Improved treatment of kinematics for gluon saturation in QCD at high energy

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h3QCD workshop

ECT*, Trento , june 17-21, 2013

Outline

- Introduction
  - Gluon saturation towards higher orders
  - Collinear resummations for BFKL
- DIS at NLO and subtraction of LL's
- Kinematical constraint for the BK equation

G.B., *to appear*

See also the summarized version: G.B., arXiv:1301.0773

Gluon saturation/CGC in dense-dilute collisions

DIS observables and forward particle production in pp/pA:

- theoretically best understood observables sensitive to gluon saturation at high energy
- abundant available or incoming data from HERA, LHC, RHIC
- successful phenomenology within the Color Glass Condensate

However: saturation effects difficult to see clearly from the most inclusive observables.

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However: saturation effects difficult to see clearly from the most inclusive observables.

$\rightarrow$ 2 directions for improvement:

1. Study to less inclusive observables, like multi-particle correlations

   cf. most talks on monday

2. Go to higher orders or other refinements for more precision

Gluon saturation/CGC at higher orders

Going to higher orders is necessary for precision studies.

For the simpler, inclusive observables, the calculation of higher order corrections has started:

- NLL corrections to the BK equation
  Balitsky, Chirilli (2008)
- NLO corrections to DIS structure functions
  Balitsky, Chirilli (2011)
  G.B. (2012)
- NLO corrections to forward single inclusive particle production in pA or pp
  Chirilli, Xiao, Yuan (2012)

Need for further resummations

However, besides running coupling effects, pathologically large corrections of two types plague higher order results and have to be resummed to obtain reliable results from BK at NLL.

- Kinematical corrections: due to a too naive treatment of the high-energy limit.
  $\rightarrow$ Main topic of the rest of this talk.
- Dynamical corrections: induced from DGLAP evolutions of the projectile and of the target, due to the duality between low x_{Bj} and high Q^2 evolutions.
  $\rightarrow$ Left for further studies.

The same problems appear in the linear regime for the BFKL equation, and the corresponding resummations have been fully performed.
Ciafaloni, Colferai, Salam, Staśto (1998-2007)
Altarelli, Ball, Forte (1999-2008)

Kinematical issues for BFKL in momentum space

Conventional derivations of the BFKL evolution require kinematical approximations for the t-channel gluons propagators, or for the energy denominators in the dipole model derivation.

Usual justification for those approximations: multi-Regge kinematics

- Strong ordering in rapidity y (or in k^+, or in k^-) of the emitted gluons
- and all $\mathbf{k}_\perp$'s of the same order

Sufficient but not necessary condition for the kinematical approximations to be valid.

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Problem: unrestricted integral over $\mathbf{k}_\perp$ in the BFKL equation
$\Rightarrow$ *A posteriori* not consistent to assume all $\mathbf{k}_\perp$'s of the same order!

Kinematical issues for BFKL in momentum space

Necessary and sufficient condition for the required kinematical approximations:

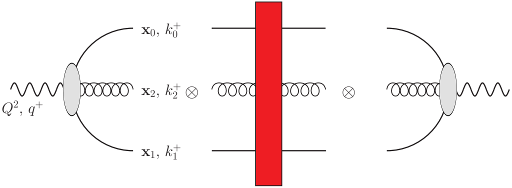
Strong ordering of the emitted gluons *both* in k^+ and in k^- simultaneously

Strong ordering is guarantied only for the evolution variable chosen for the BFKL equation: y, k^+ or k^-, depending on the factorization scheme.

$\Rightarrow$ Need to impose by hand the missing k^- and/or k^+ ordering in the equation via a kinematical constraint in the BFKL equation, by consistency.

