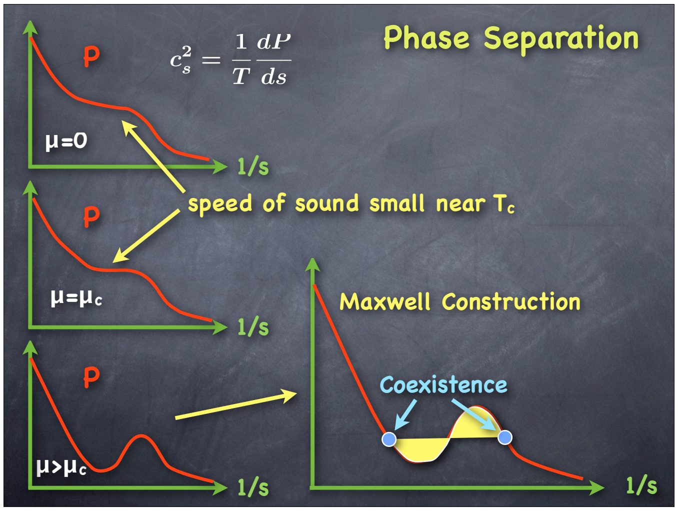
Lecture 3: Fluctuations

Physical Sources

- Droplets (Phase Separation)
- Critical Phenomena
- Isospin Dynamics
- Charge Diffusion
- Jets
- Initital Conditions
  - Random # of collisions
  - Orientation of nuclear deformation

For another lecture

Phase Separation



$$c_s^2 = \frac{1}{T}\frac{dP}{ds}$$

Lesson #1

- Fluctuation growth slow near T_c

$$\frac{d\Delta\eta}{d\tau} = \frac{c_s}{\tau}$$

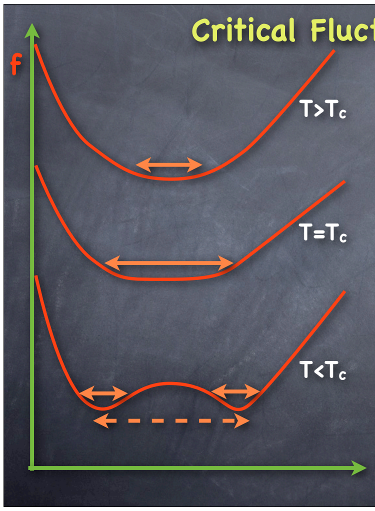
$$\Delta\eta = c_s \ln\left(\frac{\tau_f}{\tau_i}\right).$$

For c_s=0.2, τ_f=4, τ_f=8:

$$\Delta\eta = 0.14$$

- Measurements in momentum space
- All fluctuations spread out by $\Delta y_{\text{therm}} \approx$ 0.3-0.8
- Positive/Negative fluctuations correlated

Critical Fluctuations



$$\langle(\epsilon - \bar{\epsilon})^2\rangle \sim \frac{1}{(T - T_c)^\gamma}$$

Lesson #2

- Fluctuations are largest in mixed phase region
- Unstable mode growth not infinitesimally slow

Searching for Droplets

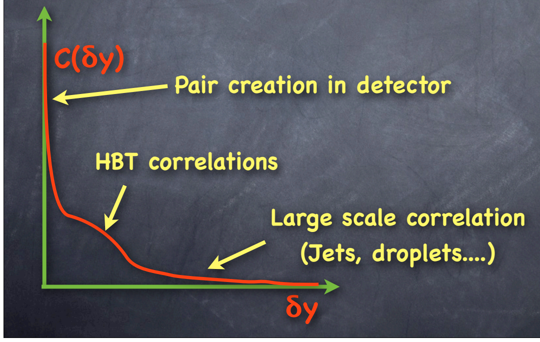
- Multiplicity Fluctuations / Rapidity Correlations
- HBT of substructure
- p_t fluctuations

Correlations/Fluctuations Equivalence

$$F_2(\Delta y) \equiv \frac{\langle N(N-1)\rangle}{\langle N\rangle^2} - 1$$
$$= \frac{2}{\Delta y}\int d\delta y \left(1 - \frac{\delta y}{\Delta y}\right)\left[C(\delta y) - 1\right]$$

Lesson #3

- Correlations are more differential than “fluctuations”
- Use whenever possible



C(δy)

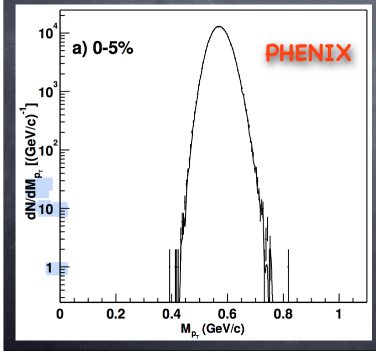
Pair creation in detector

HBT correlations

Large scale correlation
(Jets, droplets....)

