The MV Model

The early days of saturation in Minnesota

Jamal Jalilian-Marian
Baruch College, New York NY USA

1993: looking for an advisor in MN

Larry was quite new

My good friend and fellow student Alejandro Ayala had just joined him

Electroweak phase transitions

What is the origin of the matter-antimatter asymmetry in the Universe?

1. B violation
2. CP violation
3. Non-equilibrium dynamics

A.D. Sakharov,
JETP Lett. 5 (1967) 24

EW phase transition

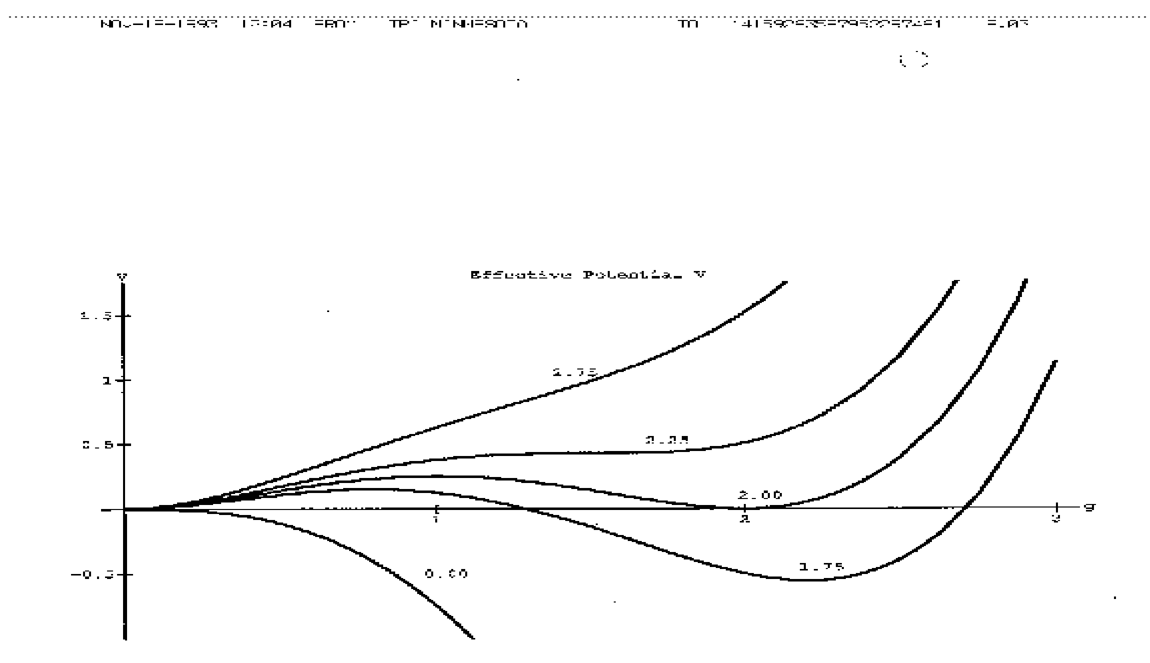


Fig 2

A "simple" problem: how do fermions scatter from the phase transition bubble walls?

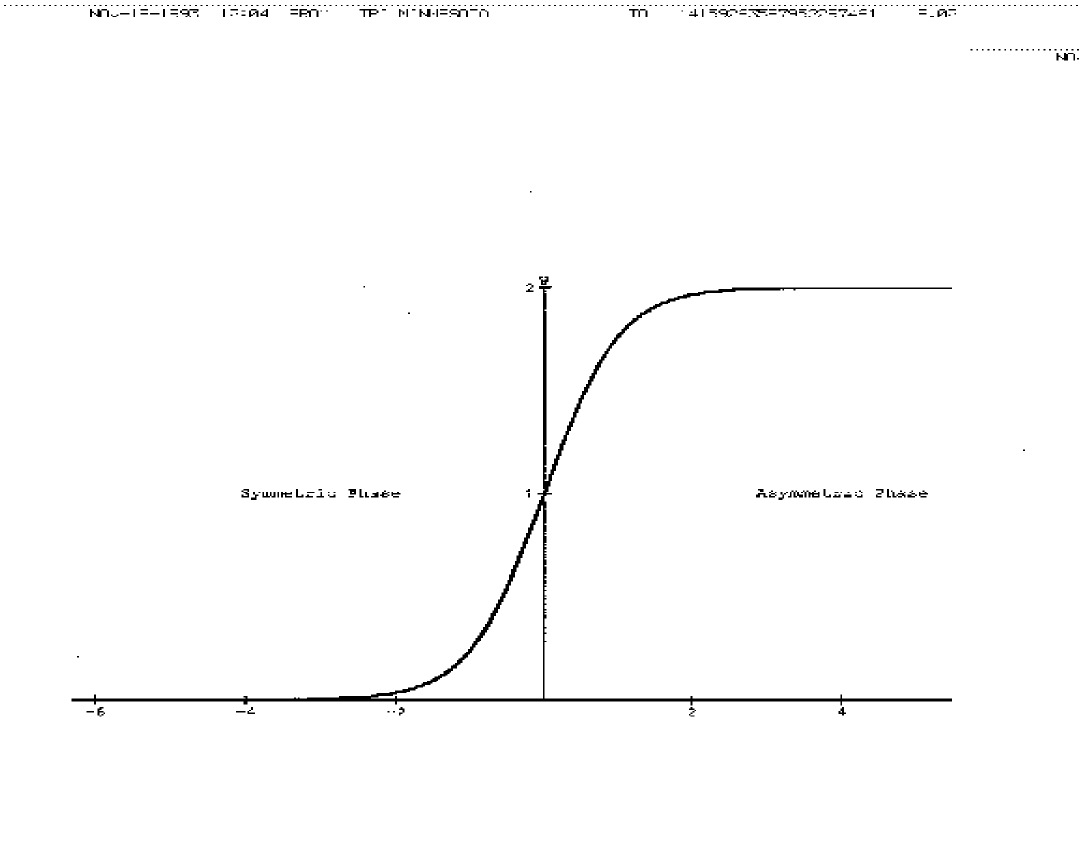


Fig. 1

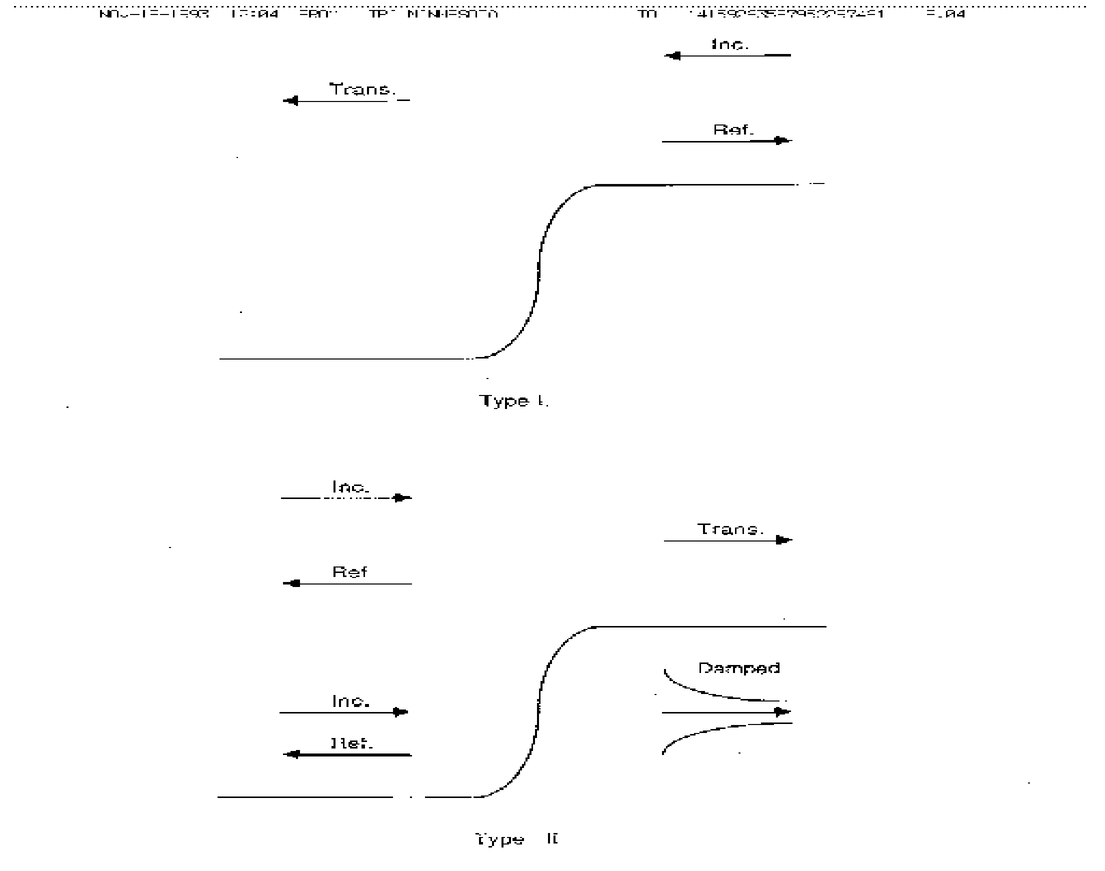


Fig. 8.

