HARD LOOP EFFECTIVE THEORY:
From QCD thermodynamics to color plasma instabilities

Anton Rebhan

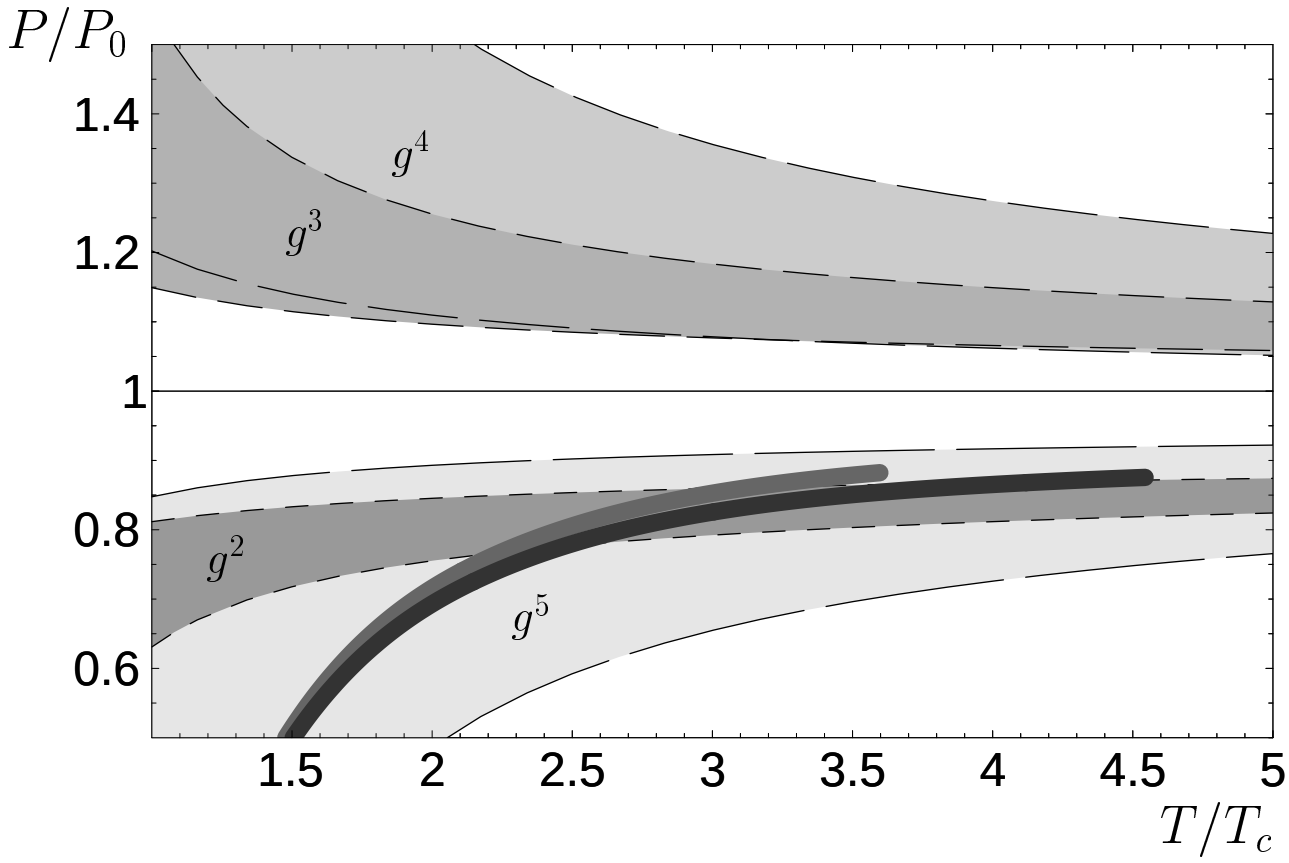
Technical University Vienna, Austria

HAPPY BIRTHDAY FROM ME, TOO!

QCD thermodynamics: Failure of strict perturbation theory

Pressure of pure-glue QCD: lattice vs. perturbative result up to order g^5 with renormalization scale dependence for $\bar{\mu}_{\rm MS} = \pi T \ldots 4\pi T$

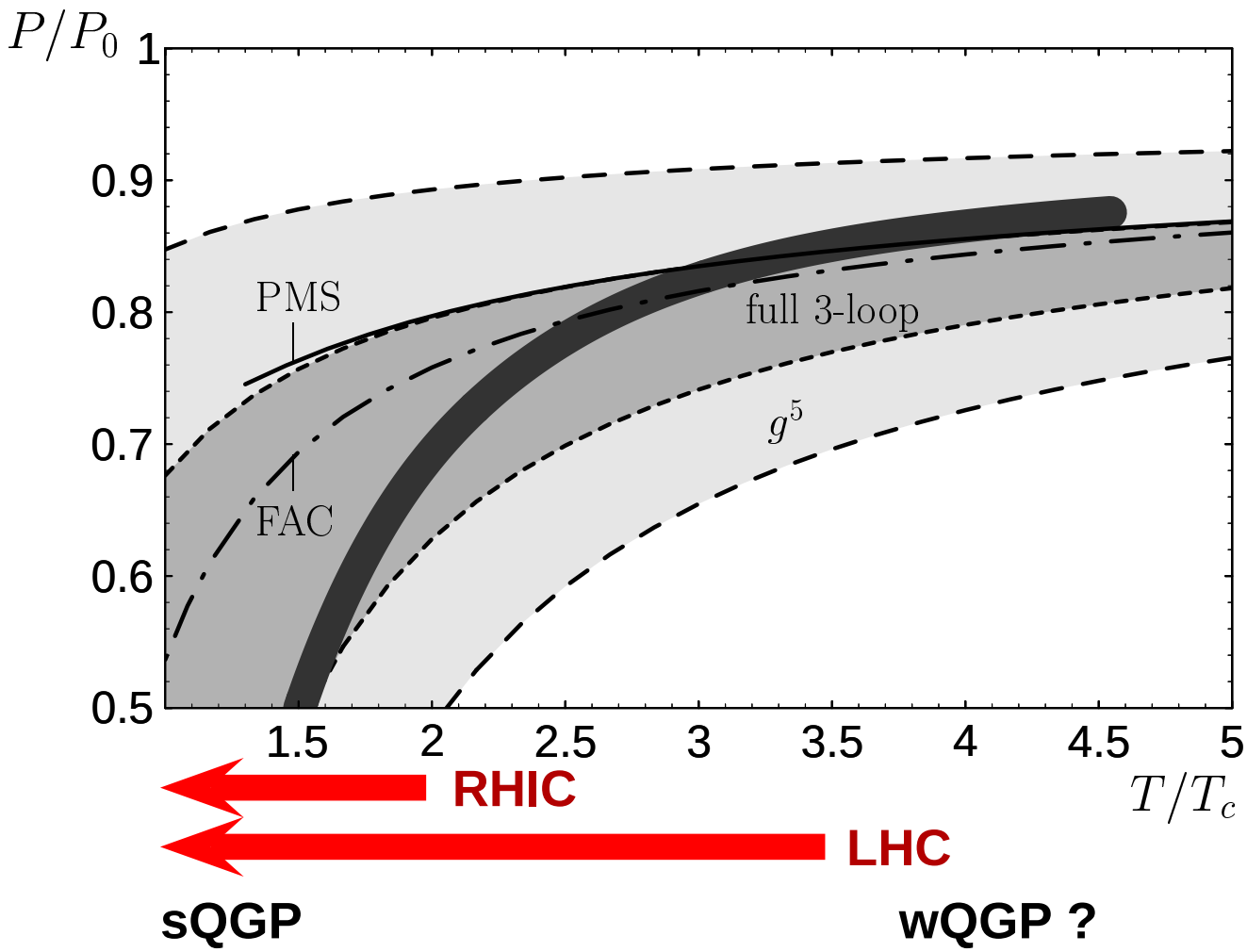
[Arnold & Zhai, PRD50('94)7603]



wQGP or sQGP?

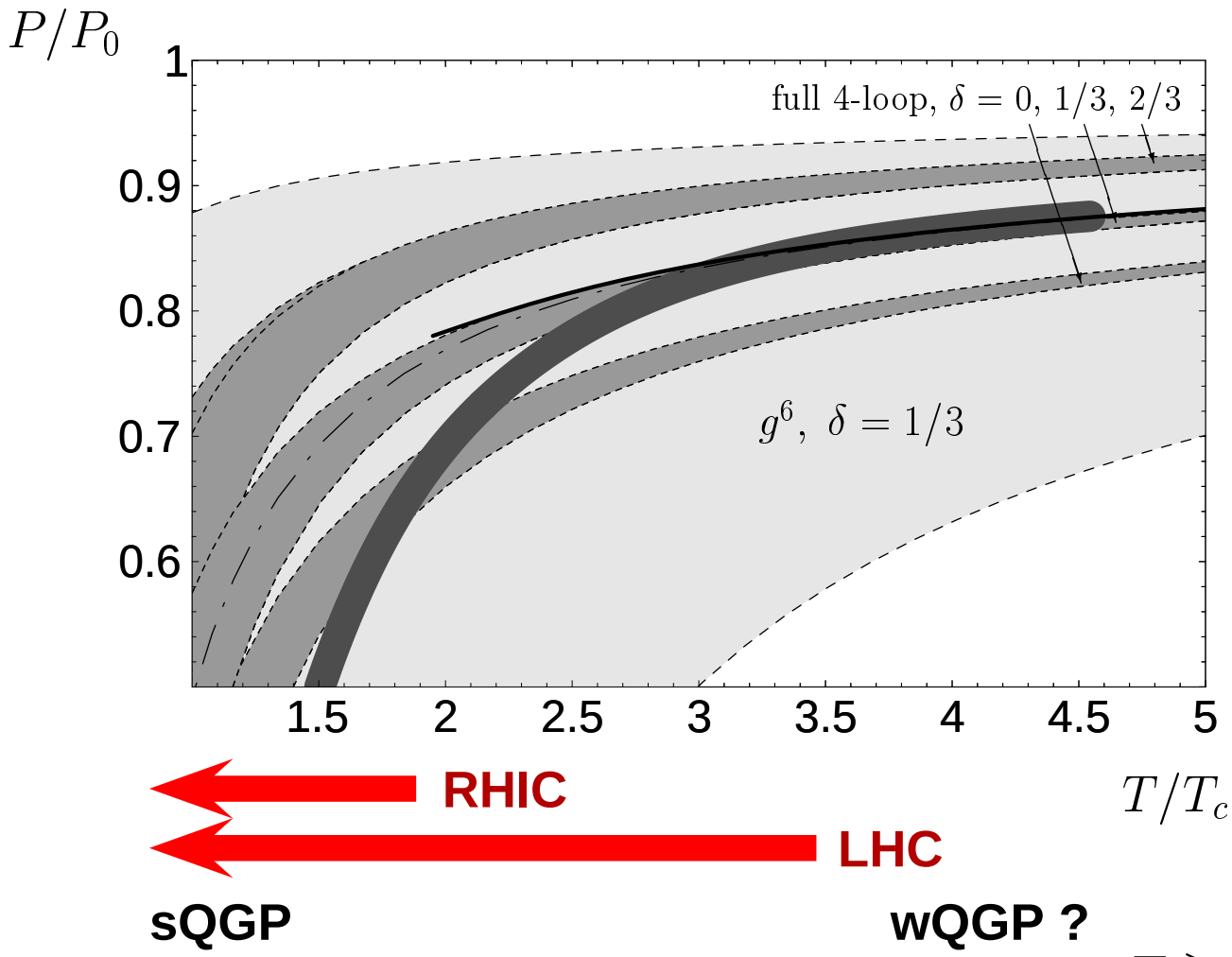
Pressure of pure-glue QCD: lattice (Bielefeld) vs. perturbative result to order g^5 with renormalization scale dependence for $\bar{\mu}_{\rm MS} = \pi T \ldots 4\pi T$

strictly pert. [Arnold & Zhai, PRD50('94)] vs. improved/optimized [Blaizot, Iancu & AR, PRD68('03)]



wQGP or sQGP?

even greater optimization effect with the perturbative $g^6[\log(g)+\delta]$ result of Kajantie, Laine, Rummukainen & Schröder (2003):



very good agreement with lattice possible for $T \gtrsim 3T_c$ for some δ

HTL-resummed entropy

Dramatic improvement already possible with result up to and including $O(g^3)$!

