Correlations and Fluctuations

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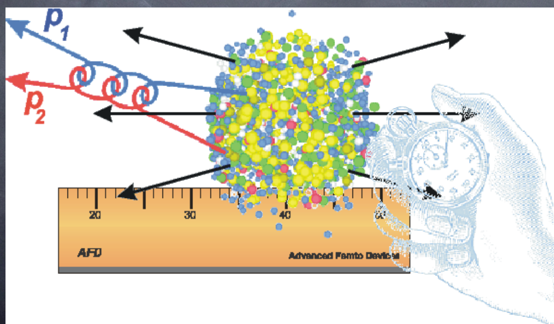
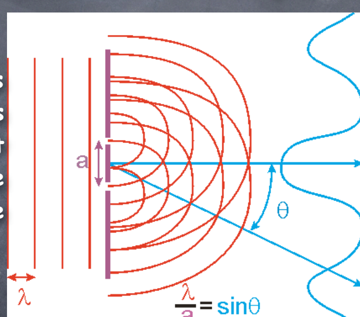


Femtoscopy Topics

- Some Perspective
- Theory
  - Koonin Equation
  - Bose-Einstein/Coulomb/Strong Interactions
  - Shapes and Sizes (Imaging)
  - Multi-particle symmetrization
- Phenomenology
  - Entropy
  - Lifetimes
  - Collective Expansion
- Testing Models
  - Blast Wave
  - Hydro/Micro
  - Eq. of State & Viscosity

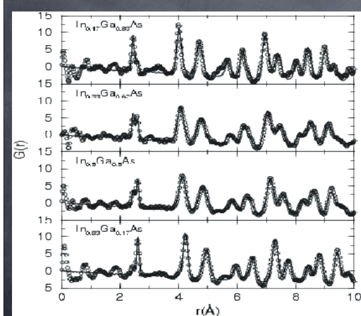
Interference from a coherent probe

Interference survives even if particles sample slit one particle at at time



Two-slit diffraction

Interference from a coherent probe



Peterson et al, cond-mat/0009364

Source could be:

- on-shell photon
- off-shell photon
- neutrons
- off-shell W/Z bosons

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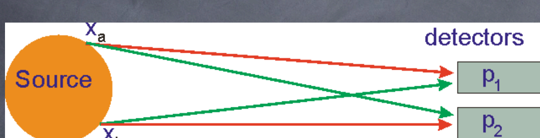
Intensity Interferometry



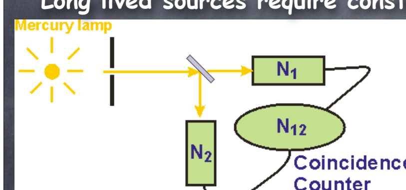
$$\mathcal{A}_{ab}(\vec{p}_1,\vec{p}_2) = \mathcal{T}_a(x_a)\mathcal{T}_b(x_b)\frac{1}{\sqrt{2}}\left\{e^{ip_1\cdot x_a+ip_2\cdot x_b} + e^{ip_1\cdot x_b+ip_2\cdot x_a}\right\}$$

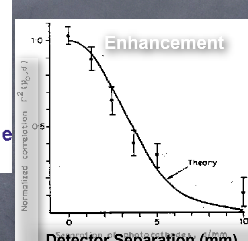
$$I(p_1,p_2) = I(p_1)I(p_2)\left\langle 1+\cos(p_1-p_2)\cdot(x_a-x_b)\right\rangle$$

Interference from identical bosons

HBT R.Hanbury-Brown and R.Twiss, Nature 170, p. 1447 (1956)

Long lived sources require constraint in $t_1 - t_2$





T_c = coincidence time

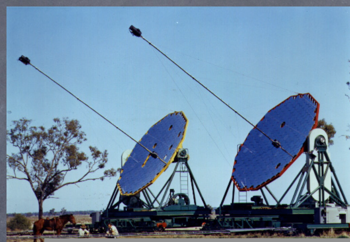
$$N_{12} = N_1 N_2 T_c\left[1+\frac{1}{2}|Y_{12}|^2\frac{T_0}{T_c}\right]$$

