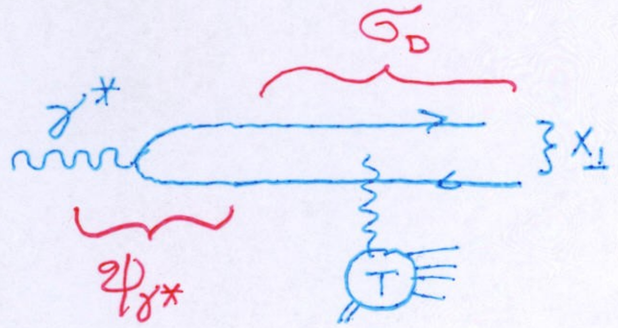
Saturation - Color Glass Condensate in DIS and Heavy Ion Reactions

I. The saturation momentum and the CGC: Generalities

DIS on a target at high energies

A. Dipole picture

$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2 \alpha_{em}} \sigma_{\gamma^*}$$



$$\sigma_{\gamma^*} = \int_0^1 dz \int d^2x_\perp \, |\psi_{\gamma^*}(z,Q,x_\perp)|^2 \, \sigma_D(x_\perp, x)$$

At high energy

$$\sigma_D \xrightarrow[x_\perp \text{small}]{} \frac{\pi^2 \alpha}{N_c} x_\perp^2 \, x G_T(x, 1/x_\perp^2)$$

Leading Twist

Q_s, the saturation momentum, is the scale where interaction becomes strong.

$$Q_s^2 = \frac{C_A}{C_F} \bar{Q}_s^2 \; ; \; \sigma_D \simeq x_\perp^2 \text{ when } \bar{Q}_s \simeq 2/x_\perp$$

B. Bjorken picture

Go to frame where dipele is at rest. Then dipole measures gluons in target. Since σ_D growth in energy stops so should growth in gluon occupation number in target.

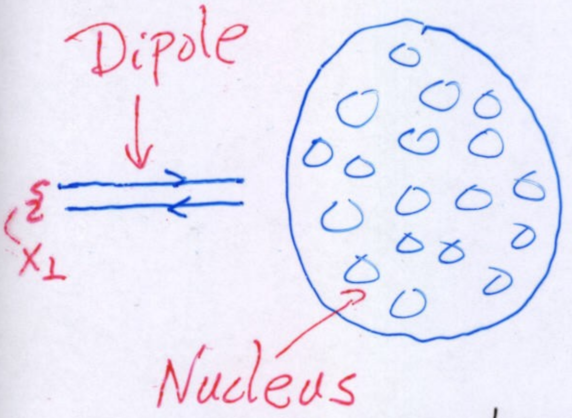
This is gluon saturation and occurs when $f_g \simeq 1/\alpha N_c$

$$f_g = \frac{(2\pi)^3}{2(N_c^2-1)} \frac{dN_g}{dy\, d^2b_\perp\, d^2p_\perp}$$ – unintegrated gluon distribution

High occupancy ~ gluon condensation in BE sense.

This is Color Glass Condensate (CGC)

2. McLerran–Venugopalan model



Dipole scattering on Nucleus

A. Furnishes precise definition of Q_s

$$S_D(b_\perp, x_\perp, x) = \exp\{-x_\perp^2 \bar{Q}_s^2/4\} \qquad S_D = 1/e \text{ when } x_\perp = 2/\bar{Q}_s$$

$$\bar{Q}_s^2 = \underbrace{\frac{C_F}{C_A} \cdot \frac{4\pi^2 \alpha N_c}{N_c^2-1} \cdot 2\sqrt{R^2-b_\perp^2}\, \rho\, x G_N}_{Q_s^2}$$

