Reaction-diffusion processes in zero transverse dimensions as toy models for high-energy QCD

Paloma Quiroga

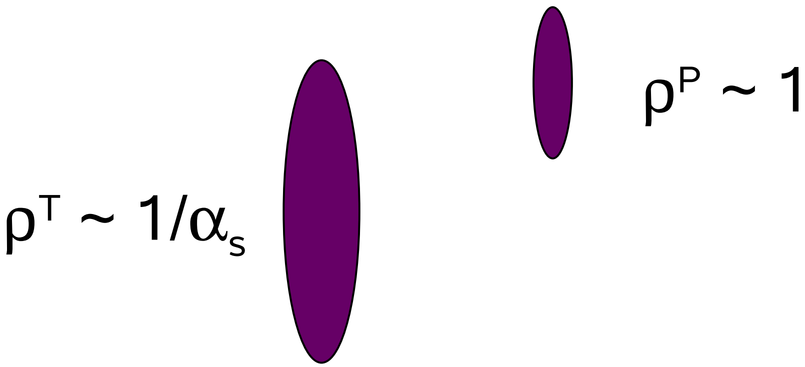
Departamento de Física de Partículas and IGFAE, Universidade de Santiago de Compostela

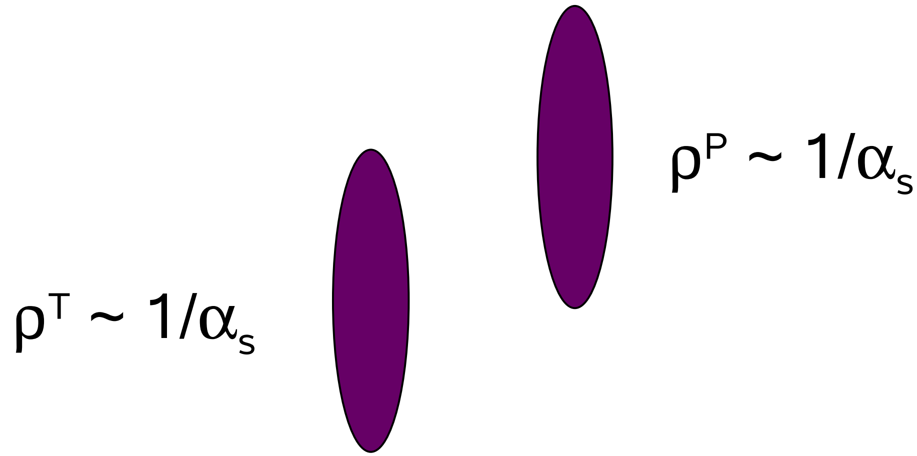
based on: N. Armesto, S. Bondarenko, J. G. Milhano and P.Q., arXiv:0803.0820[hep-ph]

OUTLINE

- High energy evolution
  - Pomeron loops
  - Reaction-Diffusion
- Our work
  - Problem and formalism
  - 3 different cases
    - Reggeon Field Theory
    - Directed Percolation
    - Reversible Process
  - Running Coupling
  - Classical Solution
- Numerical Results

High Energy Evolution (I)

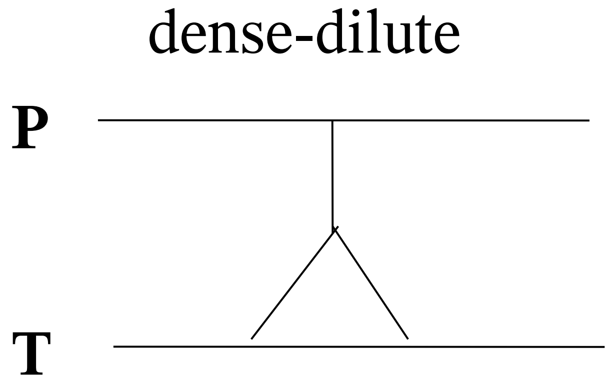


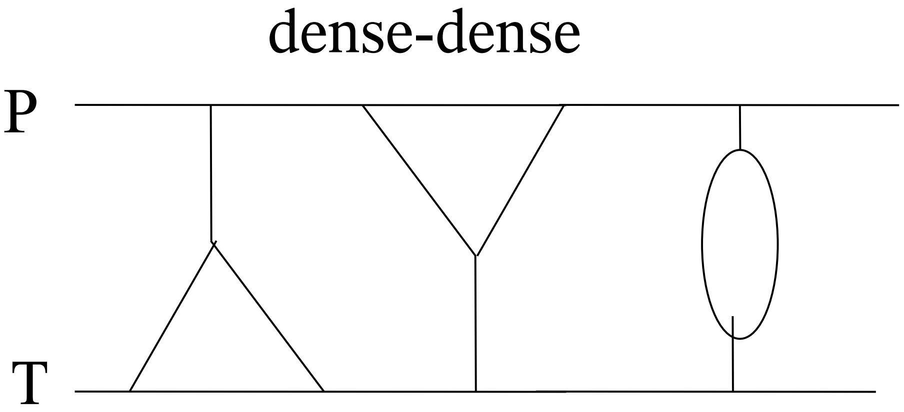


- Dense-Dilute regime (HERA?-eRHIC)
- linear in ρ^P
- only downgoing pomeron fan diagrams

- Dense-Dense regime (RHIC?-TeVatron?-LHC)
- non-linear in ρ^P and ρ^T
- up and downgoing pomeron fan diagrams: Pomeron loops

High Energy Evolution (II)





- **Problem**: solving evolution equations very complicated for Dense-Dense (missing effects in B-JIMWLK)
  - **Solutions**: Generalization of B-JIMWLK is provided by insights from statistical mechanics systems *(Iancu, Mueller and Munier, 2005)*
- **Our effort**:
  - go back to reaction-diffusion processes (as in RFT in the 70s)
  - educated guess of noise terms and compare with our general expectations for high energy evolution

Our work: problem

- Start from a differential equation in rapidity (time)

$$\frac{\partial F(y,q)}{\partial y} = -H(p,q)F(y,q)$$

H rules the evolution of an auxiliary operator which creates the evolved initial state $\Psi_i(y) \equiv F_i(y,q)\Psi_0$

- Solve the equation with initial condition

$$F(y=0,q) = 1 - \exp(-g_i q)$$ eikonal coupling

g_i: coupling parameter with the projectile

- Meaning of $F_i(y,q)$ as transition amplitude: $iA_{fi}(y) = F(y, q = g_f)$

g_f: coupling parameter with the target

Our work: formalism

Reaction Hamiltonian *(Elgart & Kamenev, 2006)*

- (p,q): canonical pair $[\bar{q},\bar{p}]=1$; $p=-\partial/\partial q\equiv-\partial_q$

  q/p will be creation/destruction from the projectile

- Reaction-diffusion system: Hamiltonian action

$$S=\int dt\int d^d x[\bar{p}\partial_t\bar{q}+D\nabla\bar{p}\nabla\bar{q}-H_R(\bar{p},\bar{q})]$$