T_0 = coherence time

< cos ΔkΔr >

HBT Hanbury-Brown and Twiss, Nature 170, p. 1447 (1956)

HBT can be applied to sources of all sizes



Narrabri Stellar Intensity Interferometer, Australia

- Measured diameter of Sirius in 1962
- Established temperature scales of hot stars

Definition of Correlation Function

$$
\begin{aligned}
C(\vec{P},\vec{q}) &= \frac{N(\vec{p}_1,\vec{p}_2)}{N(\vec{p}_1)N(\vec{p}_2)} \\
&= 1, \text{ if uncorrelated} \\
P &\equiv p_1 + p_2, \quad q \equiv \frac{m_2 p_1 - m_1 p_2}{m_1 + m_2}
\end{aligned}
$$

Usually defined in c.o.m. frame

Sources of Correlation

- Collective flow
- Jets
- Inhomogeneities & instabilities (e.g. droplets)
- Resonances
- Charge conservation
- Final-state interactions

Femtoscopy

Koonin Equation

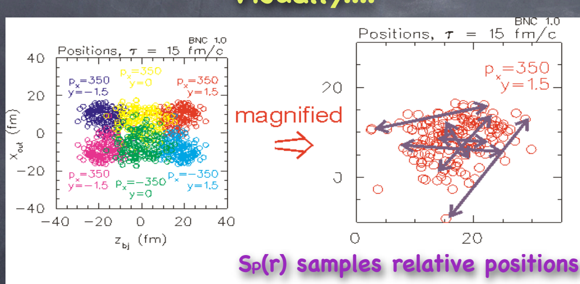
Source Function

Relative Wave Function

$$
\begin{aligned}
C(\vec{P},\vec{q}) &= \int d^3r\, S_{\vec{P}}(\vec{r})\, |\phi(\vec{q},\vec{r})|^2, \\
S_{\vec{P}}(\vec{r}) &\equiv \int d^4x_a d^4x_b\; s_a(\vec{p}_a,\vec{r}_a) s_b(\vec{p}_b,\vec{r}_b)\; \delta(\vec{r}^{\,\prime}_a - \vec{r}^{\,\prime}_b - \vec{r}), \\
\vec{p}_a &\equiv \frac{m_a \vec{P}}{m_a + m_b}, \quad \vec{p}_b = \frac{m_b \vec{P}}{m_a + m_b}
\end{aligned}
$$

- q and r evaluated in c.o.m.

Visually....



$$C(\vec{P},\vec{q}) = \int d^3r\; S_{\vec{P}}(\vec{r})\, |\phi(\vec{q},\vec{r})|^2$$

Goal: Deconvolute C(q) -> S(r)

Deriving Koonin's Equation

The WHOLE truth

$$
\begin{aligned}
N(p_a,p_b,p_c,p_d\cdots) &= \sum_{F_r} \left| \int d^4x_a d^4x_b d^4x_c \cdots T_{F,r}(x_a,x_b,x_c\cdots) \right. \\
&\left. \cdot \frac{1}{\sqrt{N!}} \left(e^{ip_a\cdot x_a + ip_b\cdot x_b + ip_c\cdot x_b} + \text{perms.}\right) \right|^2, \\
N(p_a,p_b) &= \int d^3p_c d^3p_d \cdots N(p_a,p_b,p_c\cdots)
\end{aligned}
$$

At some point, every particle acts like plane wave

Approximation #1
Ignore symmetrization with others

$$N(p_a,p_b) = \sum_{F_r}\left|\int d^4x_a d^4x_b T_{F,r}(x_a,x_b) \cdot\frac{1}{\sqrt{2}}\left(e^{ip_a\cdot x_a+ip_b\cdot x_b+ip_c\cdot x_b}+e^{ip_b\cdot x_a+ip_a\cdot x_b}\right)\right|^2$$