In momentum space:
Ciafaloni (1988)
Kwieciński, Martin, Sutton (1996)
Andersson, Gustafson, Kharraziha, Samuelsson (1996)

DIS at high energy at NLO

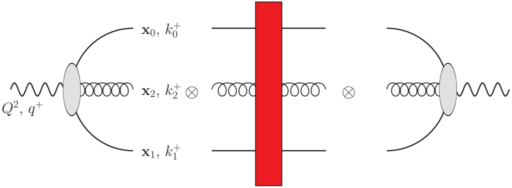


$$\sigma^{\gamma}_{T,L}(Q^2, x_{Bj}) = 2\,\frac{2N_c\,\alpha_{em}}{(2\pi)^2}\sum_f e_f^2 \int \mathrm{d}^2\mathbf{x}_0 \int \mathrm{d}^2\mathbf{x}_1 \int_0^1 \mathrm{d}z_1$$
$$\times\Bigg\{\mathcal{I}^{LO}_{T,L}(x_{01}, z_1, Q^2)\,\Big[1+\mathcal{O}(\bar{\alpha})\Big]\,\Big[1-\langle\mathcal{S}_{01}\rangle_0\Big]$$
$$+\bar{\alpha}\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\int_{k^+_{\min}/q^+}^{1-z_1}\frac{\mathrm{d}z_2}{z_2}\,\mathcal{I}^{NLO}_{T,L}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_1,z_2,Q^2)\,\langle\mathcal{S}_{01}-\mathcal{S}_{02}\,\mathcal{S}_{21}\rangle_0\Bigg\}$$

with $z_n = k_n^+/q^+$

G.B. (2012); see also Balitsky, Chirilli (2011).

DIS at high energy at NLO



$$\sigma^{\gamma}_{T,L}(Q^2, x_{Bj}) = 2\,\frac{2N_c\,\alpha_{em}}{(2\pi)^2} \sum_f e_f^2 \int \mathrm{d}^2\mathbf{x}_0 \int \mathrm{d}^2\mathbf{x}_1 \int_0^1 \mathrm{d}z_1$$

$$\times\Bigg\{ \mathcal{I}^{LO}_{T,L}(x_{01}, z_1, Q^2)\,\Big[1+\mathcal{O}(\bar{\alpha})\Big]\,\Big[1-\langle \mathcal{S}_{01}\rangle_0\Big]$$

$$+\bar{\alpha}\int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \int_{k^+_{\min}/q^+}^{1-z_1} \frac{\mathrm{d}z_2}{z_2}\, \mathcal{I}^{NLO}_{T,L}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, z_1, z_2, Q^2)\,\langle \mathcal{S}_{01} - \mathcal{S}_{02}\,\mathcal{S}_{21}\rangle_0 \Bigg\}$$

with $z_n = k_n^+/q^+$

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DIS impact factors

$$\mathcal{I}_L^{LO}(x_{01},z_1) = 4Q^2\, \mathrm{K}_0^2(QX_2)\; z_1^2(1-z_1)^2$$

$$\mathcal{I}_T^{LO}(x_{01},z_1) = Q^2 \mathrm{K}_1^2(QX_2)\, \left[z_1^2+(1-z_1)^2\right] z_1(1-z_1)$$

$$\mathcal{I}_L^{NLO}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_1,z_2) = 4Q^2\, \mathrm{K}_0^2(QX_3) \Bigg\{ (z_1+z_2)^2(1-z_1-z_2)^2 \frac{\mathcal{P}\left(\frac{z_2}{z_1+z_2}\right)}{x_{21}^2}$$

$$+z_1^2(1-z_1)^2 \frac{\mathcal{P}\left(\frac{z_2}{1-z_1}\right)}{x_{20}^2} - 2z_1(1-z_1)(z_1+z_2)(1-z_1-z_2)\left[1-\frac{z_2}{2(1-z_1)}-\frac{z_2}{2(z_1+z_2)}\right]\left(\frac{\mathbf{x}_{20}\cdot\mathbf{x}_{21}}{x_{20}^2\; x_{21}^2}\right)\Bigg\}$$

In the Bessel $\mathrm{K}_{0,1}$ functions prefactors:

$$X_2^2 = z_1\,(1-z_1)\,x_{10}^2$$

$$X_3^2 = z_1\,(1-z_1-z_2)\,x_{10}^2 + z_2\,(1-z_1-z_2)\,x_{20}^2 + z_2\,z_1\,x_{21}^2$$

DGLAP quark to gluon splitting function:

$$\mathcal{P}(z) = \frac{z}{2\,C_F}\,P_{gq}(z) = \frac{1}{2}\left[1+(1-z)^2\right]$$