- Phase space portrait determined by $H_R=0$

- Reaction Hamiltonian in a reaction is given by:

$$kA\xrightarrow{\lambda}mA, \qquad H_R(\bar{p},\bar{q})=\frac{\lambda}{k!}(\bar{p}^m-\bar{p}^k)\bar{q}^k$$

Our notation:

$$\bar{p}\rightarrow -q,\ \bar{q}\rightarrow p,\ H_R\rightarrow -H$$

Our work: H in general form

- The standard form of the Hamiltonian reads

$$H(p,q) = \alpha_1 pq - \alpha_2 qp^2 - \alpha_3 q^2 p + \alpha_4 q^2 p^2$$

parameters model-dependent: **[1/2/3/4]=[diffusion/ 2->1 / 1->2 / 2->2]**

- Property of the Hamiltonian:

$$y \to -y \ , \ q \to Ap \ , \ p \to Bq$$

$$H_{y\to -y}(Ap, Bq) = \alpha_1 BAqp - \alpha_2 B^2 Aq^2 p - \alpha_3 BA^2 qp^2 + \alpha_4 B^2 A^2 q^2 p^2$$

$$\Rightarrow \quad y \to -y \qquad q \to \frac{\alpha_2}{\alpha_3} p \qquad p \to \frac{\alpha_3}{\alpha_2} q$$

$$H(p, \partial_p) = H_{y\to -y}\left(\frac{\alpha_3}{\alpha_2} q, -\frac{\alpha_2}{\alpha_3}\partial_q\right)$$

suggestive of Dense-Dilute Duality *(Kovner & Lublinsky, 2006)*

Our work: different cases (I)

Reggeon Field Theory

- action defining the theory with triple pomeron vertex only:

(Bondarenko et al., 2006)

$$H(p,q) = \tilde{\mu} qp - \tilde{\lambda} q^2 p - \tilde{\lambda} qp^2$$

$\tilde{\mu}$ *: bare pomeron intercept*

$\tilde{\lambda}$ *: 3-pomeron vertex coupling*

$$H_R(\bar{p},\bar{q}) = (\alpha_1 - \alpha_2 \bar{q} + \alpha_3 \bar{p})\bar{p}\bar{q}$$

$$\alpha_2 = \alpha_3$$

- *zero energy lines:*

$$\bar{p} = 0, \quad \bar{q} = 0, \quad \bar{q} = \frac{\alpha_1}{\alpha_2} + \bar{p}$$

Our work: different cases (II)

Directed Percolation

- take the stochastic process: $1 \to_{\lambda} 0,\ 1 \to_{\mu} 2,\ 2 \to_{2\sigma} 1$
- obtain the Hamiltonian $H_R(\bar{p}, \bar{q}) = [(\mu - \lambda) + \mu\bar{p} - \sigma\bar{q} - \sigma\bar{p}\bar{q}]\bar{p}\bar{q}$

  the parameters are: $\alpha_1 = \mu - \lambda, \alpha_3 = \mu, \alpha_2 = \alpha_4 = \sigma$
- zero energy lines

$$\bar{p} = 0, \quad \bar{q} = 0, \quad \bar{q} = \frac{\alpha_1 + \alpha_3\bar{p}}{\alpha_2(1+\bar{p})}$$

Our work: different cases (III)

Reversible Process

- take the stochastic process: $1 \to_{\mu} 2,\ 2 \to_{2\sigma} 1$
- obtain the Hamiltonian $H_R(\bar{p},\bar{q}) = (\mu + \mu\bar{p} - \sigma\bar{q} - \sigma\bar{p}\bar{q})\,\bar{p}\bar{q}$

  parameters: $\alpha_1 = \alpha_3 = \mu, \alpha_2 = \alpha_4 = \sigma$
- zero energy lines

$$\bar{p} = 0, \quad \bar{q} = 0, \quad \bar{p} = -1, \quad \bar{q} = \frac{\alpha_1}{\alpha_2}$$

Our work: different cases (IV)

Fixing the parameters

- from **RFT** we know $$(\tilde{\mu}, \tilde{\lambda}, \tilde{\lambda}) = (1, 0.5, 0.5)$$
- **DP**:
  - compare H_R=0 with RFT: approximate the third by a straight line

$$\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4) = \frac{\lambda}{\tilde{\lambda}}(\tilde{\mu}, \tilde{\lambda}, \tilde{\mu} + \tilde{\lambda}, \tilde{\lambda}) = \frac{\lambda}{\tilde{\lambda}}(1, 0.5, 1.5, 0.5)$$

- **RP**:
  - there is no RFT limit
  - parameters are free

$$\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (\mu, \sigma, \mu, \sigma) = (1, \epsilon, 1, \epsilon)$$

  - sFKPP: Langevin equation corresponding to this reaction-diffuson problem

Running of the Couplings

- Reaction-diffusion processes: couplings fixed at this level
  - 1-dim model: RC may shift the contribution of effects of pomeron loops to higher rapidities (*Dumitru et al., 2007*)

- Heuristic procedure:
  - 1/q: some 'momentum' scale -> logarithmic running

$$\alpha_i(q) = \alpha_i \frac{\ln(Q/q)}{\ln(q_0/q)}$$

for $q<q_0$

$Q=10\ q_0$ (inverse QCD scale)

- For $q \geq q_0$

$$\alpha_i(q) = \alpha_i$$

fixed parameters

Classical Solution

- Classical solution of the Hamiltonian problem

$$\dot{p} = (-\alpha_1 + \alpha_2 p)p + 2(\alpha_3 - \alpha_4 p)qp$$
$$\dot{q} = (\alpha_1 - 2\alpha_2 p)q + (-\alpha_3 + 2\alpha_4 p)q^2$$

initial conditions: $q(y=0) = g_i \;,\; p(y=Y) = g_f$

- Classical amplitude (amplitude at tree level)