Nuclear density

Nuclear radius

impact parameter

B. For fast nucleus the density of gluons is

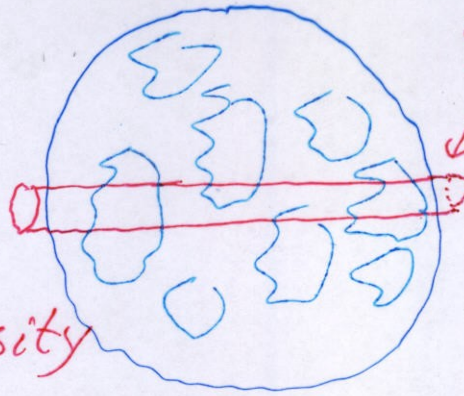
$$f_g = \frac{(2\pi)^3}{2(N_c^2-1)} \frac{dN_g}{dy\, d^2b_\perp\, d^2k_\perp} = \frac{1}{\alpha N_c} \int \frac{d^2x_\perp}{x_\perp^2}\, e^{-ik_\perp \cdot x_\perp} \left(1 - S_D(b_\perp, x_\perp, x)\right)$$

S for gluon dipole

C. Gaussian Charge (and Field) distributions

M-V basic idea

when "tube" is narrow enough charges found in tube are random



μ^2 = charge squared density at $b_\perp$ summed over length of target

$$\langle \mathcal{O}(b_\perp)\rangle = \int \mathcal{D}[\rho]\, \exp\left\{-\int \frac{\rho^2(b_\perp)}{2\mu^2(b_\perp)}\right\} \mathcal{O}[b_\perp, \rho(b_\perp)]$$

Evaluate $\mathcal{O}$ for given 2-dimensional charge distribution ρ.

For large nucleus M-V formula for $\langle \mathcal{O} \rangle$ proved by M-V and by Kovchegov.

3. Heavy Ion estimates in MV model

A. Produced transverse energy, $\frac{dE_T}{dy}$.

For central collision at RHIC

$$\frac{dE_\perp^{MV}}{dy} \simeq \frac{\pi R_A^2}{(2/Q_s)^2 \pi} \cdot \frac{1}{\alpha N_c/\pi} \cdot Q_s \simeq \frac{R_A^2 Q_s^3}{4\alpha N_c/\pi} \underset{\substack{\alpha N_c/\pi \simeq 1/3 \\ R \simeq 7\,\text{fm}}}{\simeq} 900\, Q_s^3 \Big|_{Q_s \text{ in GeV}} \text{GeV}$$

In pre-MV model Blaizot-M. (1987) found

$$\frac{dE_\perp}{dy} = \frac{16}{\pi^2}\frac{dE_\perp^{MV}}{dy} \simeq 1{,}200\,\text{GeV} \text{ with } Q_s \simeq 0.91\,\text{GeV}$$

$(Q_s^{BM})^2 = \frac{9}{8}(Q_s^{MV})^2$ using GLR equation

B. $p_\perp$-broadening and energy loss of jets

$$\frac{dp_\perp^2}{dz} = \underbrace{\hat{q}}_{\text{transport coefficient}} = Q_s^2/L$$

saturation momentum

length of material

$$-\frac{dE}{dz} = \frac{\alpha N_c}{4}\hat{q}L = \frac{\alpha N_c}{4} Q_s^2$$

BDMPS-Z using Gyulassy-Wang $\simeq$ MV model

4. Dynamics

A. JIMWLK: Evolve the charge (Field) distribution of the target.

B. Balitsky-Kovchegov: Follow scattering by evolving the elementary dipole.

Jalilian-Marian, McLerran, Leonidov, Kovner, Weigert, Iancu

A. JIMWLK

$$\langle \mathcal{O} \rangle_Y = \int \mathcal{D}[\alpha]\, W_Y[\alpha]\, \mathcal{O}(\alpha)$$

