On Possible Implications of Gluon Number Fluctuations in DIS Data

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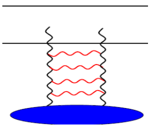
Outline

- **“Mean field equations”:**
  – Kovchegov and B-JIMWLK equations
  – Hallmark of “mean field” evolution equations:
  Geometrical scaling $T(r, Y) = T(r^2 Q_s^2(Y))$
- **Beyond mean field:**
  – Gluon number fluctuations or pomeron loops
  – Pomeron loop equations
  – Hallmark of pomeron loop equations: Diffusive scaling
  $< T(r, Y) >= T(\frac{\ln(\bar{Q}_s^2(Y) r^2)}{\sqrt{\alpha_s Y / \ln^3(1/\alpha_s^2)}})$
- **Numercial study DIS data**
- **Summary**

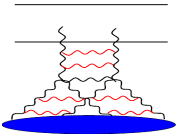
Mean field equations

- **Kovchegov equation:**

$\frac{\partial}{\partial Y} \langle T \rangle_Y \propto \bar{\alpha}_s \left[\langle T \rangle_Y - \langle T \rangle_Y \langle T \rangle_Y \right]$



  - $\langle T \rangle_Y \langle T \rangle_Y$; non-linear evolution, $\langle T \rangle_Y \leq 1$.
    The solution in saturation region
    $T(r, Y) = 1 - C_0 \exp\left[-C_1 (\rho - \rho_s(Y))^2 \right]$
    $\rho = \ln(1/r^2 Q_0^2)$, $\rho_s(Y) = \ln(Q_s^2(Y)/Q_0^2)$
  - $\langle T \rangle_Y \ll 1$; linear BFKL equation, $\frac{\partial}{\partial Y} \langle T \rangle_Y \propto \bar{\alpha}_s \langle T \rangle_Y$.
    $\langle T \rangle \sim \exp\left[c \bar{\alpha}_s Y \right] \longrightarrow$ unitarity violation!
    Solution to BFKL equation with saturation boundary($T \ll 1$ but not too samll):
    $T(r, Y) = C_2 \exp\left[-\lambda_s (\rho - \rho_s(Y)) - \frac{(\rho - \rho_s(Y))^2}{2 \bar{\alpha} \chi''(\lambda_s) Y} \right]$
    for $1 \ll \rho - \rho_s(Y) \ll 2 \chi''(\lambda_s) \bar{\alpha}_s Y$
    $\frac{(\rho - \rho_s(Y))^2}{2 \bar{\alpha} \chi''(\lambda_s) Y} \longrightarrow$ violate the Geometrical scaling



Mean field equations

Within a restricted window, $\rho - \rho_s(Y) \ll \sqrt{2\chi''(\lambda_s)\bar{\alpha}_s Y}$

$$T(r, Y) \sim C_2 \exp\left[-\lambda_s(\rho - \rho_s(Y))\right]$$

Geometrical scaling

- **B-JIMWLK equations:**

  $$\frac{\partial}{\partial Y}\langle T\rangle_Y \propto \bar{\alpha}_s\left[\langle T\rangle_Y - \langle TT\rangle_Y\right],$$
  $$\frac{\partial}{\partial Y}\langle TT\rangle_Y \propto \bar{\alpha}_s\left[\langle TT\rangle_Y - \langle TTT\rangle_Y\right], \ldots$$

  Mean field approximation: $\langle TT\rangle_Y \approx \langle T\rangle_Y \langle T\rangle_Y \rightarrow$ Kovchegov equation

- **Numerical result [Rummukainen, Weigert 04]:**

  $$\langle T\rangle_Y^{Kovchegov} \approx \langle T\rangle_Y^{B-JIMWLK}$$

Models of amplitude and Geometrical Scaling

- **GBW Model:**

  $T^{GBW}(r,x) = 1 \; - \; Exp\left\{-\,\frac{1}{4}\,Q_s^2 \cdot r^2\right\}$

  Where: $Q_s^2(x) \; = \; Q_0^2 \cdot (x_0/x)^{\lambda}$

  For DIS at HERA
  ($x \leq 10^{-2}$ and $0.045 < Q^2 < 450 GeV^2$):