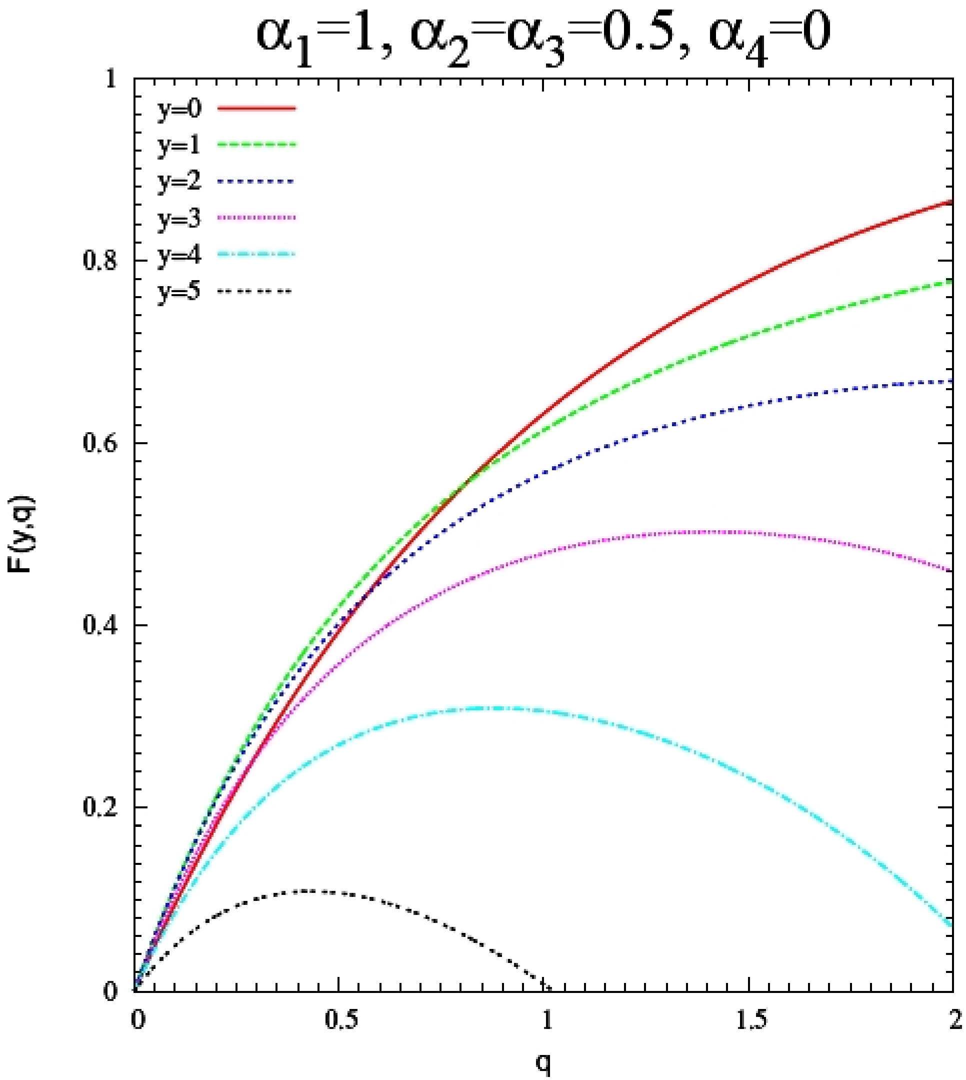
$$iA_{fi}^{clas}(y) = 1 + \sum_k \Delta_k \exp\left[-S(Y, q_k, p_k)\right]$$

$\Delta_k = \pm 1$: one symmetric (+1) and two asymmetric (-1) solutions

Numerical results
rapitity evolution of the solutions

- Reggeon Field Theory
- Directed Percolation
- Reversible Process
- Running Coupling
- Saturation Scale *(1/q, Albacete et al., 2004)*
- Classical Solution

