Critical Behavior in Strongly Interacting Matter

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Larry McLerran **60 Years** **Jean-Paul Blaizot**

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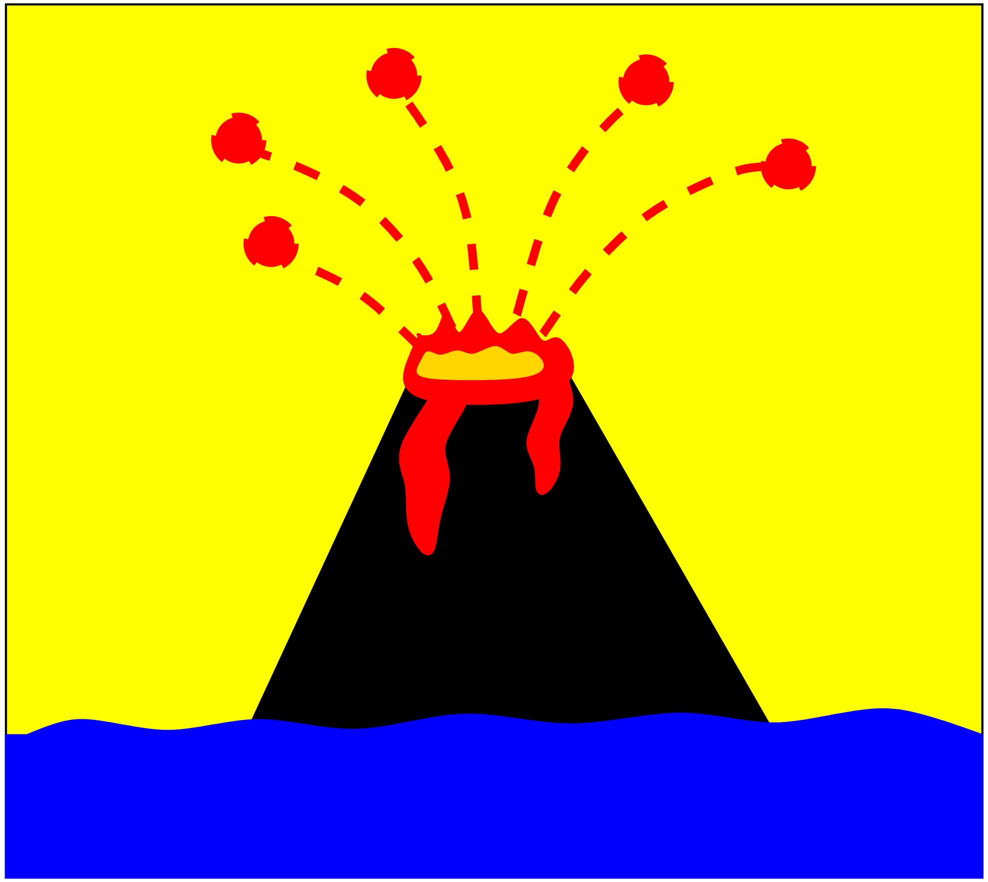
The States of Matter 500 B. C. - Experiment



The States of Matter 500 B. C. - Theory

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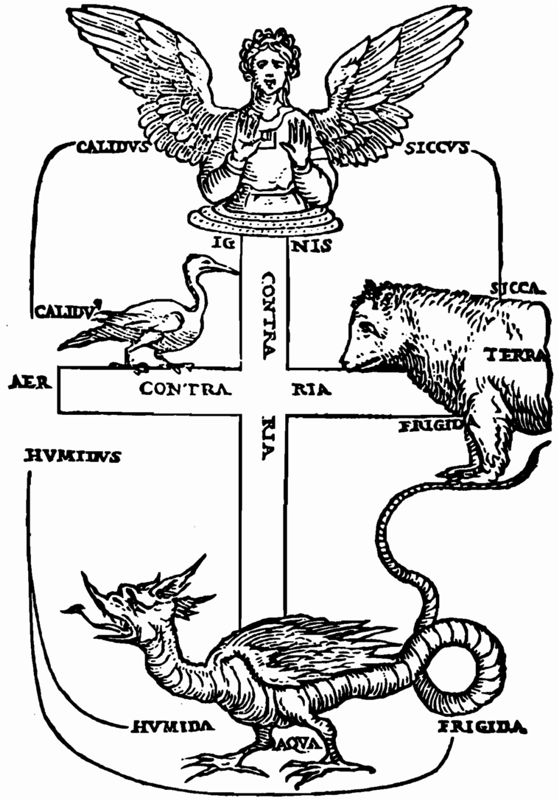

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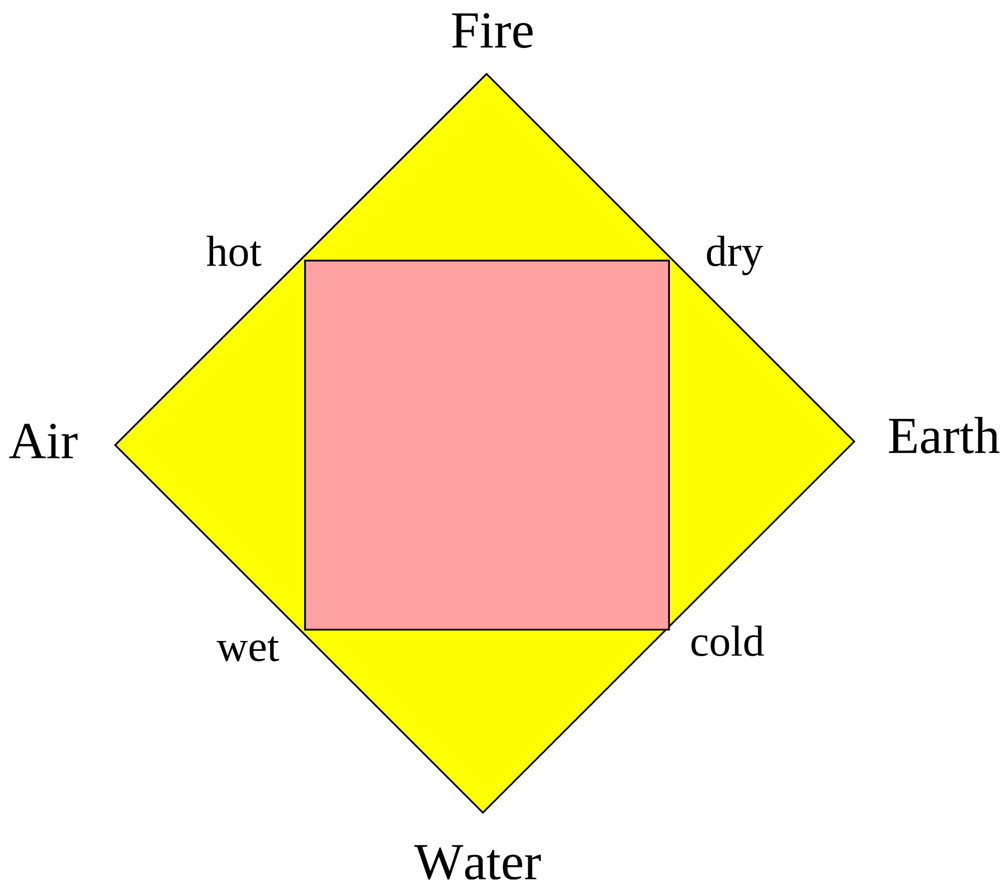