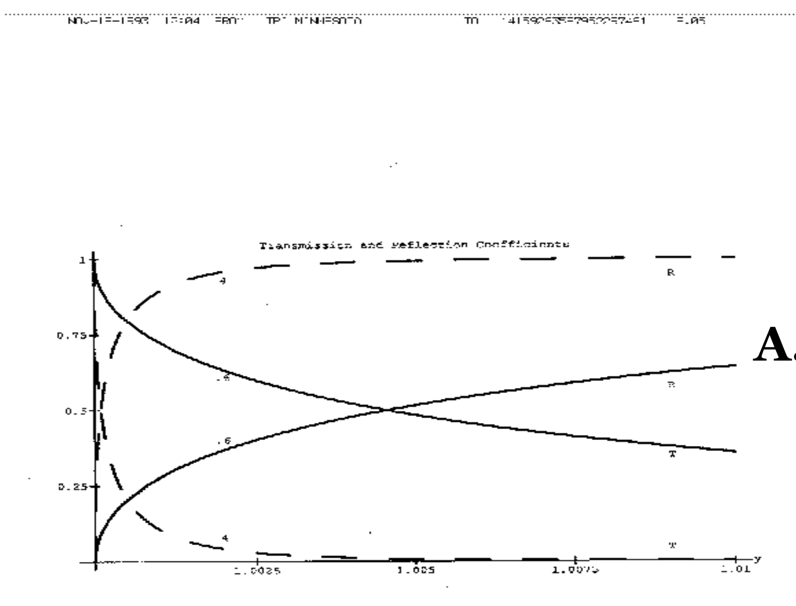


Fig. 4

A. Ayala, J. Jalilian-Marian, L. McLerran, A. Vischer
Phys.Rev.D49:5559-5570,1994

Larry and high energy QCD in MN

What is a nucleus/hadron at high energy like?

What is the nature of multi-particle production in QCD ?

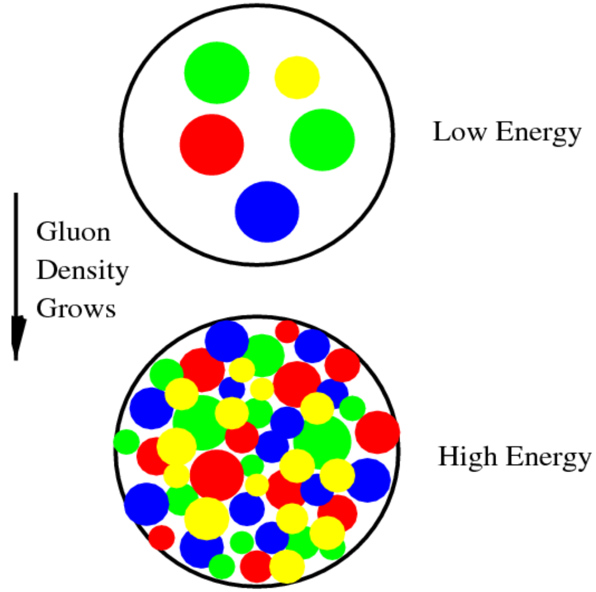
Is this important for high energy heavy ion collisions and Quark-Gluon Plasma?

Is there some universal physics involved?

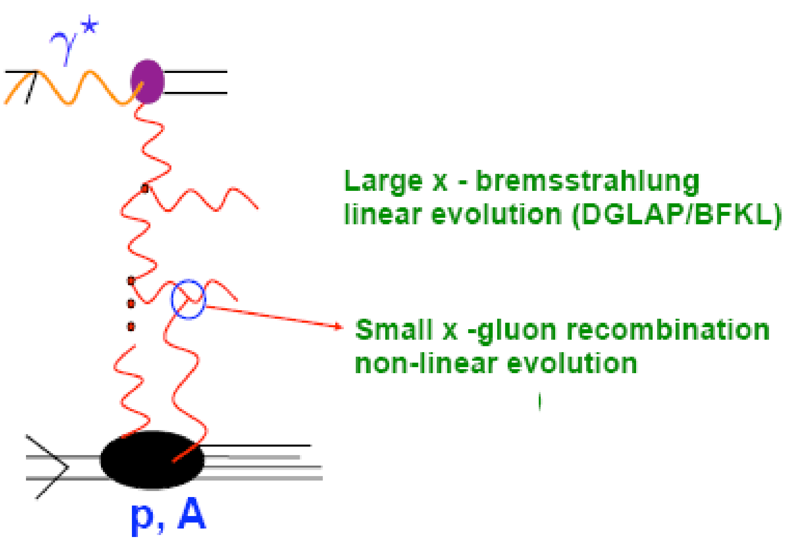
Saturation

There are many gluons in the wave function of a nucleus/hadron at high energy (small x_{Bj})

Gribov-Levin-Ryskin
Mueller-Qiu



Mechanism of gluon saturation in QCD



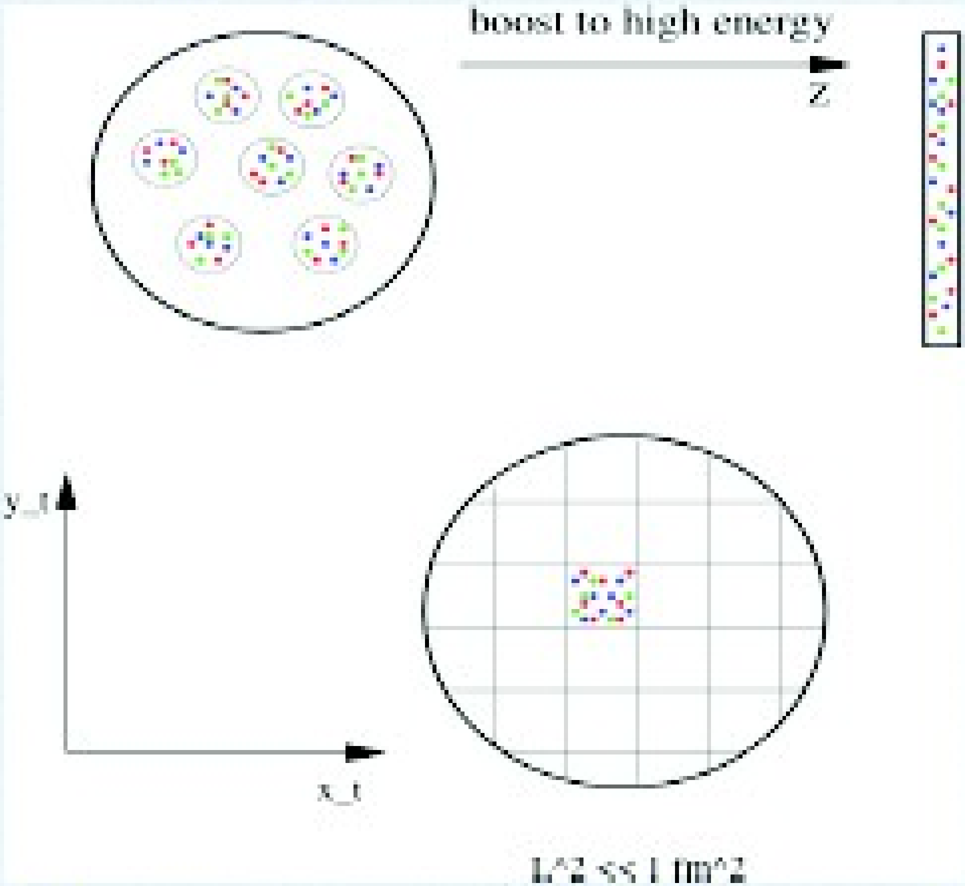
Saturation in MN

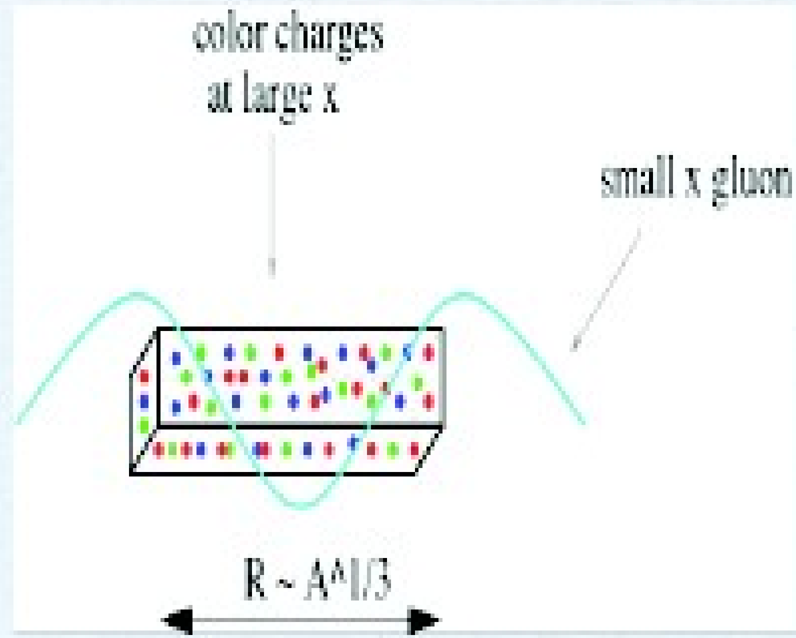
Larry & Raju:
this is a weak coupling problem