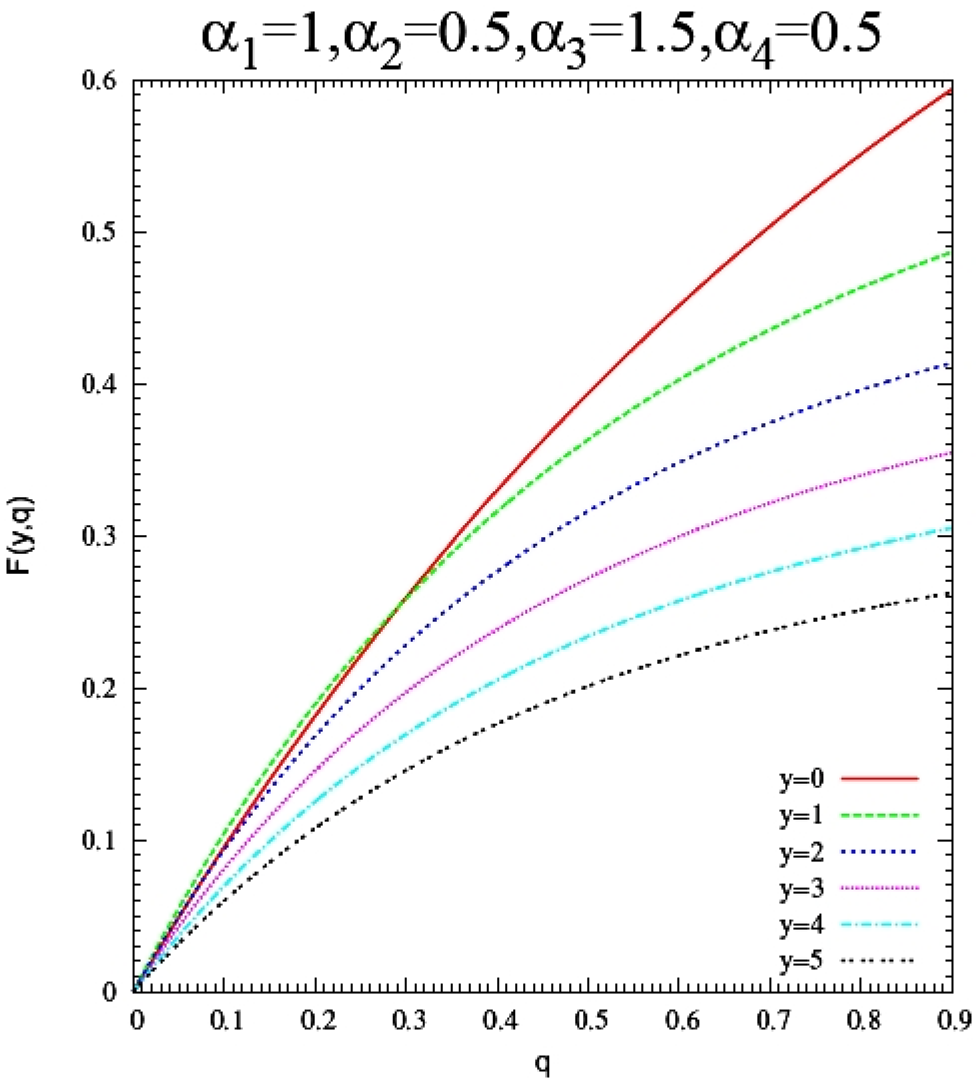
Evolution ot the solutions for RFT



- Exponential decay at high rapidity limit

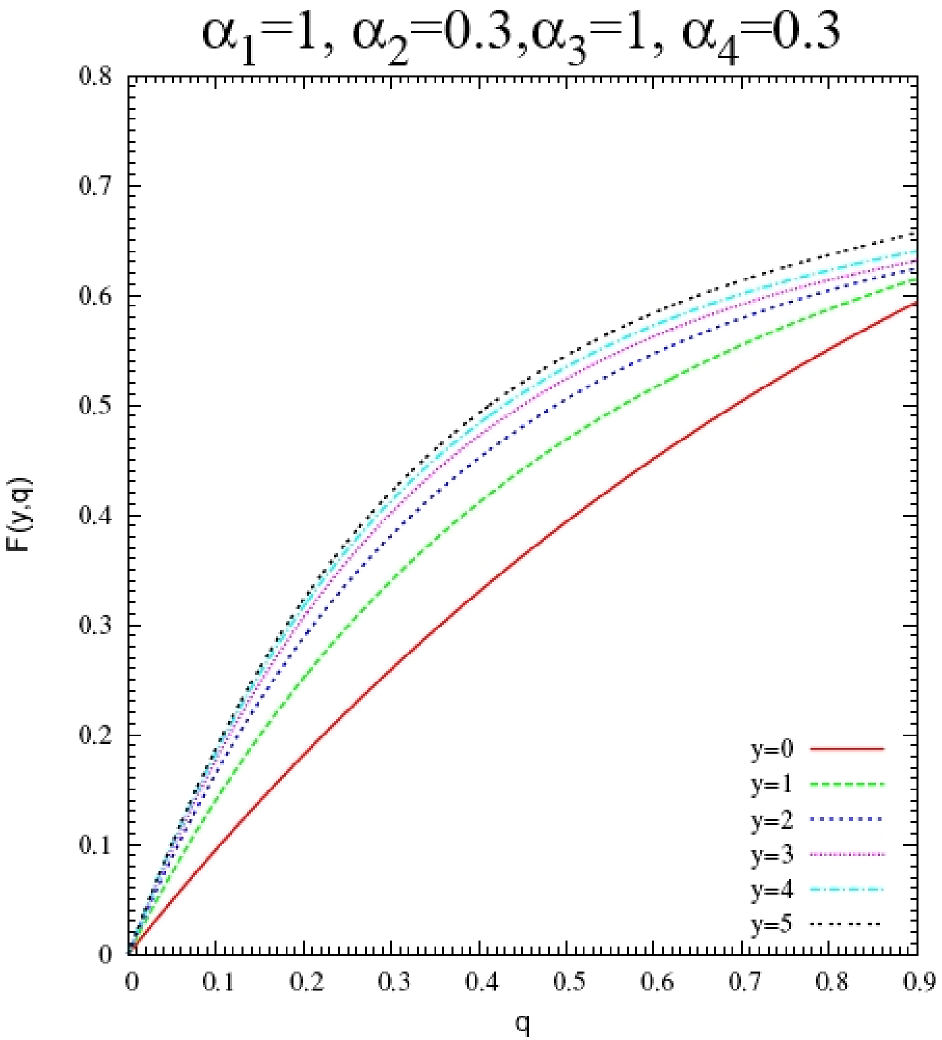
  tunneling phenomenon
  (Martin et al.,1978)
  (Alessandrini et al.,1976)
  (Ciafaloni et al., 1977)
  (Ciafaloni, 1978)

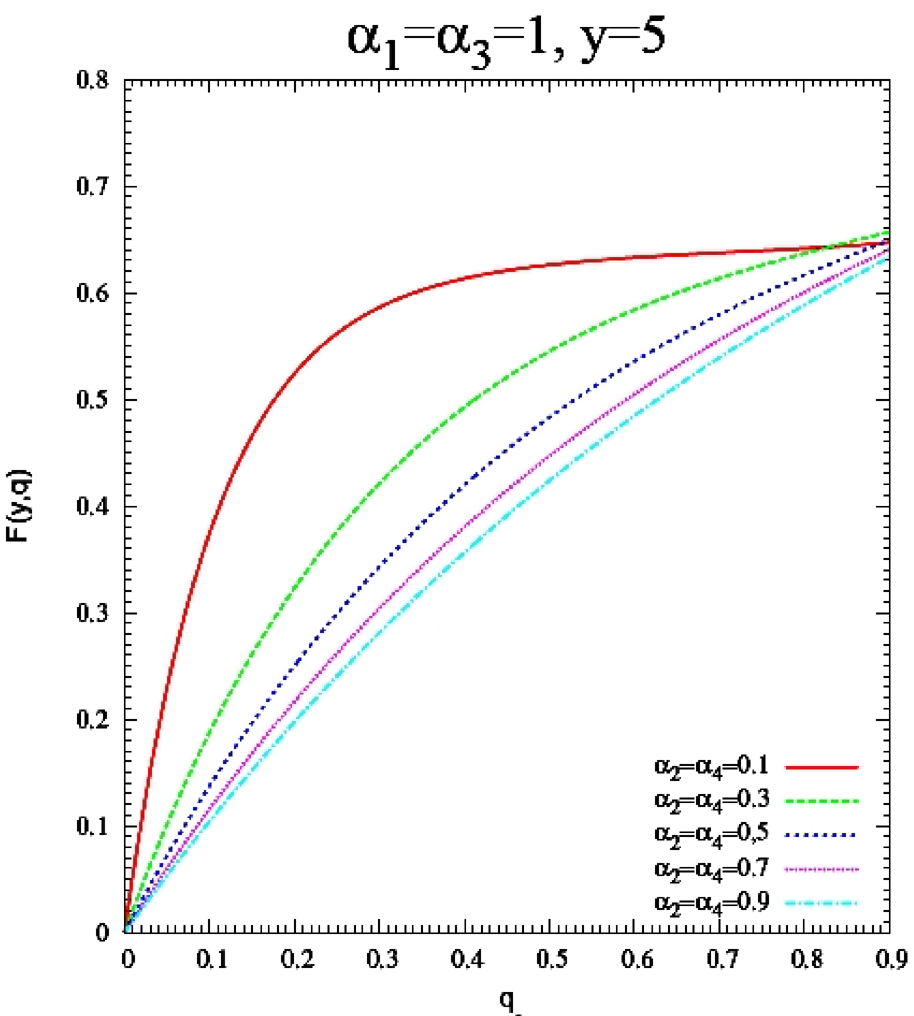
Rapidity evolution: DP



- opposite direction to what we expect in high energy
- excludes DP as a candidate reaction-diffusion model for description of HE evolution