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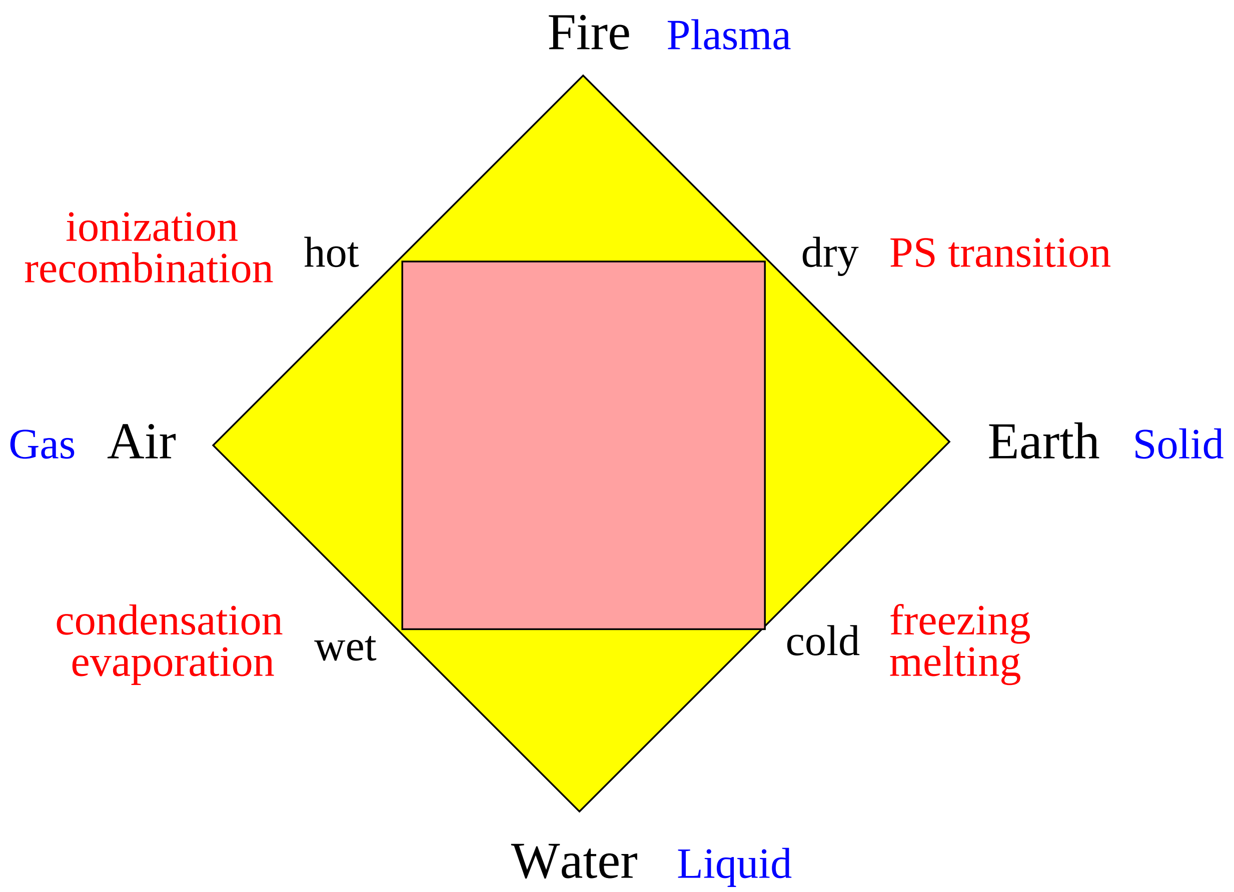
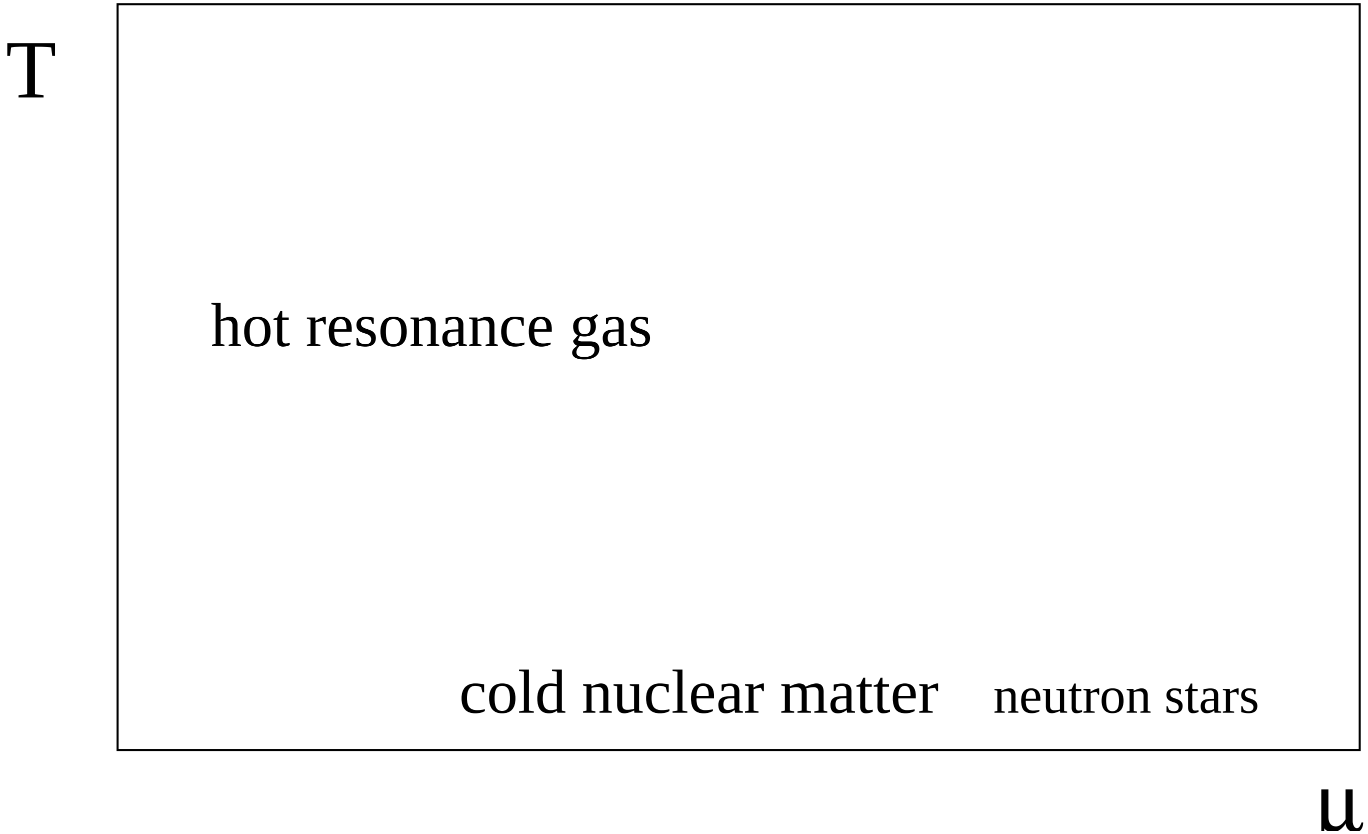


The States of Strongly Interacting Matter

1960



The States of Strongly Interacting Matter

1960

T

Zel'dovich 1959: ...use the equation of state to establish whether the different particles have a common interior and to determine how many are really elementary.