δy

$$F_{p_t} \equiv \frac{\sigma_{p_t} - \sigma_{p_t,\mathrm{mixed}}}{\sigma_{p_t,\mathrm{mixed}}}$$
$$\sigma^2_{p_t} \equiv \langle p_t^2\rangle - \langle p_t\rangle^2$$



p_t Fluctuations

- Hot drops? (T would fluctuate by few percent)
- Varying initial conditions?
- Jets?
  HIJING can explaing much of it
  (Tannenbaum and Mitchell)

Best Bet for Fluctuations

- Correlations in Δy & Δpt for Δφ≈90°
- Use same-sign heavy particles
- Be prepared for ~0.1% level effects
  - Subtract elliptic flow, jets.....

Beware of theory...

- non-quantitative (especially critical phenomena)
- ignores competing effects
- more numerology than physics

Isospin Fluctuations

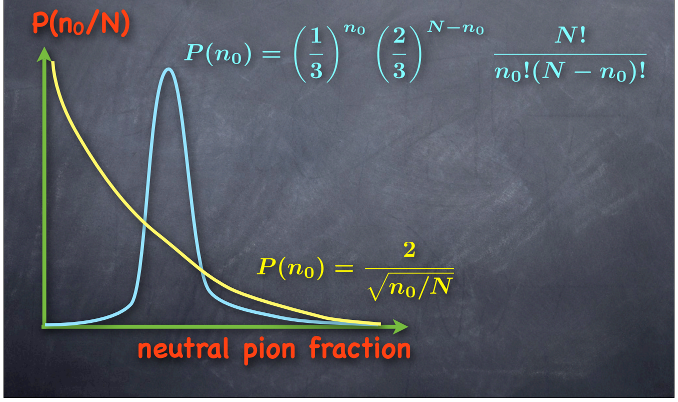
(Disoriented Chiral Condensates)

Scenario (Bjorken-Kowalski-Taylor, Wilczek-Rajagopal)
Motivated by “Centauro” cosmic ray events

- Chiral transition supercools (Baked Alaska)
- Field falls into wrong direction (quench)
- Coherent radiation

$$|\eta\rangle = \exp\left\{ i \int d^4x\, \underbrace{\boxed{\vec{j}(x)}}_{\text{classical field}} \cdot \vec{\pi}(x) \right\} |0\rangle$$

Isospin Distribution



P(n₀/N)

$$P(n_0) = \left(\frac{1}{3}\right)^{n_0}\left(\frac{2}{3}\right)^{N-n_0}\frac{N!}{n_0!(N-n_0)!}$$

$$P(n_0) = \frac{2}{\sqrt{n_0/N}}$$

neutral pion fraction

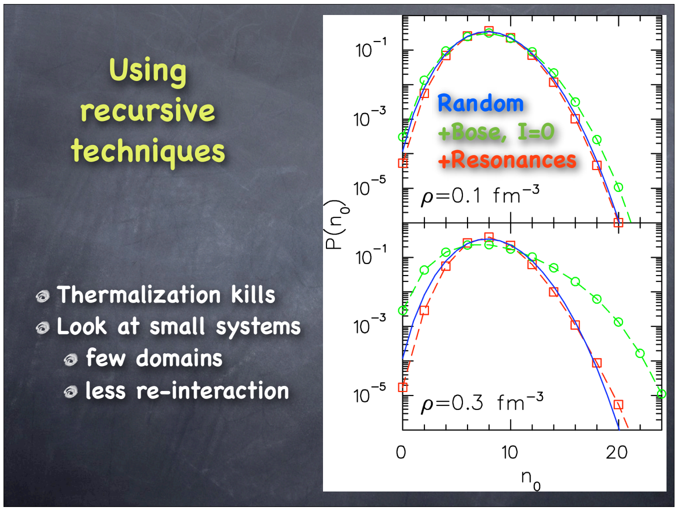
Coherence = 1-level isosinglet

$$|N\rangle = \frac{1}{\sqrt{Z}} \left\{ (a_0^\dagger)^2 + (a_x^\dagger)^2 + (a_y^\dagger)^2 \right\}^{N/2} |0\rangle$$

$$P(n_0) = \frac{2}{\sqrt{n_0/N}}$$

What if?