Semi-classical methods: effective action

A nucleus at high energy

Consider a large nucleus in the IMF frame $P^+ \to \infty$





One large component of the current-others suppressed by $\dfrac{1}{P^+}$

Wee partons see a large density of valence color charges at small transverse resolutions

The effective action

Scale separating sources & fields

$$\mathcal{Z}[j] = \int [d\rho]\, W_{\Lambda^+}[\rho] \left\{ \frac{\int^{\Lambda^+} [dA]\, \delta(A^+)\, e^{iS[A,\rho] - \int j\cdot A}}{\int^{\Lambda^+} [dA]\, \delta(A^+)\, e^{iS[A,\rho]}} \right\}$$

Gauge invariant weight functional for distribution of sources

$$S[A,\rho] = \frac{-1}{4}\int d^4x\, F_{\mu\nu}^2 + \frac{i}{N_c}\int d^2x_\perp dx^- \delta(x^-) \mathrm{Tr}\left(\rho(x_\perp) U_{-\infty,\infty}[A^-]\right)$$

Dynamical wee fields

Coupling of wee fields to sources

$$U_{-\infty,+\infty}[A^-] = \mathcal{P}\exp\left(ig\int dx^+ A^{-,a} T^a\right)$$

This action captures the remarkable properties of hadrons and nuclei at high energies

The classical solution

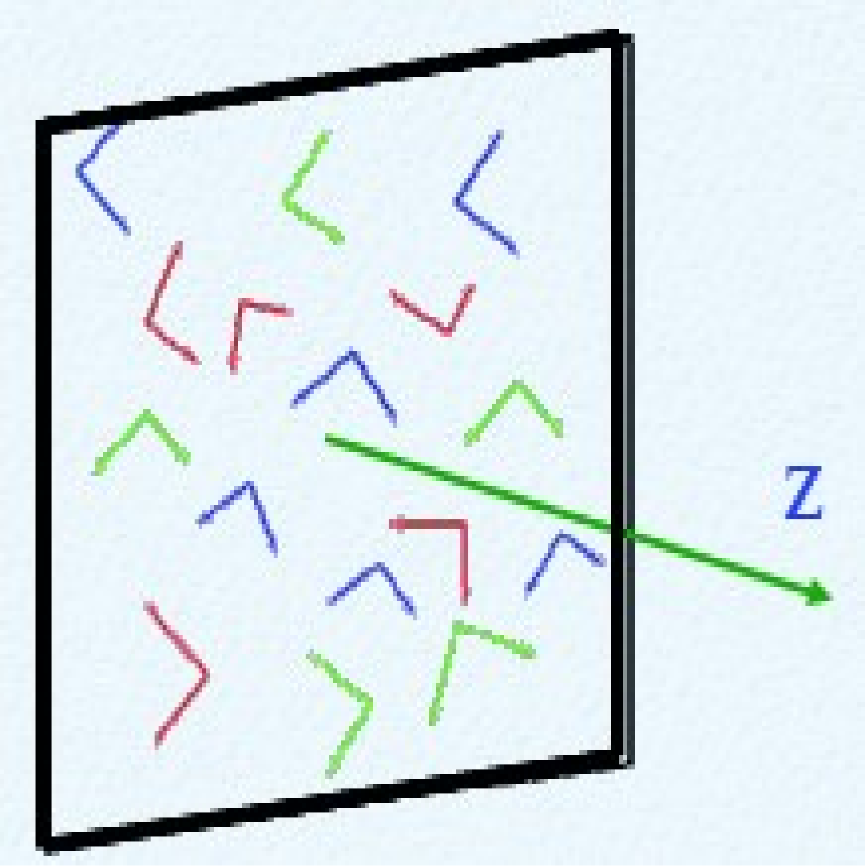
$$A^+ = A^- = 0$$

$$A_a^i = \theta(x^-)\,\alpha_a^i$$

with

$$\alpha_i \equiv \frac{i}{g}\,U\,\partial^i\,U^\dagger$$

$$\partial_i\,\alpha_i = g\,\rho$$



random color electric and magnetic fields in plane of fast moving nucleus

McLerran and Venugopalan: 1993-1994

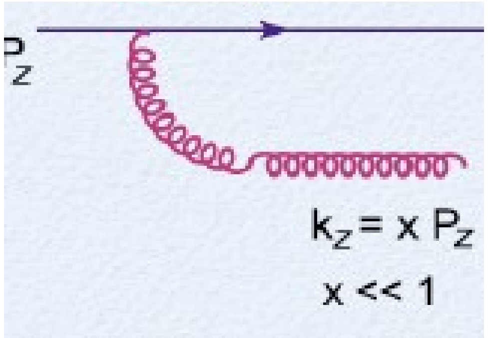
Two BIG problems!

1) gluon distribution is infrared divergent

$$\frac{dN}{dy\, d^2 k_t} \sim \frac{1}{k_t^2} \qquad \text{for all } k_t$$

what happened to non-linear, high gluon density effects!!??

2) large quantum corrections?



$$\alpha_s\, (\log 1/x) \sim 1$$

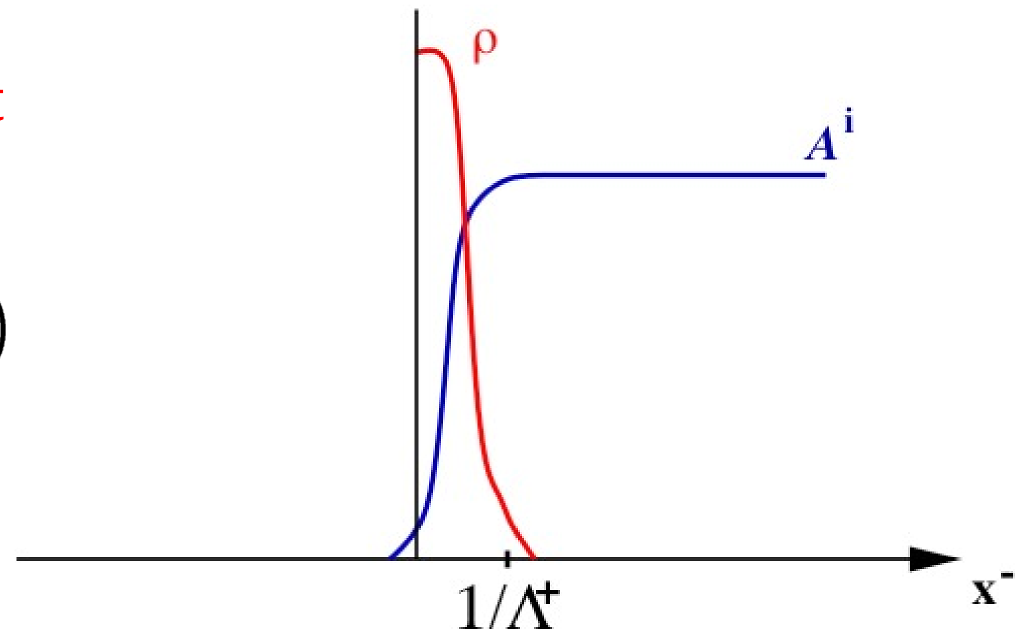
1) gluon distribution at small x

Eqs. of motion

$$\partial_i \partial_- A^i + [A_i, \partial_- A^i] = g^2 \delta(x^-) \rho(x_t)$$