HTL-resummed entropy

Dramatic improvement already possible with result up to and including $O(g^3)$!

10th birthday of: Blaizot, Iancu & AR (1999):

Φ-derivable (2PI) entropy expression:

$$\begin{aligned}\mathcal{S} &= -\,\mathrm{tr}\int_K \frac{\partial n(k_0)}{\partial T}\left[\Im m \log G^{-1} - \Im m\Pi\,\Re e G\right] \\ &\quad -2\,\mathrm{tr}\int_K \frac{\partial f(k_0)}{\partial T}\left[\Im m \log S^{-1} - \Im m\Sigma\,\Re e S\right] + \mathcal{S}_{3-\mathrm{loop}},\end{aligned}$$

HTL-resummed entropy

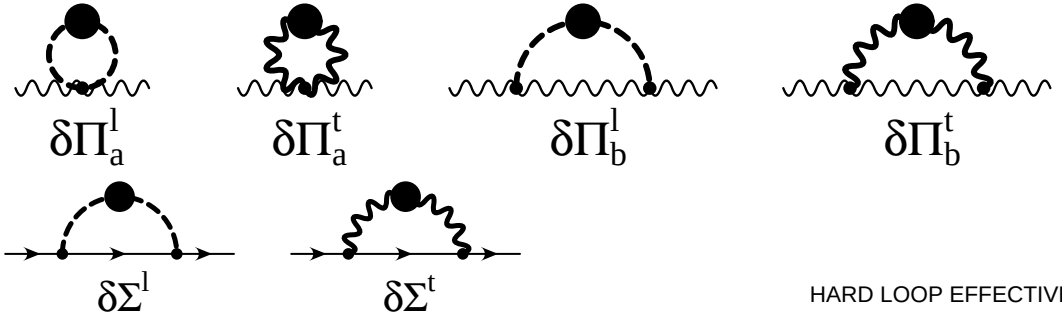
Dramatic improvement already possible with result up to and including $O(g^3)$!

10th birthday of: Blaizot, Iancu & AR (1999):

Φ-derivable (2PI) entropy expression:

$$\mathcal{S} = -\,\mathrm{tr}\int_K \frac{\partial n(k_0)}{\partial T}\left[\Im m \log G^{-1} - \Im m\Pi\,\Re e G\right]$$
$$-2\,\mathrm{tr}\int_K \frac{\partial f(k_0)}{\partial T}\left[\Im m \log S^{-1} - \Im m\Sigma\,\Re e S\right] + \mathcal{S}_{3-\mathrm{loop}},$$

- nontrivial reorganization of perturbation theory:
  - ⋆ convergence-spoiling g^3 contribution kept in nonpolynomial form;
  - ⋆ 3/4 of g^3 contribution contained in NLO correction to m_∞^2, M_∞^2 at $k \sim T$:



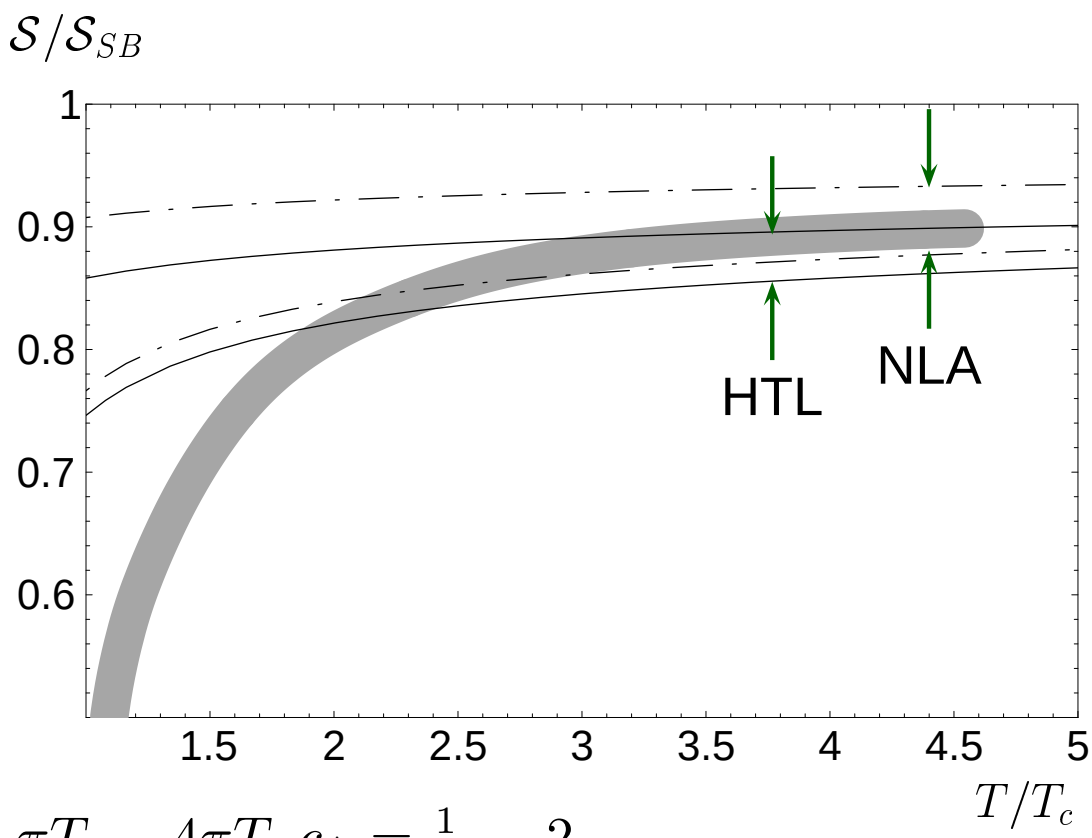
Application to pure glue QCD

Approximately self-consistent evaluations:

1) HTL self energies and propagators (full lines)

2) NLA (dash-dotted): momentum averaged NLO corrections to m_∞^2, M_∞^2, included through quadratic gap equation for hard momenta $p > \sqrt{c_\Lambda 2\pi T m_D}$

pure glue, $N_f = 0$:

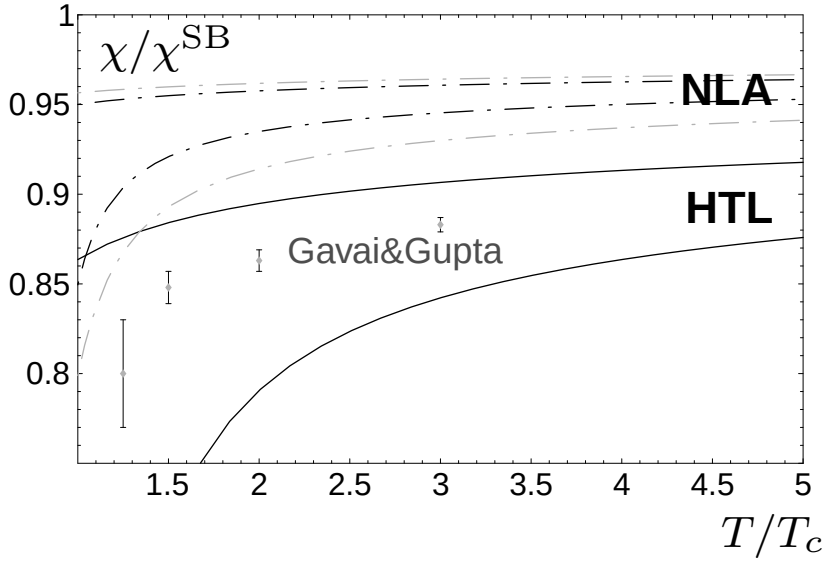


$\bar{\mu}_{MS} = \pi T \ldots 4\pi T$, $c_\Lambda = \frac{1}{2} \ldots 2$

Quark number susceptibilities

Anno 2002:

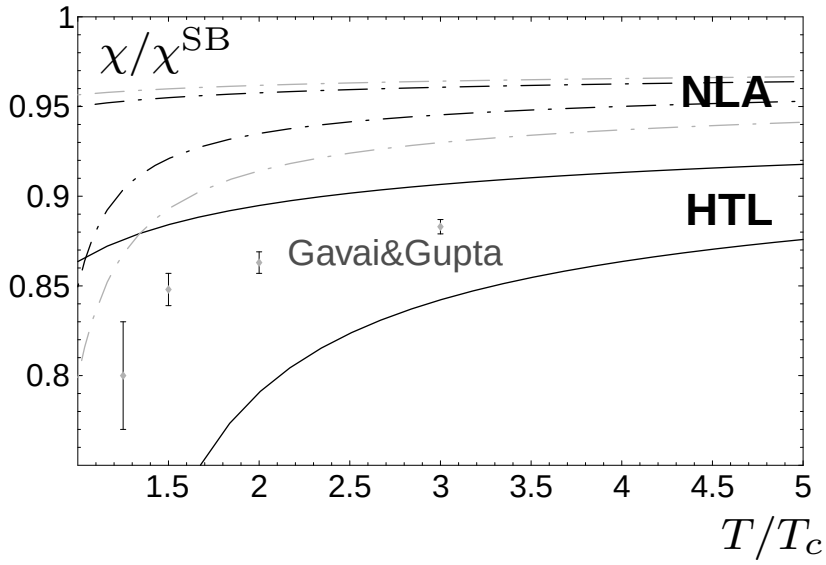
NLA appeared to spoil good agreement for HTL:



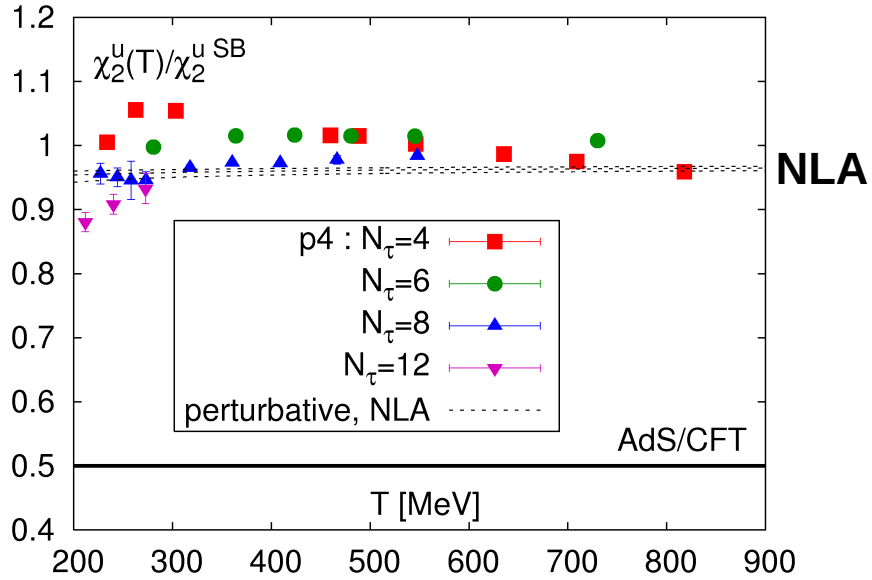
Quark number susceptibilities

Anno 2002:

NLA appeared to spoil good agreement for HTL:



P. Petrecky, QM 2009:



bad postdiction became good prediction!