  $\lambda = 0.29, \;\; x_0 = 3 \cdot 10^{-4}, \;\; Q_0 = 1 \;\; GeV.$

- **MV Model:**

$$T^{MV}(r,b,x) = 1 \; - \; Exp\left\{-\,\frac{1}{4}\,Q_s^2 \cdot r^2 \cdot \log\left(\frac{1}{r^2\Lambda^2} + e\right)\right\}$$

  - **Corrections to the region of large Q^2;**

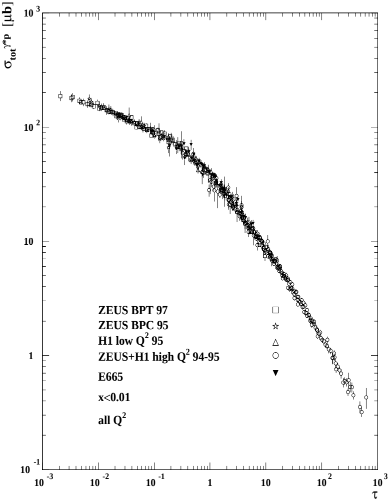
- **IIM Model(inspired by solution of BK-equation):**

$$T^{IIM}(r,x) = \begin{cases} 1 - \exp\left\{-a\ln^2(b\,r\,Q_s(x))\right\} & r\,Q_s(x) > 2 \\ N_0\left(\frac{r\,Q_s(x)}{2}\right)^{2\left(\lambda_s + \frac{\ln(2/r\,Q_s(x))}{\kappa\,\lambda\,Y}\right)} & r\,Q_s(x) < 2 \end{cases}$$

  - **Two matching parameters;**
  - **Diffusive corrections;**

Stasto, Golec-Biernat and Kwiecinski

$\tau = Q^2 R_0^2(x) \qquad R_0^2(x) \equiv (x/x_0)^{\lambda}/Q_0^2$



Geometrical Scaling

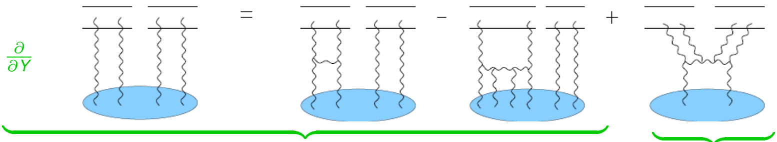
- Does the Geometrical Scaling is unique possibility to arrange the HERA data ?

  NOT

- Does the violation of the geometric scaling come from BK-diffusion term or from gluon number fluctuations(Pomeron loops) ?

Shortcomings of “mean field equations”: Pomeron loops

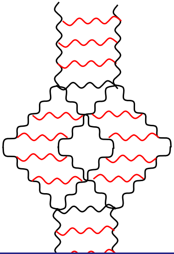
Two dipoles scattering off a target [Iancu, Triantafyllopoulos 2005]:



“mean field equations”

graph missed!

- Pomeron loops missed!



Pomeron loop equations

- Hierarchy:

$$\frac{\partial}{\partial Y}\langle T\rangle_Y \quad \propto \quad \alpha_s\,[\langle T\rangle_Y - \langle T\;T\rangle_Y]$$
$$\frac{\partial}{\partial Y}\langle T\;T\rangle_Y \quad \propto \quad \alpha_s\left[\langle T\;T\rangle_Y - \langle T\;T\;T\rangle_Y + \alpha_s^2\,\langle T\rangle_Y\right]$$

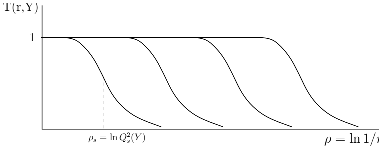
- Stochastic version:

$$\frac{\partial}{\partial Y}T_Y \quad \propto \quad \alpha_s\left[T_Y - T_Y\,T_Y + \sqrt{\alpha_s\,T}\;\nu\right]$$

- describe Gluon Number Fluctuations / Pomeron Loops;
- non-linear evolution, satisfies {s,t}-channel unitarity;