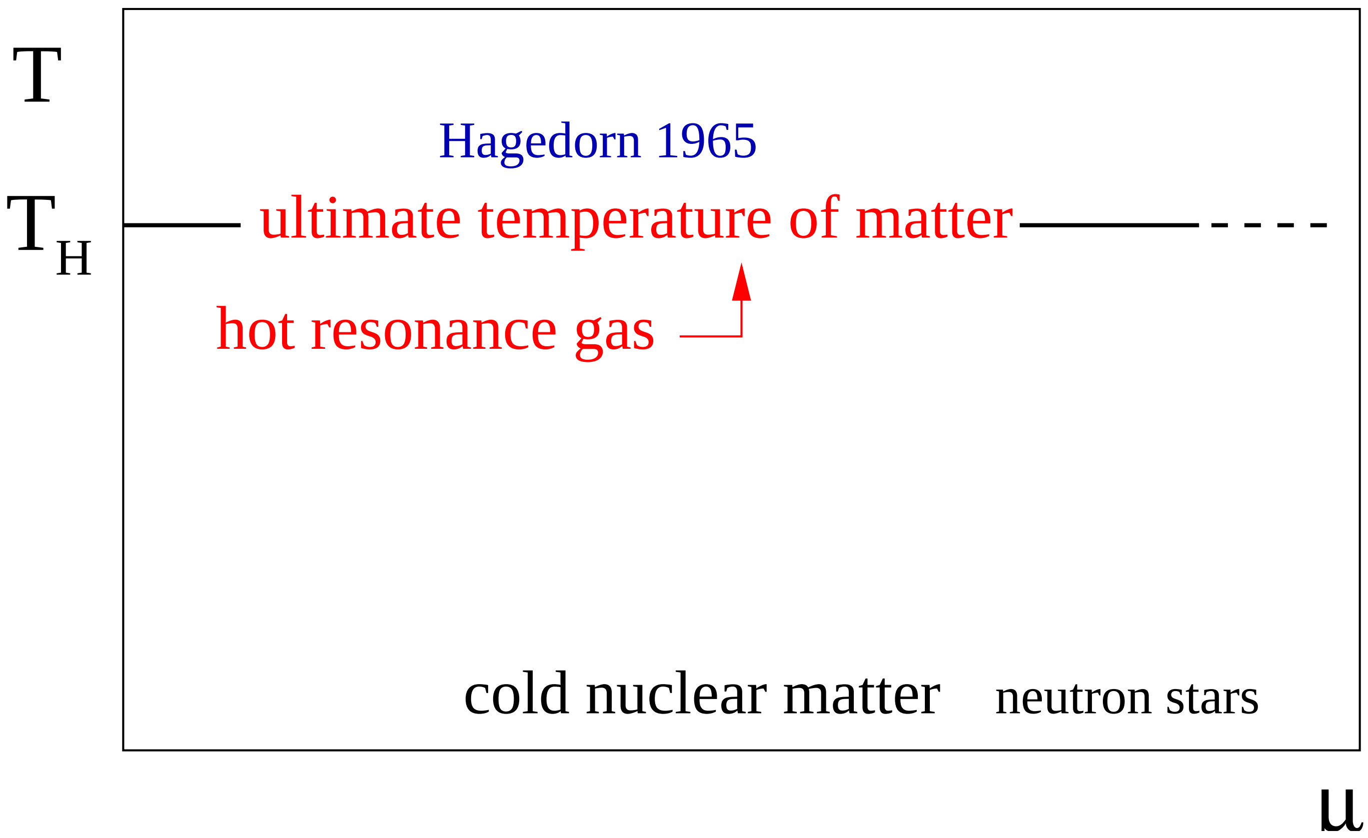
hot resonance gas

cold nuclear matter neutron stars

μ

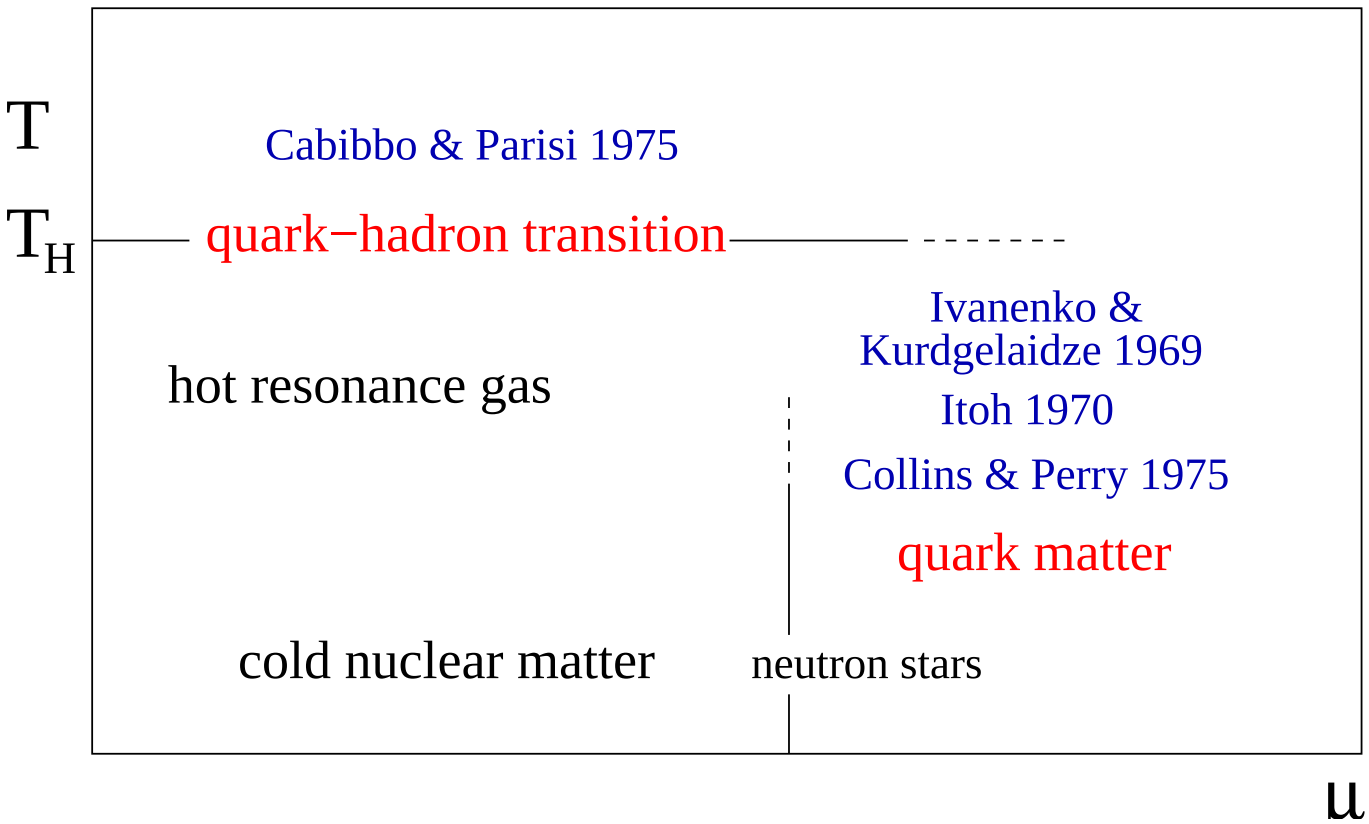
The States of Strongly Interacting Matter

