Pomeron Loops in Zero Transverse Dimensions

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Talk in High Energy QCD, From RHIC to LHC
Trento

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1. **Introduction**
   - Scattering amplitude
   - Pomeron Loops equations

2. **Solution to Pomeron Loops Equations**
   - Perturbative Solution to Hierarchy Equations
   - Non-Perturbative effects

3. **Application to diffractive scattering**
   - Hierarchy equations and AGK cutting rules
   - Diffractive scattering amplitude
1 Introduction
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   - Perturbative Solution to Hierarchy Equations
   - Non-Perturbative effects

3 Application to diffractive scattering
   - Hierarchy equations and AGK cutting rules
   - Diffractive scattering amplitude
Boost invariance of the scattering amplitude

Dipole Model, [Mueller, 1995], Onium-onium scattering (First PL calculation)

\[ S(Y) \approx \sum_{m,n=1}^{\infty} e^{-\alpha_s^2 mn} P_m \left( \frac{Y}{2} \right) P_n \left( \frac{Y}{2} \right) \]

\[ \approx \frac{1}{\alpha_s^2 e^{\alpha Y}} \ln \left[ \frac{\alpha_s^2 e^{\alpha Y}}{(1 + \alpha_s^2 e^{\alpha Y/2})^2} \right]. \]

where \( P_n(y) = \frac{1}{\exp(\alpha y)} e^{-n/\exp(\alpha y)} \) is the splitting probability without recombination.

Imposing Boost Invariance in Dipole Model, [Mueller, Salam, 1996]

\[ P_s(n, y) = \exp \left[ \alpha_s^2 n - \alpha y - \frac{1}{\alpha_s^2} \left( e^{\alpha_s^2 n} - 1 \right) e^{-\alpha y} \right], \]

\[ S(Y) \approx \frac{1}{\alpha_s^2 e^{\alpha Y}} \ln \left[ \alpha_s^2 e^{\alpha Y} \right]. \]
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\]

\[
S(Y) \simeq \frac{1}{\alpha_s^2 e^{\alpha Y}} \ln \left[ \alpha_s^2 e^{\alpha Y} \right]
\]
Boost invariant scattering amplitude

Definition

\[ S(Y) = \sum_{k=0}^{\infty} \frac{(-\alpha_s^2)^k}{k!} n^{(k)}(Y_0) n^{(k)}(Y - Y_0) \]

in which \( n^{(k)} = \langle n(n - 1) \cdots (n - k + 1) \rangle \) is the k-Pomeron amplitude.

- The k-Pomeron amplitude \( n^{(k)} \) satisfy the hierarchy equations;
- It is boost invariant, namely \( S(Y) \) is independent of \( Y_0 \), when \( \beta = \alpha \alpha_s^2 \);
- Setting \( Y_0 = 0 \), one gets

\[ S(Y) = 1 - \alpha_s^2 n^{(1)}(Y) = 1 - T \]

for particle-particle scattering.

(Other boost invariant approach, see [Blaizot, Iancu, Triantafyllopoulos, 2006])
Boost invariant scattering amplitude

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Scattering amplitude
Pomeron Loops equations

Hierarchy equations

See [Iancu, Triantafylloupoulos, 2004]

Hierarchy equations, Triple pomeron vertices only

\[
\frac{dn^{(k)}}{dy} = k \alpha n^{(k)} + k(k - 1) \alpha n^{(k-1)} - \alpha^{2} s n^{(k+1)}
\]

It has **projectile-target duality**, when \( \beta = \alpha^{2} s \). Namely, splitting term and merging term are equivalent in calculating diagrams.

Hierarchy equations, 0-dim Reaction diffusion model

\[
\frac{dn^{(k)}}{dy} = k \alpha n^{(k)} - k(k - 1) \beta n^{(k)} + k(k - 1) \alpha n^{(k-1)} - k \beta n^{(k+1)}.
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Hierarchy equations

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4-loops coupling
Langevin equations and master equations.

Langevin equations

\[
\frac{d\tilde{n}}{dy} = \alpha\tilde{n} - \beta\tilde{n}^2 + \sqrt{2}\alpha\tilde{n}\nu(y)
\]

Master equations

\[
\frac{dP_n}{dy} = \alpha(n-1)P_{n-1} - \alphanP_n + \beta(n+2)(n+1)P_{n+2} - \beta(n+1)nP_{n+1}
\]

Pathological interpretation in master equation.
Langevin equations and master equations.