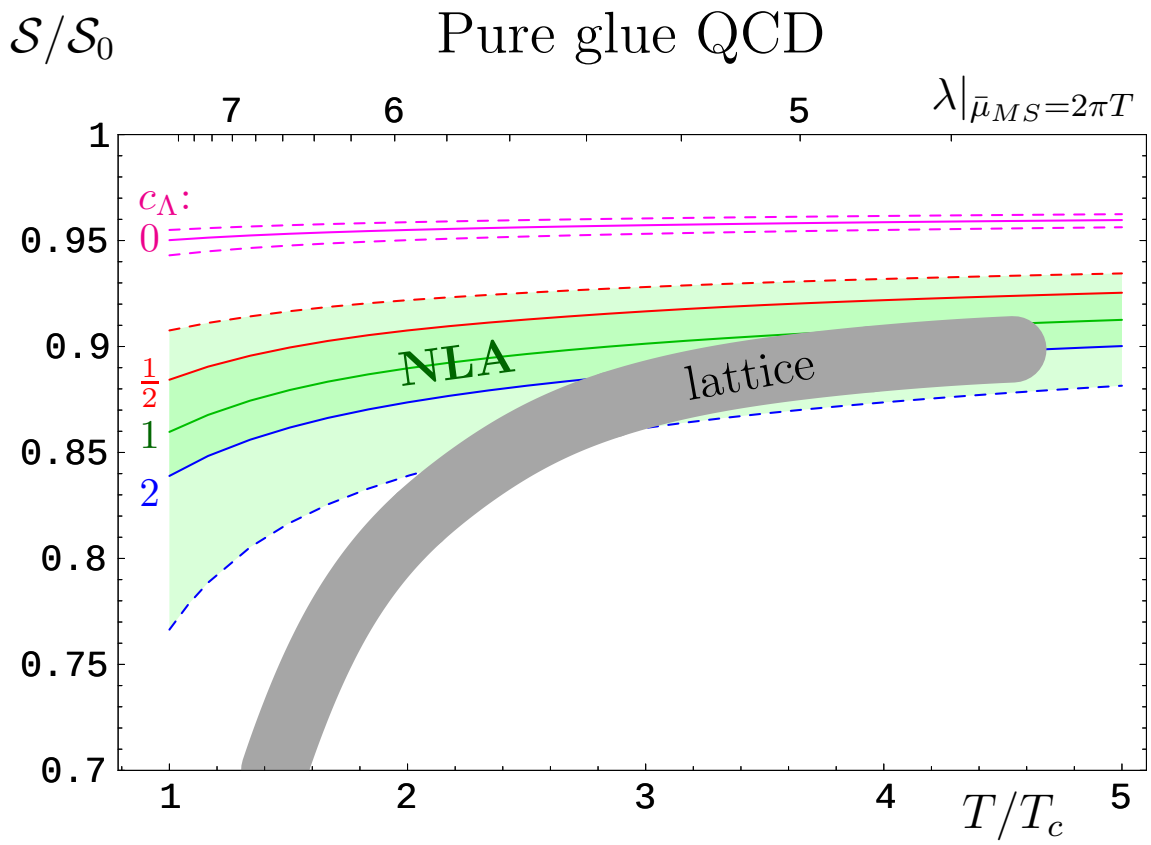
wQGP or sQGP?

Entropy in pure-glue QCD: lattice vs. **H**ard-**T**hermal-**L**oop quasiparticle entropy with

Next-to-**L**eading **A**pproximations of asymptotic thermal masses

suggestive of dominance of weakly interacting (hard) quasiparticles for $T \gtrsim 3T_c$

[Blaizot, Iancu & AR, PRD63('01)065003]



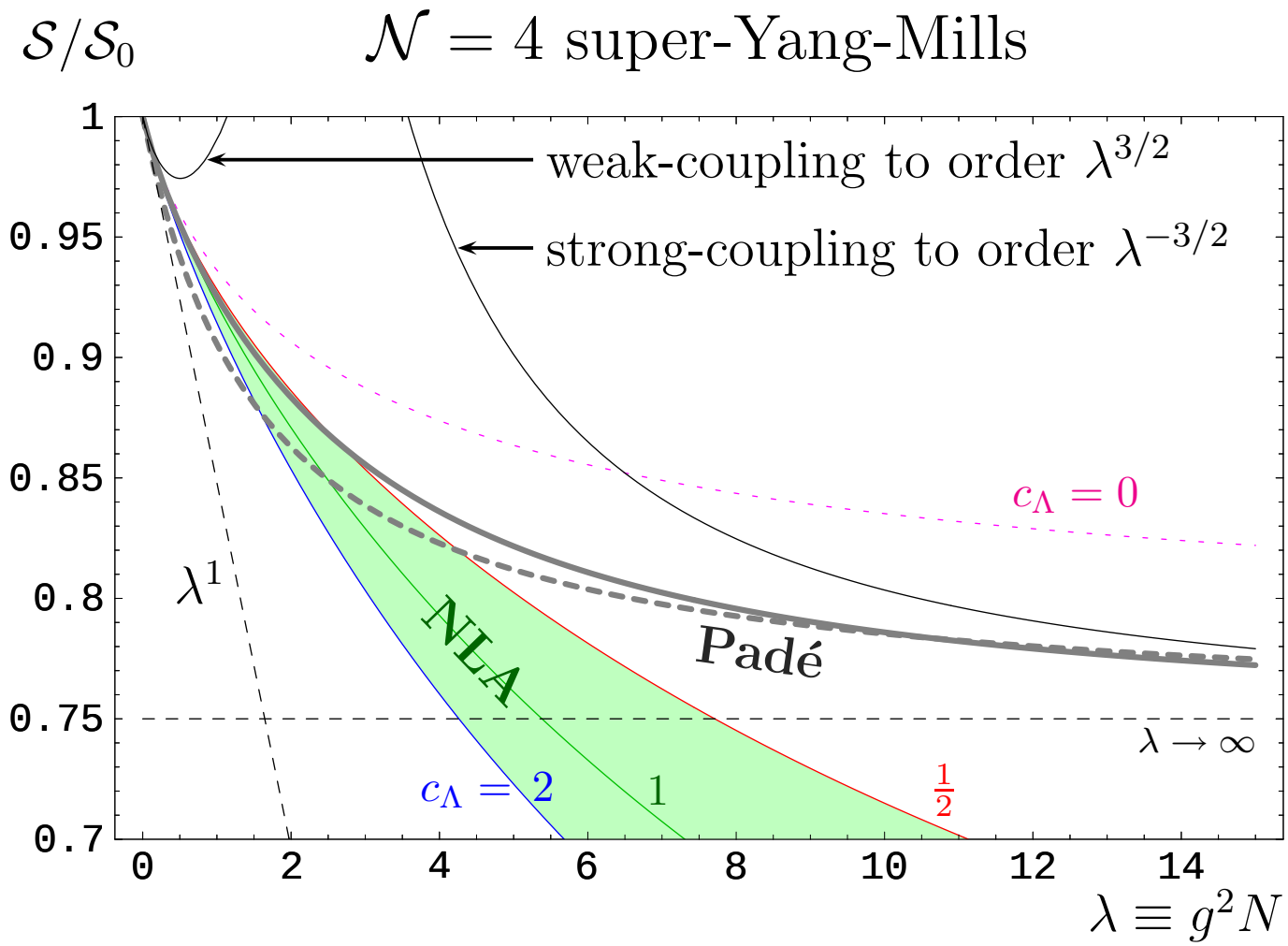
Recent improvements/fits by inclusion of (sizeable) LO quasiparticle width: Peshier, Cassing

wQGP or sQGP?

No lattice EOS results for $\mathcal{N} = 4$ SYM,

but essentially unique Padé approximant $R_{[4,4]} = \frac{1+\alpha\lambda^{1/2}+\beta\lambda+\gamma\lambda^{3/2}+\delta\lambda^2}{1+\bar{\alpha}\lambda^{1/2}+\bar{\beta}\lambda+\bar{\gamma}\lambda^{3/2}+\bar{\delta}\lambda^2}$

for known weak and strong coupling results

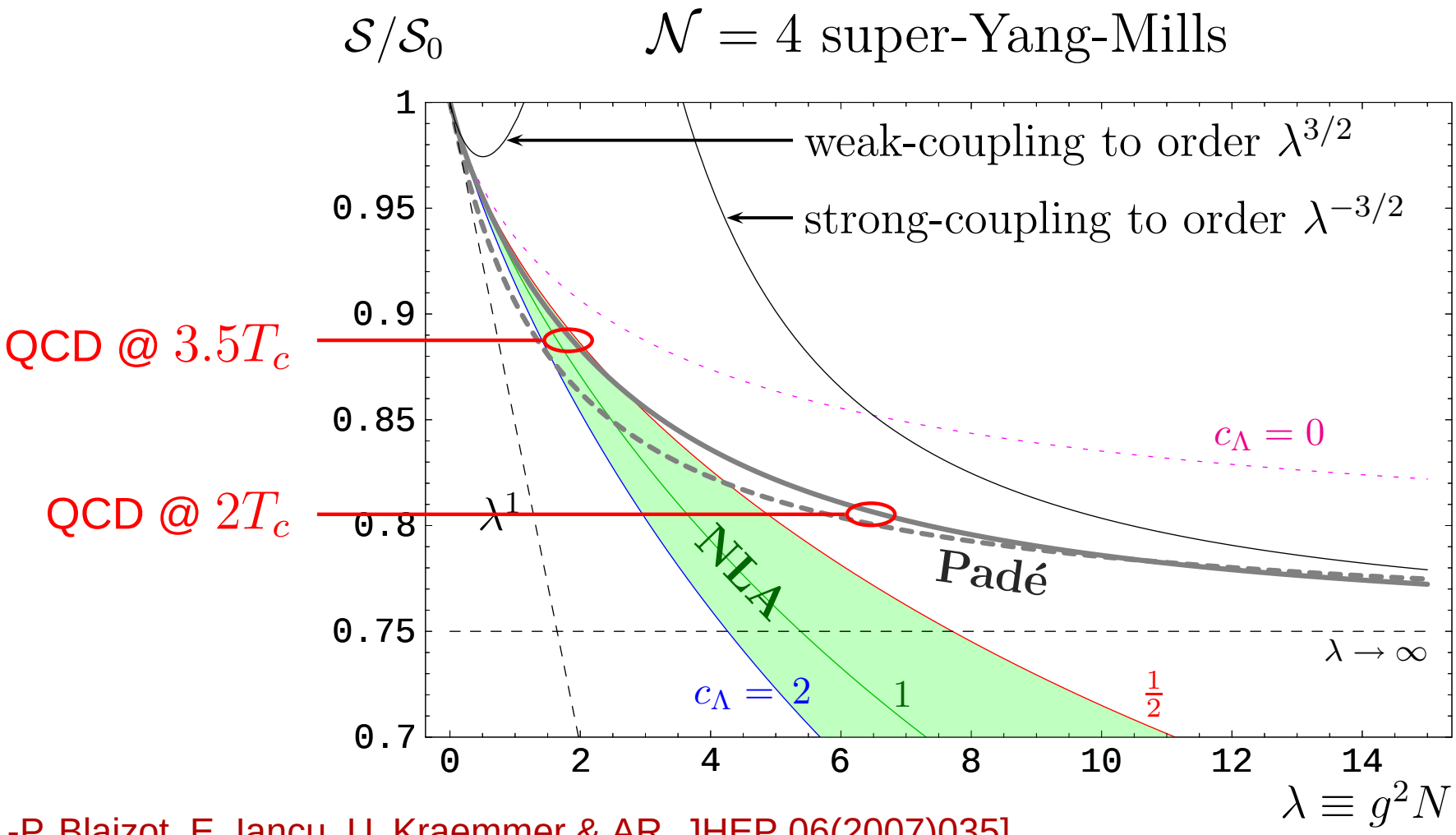


wQGP or sQGP?