- Allow many levels
- Allow Resonances
- Maintain overall isosinglet
- Maintain Bose statistics
- Conserve charges



Using recursive techniques

- Thermalization kills
- Look at small systems
  - few domains
  - less re-interaction

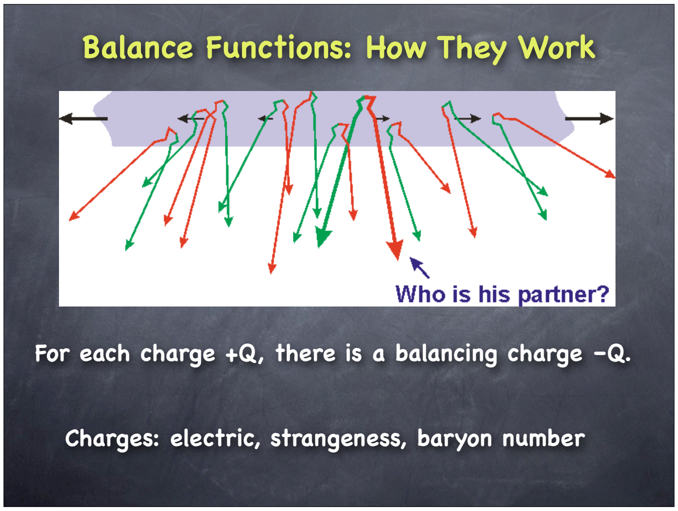
Charge correlations/fluctuations

$$\boxed{\frac{\langle Q^2\rangle}{\langle N_+ + N_-\rangle}} = 1 - \int_0^{\Delta y} d\delta y \, \boxed{B(\delta y)} \, (1 - \delta y/\Delta y)$$

Charge Fluctuations
(Jeon and Koch)

Balance Functions
(Bass, Danielewicz and S.P.)

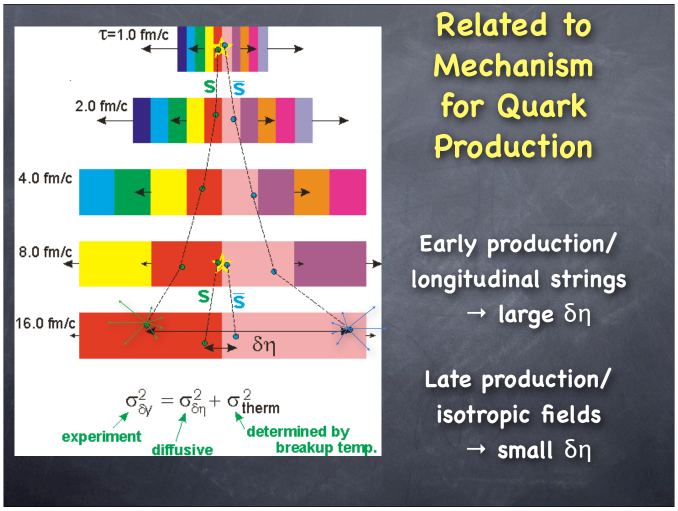
$$B(\delta y) \equiv \frac{1}{2} \left\{ \frac{N_{+-}(\delta y) - N_{++}(\delta y)}{N_+} + \frac{N_{-+}(\delta y) - N_{--}(\delta y)}{N_-} \right\}$$



Balance Functions: How They Work

For each charge +Q, there is a balancing charge -Q.