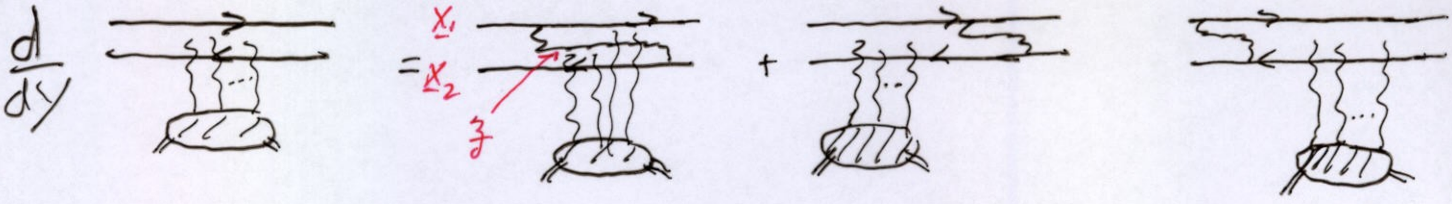
α is Field coming From charge distribution ρ

$$\frac{dW_Y[\alpha]}{dY} = \alpha_s\left[\frac{1}{2}\int d^2x\, d^2y \frac{\delta^2}{\delta\alpha^a(\underline{x})\,\delta\alpha^b(\underline{y})}\left[W_Y \eta^{ab}_{\underline{x}\underline{y}}\right] - \int d^2x \frac{\delta}{\delta\alpha^a(\underline{x})}\left[W_Y \nu^a_{\underline{x}}\right]\right]$$

Elegant Fokker-Planck equation incorporating BFKL evolution and "Pomeron mergings" starting From the target.

B. Balitsky-Kovchegov picture

Very simple! Essentially the same as JIMWLK



$$\frac{dS(x_1-x_2,Y)}{dY} = \frac{\alpha N_c}{2\pi^2}\int d^2z\,\frac{(x_1-x_2)^2}{(x_1-z)^2(z-x_2)^2}\Big[\underbrace{S^{(2)}(x_1-z,\,z-x_2,\,Y)}_{S(x_1-z,Y)\,S(z-x_2,Y)\;\longleftrightarrow\;\text{mean Field}} - S(x_1-x_2,Y)\Big]$$

$$T = 1 - S$$

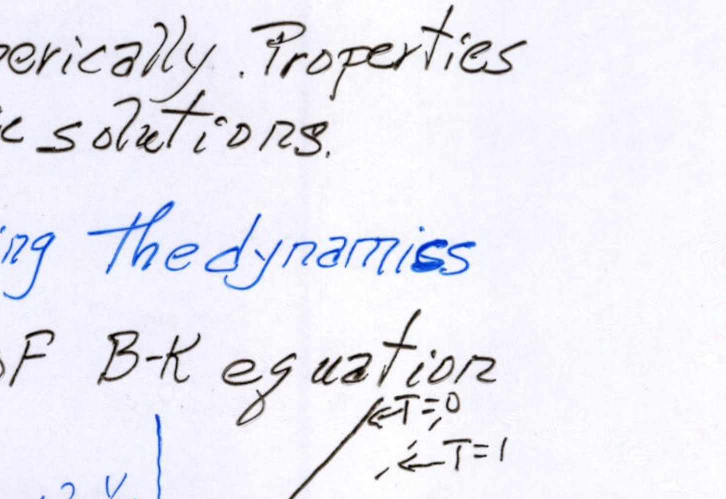
$$\frac{dT(x_1-x_2,Y)}{dY} = \frac{\alpha N_c}{2\pi^2}\int \frac{d^2z\,(x_1-x_2)^2}{(x_1-z)^2(z-x_2)^2}\Big[\overbrace{T(x_1-z,Y)+T(z-x_2,Y)-T(x_1-x_2,Y)}^{\text{BFKL}}-T(x_1-z,Y)\,T(z-x_2,Y)\Big]$$

Well studied numerically. Properties understood. No exact analytic solutions.

5. Simplifying and generalizing the dynamics

A. Simplified version of B-K equation

Drop nonlinear term, but introduce boundary requiring $T=0$. Adjust boundary so T has max value $=1$ close to boundary.



Finally take $T=1$ to left of dashed line. All this is easy to do.

Triantafyllopoulos, M.