$$A_i = \theta(x^-) \alpha_i(x_t)$$

commutator is important

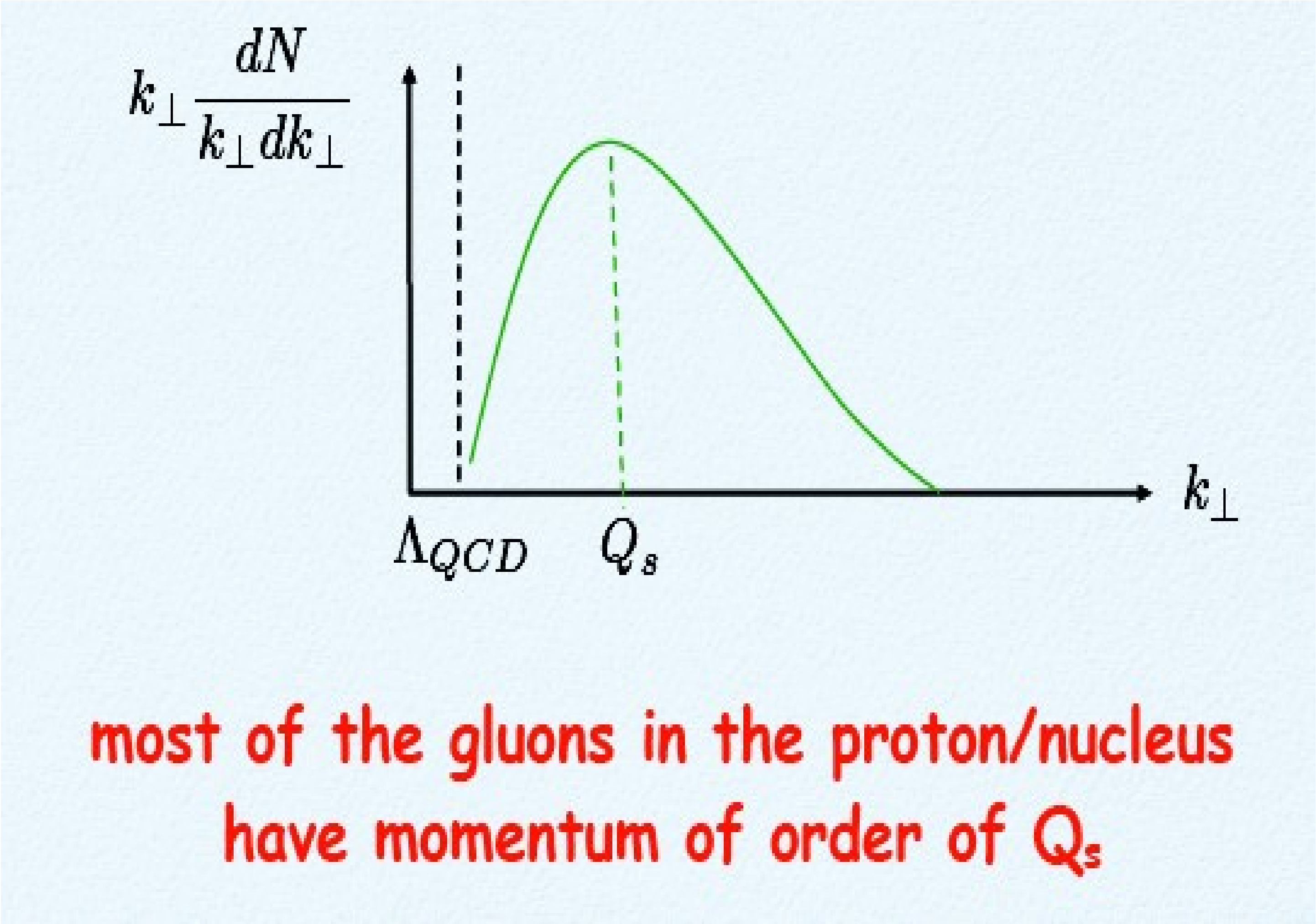


$$D_i \frac{d\,A_i}{dy} = g^2\,\rho(y, x_t)$$

compute $\frac{dN}{d^2 r_t} \sim < A_i\, A_i >$

1) gluon distribution at small x

$$\frac{dN}{dy\, d^2 r_t} \sim \left[1 - exp[-\#\, r_t^2\, Q_s^2\, log\,(r_t^2)]\right]$$



2) large quantum corrections

large logs (of 1/x) in pQCD (BFKL):
how do they affect the gluon distribution function?
how do we incorporate them in a high density environment?

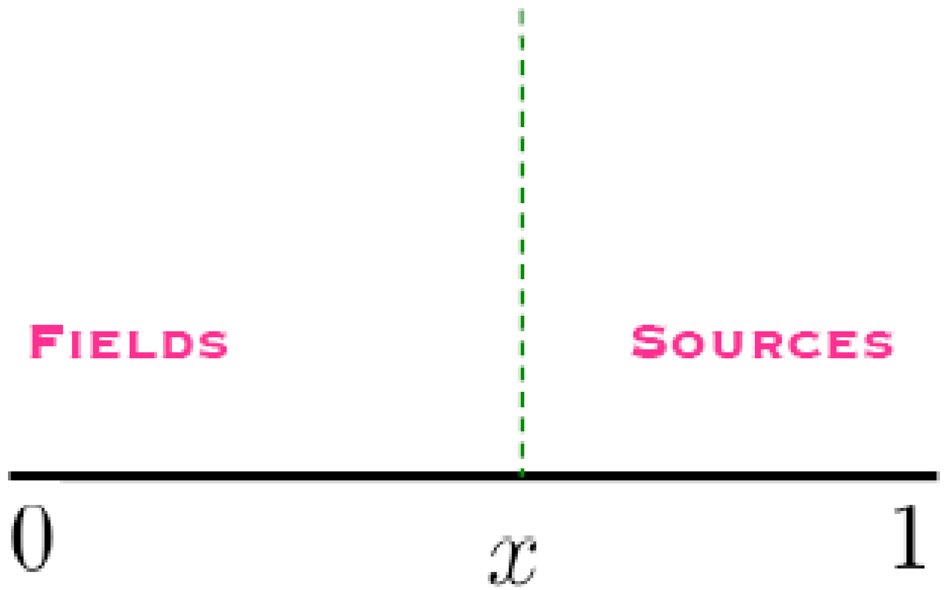
some difficulties:
gluon propagator in the background field
light cone gauge singularities

one loop correction to gluon distribution function

$$\frac{dN}{dx dk_t^2} \sim \frac{1}{x\, k_t^2}[1 + \frac{N_c\, \alpha_s}{\pi} f(k_t)\, log\, 1/x]$$