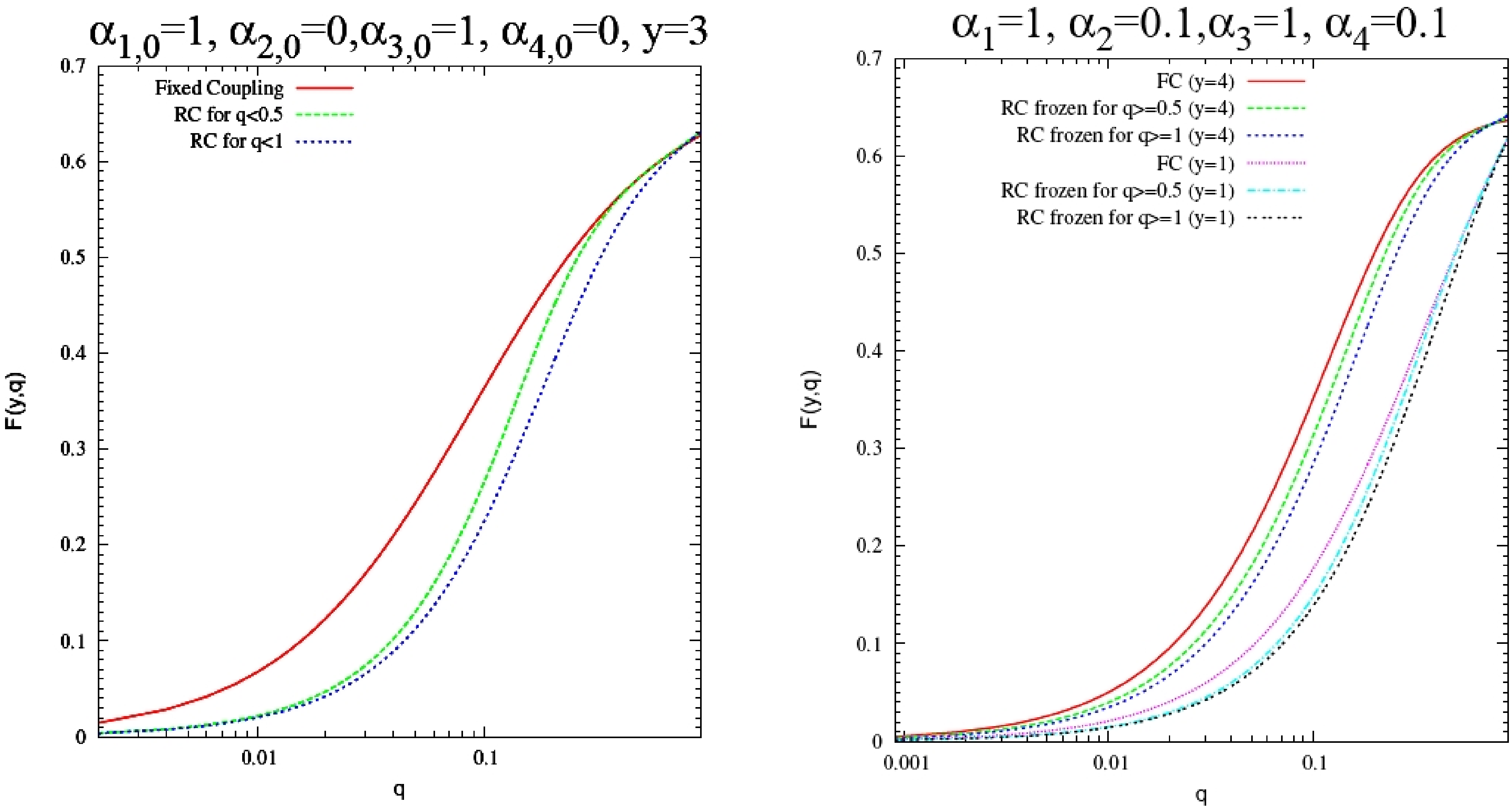
Evolution ot the solutions for RP





- Correct direction: increase y leads the front to smaller values of q
- increase α_2 =α_4 makes evolution softer

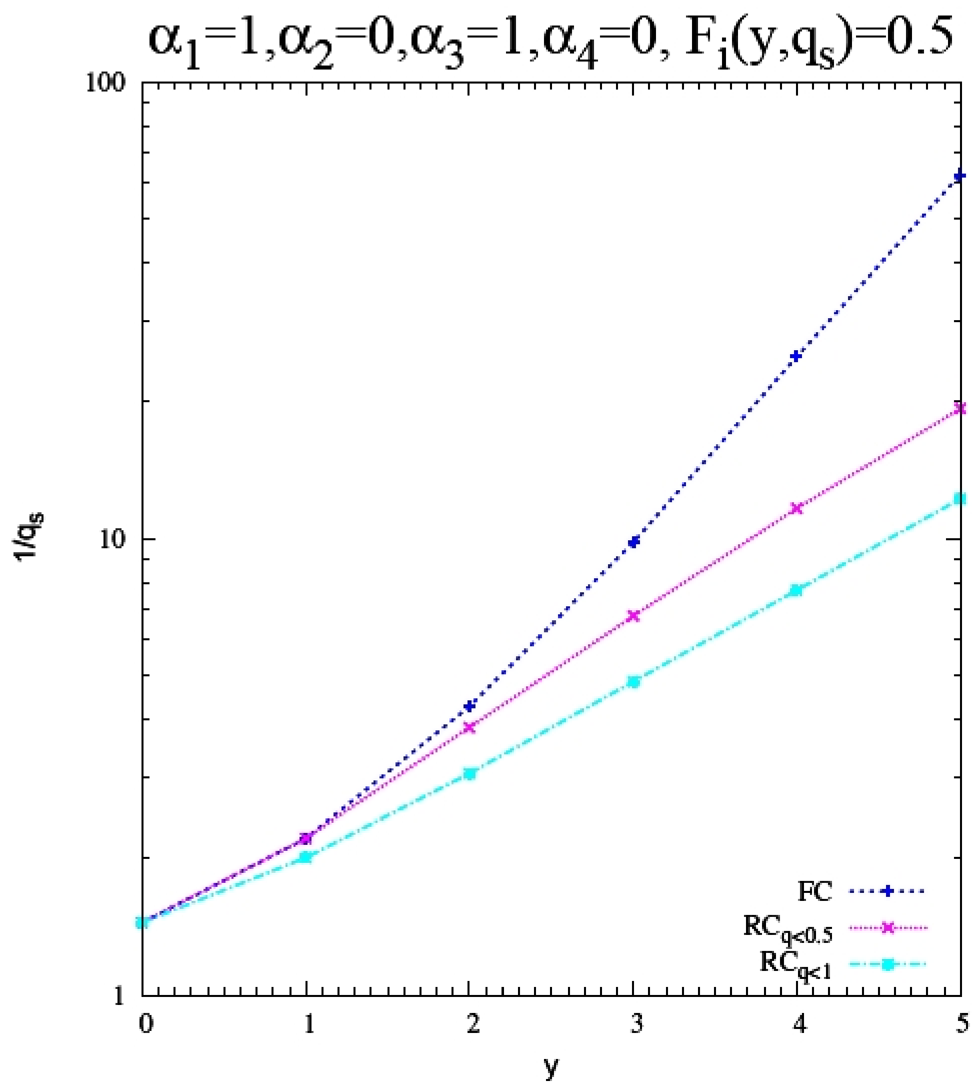
Running Coupling



- Evolution is slowed down by the running of the coupling
- Freezing point: relevant just at the beginning of the evolution *(the same in BK)*