OR

$$= \sum_{F_r}\left|\int d^4x_a d^4x_b T_{F,r}(x_a,x_b) \cdot\frac{1}{\sqrt{2}}\left[U(x_a,x_b;p_a,p_b)+U(x_a,x_b;p_b,p_a)\right]\right|^2$$

Evolution matrices

Approximation #2
Uncorrelated Emission

$$\sum_{Fr} T^*_{Fr}(x'_a,x'_b)T_{Fr}(x_a,x_b)$$

$$\rightarrow \sum_{Fa} T^*_a(x'_a)T_a(x_a)\ \sum_{Fb} T^*_b(x'_b)T_b(x_b)$$

Allows definition of s(p,x):

$$s_a(p,x) \equiv \sum_{Fa}\int d^4\delta x\ T^*_a(x+\delta x/2)T_a(x-\delta x/2)e^{ip\cdot\delta x}$$

$$N_a(\vec{p}) = \int d^4x\ \ s_a(p,x)|_{p_0=E_p}$$

Approximation #3 (identical part.s)
smoothness approx.

$$C(p_a,p_b) = 1 + \frac{\int d^4x_a d^4x_b\ s\left(\frac{p_a+p_b}{2},x_a\right) s\left(\frac{p_a+p_b}{2},x_b\right)\left[\cos(p_a-p_b)\cdot(x_a-x_b)\right]}{N(p_a)N(p_b)}$$

Smoothness approximation (q is small)

$$\frac{E_a+E_b}{2} \rightarrow E(\vec{P}/2), \qquad N(p_a)N(p_b) \rightarrow N(\vec{P}/2)$$

↓

Koonin's Equation

Approximations 3,4 (potential int.s)

3. Smoothness approximation

$$s_a\left(\frac{P}{2}+\delta p,x_a\right)s_b\left(\frac{P}{2}-\delta p,x_b\right) \approx s_a\left(E(\vec{P}/2),\vec{P}/2,x_a\right)s_b\left(E(\vec{P}/2),\vec{P}/2,x_b\right)$$

characteristic scale of $|\varphi|^2$

4. Equal time approximation

$$U(\delta t,\vec{r_a},-\delta t,\vec{r_b};F) \approx e^{-iP\cdot R}\phi(\vec{q},\vec{r})$$

↓

Koonin Equation

Validity of Koonin Equation

$$C(\vec{P},\vec{q}) = \int d^3r\ S_{\vec{P}}(\vec{r})\left|\phi(\vec{q},\vec{r})\right|^2$$

1. Ignore multi-part symmetrization:
   Excellent for low phase space density (p_t>200 MeV/c)
2. Uncorrelated emission:
   Excellent for heavy ions
3. Smoothness:
   Excellent for large hot sources (bad for pp collisions)
4. Equal times:
   irrelevant for Bose interference,
   consistent with statistical equilibrium

For RHIC/LHC: dimensions believable to:
pions ~5%, pp ~10%, $K^+K^-\leftrightarrow\Phi$ could be poor, ...

Inverting the Koonin Equation:

$$R(\vec{q}) = \int d^3r\ K(q,r,\cos\theta_{qr})S(\vec{r}),$$

$$R(\vec{q}) \equiv C(\vec{q})-1$$

$$K(q,r,\cos\theta_{qr}) \equiv \left|\phi(q,r,\cos\theta_{qr})\right|^2-1$$

Kernel

- Total momentum P suppressed
- Similar to matrix inversion

Inversion: Bose interference

$$K(q, r, cos\theta_{qr}) = \cos(\vec{q}\cdot\vec{r})$$

- Simple Fourier transform
- Both S(r) and R(q) are even functions

Inversion: potential interactions

- Typically singular
- Projection into harmonics useful
- $|\varphi|^2$ properties related to phase shifts