No lattice EOS results for $\mathcal{N} = 4$ SYM,

but essentially unique Padé approximant $R_{[4,4]} = \frac{1+\alpha\lambda^{1/2}+\beta\lambda+\gamma\lambda^{3/2}+\delta\lambda^2}{1+\bar{\alpha}\lambda^{1/2}+\bar{\beta}\lambda+\bar{\gamma}\lambda^{3/2}+\bar{\delta}\lambda^2}$

for known weak and strong coupling results



[J.-P. Blaizot, E. Iancu, U. Kraemmer & AR, JHEP 06(2007)035]

Scales of wQGP

- T: energy of hard particles
- gT: thermal masses, Debye screening mass,
  Landau damping
- g^2T: magnetic confinement, color relaxation, rate for small angle scattering
- g^4T: rate for large angle scattering; inverse shear viscosity $\eta^{-1}T^4$

Scales of wQGP

- T: energy of hard particles
- gT: thermal masses, Debye screening mass,
  Landau damping
- g^2T: magnetic confinement, color relaxation, rate for small angle scattering
- g^4T: rate for large angle scattering; inverse shear viscosity $\eta^{-1}T^4$

Effective theory at scale gT: Hard-(Thermal-)Loop Effective Action

[Frenkel, Taylor & Wong; Braaten & Pisarski 1991]

equivalent to: gauge-covariant Boltzmann-Vlasov

[Blaizot & Iancu 1993, Kelly, Liu, Lucchesi & Manuel 1994]

in particular required (to leading order!) for:

- Bottom-up thermalization [Baier, Mueller, Schiff & Son 2000]

$$t_{eq} \propto g^{-13/5}$$

Scales of wQGP

- T: energy of hard particles
- gT: thermal masses, Debye screening mass,
  Landau damping **+ plasma instabilities** [Mrówczyński 1988, 1993, . . .]
- g^2T: magnetic confinement, color relaxation, rate for small angle scattering
- g^4T: rate for large angle scattering; inverse shear viscosity $\eta^{-1}T^4$

Effective theory at scale gT: Hard-(~~Thermal~~)Loop Effective Action

[Frenkel, Taylor & Wong; Braaten & Pisarski 1991]

equivalent to: gauge-covariant Boltzmann-Vlasov

[Blaizot & Iancu 1993, Kelly, Liu, Lucchesi & Manuel 1994]

in particular required (to leading order!) for:

- Bottom-up thermalization [Baier, Mueller, Schiff & Son 2000]

$$t_{eq} \propto g^{-13/5}$$

Scales of wQGP

- T: energy of hard particles
- gT: thermal masses, Debye screening mass,
  Landau damping **+ plasma instabilities** [Mrówczyński 1988, 1993, . . .]
- g^2T: magnetic confinement, color relaxation, rate for small angle scattering
- g^4T: rate for large angle scattering; inverse shear viscosity $\eta^{-1}T^4$

Effective theory at scale gT: Hard-(~~Thermal~~)Loop Effective Action

[Frenkel, Taylor & Wong; Braaten & Pisarski 1991]

equivalent to: gauge-covariant Boltzmann-Vlasov

[Blaizot & Iancu 1993, Kelly, Liu, Lucchesi & Manuel 1994]

in particular required (to leading order!) for:

- Bottom-up thermalization [Baier, Mueller, Schiff & Son 2000]

$$t_{eq} \propto g^{-13/5} \longrightarrow g^{-?}$$

[Arnold, Lenaghan, Moore, JHEP 08 ('03) 002]

- Shear viscosity [Arnold, Moore & Yaffe]

$$(\eta/s)^{-1} = g^4 \ln(1/g) f(\ln(1/g))$$

Scales of wQGP

- T: energy of hard particles
- gT: thermal masses, Debye screening mass,
  Landau damping **+ plasma instabilities** [Mrówczyński 1988, 1993, . . .]
- g^2T: magnetic confinement, color relaxation, rate for small angle scattering
- g^4T: rate for large angle scattering; inverse shear viscosity $\eta^{-1}T^4$

Effective theory at scale gT: Hard-(~~Thermal~~)Loop Effective Action

[Frenkel, Taylor & Wong; Braaten & Pisarski 1991]

equivalent to: gauge-covariant Boltzmann-Vlasov

[Blaizot & Iancu 1993, Kelly, Liu, Lucchesi & Manuel 1994]

in particular required (to leading order!) for:

- Bottom-up thermalization [Baier, Mueller, Schiff & Son 2000]

$$t_{eq} \propto g^{-13/5} \longrightarrow g^{-?}$$ [Arnold, Lenaghan, Moore, JHEP 08 ('03) 002]

- Shear viscosity [Arnold, Moore & Yaffe]

$$(\eta/s)^{-1} = g^4 \ln(1/g) f(\ln(1/g))$$ add $(\eta/s)^{-1}_{\text{anomalous}}$!

[Asakawa, Bass & Müller, PRL 96 ('06) 252301]

Hard (Thermal) Loops = gauge covariant Boltzmann-Vlasov

With color-neutral background distribution $v \cdot \partial\, f_0(\mathbf{p}, \mathbf{x}, t) = 0$, $v^\mu = p^\mu / p^0$

$$v \cdot D\, \delta f_a(\mathbf{p}, \mathbf{x}, t) = g v_\mu F_a^{\mu\nu} \partial_\nu^{(p)} f_0(\mathbf{p}, \mathbf{x}, t) = -g(\mathbf{E}_a + \mathbf{v} \times \mathbf{B}_a) \cdot \nabla_{\mathbf{p}} f_0,$$

$$D_\mu F_a^{\mu\nu} = j_a^\nu = g \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{2p^0} \delta f_a(\mathbf{p}, \mathbf{x}, t).$$

Hard (Thermal) Loops = gauge covariant Boltzmann-Vlasov

With color-neutral background distribution $v \cdot \partial\, f_0(\mathbf{p}, \mathbf{x}, t) = 0$, $v^\mu = p^\mu / p^0$

$$v \cdot D\, \delta f_a(\mathbf{p}, \mathbf{x}, t) = g v_\mu F_a^{\mu\nu} \partial_\nu^{(p)} f_0(\mathbf{p}, \mathbf{x}, t) = -g(\mathbf{E}_a + \mathbf{v} \times \mathbf{B}_a) \cdot \nabla_{\mathbf{p}} f_0,$$

$$D_\mu F_a^{\mu\nu} = j_a^\nu = g \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{2p^0} \delta f_a(\mathbf{p}, \mathbf{x}, t).$$

- isotropic: $f_0(\mathbf{p}) = f_0(|\mathbf{p}|)$, $\nabla_{\mathbf{p}} f_0 \propto \mathbf{v}$

  $v \cdot D\, \delta f_a(\mathbf{p}, \mathbf{x}, t) = -g \mathbf{E}_a \cdot \nabla_{\mathbf{p}} f_0$ (stable)

Hard (Thermal) Loops = gauge covariant Boltzmann-Vlasov

With color-neutral background distribution $v \cdot \partial\, f_0(\mathbf{p}, \mathbf{x}, t) = 0$, $v^\mu = p^\mu / p^0$

$$v \cdot D\, \delta f_a(\mathbf{p}, \mathbf{x}, t) = g v_\mu F_a^{\mu\nu} \partial_\nu^{(p)} f_0(\mathbf{p}, \mathbf{x}, t) = -g(\mathbf{E}_a + \mathbf{v} \times \mathbf{B}_a) \cdot \nabla_{\mathbf{p}} f_0,$$

$$D_\mu F_a^{\mu\nu} = j_a^\nu = g \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{2p^0} \delta f_a(\mathbf{p}, \mathbf{x}, t).$$

- isotropic: $f_0(\mathbf{p}) = f_0(|\mathbf{p}|)$, $\nabla_{\mathbf{p}} f_0 \propto \mathbf{v}$

  $v \cdot D\, \delta f_a(\mathbf{p}, \mathbf{x}, t) = -g \mathbf{E}_a \cdot \nabla_{\mathbf{p}} f_0$ (stable)

- anisotropic $f_0(\mathbf{p})$, $\nabla_{\mathbf{p}} f_0 \not\propto \mathbf{v}$

  $v \cdot D\, \delta f_a(\mathbf{p}, \mathbf{x}, t) = -g(\mathbf{E}_a + \mathbf{v} \times \mathbf{B}_a) \cdot \nabla_{\mathbf{p}} f_0$ unstable!

Hard (Thermal) Loops = gauge covariant Boltzmann-Vlasov

With color-neutral background distribution $v \cdot \partial\, f_0(\mathbf{p}, \mathbf{x}, t) = 0$, $v^\mu = p^\mu / p^0$

$$v \cdot D\, \delta f_a(\mathbf{p}, \mathbf{x}, t) = g v_\mu F_a^{\mu\nu} \partial_\nu^{(p)} f_0(\mathbf{p}, \mathbf{x}, t) = -g(\mathbf{E}_a + \mathbf{v} \times \mathbf{B}_a) \cdot \nabla_{\mathbf{p}} f_0,$$

$$D_\mu F_a^{\mu\nu} = j_a^\nu = g \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{2p^0} \delta f_a(\mathbf{p}, \mathbf{x}, t).$$

- isotropic: $f_0(\mathbf{p}) = f_0(|\mathbf{p}|)$, $\nabla_{\mathbf{p}} f_0 \propto \mathbf{v}$

  $v \cdot D\, \delta f_a(\mathbf{p}, \mathbf{x}, t) = -g \mathbf{E}_a \cdot \nabla_{\mathbf{p}} f_0$ (stable)

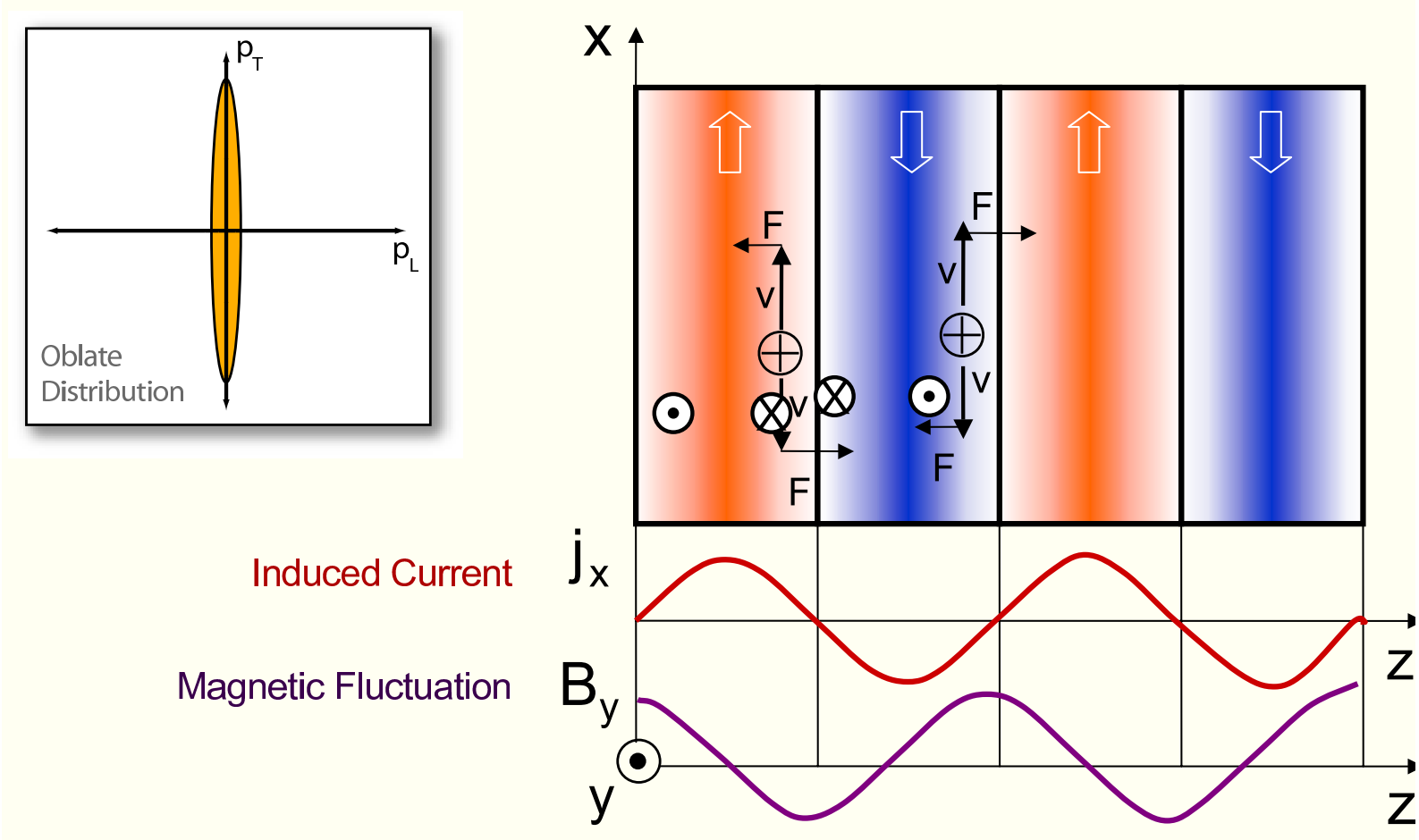
- anisotropic $f_0(\mathbf{p})$, $\nabla_{\mathbf{p}} f_0 \not\propto \mathbf{v}$