DIS impact factors

$$\mathcal{I}_T^{NLO}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_0,z_1,z_2)=\frac{Q^2\,\mathrm{K}_1^2(QX_3)}{X_3^2}\Bigg\{z_1^2(1-z_1)^2\left[z_1^2+(1-z_1)^2\right]\left(\mathbf{x}_{10}-\frac{z_2}{1-z_1}\mathbf{x}_{20}\right)^2\frac{\mathcal{P}\left(\frac{z_2}{1-z_1}\right)}{x_{20}^2}$$

$$+z_0^2(1-z_0)^2\left[z_0^2+(1-z_0)^2\right]\left(\mathbf{x}_{01}-\frac{z_2}{1-z_0}\mathbf{x}_{21}\right)^2\frac{\mathcal{P}\left(\frac{z_2}{1-z_0}\right)}{x_{21}^2}$$

$$+2z_1(1-z_1)z_0(1-z_0)\left[z_1(1-z_0)+z_0(1-z_1)\right]\left(\mathbf{x}_{10}-\frac{z_2}{1-z_1}\mathbf{x}_{20}\right)\cdot\left(\mathbf{x}_{01}-\frac{z_2}{1-z_0}\mathbf{x}_{21}\right)$$

$$\times\left[1-\frac{z_2}{2(1-z_1)}-\frac{z_2}{2(1-z_0)}\right]\left(\frac{\mathbf{x}_{20}\cdot\mathbf{x}_{21}}{x_{20}^2\,x_{21}^2}\right)$$

$$+\frac{z_0\,z_1\,z_2^2\,(z_0-z_1)^2}{(1-z_1)(1-z_0)}\,\frac{\left(\mathbf{x}_{20}\wedge\mathbf{x}_{21}\right)^2}{x_{20}^2\,x_{21}^2}+z_0\,z_1^2\,z_2\left[\frac{z_0\,z_1}{(1-z_1)}+\frac{(1-z_1)^2}{(1-z_0)}\right]\left(\mathbf{x}_{10}-\frac{z_2}{1-z_1}\mathbf{x}_{20}\right)\cdot\left(\frac{\mathbf{x}_{20}}{x_{20}^2}\right)$$

$$+z_0^2\,z_1\,z_2\left[\frac{z_0\,z_1}{(1-z_0)}+\frac{(1-z_0)^2}{(1-z_1)}\right]\left(\mathbf{x}_{01}-\frac{z_2}{1-z_0}\mathbf{x}_{21}\right)\left(\frac{\mathbf{x}_{21}}{x_{21}^2}\right)+\frac{z_0^2\,z_1^2\,z_2^2}{2}\left[\frac{1}{(1-z_1)^2}+\frac{1}{(1-z_0)^2}\right]\Bigg\}$$

Formation time interpretation of the prefactors

The DIS impact factors contain a prefactor dependent on the variables

$$X_2^2 = z_1\, z_0\, x_{10}^2 \quad (\text{with } z_0 + z_1 = 1)$$

$$X_3^2 = z_1\, z_0\, x_{10}^2 + z_2\, z_0\, x_{20}^2 + z_2\, z_1\, x_{21}^2 \quad (\text{with } z_0 + z_1 + z_2 = 1)\,.$$

$2q^+X_2^2$ and $2q^+X_3^2$ are the formation time of the $q\bar{q}$ and $q\bar{q}g$ Fock states in the photon wave-function.

$\mathrm{K}_{0,1}^2(QX_n)$ prefactors $\Rightarrow$ exponential suppression of the Fock states with formation time larger than the virtual photon lifetime $2q^+/Q^2$.

G.B. (2012)

High-energy factorization

Soft divergence of the k_2^+ integration: regulated by the physical k^+_{min} scale set by the target.
$\rightarrow$ but the integration still gives a large factor $\sim \log(q^+/k^+_{\text{min}})$.
Such LL term should be absorbed into $\langle \mathcal{S}_{01} \rangle$ to stabilize perturbation theory.