Angular projections

$$R(\vec{q}) = \sum_{\vec{\ell}} R_{\vec{\ell}}(q)\hat{q}_x^{\ell_x}\hat{q}_y^{\ell_y}\hat{q}_z^{\ell_z}$$

$$R_{\vec{\ell}}(q) = \frac{(2\ell+1)!!}{\ell!}\int\frac{d\Omega}{4\pi}R(\vec{q})\mathcal{A}_{\vec{\ell}}(\Omega)$$

$$A_{0,0,0} = 1,\quad A_{1,0,0}(\hat{n}) = \hat{n}_x,\quad A_{2,0,0}(\hat{n}) = \hat{n}_x^2 - 1/3, \cdots$$

↓

$$R_{\vec{\ell}}(q) = \int 4\pi r^2 dr\ K_\ell(q,r)S_{\vec{\ell}}(r),$$

$$K_\ell(q,r) = \frac{1}{2}\int d\cos\theta_{qr}K(q,r,\cos\theta_{qr})P_\ell(\cos\theta_{qr})$$

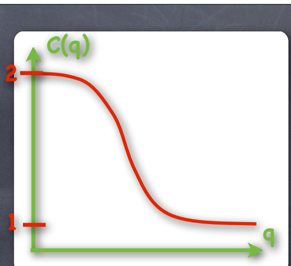
Spherical harmonics: (l_x,l_y,l_z)->(l,m)

Koonin Eq. (Projection Version)

$$R_{\vec{\ell}}(q) = \int 4\pi r^2 dr\ K_\ell(q,r)S_{\vec{\ell}}(r)$$

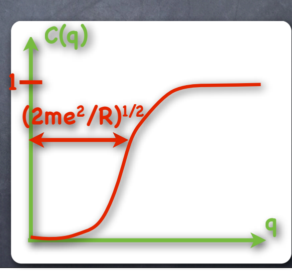
- Reduced to 1-d problems
- Convenient views of 3-d information
- Resolving power of kernels??

3 Classes of Interaction



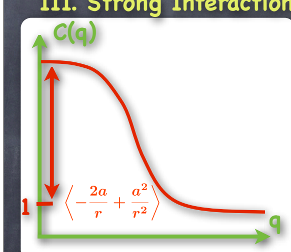
I. Bose Interference

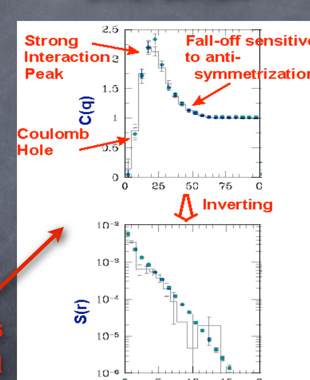
II. Coulomb



3 Classes of Interaction

III. Strong Interaction





pp correlations have it all

Kernels

- Bose Interference:
  - easily invertible

$$K_\ell(q,r) = j_\ell(qr)$$

- Strong interactions
  - depend on phase shifts

$$\phi(\vec{q},\vec{r}) = \sum_\ell (2\ell+1) i^\ell R_\ell(r) P_\ell(\cos\theta_{qr})$$

$$R_\ell(r > L_{int}) = \frac{1}{2}\left[e^{2i\delta_\ell(q)} h_\ell(qr) + h^*_\ell(qr)\right]$$

  - $K_l \neq 0$ for all l, even if $\delta_l = 0$
  - $K_l \sim r^l$
- Coulomb
  - funky quantum mechanics

Phase Shifts, Wave Functions and the Density of States

$$\phi_\ell(q,r) \sim \sin(qr+\delta), \quad \therefore qR + \delta = n\pi,$$

$$\Delta\frac{dn}{dq} = \frac{1}{\pi}\sum_\ell (2\ell+1)\frac{d\delta_\ell}{dq}$$

Furthermore

$$\int dr \left\{|\phi_\ell(q,r)|^2 - |\phi^{(0)}_\ell(q,r)|^2\right\} = \frac{1}{2}\frac{d\delta_\ell}{dq}$$