  $v \cdot D\, \delta f_a(\mathbf{p}, \mathbf{x}, t) = -g(\mathbf{E}_a + \mathbf{v} \times \mathbf{B}_a) \cdot \nabla_{\mathbf{p}} f_0$ unstable!

- anisotropic expansion: $f_0(\mathbf{p}; \mathbf{x}, t)$

Filamentation (Weibel) instabilities

E.g.: ensemble of counterstreaming currents unstable against filamentation



Abelian plasma: exponential growth of currents and magnetic fields until magnetic fields strong enough to bend trajectories $\longrightarrow$ fast isotropization

Discretized Hard Loop Effective Theory

Plasma instabilities: small fluctuations grow until linear regime is left

Nonabelian plasma instabilities: nonlinear regime entered while still in hard-loop regime!

Discretized Hard Loop Effective Theory

Plasma instabilities: small fluctuations grow until linear regime is left

Nonabelian plasma instabilities: nonlinear regime entered while still in hard-loop regime!

Useful for numerical (real-time lattice) study:

auxiliary field formulation: [Nair; Blaizot & Iancu 1994; Mrówczyński, AR & Strickland 2004]

$$\delta f^a(x;p) = -gW^a_\mu(t,\mathbf{x};\mathbf{v})\partial^\mu_{(p)} f_0(\mathbf{p})$$

$$\boxed{[v\cdot D(A)]W_\mu(x;\mathbf{v}) = F_{\mu\gamma}(A)v^\gamma} \quad v^\mu \equiv p^\mu/|\mathbf{p}| = (1,\mathbf{v})$$

$$\boxed{D_\rho(A)F^{\rho\mu} = j^\mu(x) = -g^2\int \frac{d^3p}{(2\pi)^3}\frac{1}{2|\mathbf{p}|}\,p^\mu\,\frac{\partial f(\mathbf{p})}{\partial p^\nu}W^\nu(x;\mathbf{v})}$$

Hard Loop effective theory: (hard) scale $|\mathbf{p}|$ can be integrated out

Auxiliary field version: local in terms of field living also on velocity space S_2

Discretized Hard Loop Effective Theory

Plasma instabilities: small fluctuations grow until linear regime is left

Nonabelian plasma instabilities: nonlinear regime entered while still in hard-loop regime!

Useful for numerical (real-time lattice) study:

auxiliary field formulation: [Nair; Blaizot & Iancu 1994; Mrówczyński, AR & Strickland 2004]

$$\delta f^a(x;p) = -gW^a_\mu(t,\mathbf{x};\mathbf{v})\partial^\mu_{(p)} f_0(\mathbf{p})$$

$$\boxed{[v\cdot D(A)]W_\mu(x;\mathbf{v}) = F_{\mu\gamma}(A)v^\gamma} \quad v^\mu \equiv p^\mu/|\mathbf{p}| = (1,\mathbf{v})$$

$$\boxed{D_\rho(A)F^{\rho\mu} = j^\mu(x) = -g^2\int \frac{d^3p}{(2\pi)^3}\frac{1}{2|\mathbf{p}|}\,p^\mu\,\frac{\partial f(\mathbf{p})}{\partial p^\nu}W^\nu(x;\mathbf{v})}$$

Hard Loop effective theory: (hard) scale $|\mathbf{p}|$ can be integrated out

Auxiliary field version: local in terms of field living also on velocity space S_2

$\longrightarrow$ discretize also velocity space

$$D_\rho(A)F^{\rho\mu} = j^\mu(x) = \frac{1}{\mathcal{N}}\sum_{\mathbf{v}} v^\mu \mathcal{W}_{\mathbf{v}}(x)$$

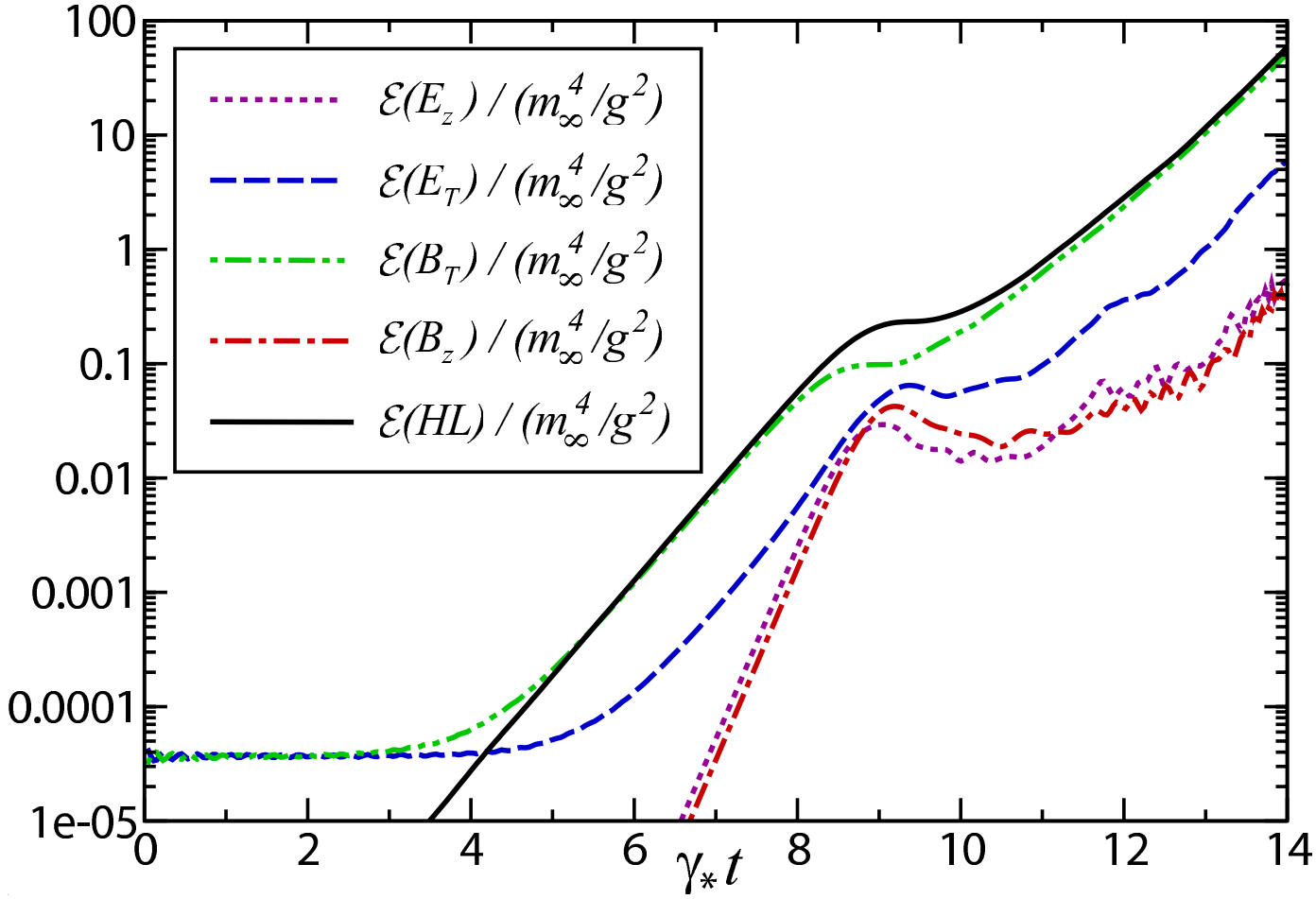
"disco balls"

Transversely constant modes: 1D+3V

Most unstable modes in linear response: $\mathbf{k} \parallel \mathbf{n}$

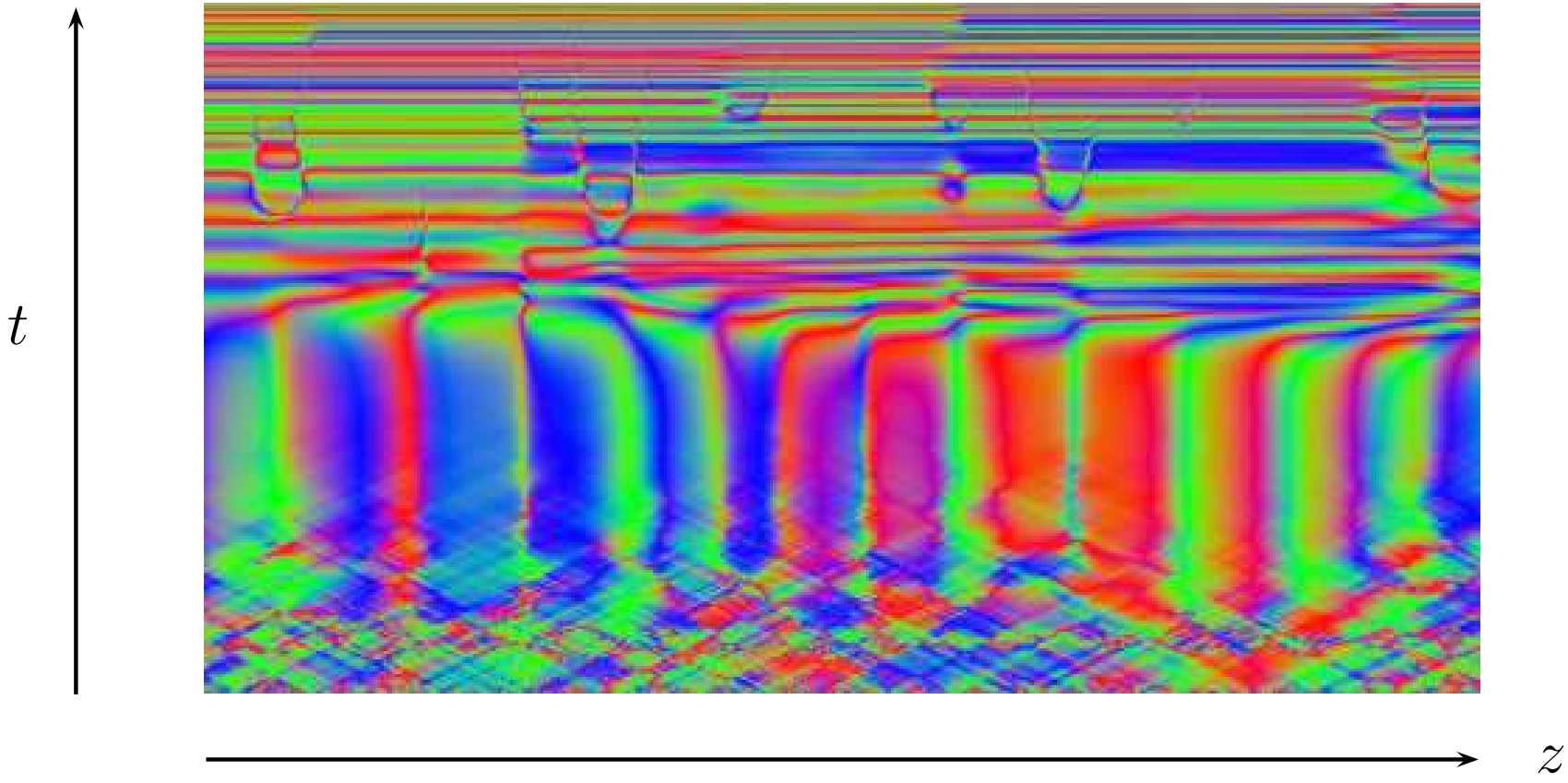
$\Longrightarrow$ no dependence on transverse coordinates;
dimensional reduction to 1 spatial dimension