Langevin equations

\[ \frac{d\tilde{n}}{dy} = \left( \alpha \tilde{n} - \beta \tilde{n}^2 + \sqrt{2\alpha \tilde{n}} \nu(y) \right) \]

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\[ \begin{array}{c}
1 \rightarrow 2 \\
2 \rightarrow 0 \\
2 \rightarrow 1 (-)
\end{array} \]

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Pathological interpretation in master equation.
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Perturbative Solution to Hierarchy Equations

Non-Perturbative effects

Application to diffractive scattering

Hierarchy equations and AGK cutting rules

Diffractive scattering amplitude

\[
\frac{dn^{(k)}}{dy} = k\alpha n^{(k)} + k(k - 1)\alpha n^{(k-1)} - k\beta n^{(k+1)}
\]

- Initial condition \( n_0^{(1)}\big|_{y=0} = 1 \) and \( n_0^{(k)}\big|_{y=0} = 0 \);
- Solve the hierarchy equation without recombination term \( k\beta n^{(k+1)} \) exactly, \( n_0^{(k)}(y) = k! e^{k\alpha y} (1 - e^{-\alpha y})^{k-1} \);
- Treat the recombination term as perturbation;
- Use graphs to visualize solutions, organize graphs into categories such as: LO, NLO, etc.
Perturbative calculation of the hierarchy equations


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• **LO graphs and NLO graphs**

• **Leading Order contributions:**

\[
\alpha_s^2 n_{LO}^{(1)}(Y) = \sum_{k=1}^{\infty} (-1)^{k+1} \cdot k! \left( \alpha_s^2 e^{\alpha Y} \right)^k;
\]

• **Next-Leading Order contributions. Two types contribution; graphic interpretation.**

\[
\alpha_s^2 n_{NLO}^{(1)}(Y) = \sum_{k=1}^{\infty} (-1)^k k! \left( \alpha_s^2 e^{\alpha Y} \right)^k \frac{(k-1)(3k-4)\alpha Y}{k} e^{\alpha Y}
\]

\[
= \sum_{k=0}^{\infty} (-1)^{k+1} k! \left( \alpha_s^2 e^{\alpha Y} \right)^k k(3k-1)\alpha_s^2 \alpha Y
\]
LO vs NLO, etc.

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**LO vs NLO, etc.**

- LO graphs and NLO graphs
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  \]
Scattering amplitude

Using Borel resummation, scattering amplitude reads

\[ S(Y) \approx \frac{1}{\alpha_s^2 e^{\alpha Y}} \ln \left[ \alpha_s^2 e^{\alpha Y} \right] \left[ 1 + 4 \alpha_s^2 \alpha Y + \text{higher - corrections} \right] \]

- Leading order result agrees with [Mueller, Salam 1996].
- Same NLO result is found in [Shoshi, Xiao, 2005] by using another method, it arises from the Hierarchy equations.
- When \( y \) is of order \( \frac{1}{\alpha_s^2} \), the NLO contribution is as large as the LO. This is a critical rapidity which indicates that all the categories of graphs (LO, NLO, etc.) should be considered and properly resummed.
- Namely, the perturbative calculation breaks down, and non-perturbative effects may come into play.
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Non-perturbative effects

[Ciafaloni, Le Bellac, Rossi, 1977]  
[Bondarenko, Motyka, Mueller, Shoshi, Xiao, 2005]

- Ciafaloni et al found the tunnelling effect in the scattering amplitude,

\[ T(Y) \simeq \exp(-E_0 Y), \]

where \( E_0 = \frac{\alpha}{\alpha_s \sqrt{2\pi}} \exp\left(-\frac{1}{\alpha_s^2}\right). \)

- The same result emerges from fixed point calculations of the master equations,

\[ 1 - P_0(Y) \simeq \exp\left(-\frac{\alpha Y}{\alpha_s \sqrt{2\pi}} \exp\left(-\frac{1}{\alpha_s^2}\right)\right) \]

- Estimation shows that the rapidity for building this meta-stable probability distribution is \( \frac{1}{\alpha_s^2}. \)
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3. Application to diffractive scattering
   - Hierarchy equations and AGK cutting rules
   - Diffractive scattering amplitude
The solutions to the hierarchy equations admit direct graph interpretation. Together with the AGK cutting rule, the case of multiple pomeron exchange, one can calculate the single, double and central diffractive scattering amplitude.