1965



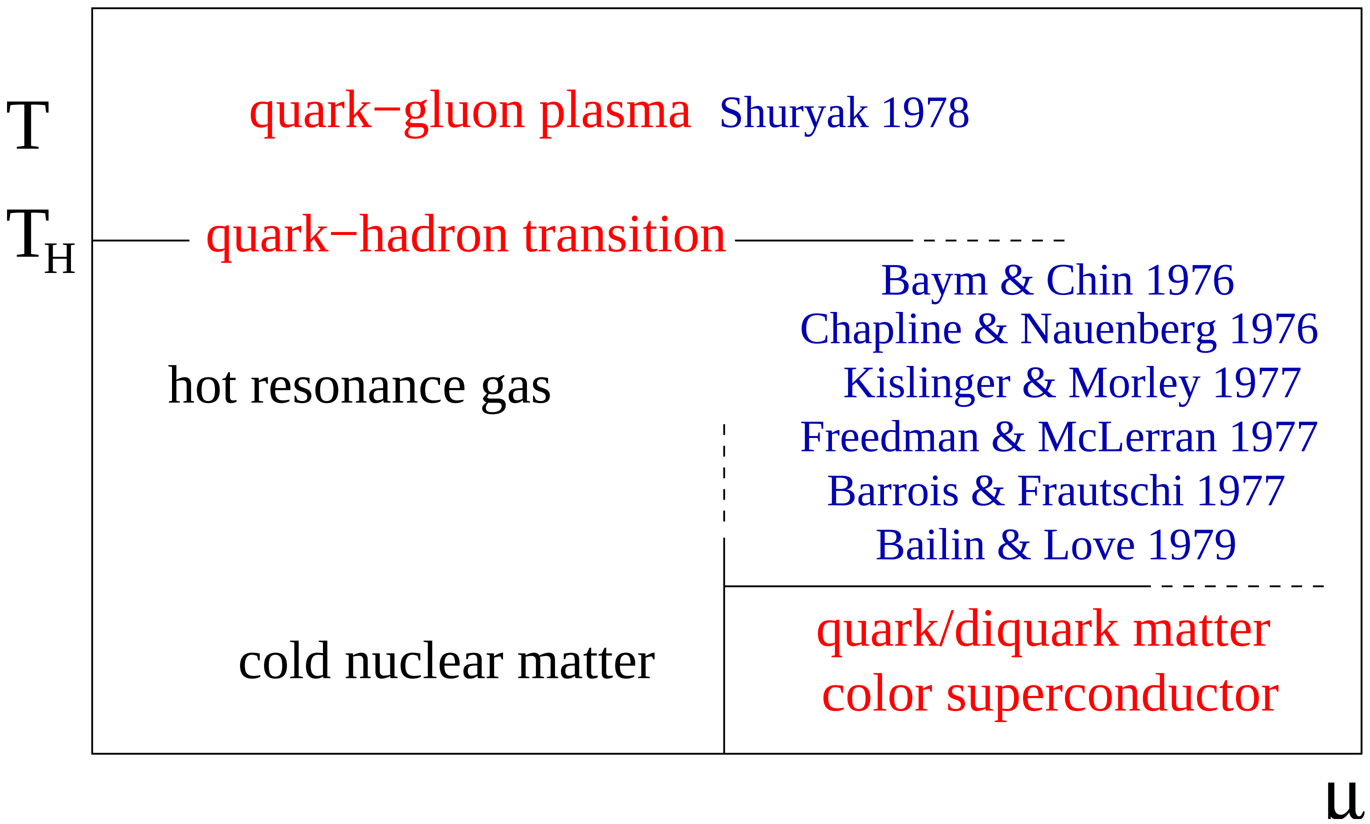
The States of Strongly Interacting Matter

1975



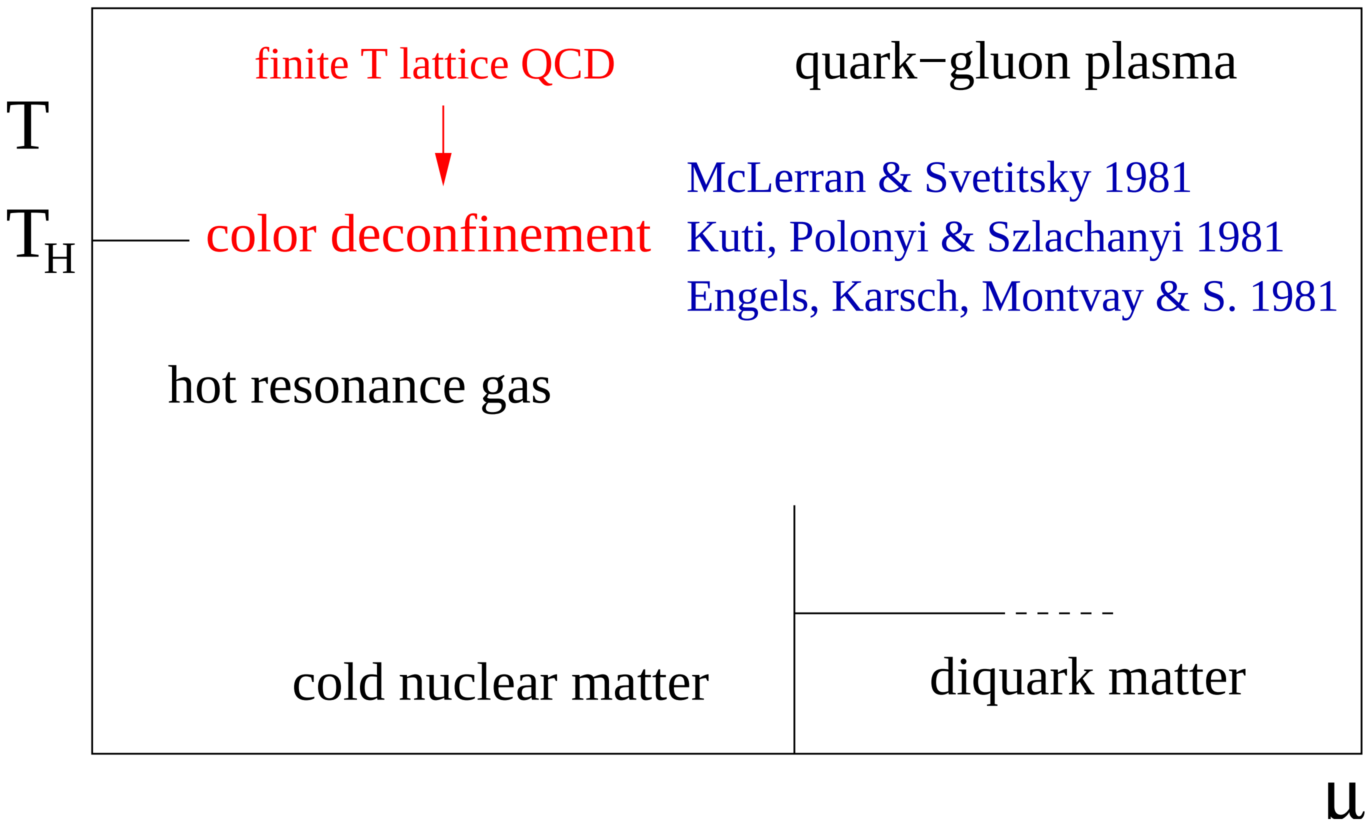
The States of Strongly Interacting Matter