B. Generalizing the discussion

Munier and Peschanski recognized that B-K equation, and the simplified version above, are in the same universality class as the FKPP equation

$$\frac{\partial}{\partial t} u(t,x) = \underbrace{\frac{\partial^2}{\partial x^2} u(t,x)}_{\text{diffusion}} + \underbrace{u(t,x)}_{\text{growth}} - u^2(t,x)$$

$u=0$ unstable fixed point

$u=1$ = Fixed point (stable)

General behaviors:

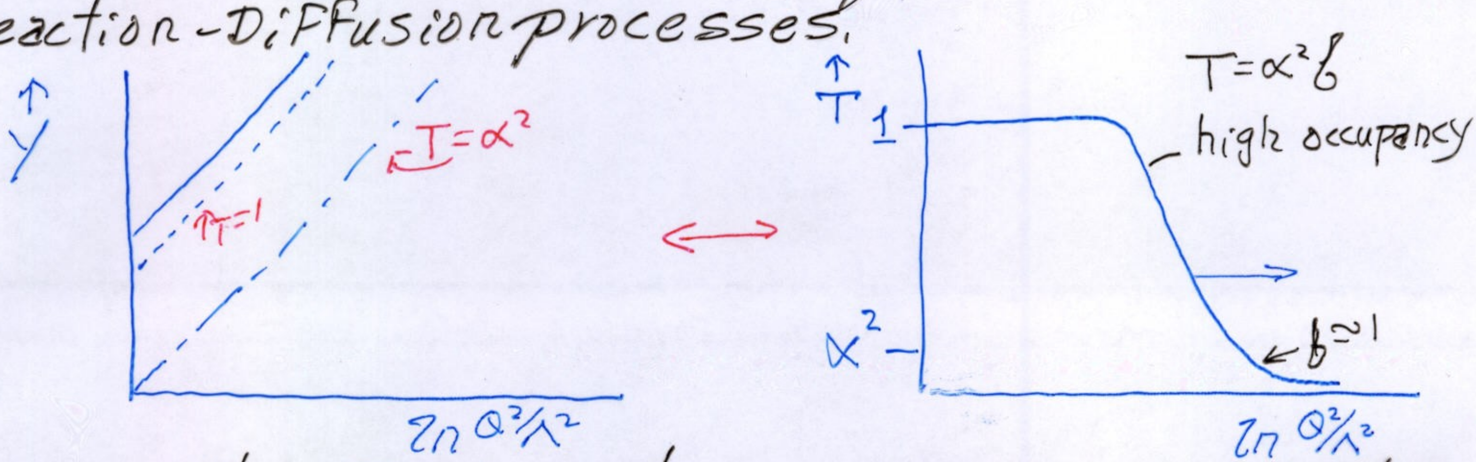
$$Q_s^2(Y) \simeq \mu^2 [\alpha Y]^{-\frac{3}{2(1-\lambda_0)}} \exp\left\{ \frac{2\alpha N_c}{\pi} \frac{\chi(\lambda_0)}{1-\lambda_0} Y \right\}$$

$$T \simeq T_0 \left(Q^2/Q_s^2 \right)^{-(1-\lambda_0)} \left[\ln Q^2/Q_s^2 + \text{const} \right] \qquad Q^2/Q_s^2 > 1$$

geometric scaling

6. A Complication

The JIMWLK-BK equations are mean field equations for the exact evolution in the same sense that FKPP is mean field equation for Reaction-Diffusion processes.



When $T \simeq \alpha^2$ parton occupancies are of order one. Mean field picture breaks down. In statistical mechanics noisy travelling wave obeys sFKPP equation.

When $T \lesssim \alpha^2$ one does not expect the mean field picture to be valid as parton (dipole) occupations are not large. In statistical physics travelling waves become noisy as exemplefied by the sFKPP equation. In QCD the stochasticity comes from the quantum mechanics of dipole splitting.