Convenient (but not unique) choice of factorization scheme:

- only gluons with $k^+ < k_f^+$ are included into the shockwave field $\mathcal{A}$ of the target
- other gluons appear in the N^{n}LO corrections to the considered observable

see *e.g.* Balitsky, Chirilli (2007)

LL BK evolution for $\langle \mathcal{S}_{01} \rangle$

In the LO contribution to $\sigma^{\gamma}_{T,L}$:

$\rightarrow$ replace the *classical* $\langle \mathcal{S}_{01} \rangle_0$ by the evolved one $\langle \mathcal{S}_{01} \rangle_{Y_f^+}$,

thanks to the integrated version

$$1 - \langle \mathcal{S}_{01} \rangle_0 = 1 - \langle \mathcal{S}_{01} \rangle_{Y_f^+} - \bar{\alpha} \int_0^{Y_f^+} \mathrm{d} Y_2^+ \int \frac{\mathrm{d}^2 \mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2 \, x_{21}^2} \, \langle \mathcal{S}_{01} - \mathcal{S}_{02} \mathcal{S}_{21} \rangle_{Y_2^+}$$

of the LL BK equation

$$\partial_{Y^+} \langle \mathcal{S}_{01} \rangle_{Y^+} = \bar{\alpha} \int \frac{\mathrm{d}^2 \mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2 \, x_{21}^2} \, \langle \mathcal{S}_{02} \mathcal{S}_{21} - \mathcal{S}_{01} \rangle_{Y^+}$$

Low-x_{Bj} DIS at NLO and LL accuracy

$$\sigma^{\gamma}_{T,L} = 2\,\frac{2N_c\,\alpha_{em}}{(2\pi)^2}\sum_f e_f^2 \int \mathrm{d}^2\mathbf{x}_0 \int \mathrm{d}^2\mathbf{x}_1 \int_0^1 \mathrm{d}z_1 \Bigg\{ \mathcal{I}^{LO}_{T,L}(x_{01}, z_1)$$
$$\times \Bigg[1 - \langle \mathcal{S}_{01} \rangle_{Y_f^+} - \bar{\alpha} \int_{k^+_{min}/q^+}^{k_f^+/q^+} \frac{\mathrm{d}z_2}{z_2} \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\, \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\, \langle \mathcal{S}_{01} - \mathcal{S}_{02}\mathcal{S}_{21} \rangle_{\log(z_2\, q^+/k^+_{min})} \Bigg]$$
$$+ \bar{\alpha} \int_{k^+_{min}/q^+}^{1-z_1} \frac{\mathrm{d}z_2}{z_2} \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\, \mathcal{I}^{NLO}_{T,L}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, z_1, z_2)\, \langle \mathcal{S}_{01} - \mathcal{S}_{02}\,\mathcal{S}_{21} \rangle_0 \Bigg\}$$

The LL contributions $(\bar{\alpha}\, Y_f^+)^n$ essentially cancel between the last two terms, and remain only in $\langle \mathcal{S}_{01} \rangle_{Y_f^+}$, because

$$\mathcal{I}^{NLO}_{T,L}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, z_1, z_2 = 0) = \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\;\; \mathcal{I}^{LO}_{T,L}(x_{01}, z_1)$$

Available range for the evolution of the target

The presence of the target sets a physical lower bound k^+_{min} on the k^+ of the relevant gluons and on the factorization scale k^+_f:

$$k^+_f \geq k^+_{min} = \frac{Q_0^2}{2x_0 P^-} = \frac{x_{Bj}\, Q_0^2}{x_0\, Q^2}\, q^+$$

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$\Rightarrow$ Range for LL evolution from the target to the factorization scale in DIS:

$$Y^+_f = \log\left(\frac{k^+_f}{k^+_{\textrm{min}}}\right) = \log\left(\frac{x_0\, Q^2\, k^+_f}{x_{Bj}\, Q_0^2\, q^+}\right)$$

$\rightarrow$ Not a rapidity range, and not $\log(x_0/x_{Bj})$ either, beyond LL accuracy.

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$\rightarrow$ Not a rapidity range, and not $\log(x_0/x_{Bj})$ either, beyond LL accuracy.

Reasonable choice factorization scale choice for DIS:

$k_f^+ = z_1\,(1-z_1)\,q^+$.