A. Ayala, J. Jalilian-Marian, L. McLerran, R. Venugopalan, PRD53 (1996) 458

Renormalization Group Equations

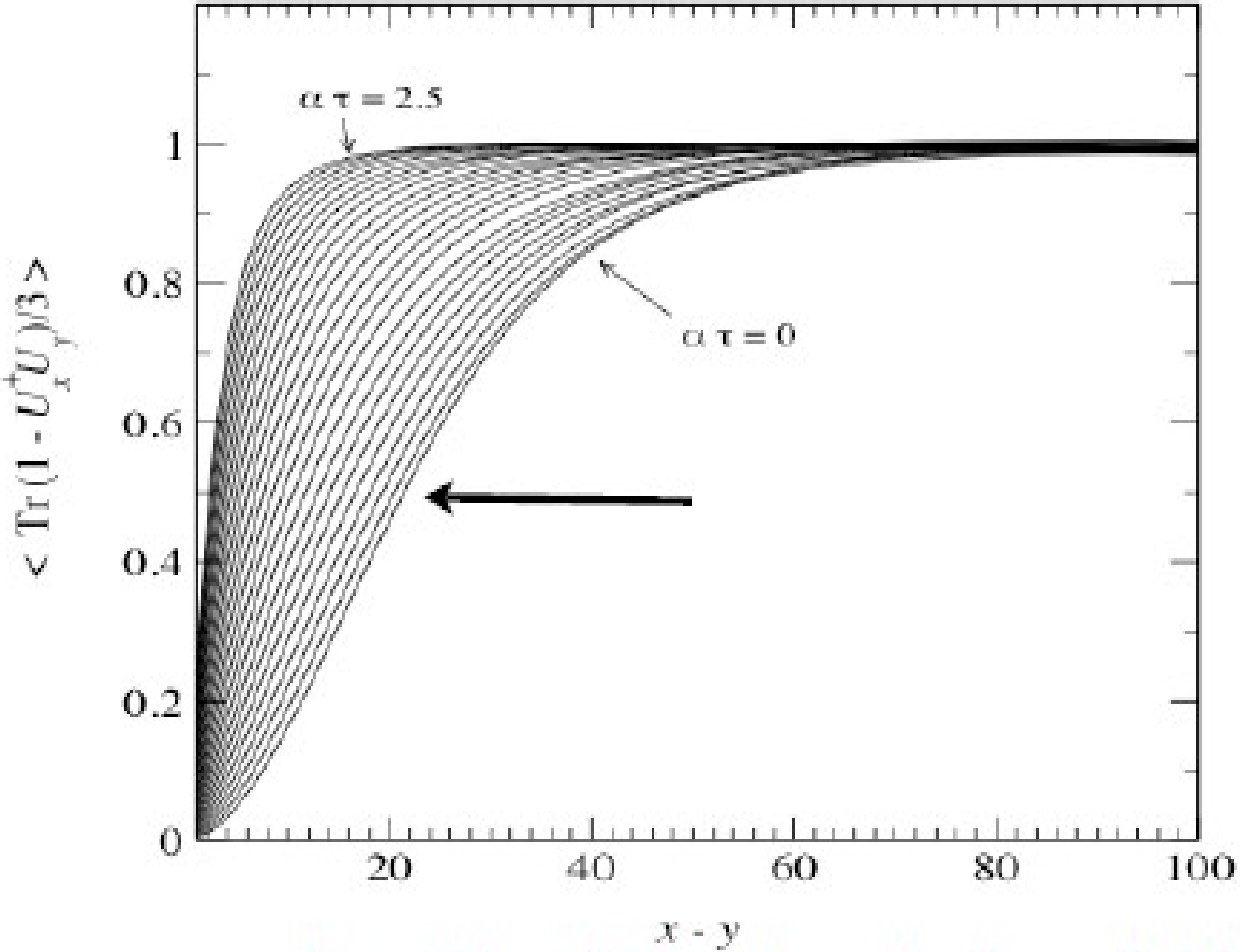


$$\frac{\partial}{\partial y} W = H\, W$$

JIMWLK equation

the 2-point function $T(x_t, y_t) = 1/N_c \, \mathrm{Tr}\,[1 - U^+(x_t)\, U(y_t)]$

(probability for scattering of a quark-anti-quark dipole on a target)



define $Q_s = 1/r_t$

when $T(r_t) = 1/2$

it grows with energy

pQCD:

color transparency

$$T \sim r_t^2\, xG(x, \frac{1}{r_t})$$

non-linearities unitarize the scattering probability

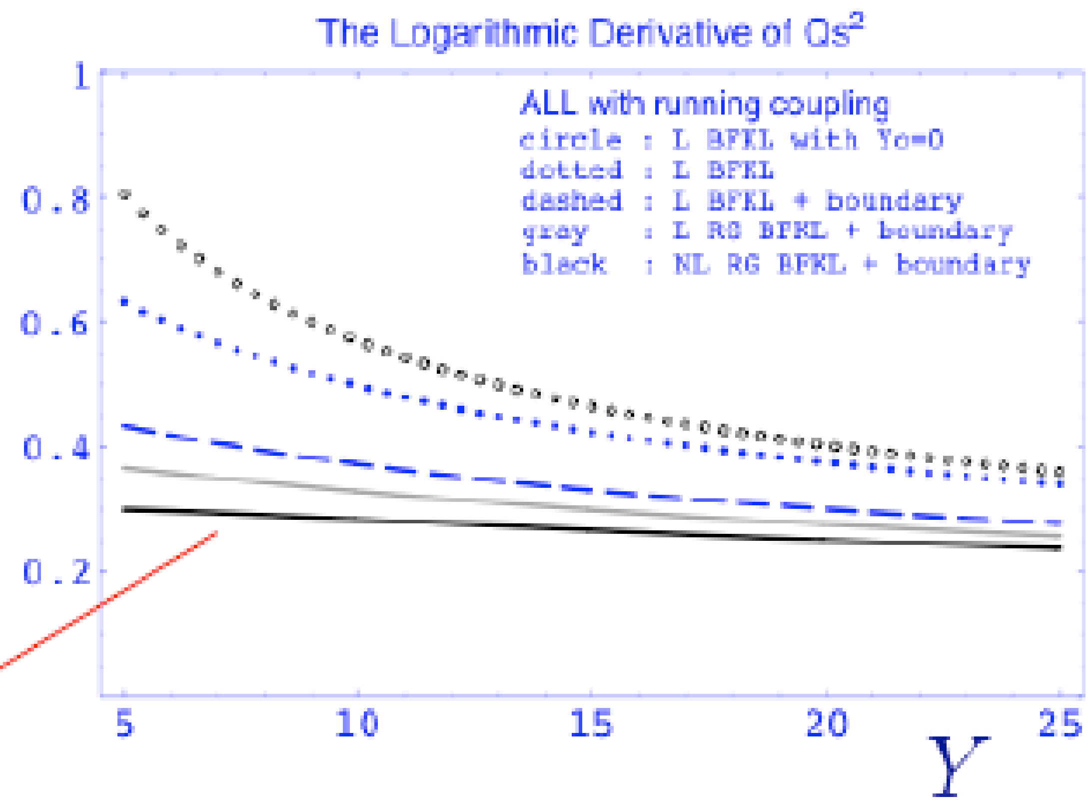
How does Q_s behave as function of Y?

Fixed coupling LO BFKL: $Q_s^2 = Q_0^2 e^{c\,\bar{\alpha}_s Y}$

LO BFKL+ running coupling: $Q_s^2 = \Lambda_{\rm QCD}^2 e^{\sqrt{2b_0 c(Y+Y_0)}}$

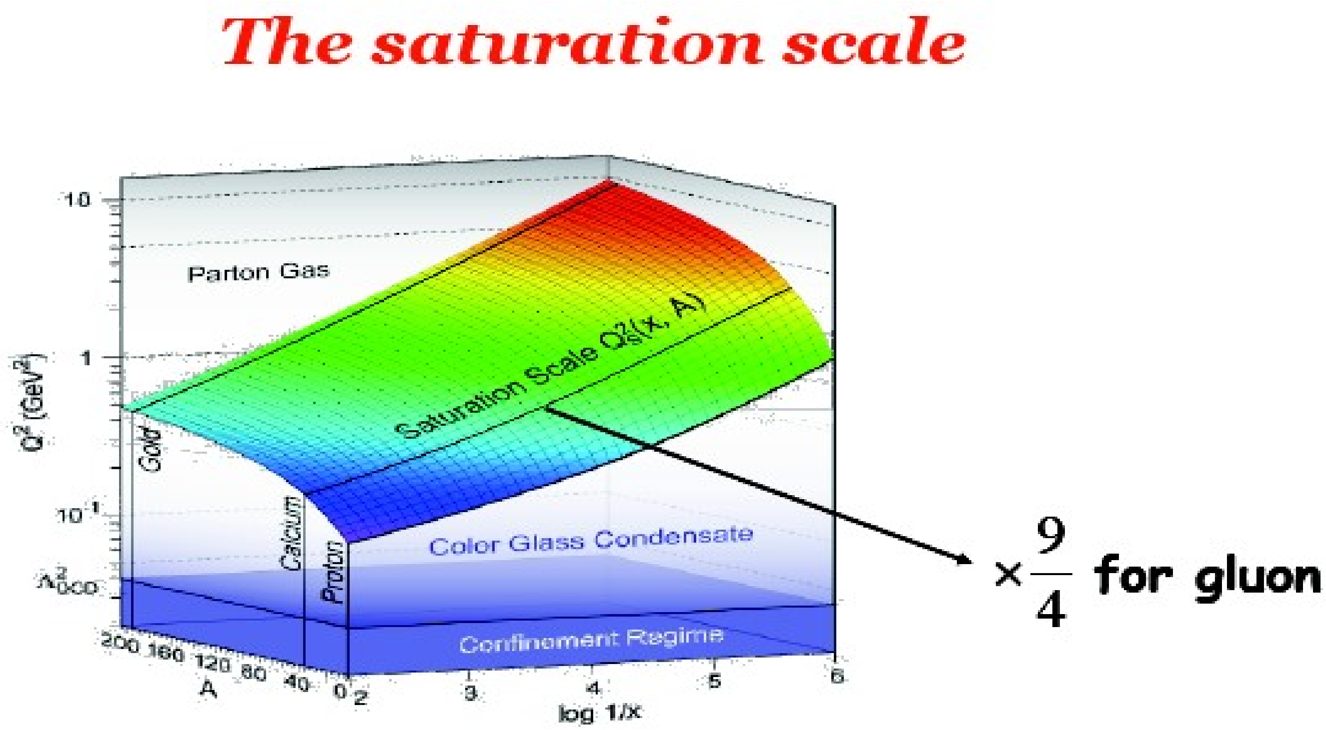
Re-summed NLO BFKL + CGC:

$$\lambda \equiv \frac{d\ln Q_s^2}{dY}$$



Very close to HERA result!