Saturation Scale (I)

$F(y,q_s(y))=k$



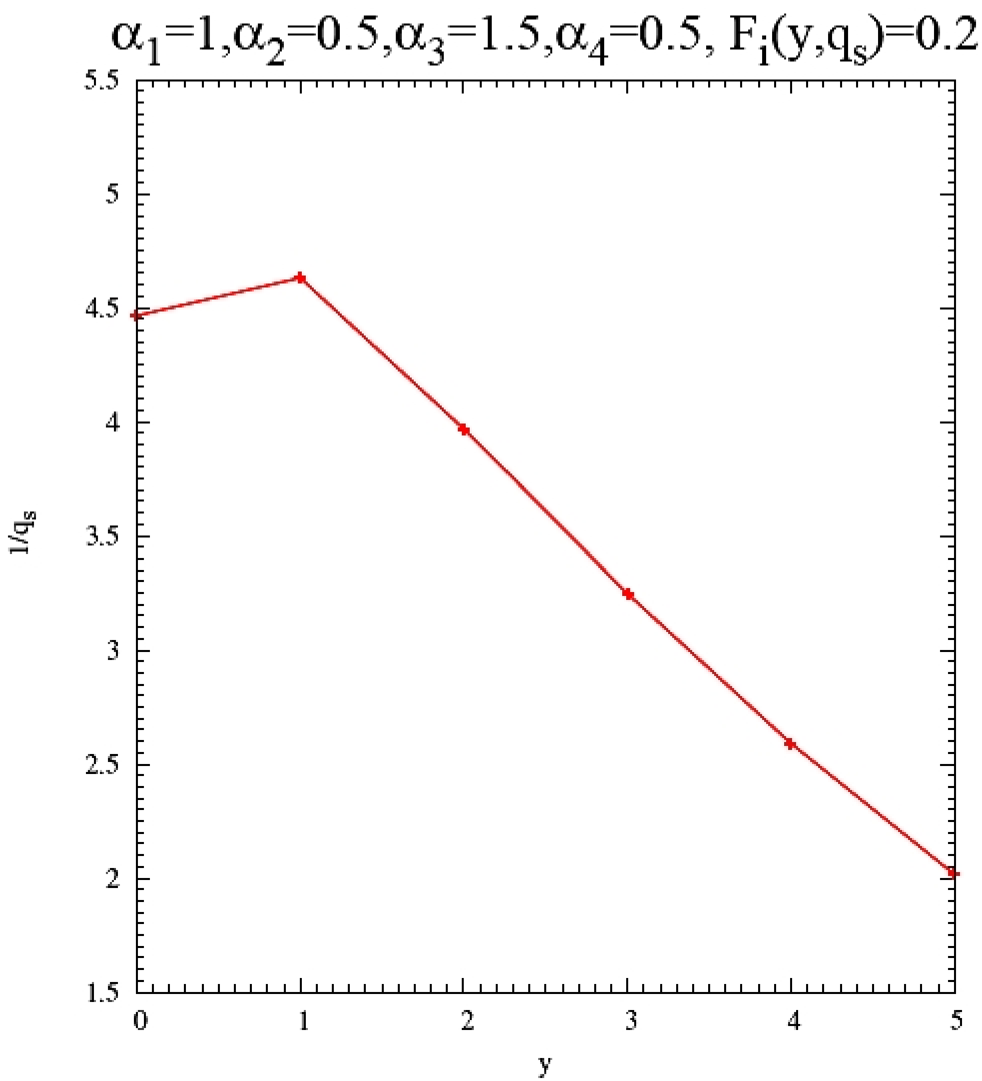
Fan case

- running of α_i: slower increase of $1/q_s$

as in BK

Saturation Scale (II)