1980



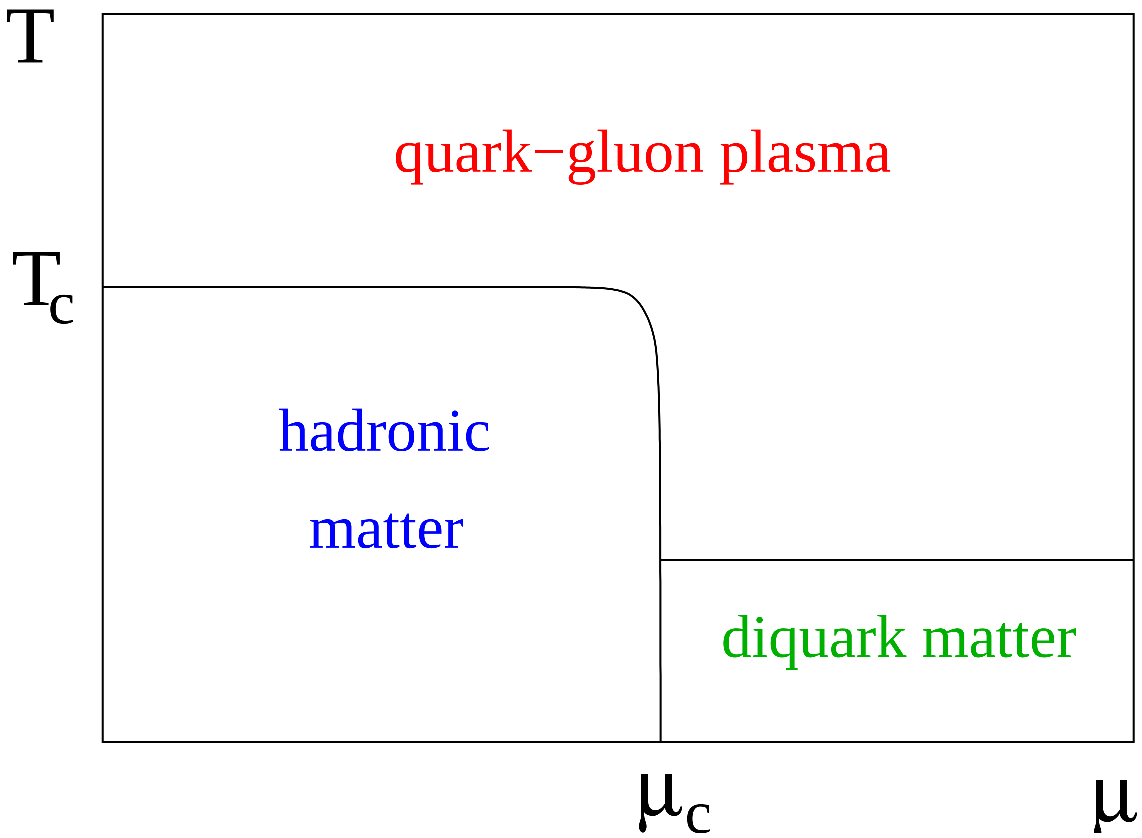
The States of Strongly Interacting Matter

1985



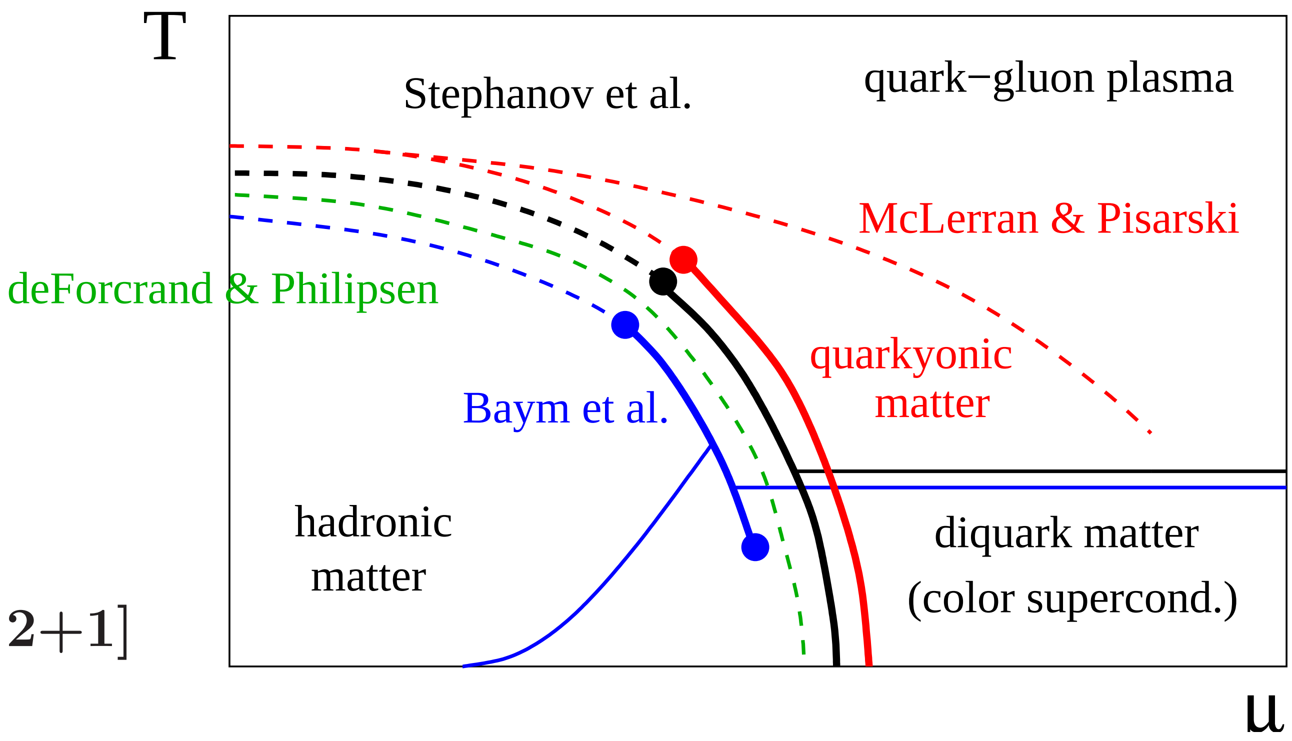
The States of Strongly Interacting Matter

1990



The States of Strongly Interacting Matter

2009



[for 2+1]

that's not nearly all: N-JL, P-N-JL, CFL etc., quark crystals...