According to AGK cutting rules, one gets

\[
\frac{d\sigma_{SD}(Y, Y_0)}{dY_0} = \sum_{n=2}^{\infty} (-1)^n 2^{n-1} F^n(Y, Y_0),
\]

where \(2^{n-1}\) counts the number of graphs when cut is made. \(F^n(Y, Y_0)\) is the amplitude in the case \(n\)-pomeron exchange.
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Diffractive scattering amplitude

Single diffractive scattering

[Shoshi, Xiao 2006] Other approach, see [Kozlov, Levin, Khachatryan, Miller, 2006]

Example

When the diffractive mass and the rapidity gap \((Y - Y_0)\) are kept large, one finds

\[
\frac{d\sigma_{SD}(Y, Y_0)}{dY_0} \sim \frac{1}{2} e^{-\alpha Y_0} \sim \frac{1}{2} \alpha \left( \frac{m_p^2}{M_x^2} \right)^\alpha,
\]

Comments:
- Unitarised single diffractive cross section after re-summation.
- Above calculation applies when the rapidity \(Y \gg \frac{1}{\alpha} \ln \left( \frac{1}{\alpha^2} \right) \rightarrow \) Loop correction is important. This perturbative calculation is only a LO calculation, it breaks down when rapidity goes higher than \(\frac{1}{\alpha^2 s}\).

Therefore, \(\frac{1}{\alpha} \ln \left( \frac{1}{\alpha^2} \right) \ll Y \ll \frac{1}{\alpha^2 s} \).
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\frac{d\sigma_{SD}(Y, Y_0)}{dY_0} \simeq \frac{1}{2} e^{\alpha Y_0} \simeq \frac{1}{2} \alpha \left( \frac{m_P^2}{M^2_X} \right)^{\alpha},
\]

Comments:

- Unitarised single diffractive cross section after re-summation.
- Above calculation applies when the rapidity \(Y \gg \frac{1}{\alpha} \ln \left( \frac{1}{\alpha_s^2} \right) \) Loop correction is important. This perturbative calculation is only a LO calculation, it breaks down when rapidity goes higher than \(\frac{1}{\alpha_{s}^2} \alpha \).

Therefore, \(\frac{1}{\alpha} \ln \left( \frac{1}{\alpha_s^2} \right) \ll Y \ll \frac{1}{\alpha_s^2} \alpha\).
Double diffractive scattering

Example

Keeping the diffractive masses large, one finds

\[ \frac{d\sigma_{DD}(Y, Y_0, Y_1)}{dY_0 dY_1} \approx \frac{1}{4} \alpha^2 e^{-\alpha Y_0 - \alpha Y_1} \]

\[ \approx \frac{1}{4} \alpha^2 \left( \frac{m_p^2}{M_x^2} \right)^\alpha \left( \frac{m_p^2}{M_x'^2} \right)^\alpha \]

Comments: \( d\sigma_{DD}/dY_0 dY_1 \sim [d\sigma_{SD}/dY_0 d\sigma_{SD}/dY_1]/\sigma_{tot} \), fits the naive expectation.
Keeping the diffractive masses large, one finds

\[
\frac{d\sigma_{DD}(Y, Y_0, Y_1)}{dY_0 dY_1} \approx \frac{1}{4} \alpha^2 e^{-\alpha Y_0 - \alpha Y_1}
\]

\[
\approx \frac{1}{4} \alpha^2 \left( \frac{m_p^2}{M_x^2} \right) \left( \frac{m_p^2}{M'_x^2} \right) \alpha^2
\]

Comments: \(d\sigma_{DD}/dY_0 dY_1 \approx [d\sigma_{SD}/dY_0 d\sigma_{SD}/dY_1]/\sigma_{tot}\), fits the naive expectation.
For high energy scattering, and large

\[ Y_1 - Y_0 = \delta Y = \ln \left( \frac{M_X^2}{m_p^2} \right), \]

one finds

\[
\frac{d\sigma_{CD}(Y, Y_0, Y_1)}{dY_0 dY_1} \approx \alpha^2 \alpha_s^2 e^{-\alpha \delta Y} = \alpha^2 \alpha_s^2 \left( \frac{m_p^2}{M_X^2} \right)^\alpha,
\]
The End