The structure of QCD and the sFKPP equation are believed to be in the same universality class. Iancu, Munier, M
Munier review

7. The search for the "complete" equation at small-x

Goal is to find equation which has BFKL evolution along with (i) unitarity (nonlinear) term at high dipole density and (ii) stochasticity at low density and which can be numerically simulated or approximately solved.

In Dipole-Reaction-Diffusion language

Blaizot, Iancu, Itakura, Triantafyllopoulos wrote an interesting equation

$$H = \underset{\text{BFKL}}{H_0} + \underset{\text{Dipole splitting}}{H_{1\to 2}} + \underbrace{H_{2\to 1}}_{\text{"Dipole" mergings}}$$

Formally this has the essence of the physics, but $H_{2\to 1}$ is not positive.

Reggeon Field theory in zero transverse dimensions, with only 3-P coupling, is very similar. From reaction-diffusion (s-channel) point of view there are vertices

$$H = H_{1\to2} + H_{2\to0} + H_{2\to1}$$

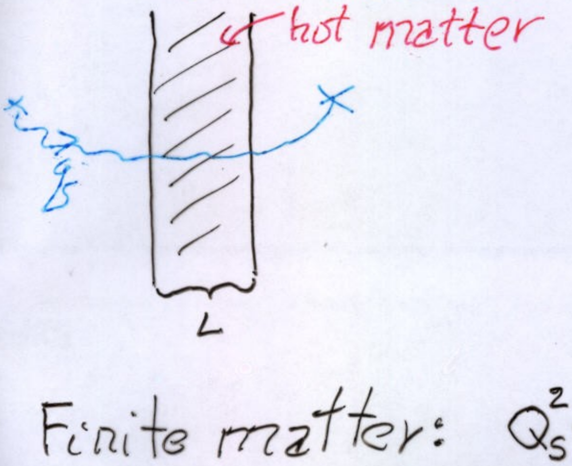
Prob α β $-\beta$

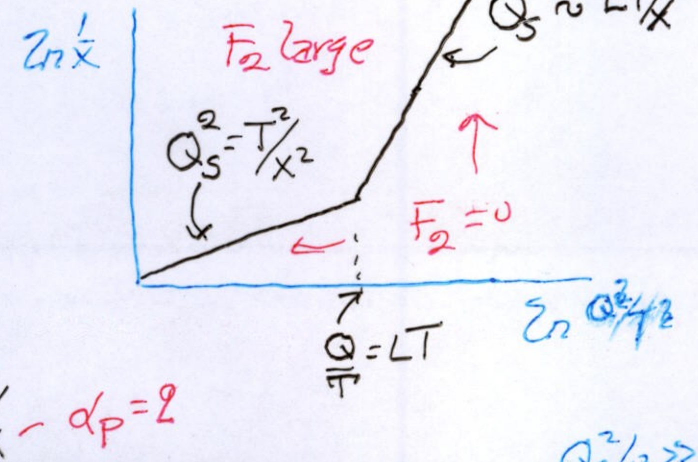
This is much like H_{BIIT} however here we can solve the problem analytically. Attempts at numerical solutions show instability.

8. $N=4$ SYM theory at large $g^2N=\lambda$.

Gives interesting but, in many respects, not realistic picture of scattering.

Conformal symmetry preserved in DIS on hot matter





Finite matter: $Q_s^2 \sim LT^3/x$ – $\alpha_P = 2$

Infinite matter: $Q_s^2 \sim (T/x)^2$ $\quad F_2 \sim N^2 Q^2 \ln Q_s^2/Q^2$ $\quad Q_s^2/Q^2 \gg 1$, $TL \ll 1/x$

Formulas for $p_\perp$-broadening and energy loss for heavy quarks look like those of QCD, in terms of Q_s and with $\alpha N_c \to \sqrt{\lambda}$

$$\frac{dp_\perp^2}{dz} \sim \sqrt{\lambda}\, Q_s T^2 \qquad \Delta p_\perp^2 \sim L^2$$

$$-\frac{dE}{dz} \sim \sqrt{\lambda}\, Q_s^2 \qquad \Delta E \sim L^3 \;\; !!$$