Incorrect subtraction of Leading Logs

Low z_2 contribution to σ_L^γ at NLO (for $z_2 \ll z_1,\, 1-z_1$):

$$\propto \quad \bar{\alpha}\frac{\mathrm{d}z_2}{z_2} \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\, \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\, \mathrm{K}_0^2(QX_3)\, \langle \mathcal{S}_{01} - \mathcal{S}_{02}\, \mathcal{S}_{21}\rangle \dots$$

Low z_2 term used to subtract the LL's from σ_L^γ at NLO:

$$\propto \quad \bar{\alpha}\frac{\mathrm{d}z_2}{z_2}\, \mathrm{K}_0^2(QX_2) \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\, \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\, \langle \mathcal{S}_{01} - \mathcal{S}_{02}\, \mathcal{S}_{21}\rangle \dots$$

At low z_2: $X_3 \simeq X_2$ in most of the $\mathbf{x}_2$ plane.

But mismatch in the regime $z_1(1-z_1)x_{01}^2 < z_2 x_{02}^2 \simeq z_2 x_{12}^2$ where $X_3^2 \simeq z_2 x_{02}^2 \simeq z_2 x_{12}^2 > X_2^2 = z_1(1-z_1)x_{01}^2$:
$\rightarrow$ gluon emitted at very large transverse distance from the $q\bar{q}$ dipole.

Incorrect subtraction of Leading Logs

In the regime $z_2 \ll z_1,\ 1-z_1$ and $z_1(1-z_1)x_{01}^2 \ll z_2 x_{02}^2 \simeq z_2 x_{12}^2$:

- $\mathrm{K}_0(QX_3)$ is exponentially smaller than $\mathrm{K}_0(QX_2)$
- No contribution to LL's is present in σ_L^γ at NLO!

$\Rightarrow$ More LL's subtracted with the BK equation than present in σ_L^γ (and σ_T^γ).

Incorrect treatment in a kinematical regime parametrically narrow, but quantitatively important:

- LL subtraction with the standard BK equation spoils the suppression of Fock states with too large formation time
- subtraction term not only larger than the *bare* NLO terms but also than the LO term in the collinear regime
- spoils the DGLAP DLL collinear limit.

Corrected real gluon emission kernel

Real emission contribution to the usual LL:

$$\bar{\alpha}\,\frac{\mathrm{d}z_2}{z_2}\,\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\frac{x_{01}^2}{x_{02}^2 x_{21}^2}\,\left\langle \mathcal{S}_{02}\,\mathcal{S}_{21} - \frac{1}{N_c^2}\mathcal{S}_{01}\right\rangle_{Y_2^+}$$

Ordering in $k^+ = z\,q^+$ guarantied by the choice of factorization scheme/evolution in k^+.

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Modification: forbid gluon emission in the regime
$z_1(1-z_1)x_{01}^2 \ll z_2 x_{02}^2 \simeq z_2\,x_{12}^2$
$\to$ Mixed-space analog of the k^- ordering (kinematical constraint).

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$\rightarrow$ Mixed-space analog of the k^- ordering (kinematical constraint).

$\Rightarrow$ Multiply the real contribution by $\theta\left(z_f\,x_{01}^2 - z_2\,\min(x_{02}^2, x_{21}^2)\right)$

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Real emission contribution to the usual LL:

$$\bar{\alpha}\, \frac{\mathrm{d}z_2}{z_2}\, \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\, \frac{x_{01}^2}{x_{02}^2 x_{21}^2}\, \left\langle \mathcal{S}_{02}\, \mathcal{S}_{21} - \frac{1}{N_c^2}\mathcal{S}_{01} \right\rangle_{Y_2^+}$$

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Same general idea as in the previous study in mixed space:
Motyka, Staśto (2009)
However: inappropriate treatment of virtual corrections there.

Calculating virtual corrections from unitarity

Assume the kinematical constraint to preserve the probabilistic interpretation of the parton cascade.
Evolution of $\langle S_{01}\rangle$ over a finite range $Y_f^+ = \log(k_f^+/k_{\min}^+)$:

$$\begin{aligned}\langle S_{01}\rangle_{Y_f^+} \quad = \quad & \langle S_{01}\rangle_0 \;\; D_{01}(Y_f^+) + \bar{\alpha}\int_0^{Y_f^+} \mathrm{d}Y_2^+ \;\; D_{01}(Y_f^+ - Y_2^+) \\ & \times \int \frac{\mathrm{d}^2\mathbf{x_2}}{2\pi}\,\frac{x_{01}^2}{x_{02}^2 x_{21}^2}\,\theta\left(Y_f^+ - Y_2^+ - \log\left(\frac{\min(x_{02}^2, x_{21}^2)}{x_{01}^2}\right)\right) \\ & \times \left\langle \mathcal{S}_{02}\,\mathcal{S}_{21} - \frac{1}{N_c^2}\mathcal{S}_{01}\right\rangle_{Y_2^+}\end{aligned}$$

with the probability $D_{01}(Y^+)$ of no splitting for the dipole 01 in the range Y^+.