For qR>>1, Koonin Eq. equivalent to thermal equilibrium

Funky Coulomb Stuff

- Non-analytic for q,r -> 0
- Coulomb effects can be approximately factored away for ππ,

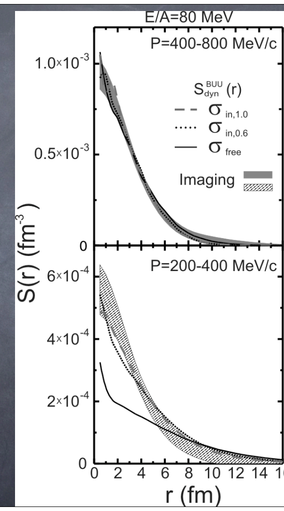
$$\text{For } R << a_0 \equiv \frac{1}{me^2},$$

$$|\phi|^2 \to G(\eta)|\phi_0|^2,$$

$$G(\eta) = \frac{2\pi\eta}{e^{2\pi\eta}-1}, \quad \eta = 1/qa_0$$

Imaging (Resolving Power)

- Determining $S_0(r)$
  - ππ, KK ~ 3-4 parameters
  - others ~ 2-3 parameters
- Determining Shape
  - ππ, l=0,2,4,6
  - others, l=0,1,2(3)



Random Topic #1
A Classical Perspective for Wave Functions

$$|\phi(q,r)|^2 \approx \frac{d^3q_0}{d^3q}, \quad \text{as } qr >> 1$$

$$\frac{q_0^2}{2m} + V(r) = \frac{q^2}{2m}$$

$$\frac{q_0^2 dq_0}{q^2 dq} = \sqrt{1 - \frac{2mV(r)}{q^2}}$$

Random Topic #2
Multi-Particle Symmetrization

Consider system with A bosons

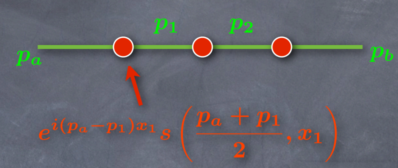
$$N(p_1\cdots p_A) = \sum_{Fn}\left|\int dx_1\cdots dx_n T_1(x_1)\cdots T_A(x_A)\right.$$

$$\left.\frac{1}{A!}\sum_{\text{Perm.s } P}\left\{e^{ip_1x_1+\cdots p_Ax_A} + \cdots e^{ip_1x_1^{(P)}+\cdots p_Ax_A^{(P)}}\right\}\right|^2$$

100! terms YIKES!!!!!!

$$N(p_1) = \int dp_2\cdots dp_A N(p_1\cdots p_A)$$

Permutation Diagrams

$G_3(p_a, p_b) =$ 

$C_3 =$ 

$$Z_A = \sum_n \frac{1}{n} C_n Z_{A-n}$$

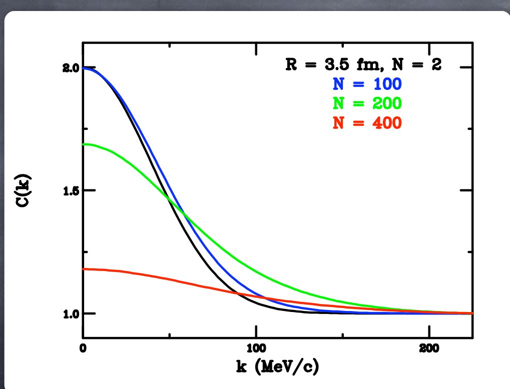
Analytic integrals for Gaussian or box

Spectra and Correlations

$$N(p) = \sum_n \frac{G_n(p) Z_{A-n}}{Z_A}$$

$$N(p_1, p_2) = \sum_{m,n} \frac{G_m(p_1) G_n(p_2) Z_{A-m-n}}{Z_A}$$

Distortions to Correlations



Distorted Multiplicity Distribution