[AR, Romatschke & Strickland, PRL 94 ('05) 102303]



Transversely constant modes: 1D+3V

Evolution of color degrees of freedom:
(parallel-transported color from fixed spatial point)



Late-time (non-linear) regime: Abelianization over extended spatial domains
– responsible for continued Abelian-like growth in non-linear regime

3D+3V

However: local Abelianization can be destroyed by interactions with not perfectly transversely constant modes

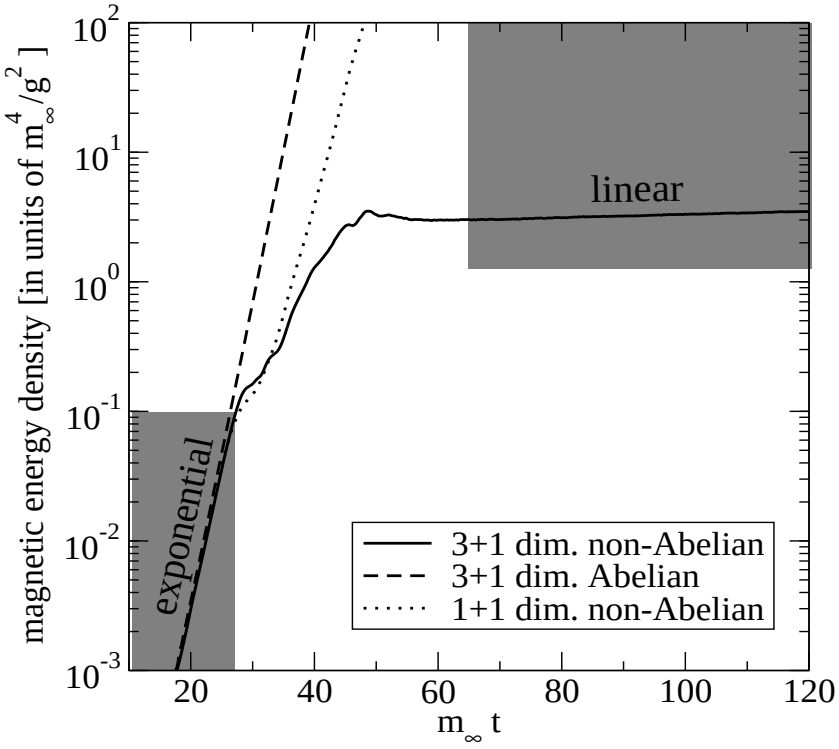
3D+3V

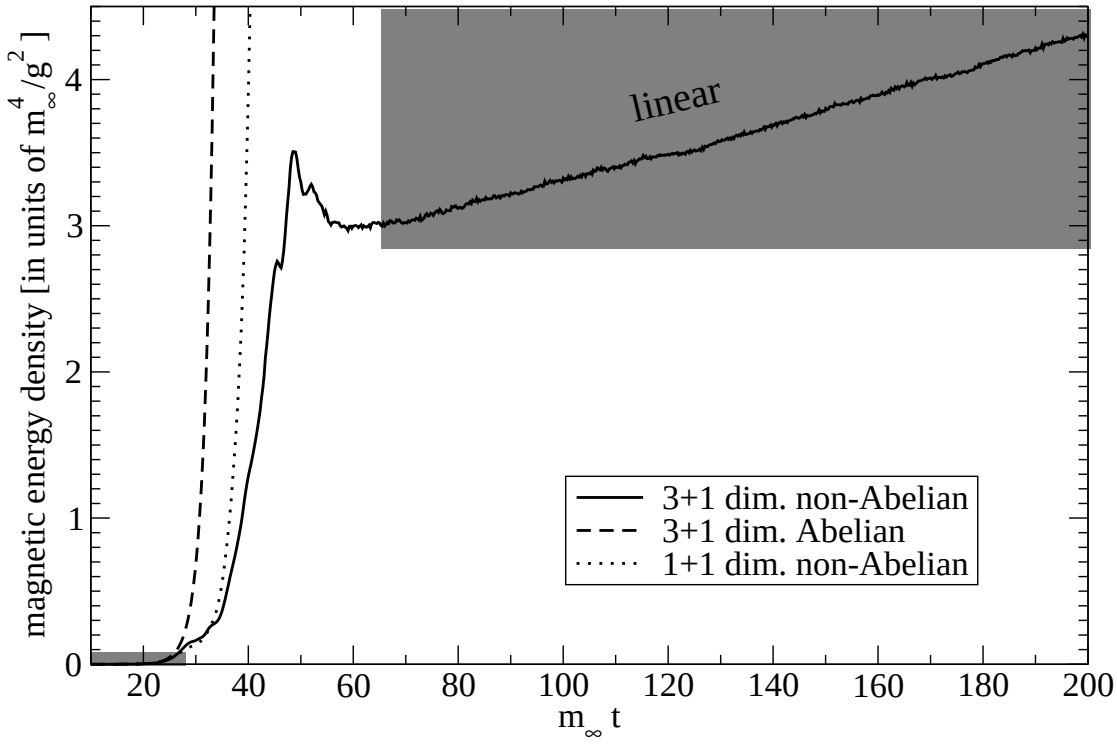
However: local Abelianization can be destroyed by interactions with not perfectly transversely constant modes

$\longrightarrow$ attenuation of exponential growth to only linear one:

[Arnold, Moore & Yaffe, PRD72 ('05); AR, Romatschke & Strickland, JHEP 09 ('05)]

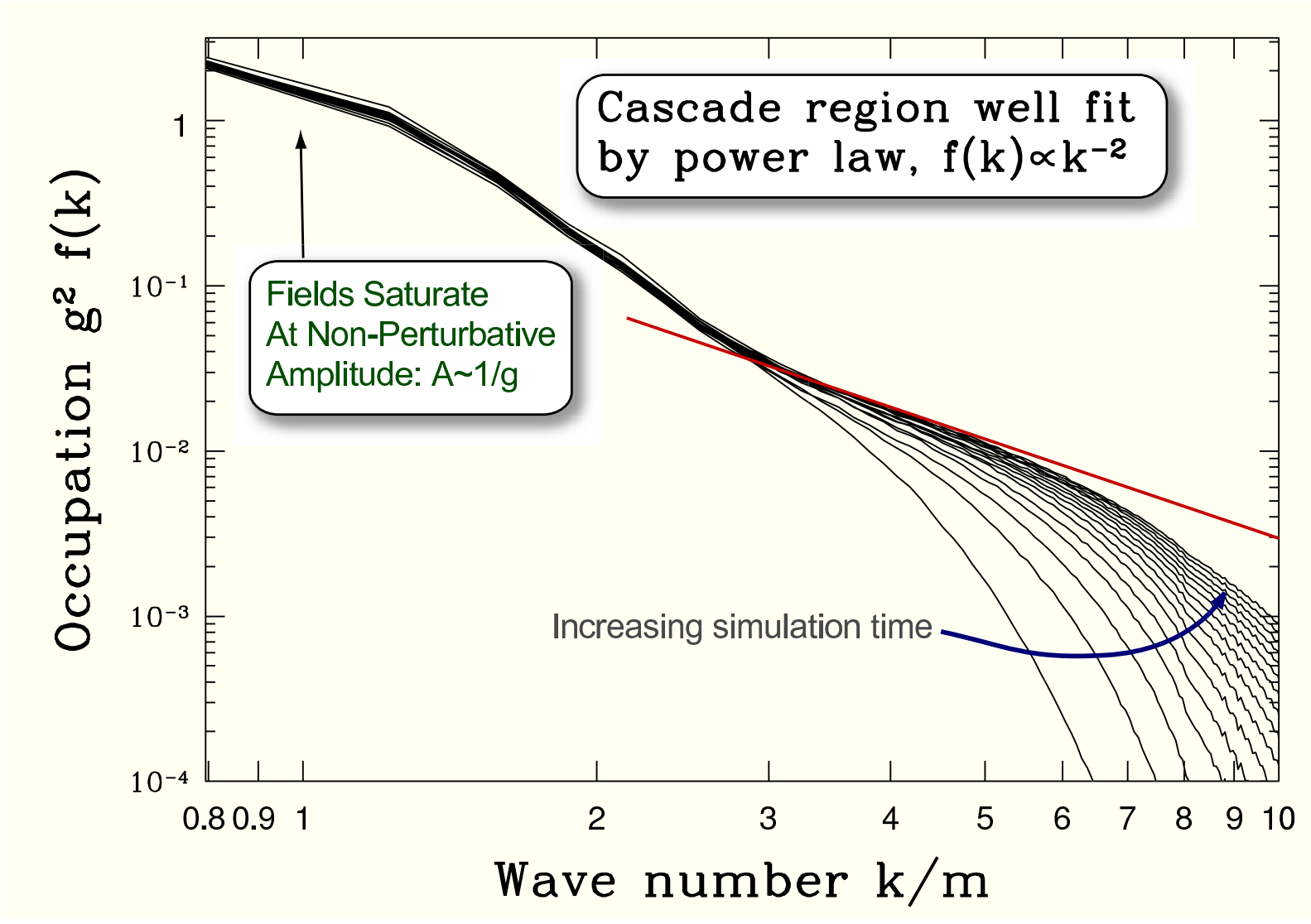
[Bödeker & Rummukainen, JHEP 07 ('07)]





(btw different discretization method: finite number of spherical harmonics W_{lm})

Cascade


Cascade region well fit
by power law, f(k)∝k⁻²
Fields Saturate
At Non-Perturbative
Amplitude: A~1/g
Increasing simulation time
Occupation g² f(k)
1
10⁻¹
10⁻²
10⁻³
10⁻⁴
0.8 0.9 1 2 3 4 5 6 7 8 9 10
Wave number k/m

Plasma instabilities in Bjorken expansion

Longitudinal (Bjorken) expansion: Competition between

- increasing anisotropy (more and more modes become unstable)
- and decreasing density ($\leftrightarrow$ growth rate)

Plasma instabilities in Bjorken expansion

Longitudinal (Bjorken) expansion: Competition between

- increasing anisotropy (more and more modes become unstable)
- and decreasing density ($\leftrightarrow$ growth rate)

Already Abelian (linear) regime requires numerical work

[Romatschke & AR, PRL 97 (2006)]

Asymptotics $\propto {}_2F_3(\ldots;-\mu\tau) \sim \exp(\sqrt{\frac{\pi}{2}m_D^2\tau_{\rm iso}\tau})$

