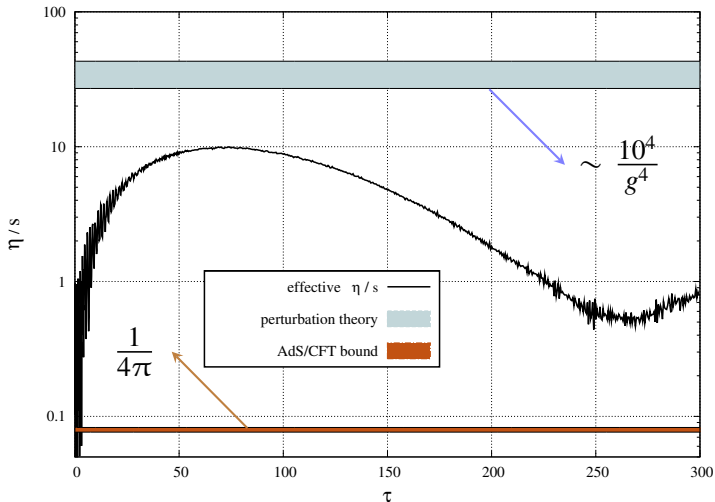
COMPARISON WITH HYDRODYNAMICS: VISCOSITY

$$P_T - P_L = \frac{2\eta}{\tau}$$



COMPARISON WITH HYDRODYNAMICS: VISCOSITY

see also [ASAKAWA, BASS, MULLER (2006-07)]

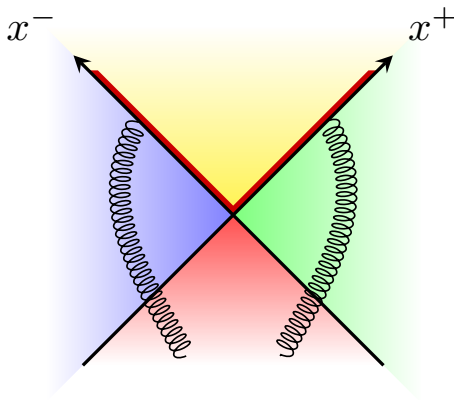


5 QCD CASE

Spectrum of fluctuations

HOW TO CONSTRUCT THE SPECTRUM OF FLUCTUATIONS?

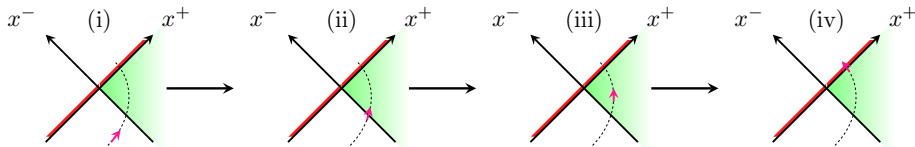
[TE, GELIS (in preparation)]



$$a_{\mathbf{k}\mu\lambda}^a = \epsilon_{\mu\mathbf{k}\lambda}^a e^{i\mathbf{k}x}$$

HOW TO CONSTRUCT THE SPECTRUM OF FLUCTUATIONS?

The whole evolution



$$A^{\mu a}(\tau_0, \eta, \mathbf{x}_\perp) = \mathcal{A}^{\mu a}(\mathbf{x}_\perp) + \text{Re} \sum_{\lambda, c} \int_{\mathbf{k}} c_{k\lambda c} a_{\mu k\lambda c}^a(\tau_0, \eta, \mathbf{x}_\perp)$$

Lattice simulations \Rightarrow [TE, GELIS (work in progress)]

HOW TO CONSTRUCT THE SPECTRUM OF FLUCTUATIONS?

A side remark

arXiv:1303.5650 :

$$\langle A \rangle, \langle E \rangle = 0$$

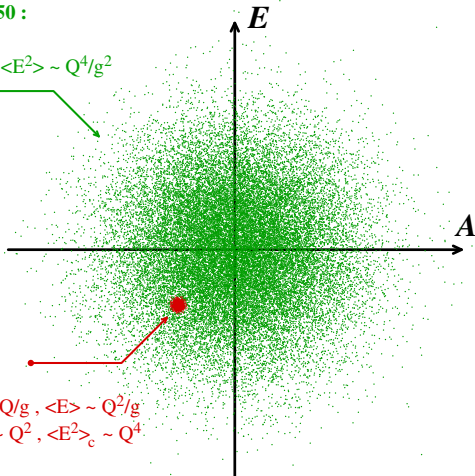
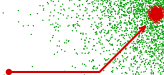
$$\langle A^2 \rangle \sim Q^2/g^2, \langle E^2 \rangle \sim Q^4/g^2$$



CGC :

$$\langle A \rangle \sim Q/g, \langle E \rangle \sim Q^2/g$$

$$\langle A^2 \rangle_c \sim Q^2, \langle E^2 \rangle_c \sim Q^4$$



FIXED VOLUME

Instabilities



Decoherence



Equation of state $\epsilon = 3P$ (isotropy by construction)



BOSE-EINSTEIN condensate



$$f_k \propto \frac{T}{\omega_k - m} - \frac{1}{2}$$

EXPANDING SYSTEM

Instabilities [BERGES, BOGUSLAVSKI, SCHLICHTING (2012)]



Decoherence



Equation of state $\epsilon = 2P_T + P_L$



Isotropization

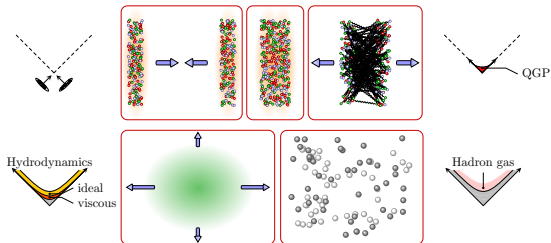


Thermal distribution



BOSE-EINSTEIN condensate?

Thank You!



NON-ZERO INITIAL MODE: $\varphi_0 \sim \cos k \cdot x$