A hadron/nucleus at high energy is a Color Glass Condensate



$\alpha_s(Q_s^2) << 1$

Below Q_s: strong color fields

Above Q_s: Anomalous dimension: LT shadowing

Probing CGC at RHIC

Back with Larry in BNL

Heavy ion collisions: hot nuclear matter - QGP

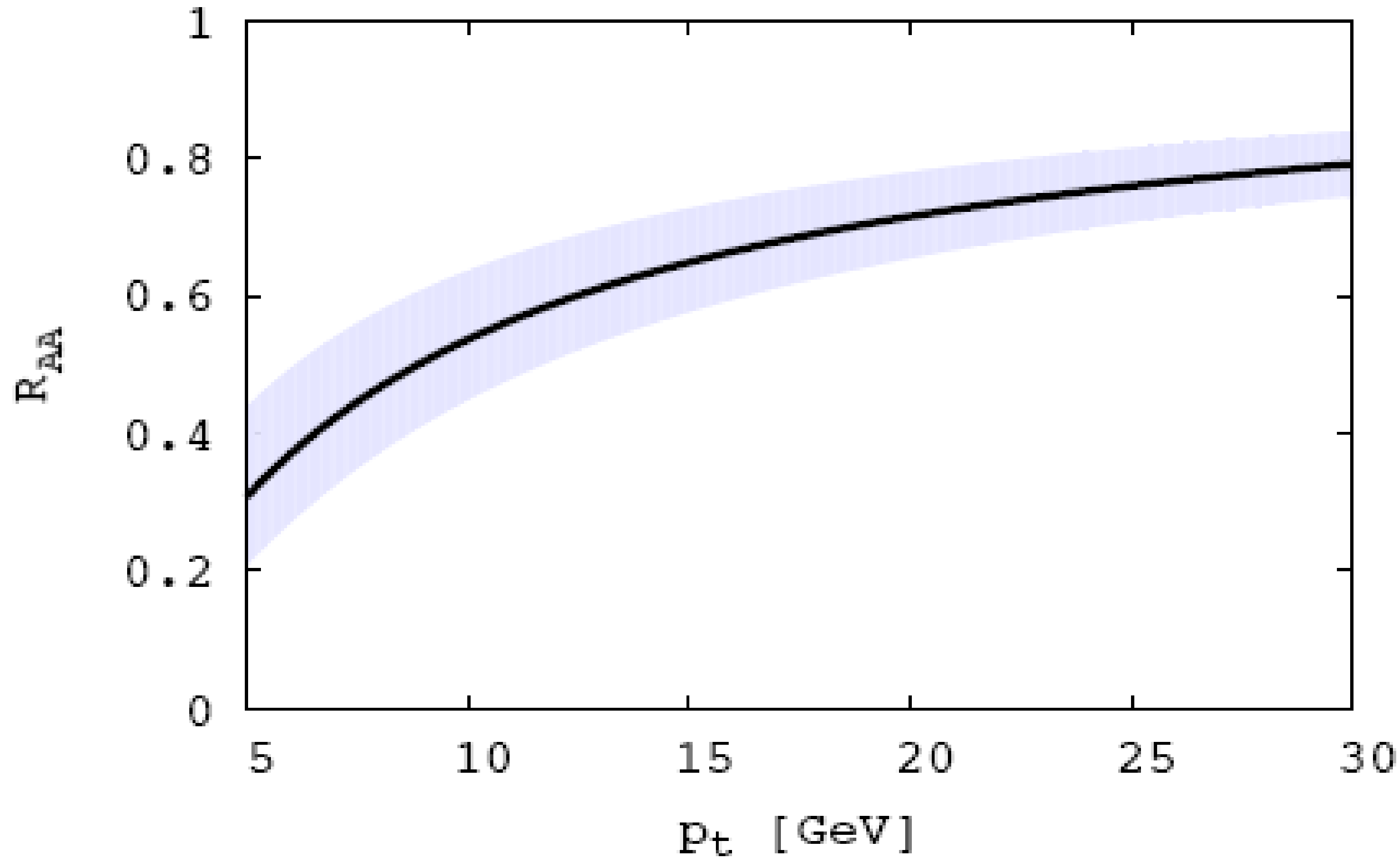
toward initial conditions
strong fields - glasma tubes at early times
instabilities - thermalization?

Proton-nucleus collisions: cold nuclear matter

cleaner probe of CGC – no medium effects
forward rapidity – smallest x in target nucleus
under better theoretical control

E-Loss from CGC?

Schenke, Strickland, Dumitru, Nara, Greiner, PRC79, 034903 (2009)



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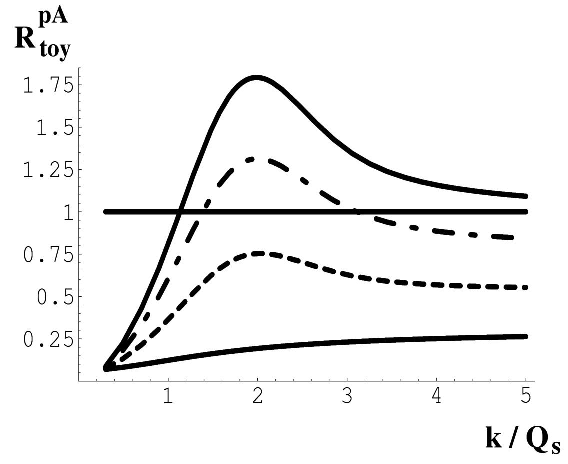
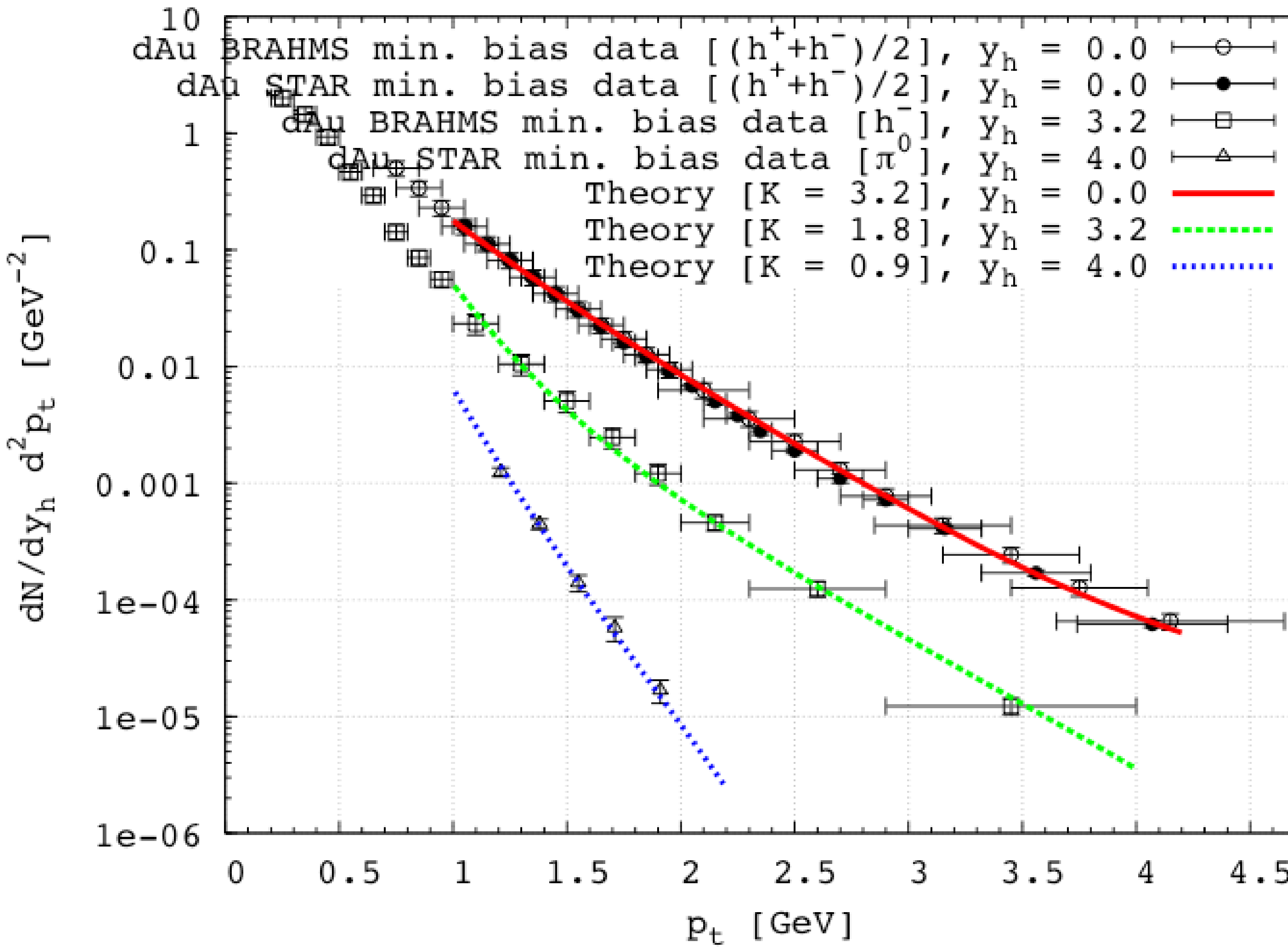
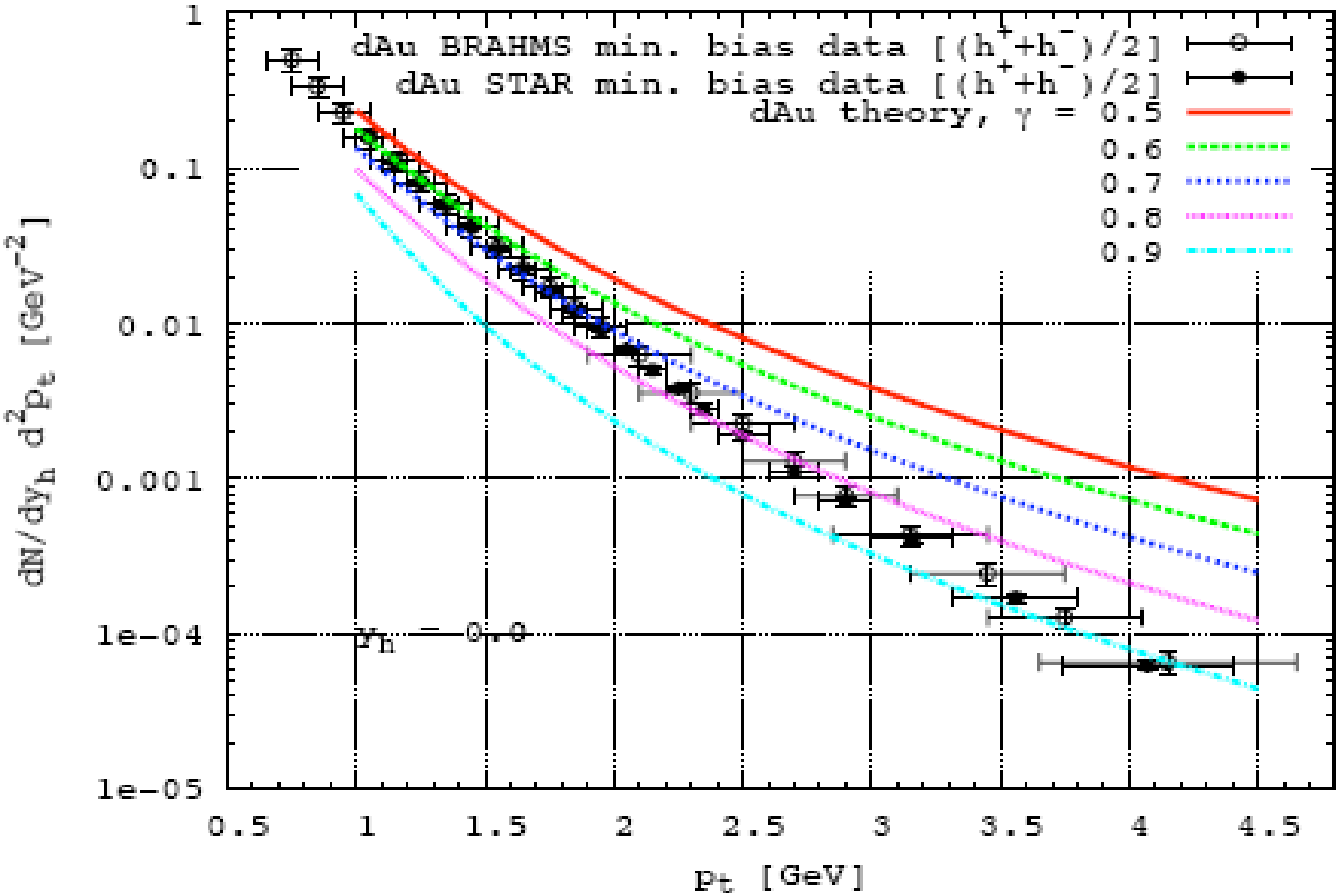