Effects of Pomeron Loops/Gluon Nnumber Fluctuations

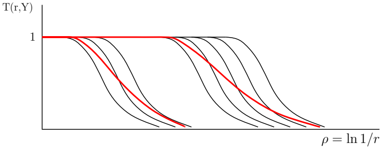
- Single events



Geometric scaling

$$T(r, Y) = T(r^2\, Q_s^2(Y))$$

- Average over events



Diffusive scaling

$$\langle T(r, Y)\rangle = \mathcal{F}\left(\frac{\ln(\bar{Q}_s^2(Y)\; r^2)}{\sqrt{D\,Y}}\right)$$

- Statistical physics $\Longleftrightarrow$ hdQCD: $\langle T(\rho - \rho_s)\rangle = \int d\rho_s\, P(\rho_s)\; T(\rho - \rho_s)$

[Iancu,Mueller,Munier (2004)]

where $P(\rho_s) = \frac{1}{\sqrt{2\pi\sigma^2}}\, \exp[\frac{(\rho_s - \langle\rho_s\rangle)^2}{2\sigma^2}]$, $\quad \sigma^2 = \langle\rho_s^2\rangle - \langle\rho_s\rangle^2 = D\,Y$

$\Longrightarrow$ shape of $\langle T\rangle$ becomes flatter with increasing Y

Numerical Evaluation

- **Fit for the F_2 structure function**
  (**ZEUS data in the kinematical range:** $x \leq 10^{-2}$ and $0.045\,\mathrm{GeV}^2 < Q^2 < 50\,\mathrm{GeV}^2$)

$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}\left(\sigma_T(x,Q^2) + \sigma_L(x,Q^2)\right)$$

$$\sigma_{T,L}(x,Q^2) = \int dz\, d^2r\, |\psi_{T,L}(z,r,Q^2)|^2\, \sigma_{dip}(x,r) \qquad \sigma_{dip} = 2\pi R^2 \begin{cases} T(r,x) \\ \langle T(r,x)\rangle \end{cases}$$

$$|\Psi_T|^2 = \tfrac{3\,\alpha_{em}}{2\pi^2} \textstyle\sum_f e_f^2 \left\{[z^2 + (1-z)^2]\widetilde{Q}_f^2 K_1^2(\widetilde{Q}_f r) \,+\, m_f^2\, K_0^2(\widetilde{Q}_f r)\right\}$$

$$|\Psi_L|^2 = \tfrac{3\,\alpha_{em}}{2\pi^2} \textstyle\sum_f e_f^2 \left\{4Q^2 z^2 (1-z)^2 K_0^2(\widetilde{Q}_f r)\right\}, \qquad \widetilde{Q}_f^2 = z(1-z)Q^2 + m_f^2$$

- **Parameters are fixed via minimization of the χ^2**

$$\chi^2 = \sum_i \frac{\left(F_i^{\mathrm{mod}}(p_1,\ldots,p_n) - F_i^{\mathrm{exp}}\right)^2}{\mathrm{err}_i^2} \qquad \mathrm{err}^2 = \mathrm{err}_{sys}^2 + \mathrm{err}_{sta}^2$$

Numerical Evaluation

GBW model:

The parameters of the event-by-event T^{GBW} and of the physical $\langle T^{\text{GBW}} \rangle$ amplitude.

	χ^2	χ^2/d.o.f	x_0 ($\times 10^{-4}$)	λ	R(fm)	D
T^{GBW}	266.22	1.74	4.11	0.285	0.594	0
$\langle T^{\text{GBW}} \rangle$	173.39	1.14	0.0546	0.225	0.712	0.397

IIM model:

The parameters of the event-by-event T^{IIM} and of the physical $\langle T^{\text{IIM}} \rangle$ amplitude.

	χ^2	χ^2/d.o.f	x_0 ($\times 10^{-4}$)	λ	R(fm)	D
T^{IIM}	150.45	0.983	0.5379	0.252	0.709	0
$\langle T^{\text{IIM}} \rangle$	122.62	0.807	0.0095	0.198	0.812	0.325