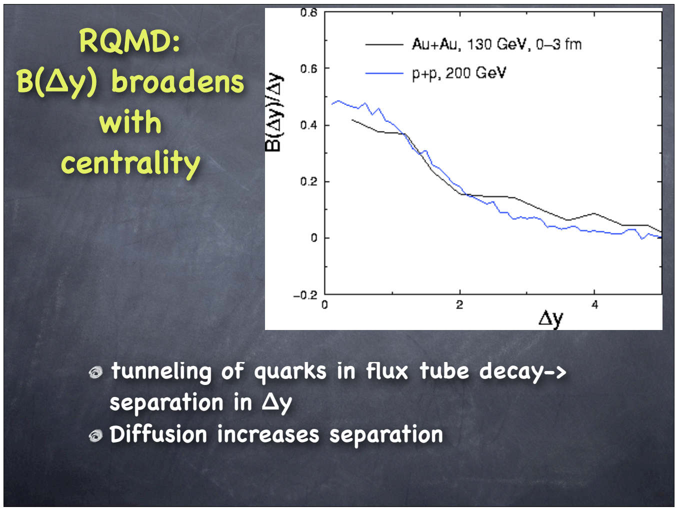
Charges: electric, strangeness, baryon number



Related to Mechanism for Quark Production

Early production/
longitudinal strings
→ large δη

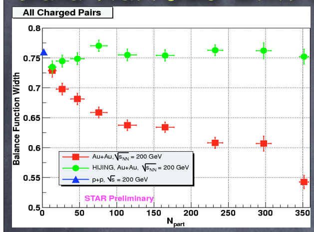
Late production/
isotropic fields
→ small δη

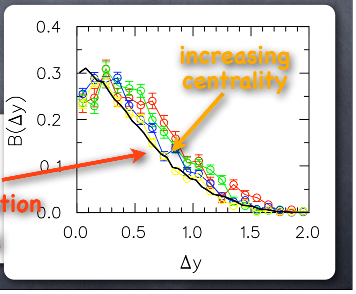


RQMD: B(Δy) broadens with centrality

- tunneling of quarks in flux tube decay-> separation in Δy
- Diffusion increases separation

Data Narrows with Centrality





"canonical"
blast-wave
charge conservation
remains local

Explanation?

- Modified initial gluon-dynamics,
- quarks produced into non-diffusive medium

To Clarify:

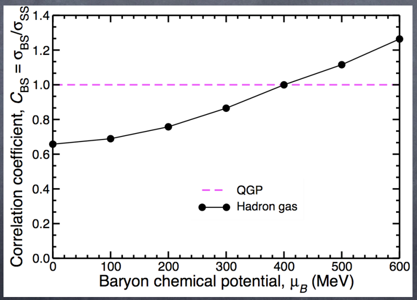
- 3-d analysis
- kaon and proton balance functions

- Requires STAR time-of-flight wall

Strangeness-Baryon Correlations

(Koch,Majumder,Randrup)

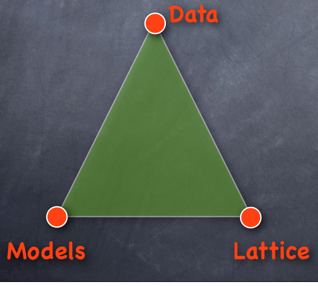
$$C_{BS} \equiv -3\frac{\langle BS\rangle - \langle B\rangle\langle S\rangle}{\langle S^2\rangle - \langle S\rangle^2}$$
$$= 1, \text{ for QGP}$$



Can also be applied to lattice!

Brings up another lesson...

- Don't compare Grand-Canonical models to data
- Simple GC fluctuation models can provide invaluable insight to lattice



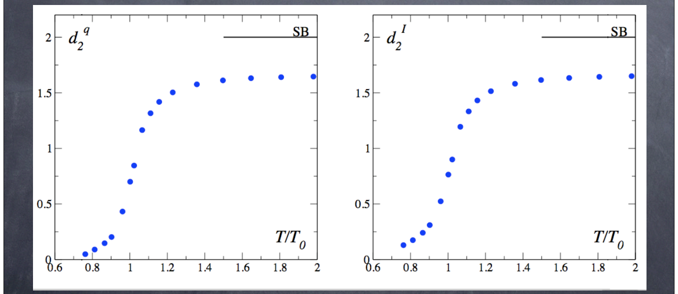
Fluctuations on the Lattice

(Ejiri,Karsch,Redlich)