Calculating virtual corrections from unitarity

In the vacuum (absence of target), $S_{01} = S_{02} = S_{21} = 1$.
$\rightarrow$ equation determining $D_{01}(Y^+)$.
Solution:

$$D_{01}(Y^+) = \exp\left[-\bar{\alpha}\,\frac{2C_F}{N_c}\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\frac{x_{01}^2}{x_{02}^2 x_{21}^2}\,\left(Y^+ - \Delta_{012}\right)\,\theta\left(Y^+ - \Delta_{012}\right)\right]$$

with the notation

$$\Delta_{012} = \max\left\{0,\ \log\left(\frac{\min(x_{02}^2, x_{21}^2)}{x_{01}^2}\right)\right\}$$

Typical behavior:

$$\Delta_{012} = 0 \quad \text{for} \quad x_{02}^2 \ll x_{01}^2 \quad \text{or} \quad x_{21}^2 \ll x_{01}^2$$

$$\Delta_{012} \sim \log\left(\frac{x_{02}^2}{x_{01}^2}\right) \sim \log\left(\frac{x_{21}^2}{x_{01}^2}\right) \quad \text{for} \quad x_{01}^2 \ll x_{02}^2 \sim x_{21}^2$$

Kinematically constrained BK equation (kcBK)

Rewriting the new evolution equation as a differential equation and discarding irrelevant terms explicitly of order NLL:

$$\partial_{Y^+} \langle \mathcal{S}_{01} \rangle_{Y^+} = \bar{\alpha} \int \frac{d^2\mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\, \theta(Y^+ - \Delta_{012}) \times \left\{ \left\langle \mathcal{S}_{02}\mathcal{S}_{21} - \frac{1}{N_c^2}\mathcal{S}_{01} \right\rangle_{Y^+ - \Delta_{012}} - \left(1 - \frac{1}{N_c^2}\right) \langle \mathcal{S}_{01} \rangle_{Y^+} \right\}$$

G.B., *to appear*

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G.B., *to appear*

Each of the two modifications should slow down BK evolution:

- Restriction of phase space by the theta function
- Shift of the Y^+ argument of the dipole amplitude in the real term but not in the virtual term.

Large effect especially at small Y^+.

Kinematically constrained BK equation (kcBK)

$$\partial_{Y^+}\left\langle \mathcal{S}_{01}\right\rangle_{Y^+} = \bar{\alpha}\int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\frac{x_{01}^2}{x_{02}^2\,x_{21}^2}\,\theta(Y^+ - \Delta_{012})$$

$$\times\left\{\left\langle \mathcal{S}_{02}\mathcal{S}_{21} - \frac{1}{N_c^2}\mathcal{S}_{01}\right\rangle_{Y^+-\Delta_{012}} - \left(1-\frac{1}{N_c^2}\right)\left\langle \mathcal{S}_{01}\right\rangle_{Y^+}\right\}$$

That modification of the LL BK equation resums precisely the largest and most pathological corrections appearing in the known NLL BK equation.

$\Rightarrow$ Necessary step towards a stable and reliable version of the NLL BK equation.

When regularizing the NLO DIS impact factors and removing the LL contribution using that kcBK equation:
fully correct subtraction the LL contributions, with no mismatch in the collinear regime, by contrast to the standard LL BK case.

Conclusions

Kinematical constraint in BK/BFKL/... :

- Prevent the appearance of large *kinematical* corrections at higher orders in the evolution kernel and in the process-dependent impact factors in any collinear limit.
- Restore the intuitive ordering in *formation time*

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Future developments:

- Kinematical constraint for JIMWLK?
- Resummation of large *dynamical* higher order corrections for BK, to get a full collinear-resummed BK ?