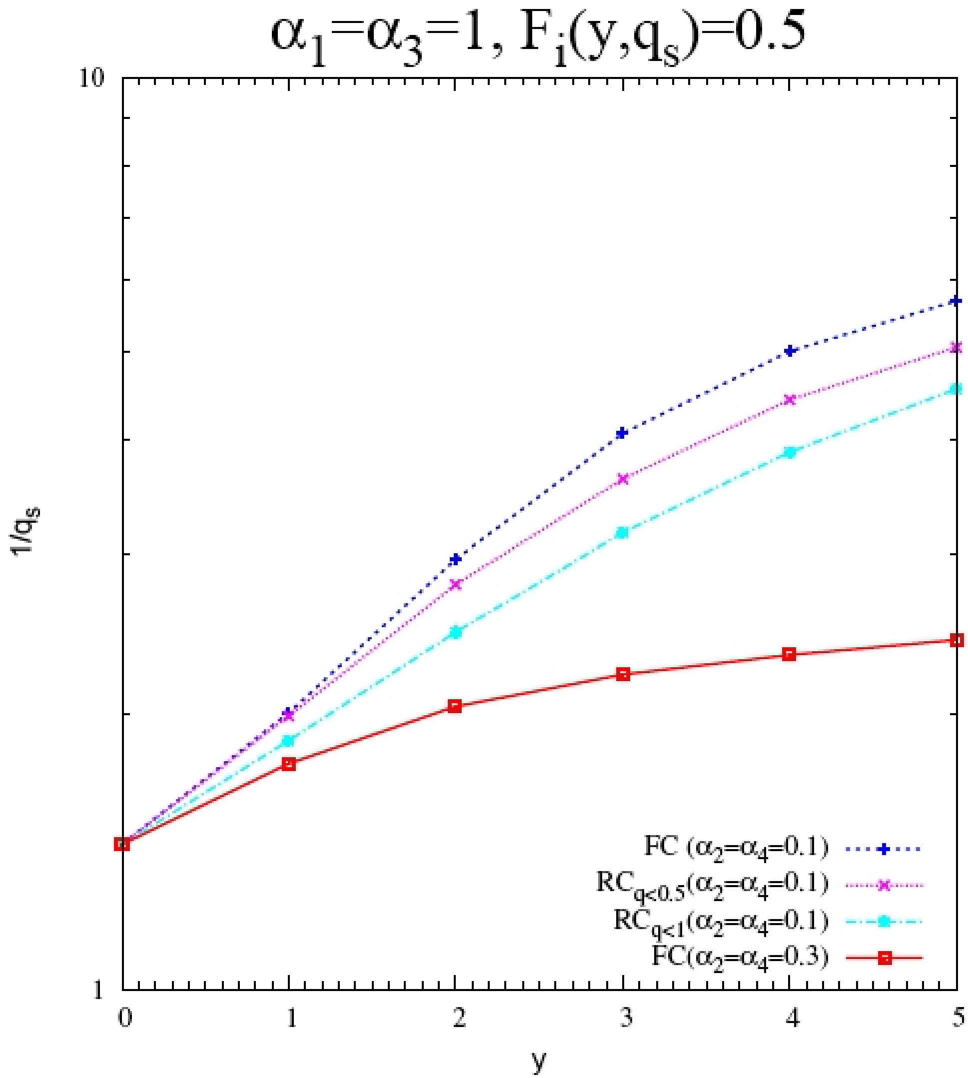
$F(y,q_s(y))=k$



Directed Percolation

- Decrease of $1/q_s$ as expected from the evolution direction

Saturation Scale (III) $F(y,q_s(y))=k$



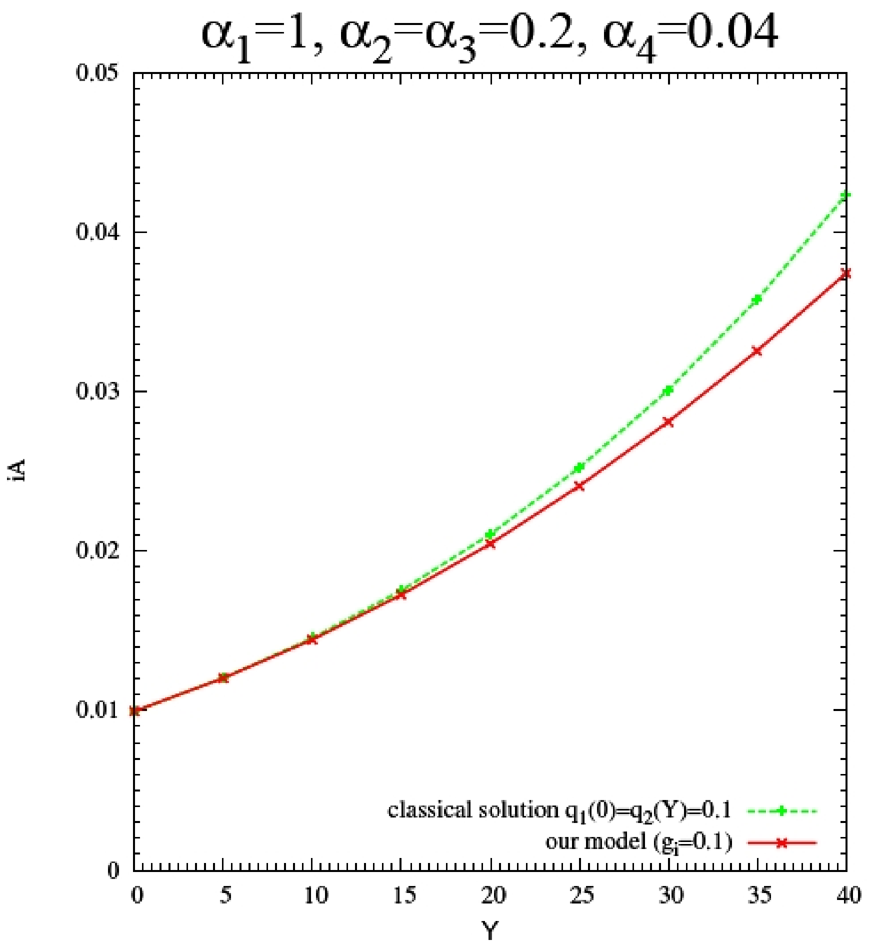
Reversible Process

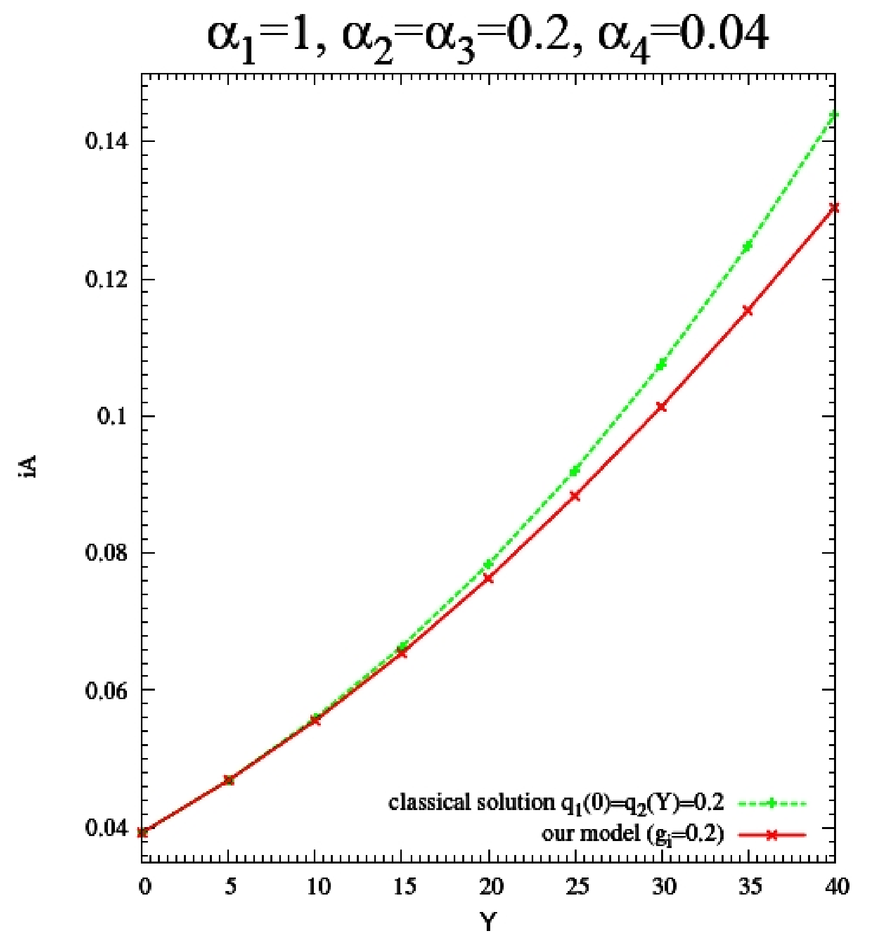
- Increase of strengths2->1 and 2->2 and RC slow down the increase of the saturation scale

- Competition between *"Pomeron loops" and RC* *(Dumitru et al., 2007)*

Classical Solution

- Compare the full quantum and the symmetrical classical solutions
- Classical solution always above the quantum one
  - Difference growing with increasing rapidity





FINAL REMARKS

- RFT: known behaviour of vanishing amplitude with increasing y
- DP: solutions decreasing with increasing y
- **RP show the right characteristics**
- 2->1/2->2 terms and running α_s slow the evolution *(Iancu et al., 2007)*
- Quantum effects tend to slow down the evolution
- Our work is in 0-dim. Evolution can change a lot if we go to1 or 2 transverse dimensions (diffusion)

Many thanks to Sergey Bondarenko, Misha Braun, Edmond Iancu, Alex Kovner, Carlos Pajares and Alex Prygarin.