Conventional Basis of Critical Behavior

- confinement/deconfinement $\sim$ spontaneous Z_3 symmetry breaking
  McLerran & Svetitsky 1981, Svetitsky & Yaffe 1982

- dynamical mass generation $\sim$ spontaneous chiral symmetry breaking
  Pisarski & Wilczek 1984

consider phase structure for $\mu = 0$:

genuine thermal phase transitions

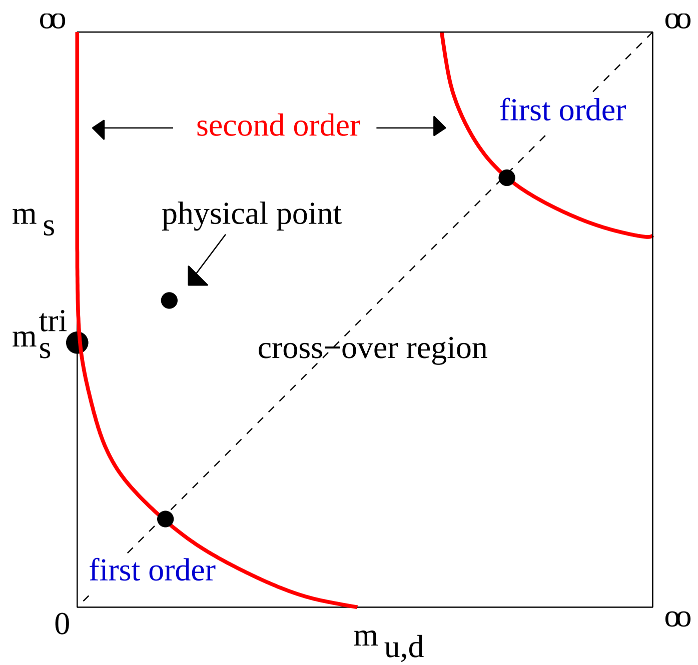
(singularities in partition function)

only for special values of $m_{u,d}, m_s$

but always $\exists$ "transition region"

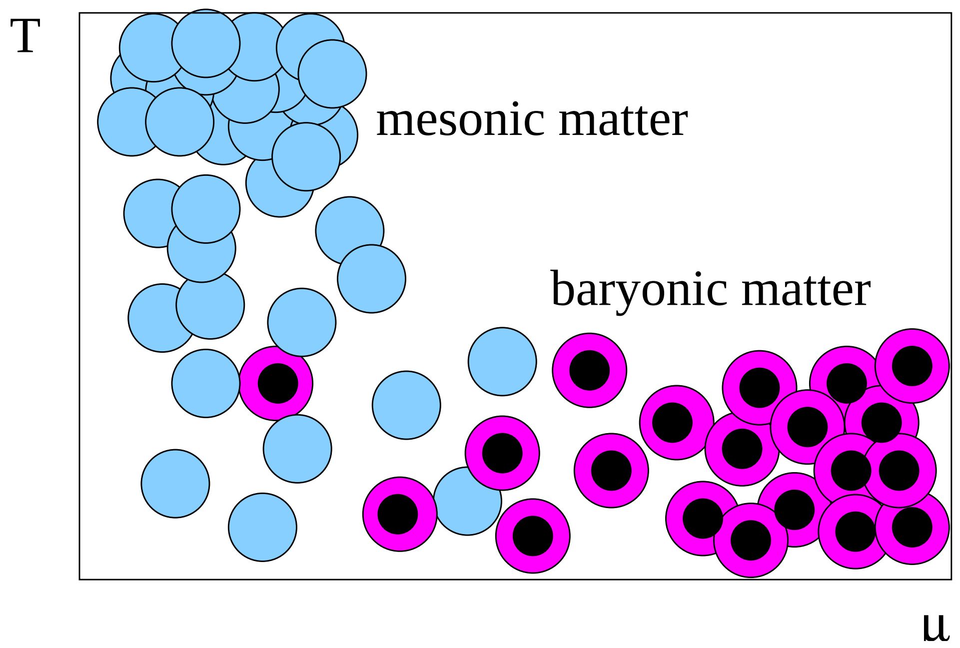
with sharp variation of thermal

observables: "rapid cross-over"



how to understand?

Constituent Structure of Hadronic Matter



- low μ: with increasing T, mesonic medium of increasing density, arbitrary amount of overlap (interpenetration)

- low T: with increasing μ, baryonic medium of increasing density, hard core repulsion limits overlap

In both cases, $\exists$ clustering

Is there a relation between clustering and critical behavior?

Frenkel 1939, Essam & Fisher1963

consider spin systems, e.g., Ising model

- for $H = 0$,
  spontaneous Z_2 symmetry breaking $\longrightarrow$ magnetization transition

- but this can be translated into cluster formation and fusion
  critical behavior via cluster fusion: percolation $\equiv$
  critical behavior via spontaneous symmetry breaking

  Fisher 1967, Fortuin & Kasteleyn 1972, Coniglio & Klein 1980

- for $H \neq 0$, Isakov 1984
  partition function is analytic, no thermal critical behavior
  but clustering & percolation persists Kertész 1989

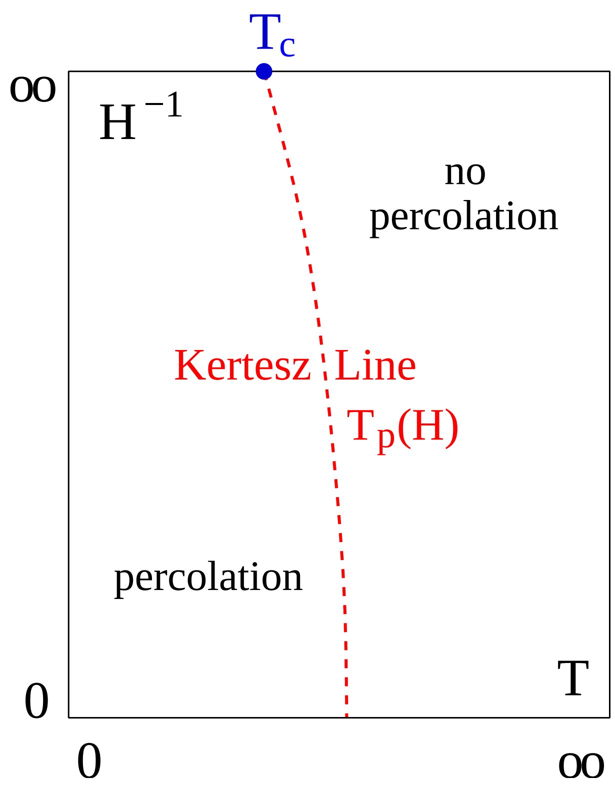
  $\exists$ geometic critical behavior

In spin systems,

$\exists$ geometric critical behavior for all values of H;

for $H = 0$, this can become identical to thermal critical behavior, with non-analytic partition function & Z_2 exponents

for $H \neq 0$, $\exists$ Kertész line geometric transition with singular cluster behavior & percolation exponents



For spin systems,

thermal critical behavior $\subset$ geometric critical behavior

Also in QCD? Hadrons have intrinsic size, with increasing density they form clusters & eventually percolate

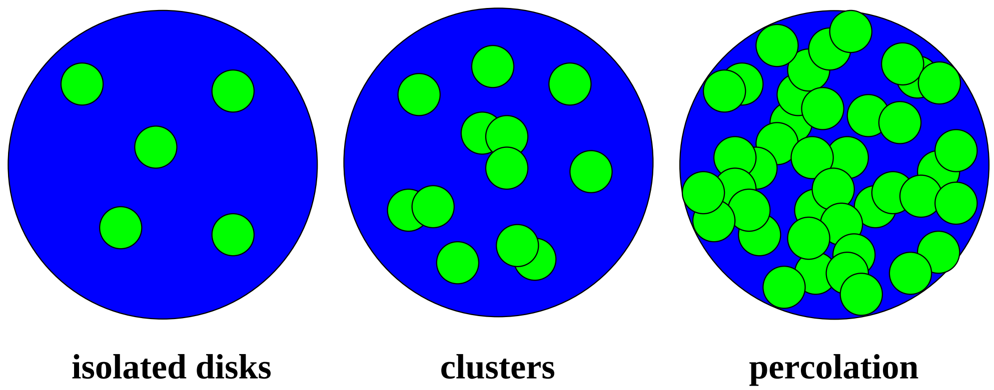
Hadron Percolation ∼ Color Deconfinement