true predictions for y = 4, y = 0



From BFKL to DGLAP: *the rise of the anomalous dimension*



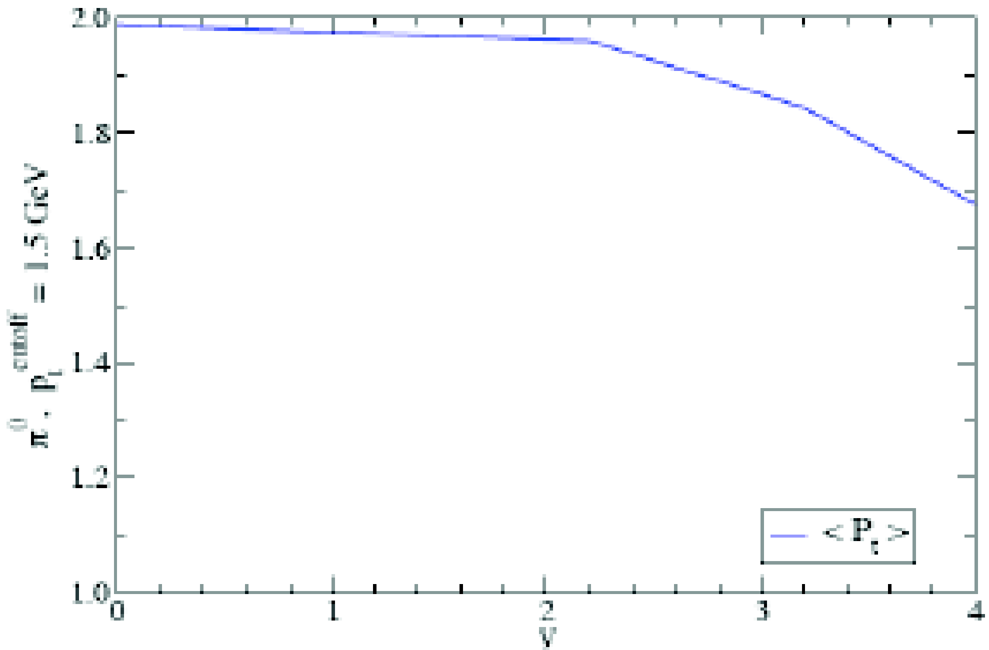
Hadron production at RHIC

Caveats:

Large (y dependent) K factor

define average p_t with a cutoff

$$< p_t > \equiv \frac{\int_{p_t^{min}} d^2p_t\, p_t\, \frac{d\sigma^{p(d)A\to\pi^0(p_t,y_h)\,X}}{d^2p_t\, dy_h}}{\int_{p_t^{min}} d^2p_t\, \frac{d\sigma^{p(d)A\to\pi^0(p_t,y_h)\,X}}{d^2p_t\, dy_h}}$$



$$N_{F,A} \to \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right]^1 \qquad Q_s^2/\Lambda_{QCD} << p_t$$

$$N_{F,A} \to \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right]^\gamma \qquad Q_s << p_t << Q_s^2/\Lambda_{QCD}$$