THE END

BACKUP SLIDES

Numerical method

- Solve equation: second order Runge-Kutta technique by hand treating specially ends of the grid
- Discretized q range: 500 points per unit (precision few %)
- Rapidity region studied:
  - y between 0 and 5 in general
  - up to y=40 when comparing with classical solution
- Step in rapidity: $h=6.25*10^{-6}$
- $g_i=1$
- Classical equations: solved by the shooting method

Numerical results: check

Fan case: analytical solution

- solve the evolution equation in a simple way obtaining

$$F_{fan}(y,\bar{q}) = 1 - exp\left[-\frac{g_i\bar{q}e^{\alpha_1 y}}{1+\frac{\bar{q}\alpha_3}{\alpha_1}(e^{\alpha_1 y}-1)}\right]$$

  same result as solving numerically

- single coupling: $F_i(0,\bar{q}) = 1 - e^{-g_i\bar{q}} \simeq g_i\bar{q}$

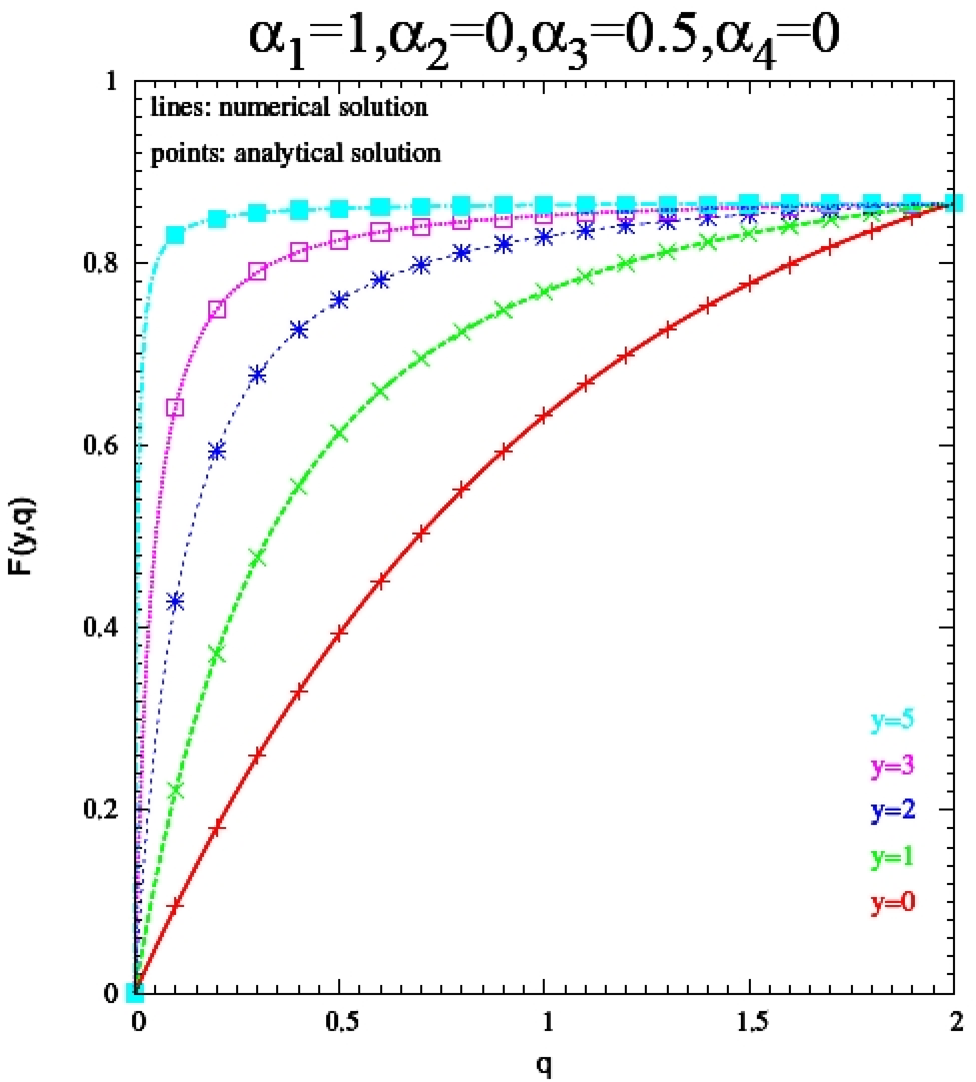
  defining a new variable: $dz = \frac{d\bar{q}}{\alpha_1\bar{q}-\alpha_3\bar{q}^2}$

$$z = \frac{1}{\alpha_1}\ln\frac{\bar{q}}{\bar{q}-\alpha} \qquad \bar{q} = \frac{\alpha}{1-e^{\alpha_1 z}}$$

  we find:

$$\left(\frac{\partial}{\partial y}-\frac{\partial}{\partial z}\right)F(y,z) = 0 \qquad F(y,z) = f(y+z) \qquad F^{hA}_{fan} = \frac{q_i\bar{q}e^{\alpha_1 y}}{1+\frac{\bar{q}}{\alpha}(e^{\alpha_1 y}-1)}$$

Numerical results: check



Fan case

- We compare the analytical and our numerical solutions
- We find a very good agreement

Numerical results: Fan case invariance

- Hamiltonian in this particular case: $\alpha_2 = \alpha_4 = 0$
- apply to this H the change

$$t \rightarrow -t$$
$$\bar{q} \rightarrow \frac{\alpha_2}{\alpha_3}\bar{p}$$
$$\bar{p} \rightarrow \frac{\alpha_3}{\alpha_2}\bar{q}$$

$$H_{fan}(\bar{p},\bar{q}) = \alpha_1 \bar{q}\bar{p} - \alpha_3 \bar{q}^2 \bar{p}$$
$$\downarrow$$
$$H_{t\rightarrow -t}\left(\bar{p} \rightarrow \frac{\alpha_3}{\alpha_2}\bar{q}, \bar{q} \rightarrow \frac{\alpha_2}{\alpha_3}\bar{p}\right) = \alpha_1 \bar{q}\bar{p} - \alpha_2 \bar{q}\bar{p}^2$$

- splitting term in P-T direction → splitting in T-P one