Model independent approach

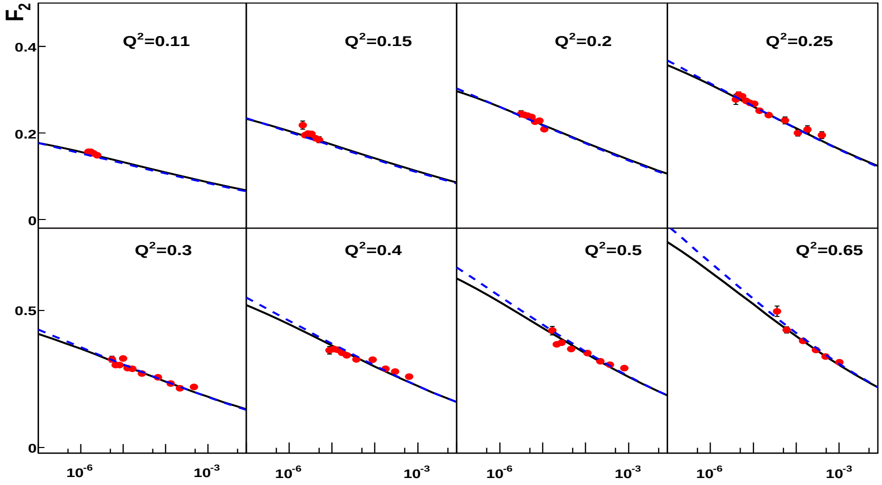
In case fluctuations are important in the range of HERA data, one finds the diffusive scaling behavior:

$$\langle T(r,Y)\rangle = \bar{T}(r,Y) = \bar{T}\left(\frac{\ln(\frac{1}{r^2 Q_s^2})}{\sqrt{DY}}\right)$$

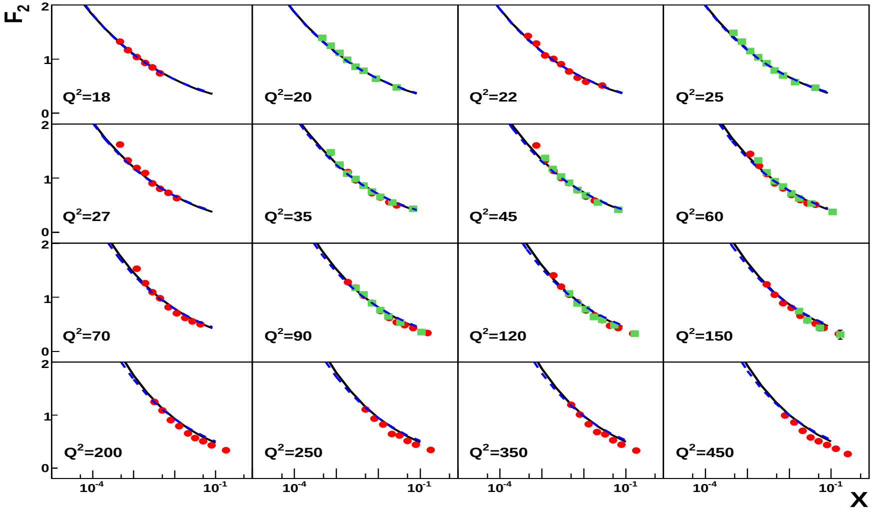
- Quality factor[Gelis,Peschanski,Soyez,Schoeffel]:
  $\mathcal{O}(\lambda) = \sum_i \frac{(\sigma_i - \sigma_{i-1})^2}{(\tau_i - \tau_{i-1})^2 + \varepsilon^2}$,
  with $\sigma = 4\pi^2 \alpha_{em} F_2(x, Q^2)/Q^2$, $\tau = \ln(1/r^2 Q_s^2)/\sqrt{DY}$
  and $\varepsilon = 1/n$

We got $\lambda = 0.215$ for the input-values $0.01 \leq D \leq 0.7$, which seems tell us that gluon number fluctuations are relevent in the range of HERA data.

Geometrical versus Diffusive Scaling - small Q^2



Geometrical versus Diffusive Scaling - large Q^2



Summary

- Description of the DIS data is improved once gluon number fluctuations are taken into account.
- This outcome seems to indicate the evidence of geometric scaling violations, and a possible implication of gluon number fluctuations, in the DIS data.
- However, looking only on HERA data one cannot exclude the possibility that the scaling violations may also come from the diffusion part of the solution to the BK-equation.

For the technical details: Kozlov, Shoshi, Xiang arXiv:0707.4142