Pomeranchuk 1951

Baym 1979, Çelik, Karsch & S. 1980

Recall percolation

- 2-d, with overlap:
  lilies on a pond



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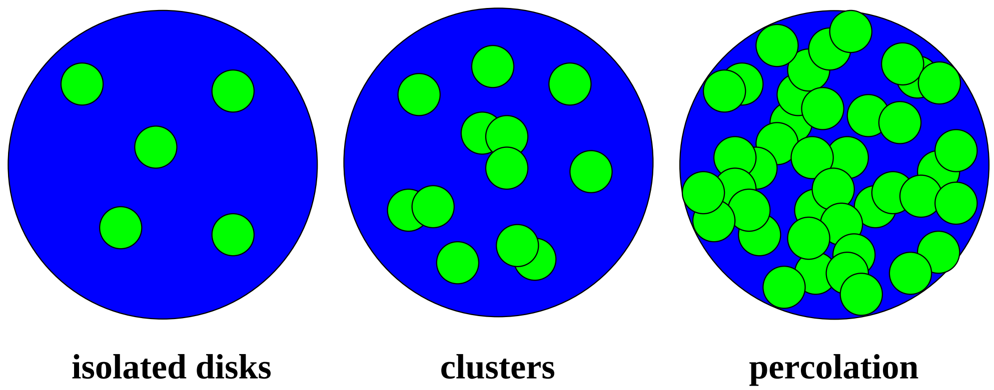

- 2-d, with overlap:
  lilies on a pond

here: green lilies, all same size

small sidetrack:

try to imagine if they came in all colors and sizes...

Bharti Kher: "Think of something great"

How is the color glass formed?

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Exercise:

Determine the onset of percolation in two dimensions for discs of varying size, with a density-dependent disc size. Specifically, at low density the disc radius is $r_0 \simeq 1$ fm, with increasing density it decreases, so that at high densities more and more smaller and smaller discs fill up the "holes".

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The percolation density defines

onset of color glass formation

.

end of sidetrack...

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Recall percolation

- 2-d, with overlap:
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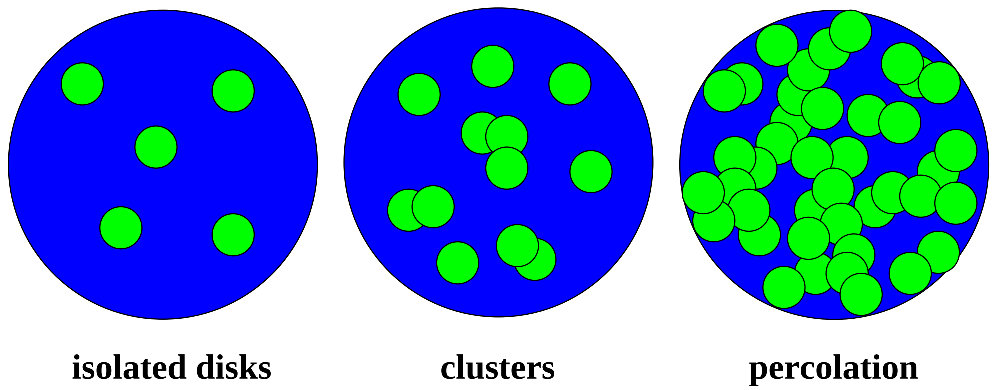


- 3-d: N spheres of volume V_h in box of volume V, with overlap
  increase density $n = N/V$ until largest cluster spans volume:
  <u>percolation</u>
  critical percolation density $n_p \simeq 0.34/V_h$
  at $n = n_P$, 30 % of space filled by overlapping spheres,
  70 % still empty

how dense is the percolating cluster? Digal, Fortunato & S. 2004

critical cluster density $n_m \simeq 1.2/V_h$

$$R_h \simeq 0.8 \text{ fm} \;\Rightarrow\; n_m \simeq \frac{0.6}{\text{fm}^3} \quad \text{as deconfinement density}$$

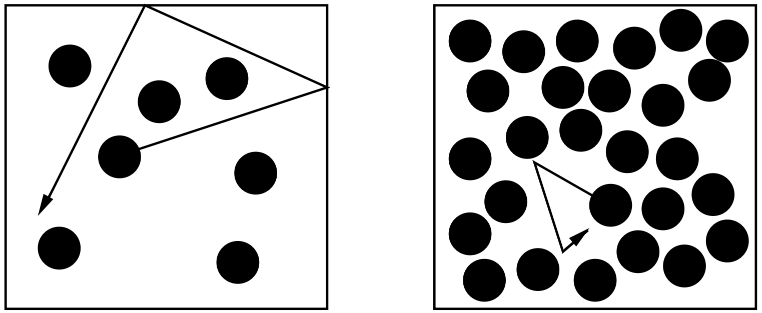
so far, cluster constituents were allowed arbitrary overlap

what if they have a hard core?

then $\exists$ jamming

at high density, constituents

have restricted spatial mobility



$\exists$ jamming transition

with mobility $\sim$ order parameter

Karsch & S. 1980

percolation for spheres of radius R_0
with a hard core of radius $R_{hc} = R_0/2$ Kratky 1988

hard cores tend to prevent dense clusters;

higher density needed to achieve percolating jammed clusters

$$n_b \simeq \frac{2.0}{V_0} = \frac{0.25}{V_{hc}} \simeq \frac{1.0}{\mathrm{fm}^3} \simeq 6\ \mathrm{n}_0$$

for the deconfinement density of baryonic matter

NB: additional uniform attractive potential
$\longrightarrow$ first order thermal transition

$\exists$ two percolation thresholds in strongly interacting matter:

- mesonic matter, full overlap: $n_m \simeq 0.6/\mathrm{fm}^3$
- baryonic matter, hard core: $n_b \simeq 1.0/\mathrm{fm}^3$

now apply to determine critical behavior

If interactions are resonance dominated,

interacting medium $\equiv$ ideal resonance gas

Beth & Uhlenbeck 1937; Dashen, Ma & Bernstein 1969

consider ideal resonance gas of all PDG states for $M \leq 2.5$ GeV

partition function

Castorina, Redlich & S. 2008

$$\ln\ Z(T,\mu,\mu_S,V) = \ln\ Z_M(T,\mu_S,V)\ +\ \ln\ Z_B(T,\mu,\mu_S,V)$$

with

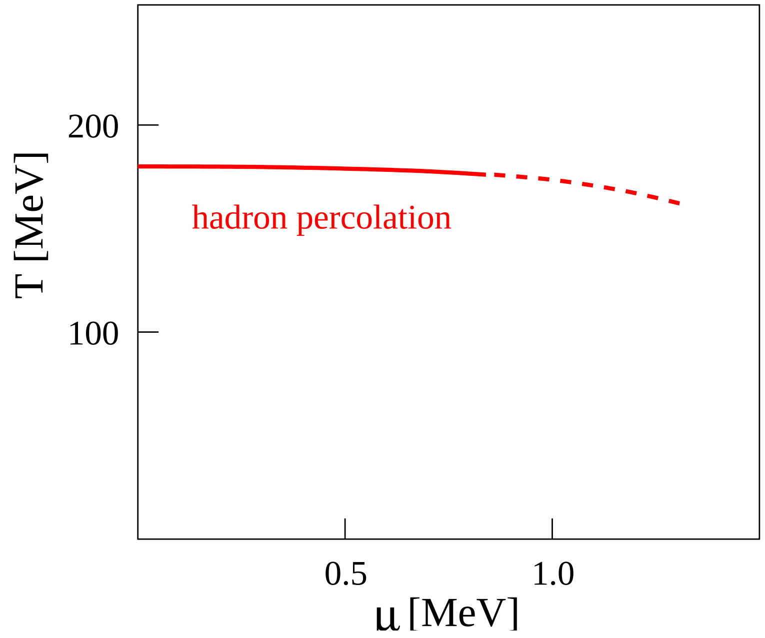
$$\ln\ Z_M(T,V,\mu_S) = \sum_{\text{mesons i}} \ln\ Z_M^i(T,V,\mu_S)$$