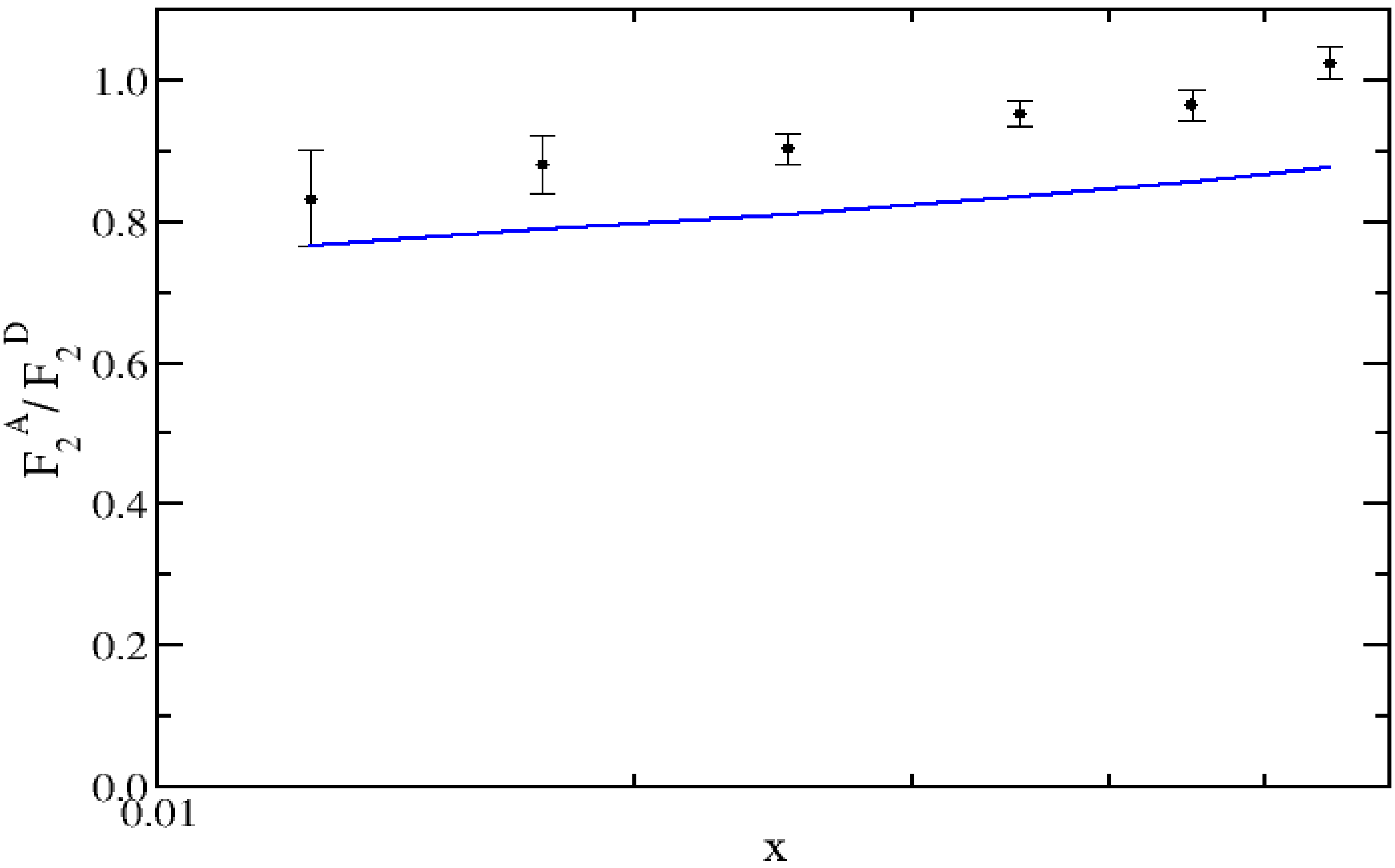
$$N_{F,A} \to \frac{1}{p_t^2}\ln\left[\frac{Q_s^2}{p_t^2}\right] \qquad p_t << Q_s$$

$$R \equiv \frac{< p_t >}{\Lambda_{cutoff}}$$

$$\frac{1}{m+1} < R - 1 < \frac{1}{m-1}$$

Nuclear Shadowing: F_2

NMC

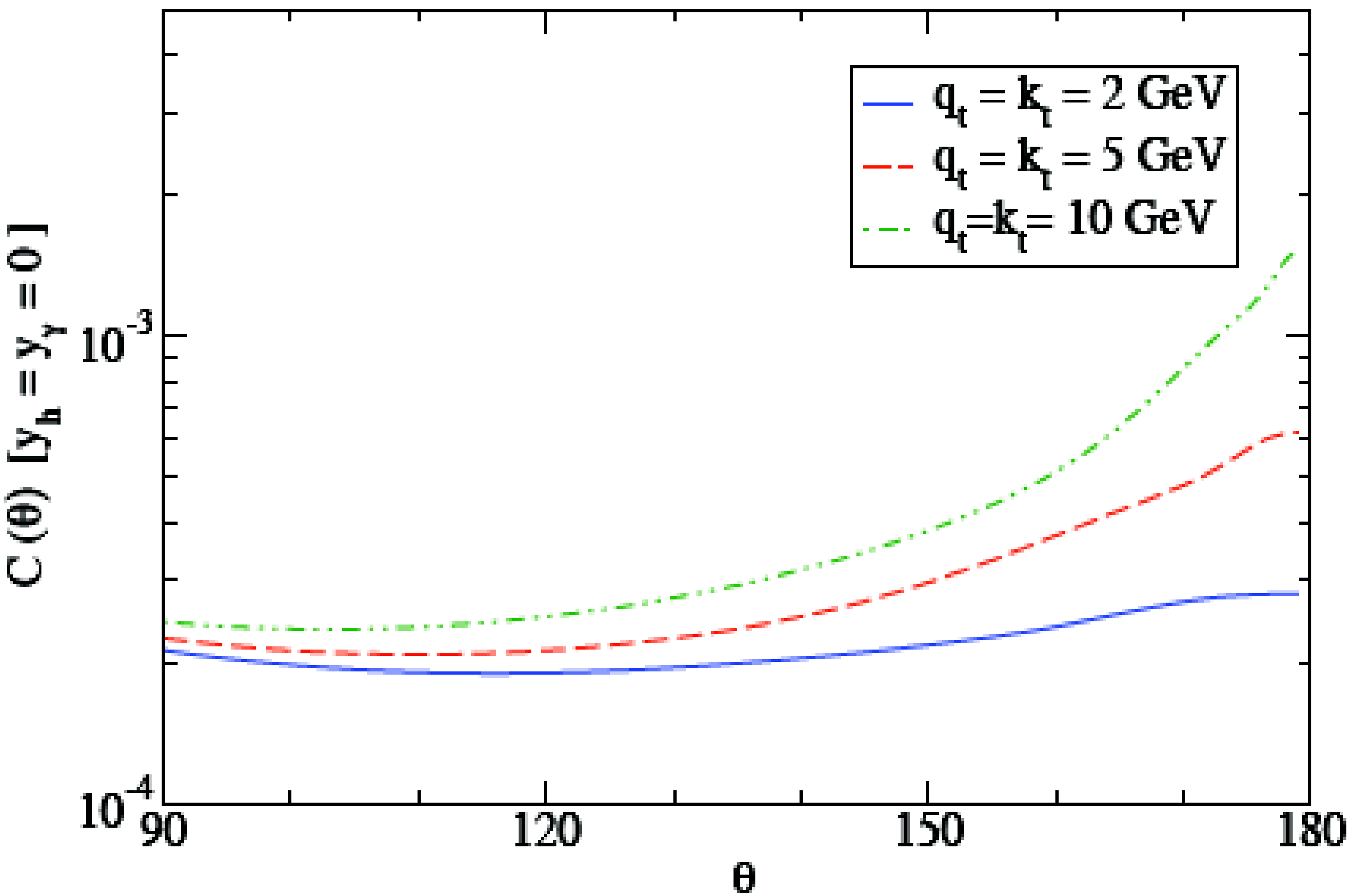


Same degrees of freedom: N_F

JJM, in progress
gluons?

Pion - photon correlation

RHIC



$$x_g = \frac{1}{\sqrt{s}} \left[\frac{q_t}{z} e^{-y_\pi} + k_t e^{-y_\gamma}\right]$$

QCD at high energy: CGC

Provides a first principle, self consistent approach to high energy QCD processes

New developments/significant progress in CGC theory

Evidence for CGC at RHIC

many thanks

Single Hadron Production in pA

$$\frac{d\sigma^{pA\rightarrow hX}}{dY\, d^2P_t\, d^2b} = \frac{1}{(2\pi)^2}\int_{x_F}^{1} dx\, \frac{x}{x_F}$$

$$\Big\{ f_{q/p}(x,Q^2)\;\; N_F[\frac{x}{x_F}P_t, b, y]\;\; D_{h/q}(\frac{x_F}{x}, Q^2) +$$

$$f_{g/p}(x,Q^2)\;\; N_A[\frac{x}{x_F}P_t, b, y]\;\; D_{h/g}(\frac{x_F}{x}, Q^2) \Big\}$$

N_F, N_A are dipoles in fundamental and adjoint representation and satisfy nonlinear evolution equations the 2-point function $T(x_t, y_t) = 1/N_c \, \mathrm{Tr}\,[1 - U^+(x_t)\, U(y_t)]$

(probability for scattering of a quark-anti-quark dipole on a target)

A closed form equation

$$\boxed{\partial_Y \langle T_{\mathbf{xy}} \rangle = \frac{\bar{\alpha}}{2\pi} \int d^2z \, \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2 (\mathbf{z}-\mathbf{y})^2} \left[\langle T_{\mathbf{xz}} \rangle + \langle T_{\mathbf{zy}} \rangle - \langle T_{\mathbf{xy}} \rangle - \langle T_{\mathbf{xz}} \rangle \langle T_{\mathbf{zy}} \rangle \right]}$$

JIMWLK + large N_c + mean field approx. = BK equation

The simplest equation to include unitarity: $T < 1$

Exhibits geometric scaling

$$\mathbf{T}(\mathbf{x}, \mathbf{r_t}) \rightarrow \mathbf{T}[\mathbf{r_t} \mathbf{Q_s}(\mathbf{x})]$$

for

$$\mathbf{Q_s} < \mathbf{Q} < \frac{\mathbf{Q_s^2}}{\Lambda_{\mathbf{QCD}}}$$

$$r_t \ll 1/Q_s \qquad T \sim (r_t\, Q_s)^{\gamma_s}$$

$$r_t \gg 1/Q_s \qquad T \sim ln^2(r_t\, Q_s)$$