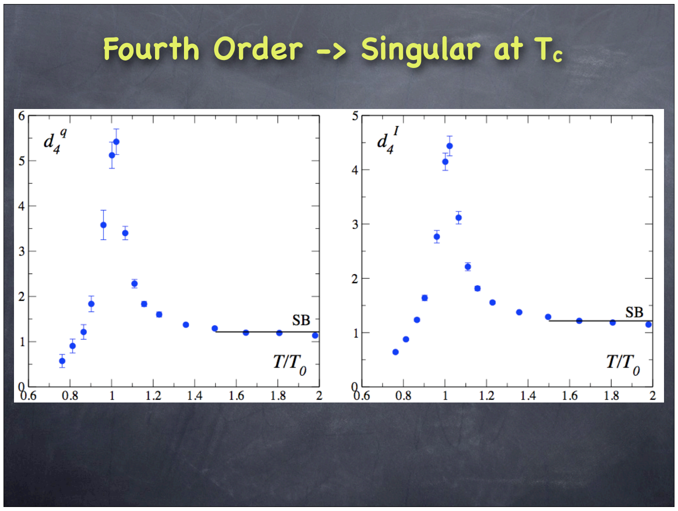
$$VT^3 d^{(2)} \equiv \langle \delta q^2 \rangle = \frac{\partial^2}{\partial(\mu/T)^2} \ln Z$$
$$VT^3 d^{(4)} \equiv \langle \delta q^4 - 3\langle \delta q^2 \rangle^2 = \frac{\partial^4}{\partial(\mu/T)^4} \ln Z$$

Charges: electric, quark #, I_3

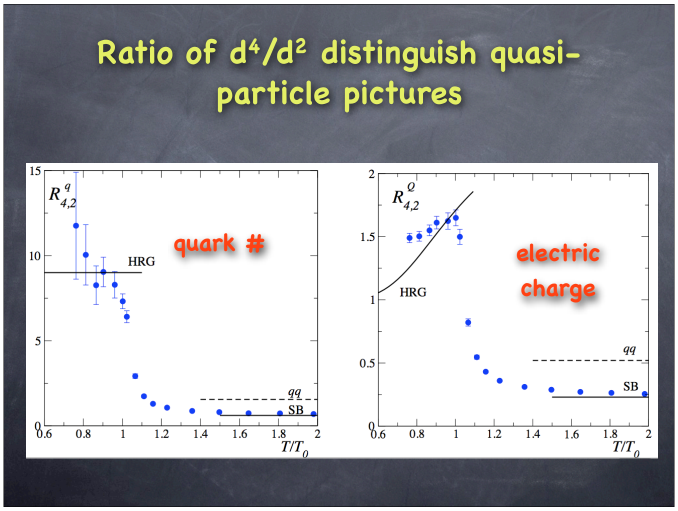
Fluctuations increase at T_c



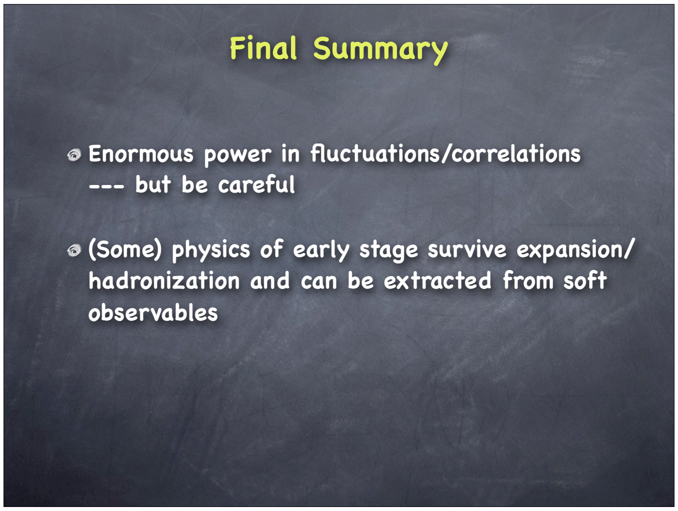
somewhat < simple quark picture



Fourth Order -> Singular at T_c



Ratio of d^4/d^2 distinguish quasi-particle pictures



Final Summary

- Enormous power in fluctuations/correlations --- but be careful
- (Some) physics of early stage survive expansion/ hadronization and can be extracted from soft observables