Plasma instabilities in Bjorken expansion

Longitudinal (Bjorken) expansion: Competition between

- increasing anisotropy (more and more modes become unstable)
- and decreasing density ($\leftrightarrow$ growth rate)

Already Abelian (linear) regime requires numerical work

[Romatschke & AR, PRL 97 (2006)]

Asymptotics $\propto {}_2F_3(\ldots;-\mu\tau) \sim \exp(\sqrt{\frac{\pi}{2}m_D^2\tau_{\rm iso}\tau})$

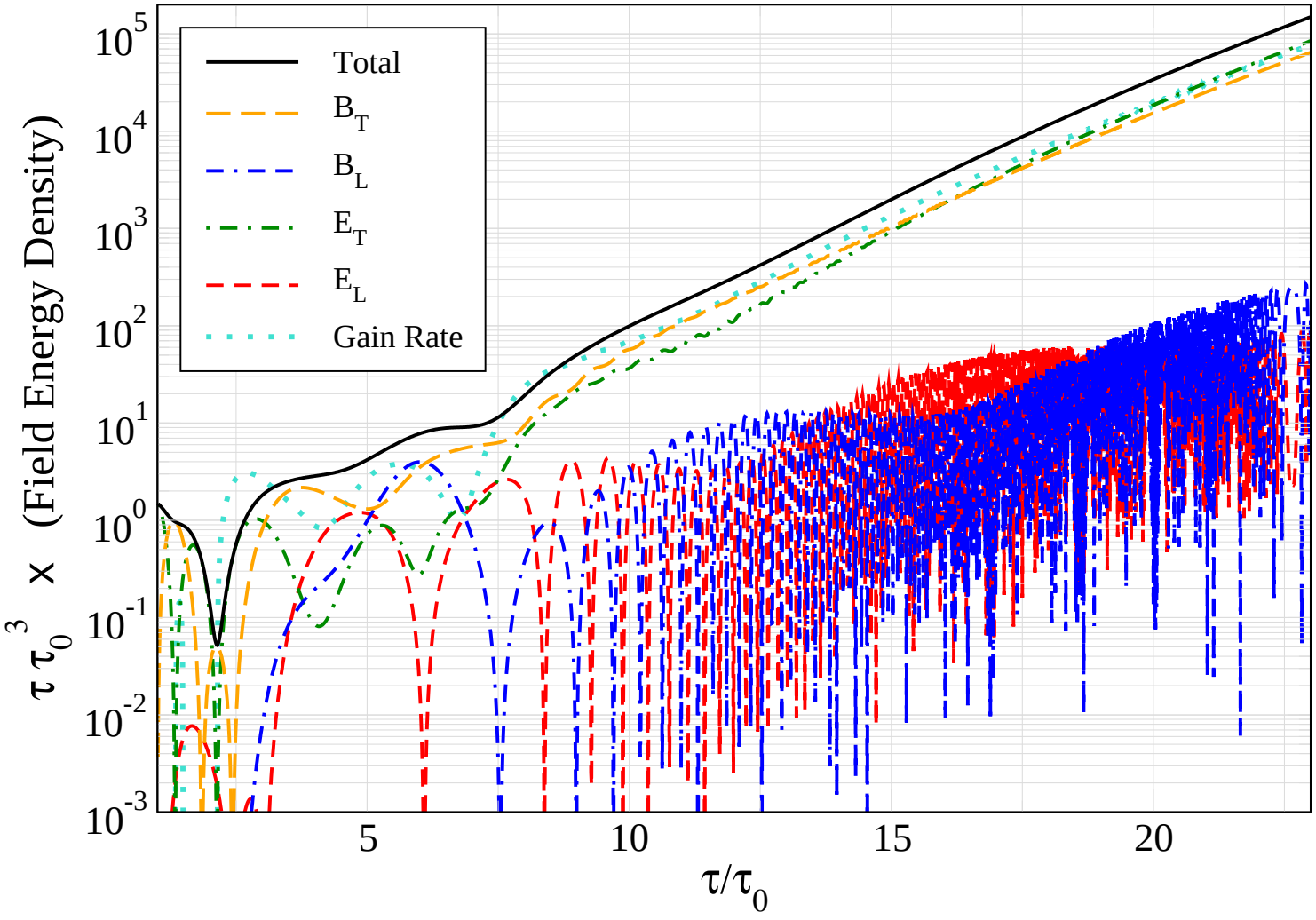
Non-Abelian: discretized Hard-Expanding-Loop theory (HEL)

(discoballs $\longrightarrow$ disco-cylinders in momentum rapidity and azimuth angle)

[AR, Strickland & Attems, PRD78 (2008)]

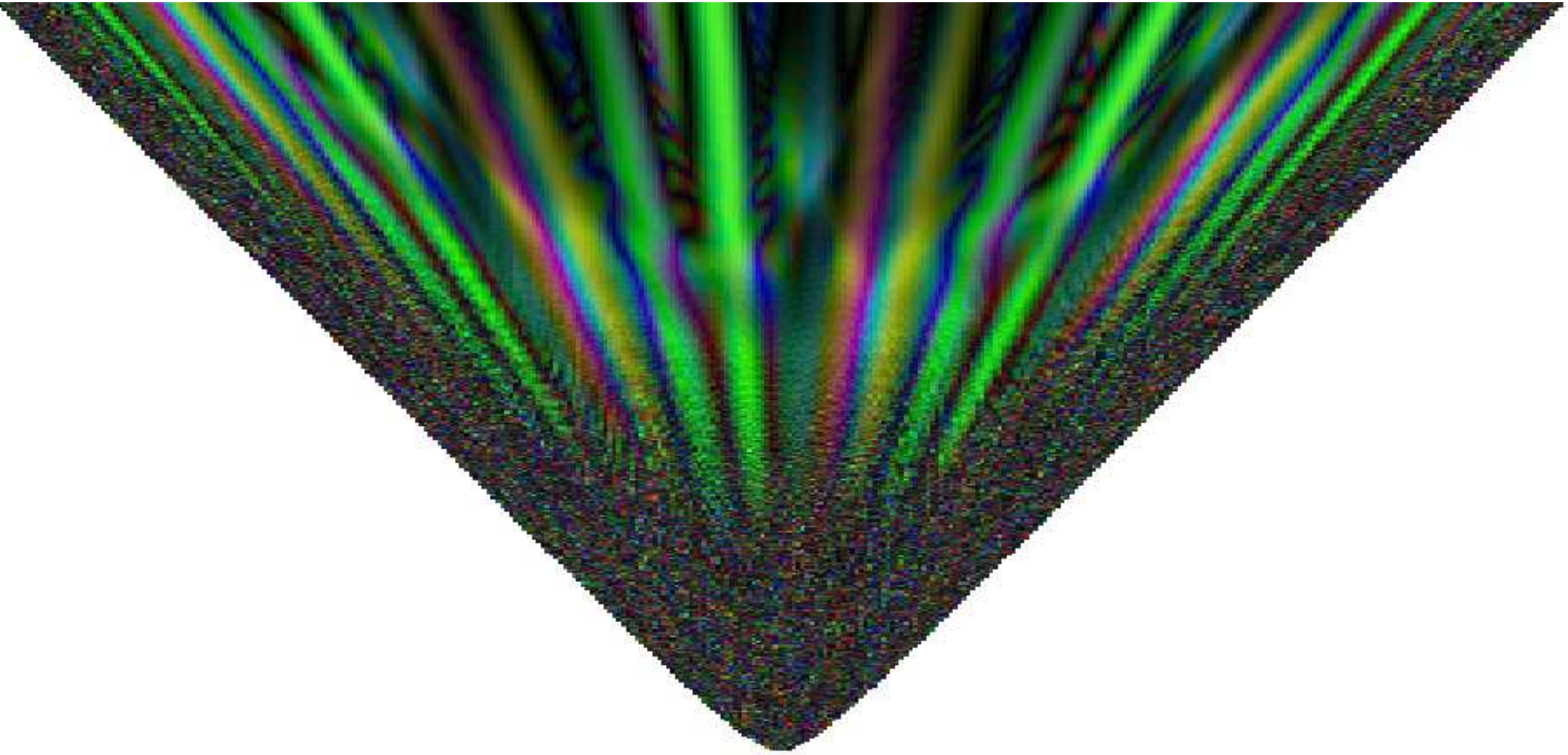
Non-Abelian Discretized HEL

Single nonabelian mode (transversely constant): (unrealistically dense plasma)



Non-Abelian Discretized HEL — Visualization in Lab Frame

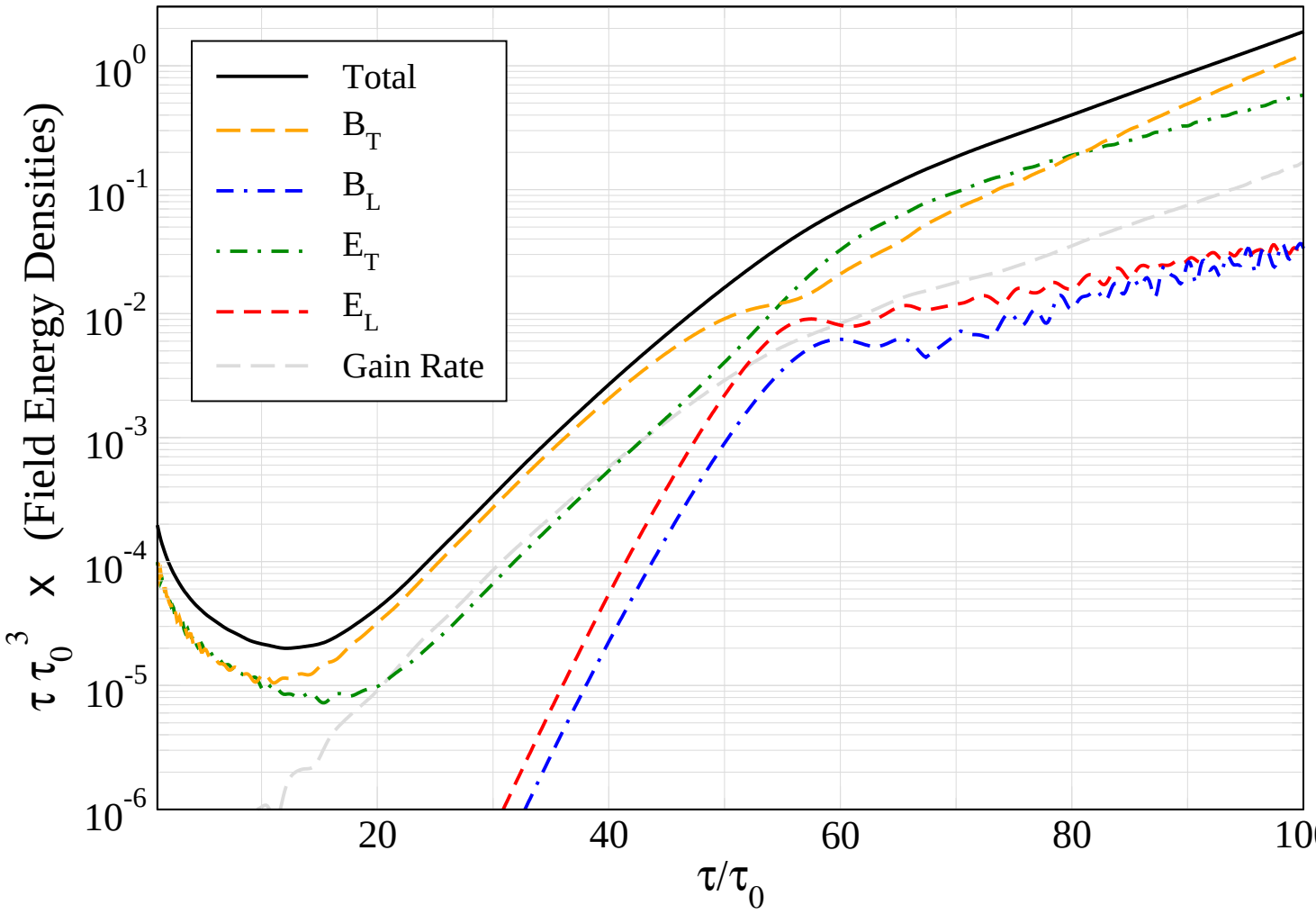
Transversely constant color plasma instabilities from tiny initial rapidity fluctuations:



Non-Abelian Discretized HEL – Chromofield energy densities

Hard gluon number density from **CGC** and weak initial fluctuations(!):

Effective growth rates up to $(0.7\,\mathrm{fm}/c)^{-1}$ (RHIC), $(0.3\,\mathrm{fm}/c)^{-1}$ (LHC)
BUT: uncomfortable delay of onset of exponential (in $\sqrt{\tau}$) growth



$= \begin{cases} 20 \text{ fm/c (RHIC)} \\ 7 \text{ fm/c (LHC)} \end{cases}$

Conclusions and Outlook

- wQCD may become relevant at $T \gtrsim 3T_c$ (LHC?)