$$\ln\ Z_B(T,\mu,\mu_S,V) = \sum_{\text{baryons i}} \ln\ Z_B^i(T,\mu,\mu_S,V)$$

for mesonic and baryonic contributions; enforce $S = 0$

• low baryon-density limit: percolation of overlapping hadrons

$$n_h(T_h, \mu) = \frac{\ln Z(T, \mu, V)}{V} = 0.6/\mathrm{fm}^3$$



Obtain at $\mu = 0$

$$T_h \simeq 180 \text{ MeV}$$

deconfinement temperature

based on hadron percolation

baryons included, but hard core effects ignored

slow decrease of transition temperature with μ,
due to associated production

• high baryon-density limit:

percolation/jamming of hard-core baryons

density of pointlike baryons

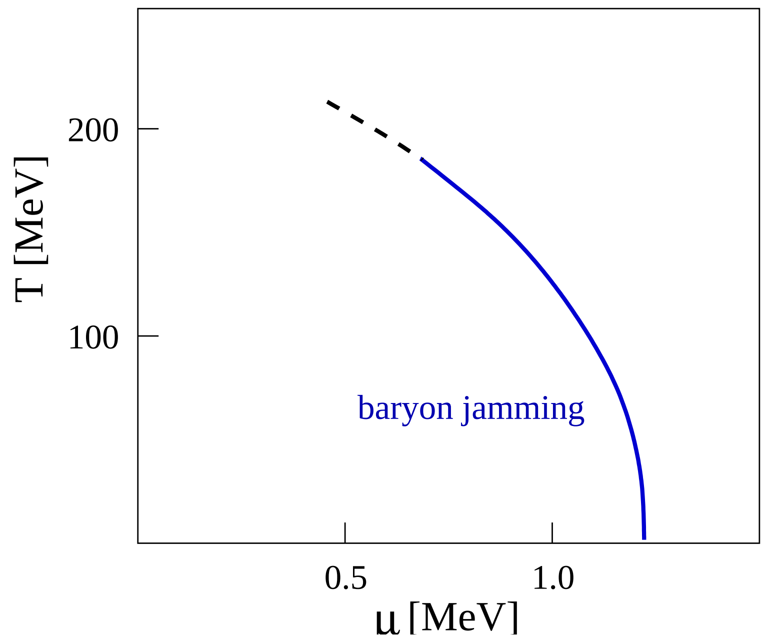
$$n_b^0 = \frac{1}{V}\left(\frac{\partial\ T \ln Z_B(T,\mu,V)}{\partial\mu}\right)$$

hard core ⇒ excluded volume
(Van der Waals)

$$n_b = \frac{n_b^0}{1 + V_{hc} n_b^0}$$

jamming threshold
→ transition line

$$n_b^c(T,\mu) = \frac{2.0}{V_0} = \frac{1.0}{\mathrm{fm}^3}$$



combine the two mechanisms:

phase diagram of hadronic matter

- **low baryon density:**

percolation of overlapping hadrons

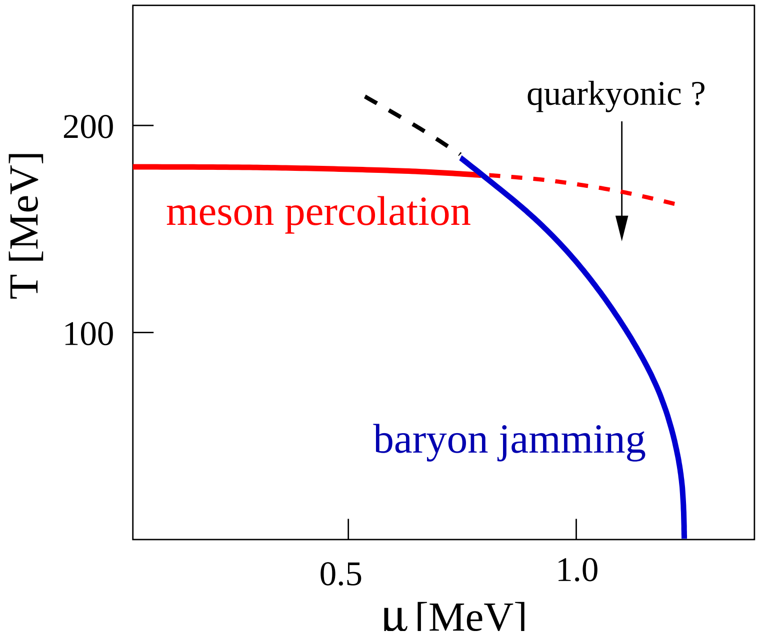
clustering $\sim$ attraction

- **high baryon density:**

percolation of hard-core baryons

clustering $\sim$ attraction, hard-core $\sim$ repulsion

$\longrightarrow$ 1^{st} order transition



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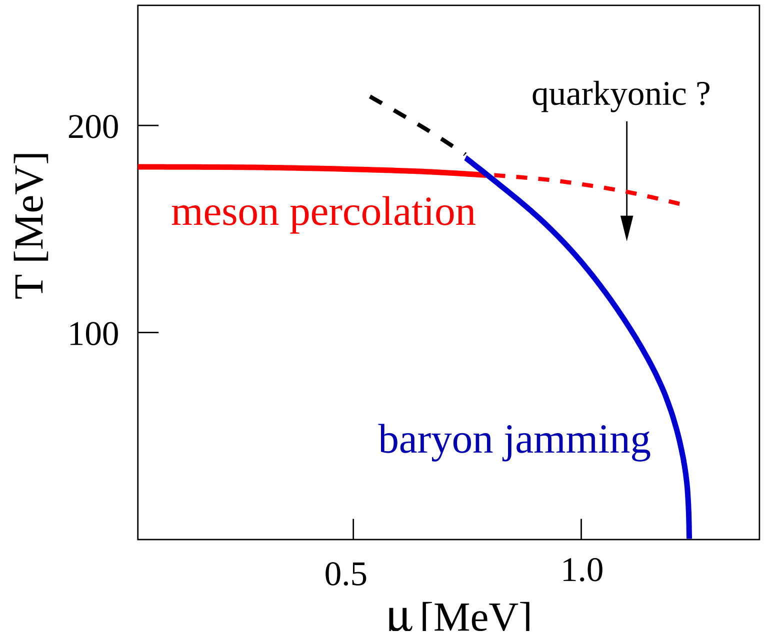

• low baryon density:

percolation of overlapping hadrons

clustering $\sim$ attraction

• high baryon density:

percolation of hard-core baryons

clustering $\sim$ attraction, hard-core $\sim$ repulsion

$\longrightarrow$ 1^{st} order transition

clustering and percolation can provide

a conceptual basis for the QCD phase diagram

which is more general than symmetry breaking