Conclusions and Outlook

- wQCD may become relevant at $T \gtrsim 3T_c$ (LHC?)
- Plasma instabilities are parametrically dominant phenomenon in anisotropic wQCD plasmas with interesting characteristic time scales
- Uncomfortably long delay of onset of plasma instabilities in Bjorken expansion to explain early isotropization at RHIC
- More important role for plasma instabilites at LHC?

Conclusions and Outlook

- wQCD may become relevant at $T \gtrsim 3T_c$ (LHC?)
- Plasma instabilities are parametrically dominant phenomenon in anisotropic wQCD plasmas with interesting characteristic time scales
- Uncomfortably long delay of onset of plasma instabilities in Bjorken expansion to explain early isotropization at RHIC
- More important role for plasma instabilites at LHC?

Forthcoming:

- Full 3D+3V (needed for analysis of generic large initial fields)

Open challenge:

- Complete perturbative bottom-up thermalization scenario

The party must go on!

Non-Abelian Discretized HEL

Hard gluon number density and initial fluctuation spectrum from **CGC** $\longrightarrow$

Parameters from saturation scenario $\tau_0 \simeq Q_s^{-1}$:

$n(\tau_0) = c\frac{(N_c^2-1)Q_s^3}{4\pi^2 N_c \alpha_s (Q_s\tau_0)}$

with gluon liberation factor $c = \begin{cases} 0.5 & \text{Krasnitz '99 et al.} \longrightarrow \text{1.1 Lappi '07 (numerical)} \\ \underline{2\ln 2} \approx 1.39 & \text{Kovchegov (analytical estimate)} \end{cases}$

$f_{\rm iso} = \mathcal{N} f_{\rm thermal}$ with (transverse) temperature $T = 0.47 Q_s$ [Krasnitz et al.]

$$\text{pure glue} \qquad \longrightarrow \qquad \mathcal{N} = \frac{1}{\alpha_s}\frac{c}{8N_c(0.47)^3\zeta(3)}\frac{\tau_0}{\tau_{iso}}\frac{1}{Q_s\tau_0}$$

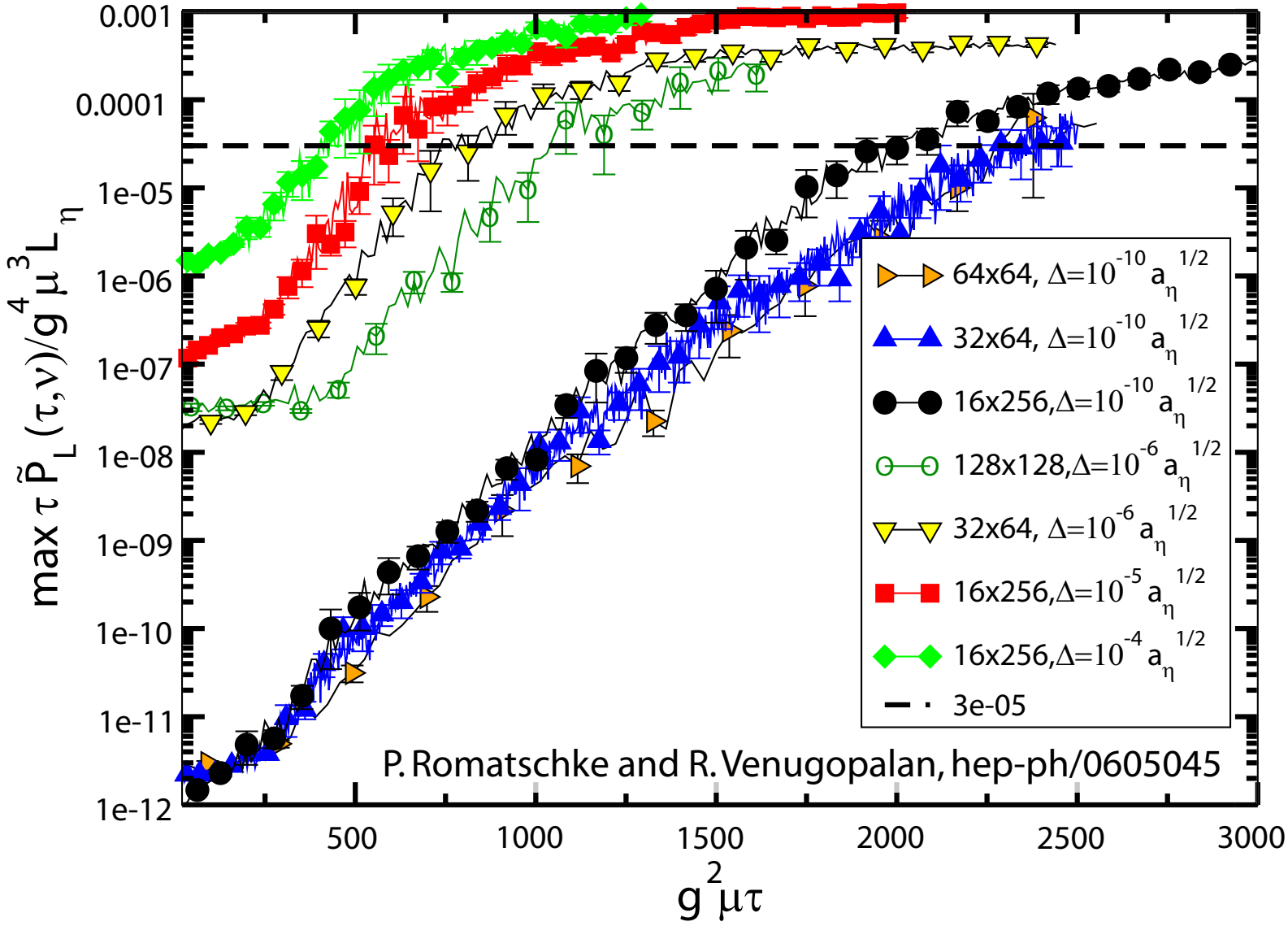
$$\longrightarrow \qquad \frac{\mu}{Q_s} = \frac{1}{8}m_D^2\pi\tau_{\rm iso}Q_s^{-1} = \frac{\pi^2}{48\cdot 0.47\cdot\zeta(3)}\,c \approx \begin{cases} 0.182 & (c=0.5) \\ \underline{0.505} & (c = 2\ln 2) \end{cases}$$

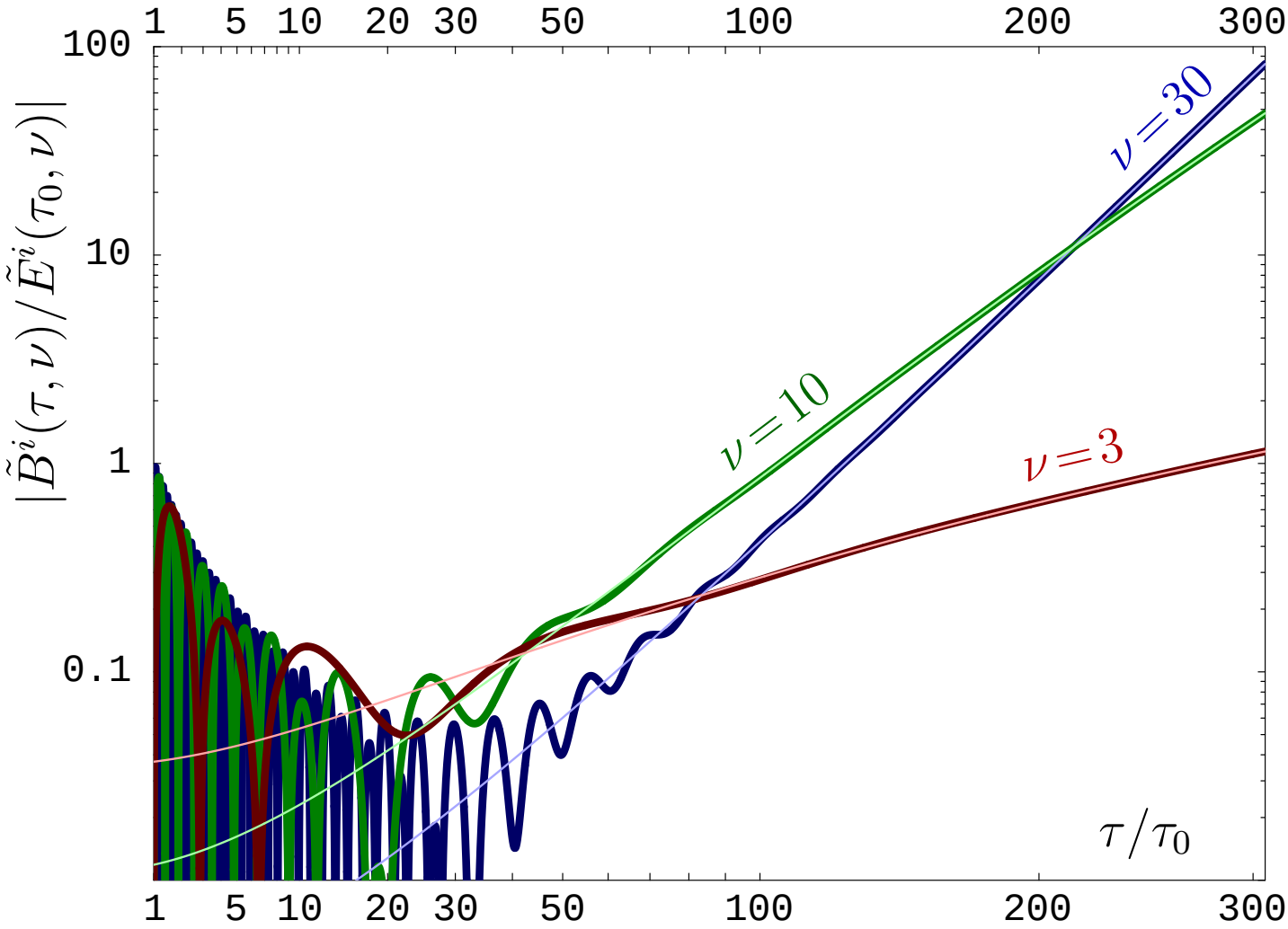
$Q_s \simeq 1$ GeV (RHIC) . . . 3 GeV (LHC) ?

+ form of initial fluctuation spectrum from Fukushima, McLerran & Gelis 2007

Unstable glasma

P. Romatschke and R. Venugopalan, PRL 96, PRD 74 (2006)



Transversely constant modes in linear regime: *Numerical results*

Numerical result *vs.* asymptotic ${}_2F_3$ behavior (thin bright lines) $(c = 0.5)$