Thermodynamics and phase transitions in effective models for cold and dense matter

Letícia F. Palhares
Eduardo S. Fraga

(IF-UFRJ)

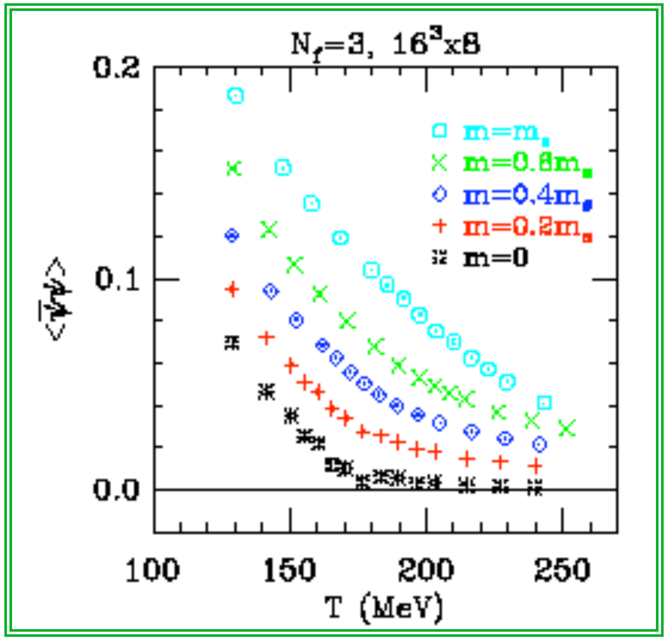
Quark masses bring important modifications to the QCD phase diagram:

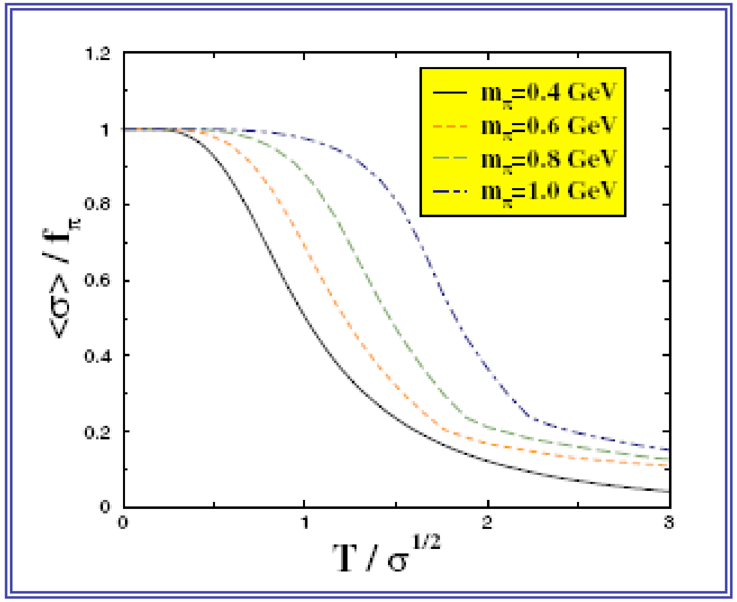
⇒ in the critical region **[Stephanov (2006)]**

⇒ in color superconductivity **[Alford *et al* (2008)]**

Lattice ⇒ sensible corrections in the chiral condensate

[Bernard *et al* – MILC collab. (2003)]





Effective models ⇒ quark mass effects are also relevant for the chiral transition

$$T_{\mathbf{crossover}}(m_q)$$

[Dumitru, Röder & Ruppert (2004)]

What about the μ axis? $\mu_{1^{st}\text{order}}(m_q)$?

➔ **possible important astrophysical implications**

[Fraga, Pisarski & Schaffner-Bielich (2001/2002)]

- **Perturbative QCD thermodynamics**:

  2-loop pressure in QCD at finite mu:

  $\Rightarrow$ Quark mass and RG corrections up to ~25%

  [Fraga & Romatschke (2005)]

- **Complementary study:**

  Low energy effective models at finite density

  $\Rightarrow$ perturbative treatment

  $\Rightarrow$ Thermodynamics + Phase Transition (condensate)

  $\Rightarrow$ mass and scale dependence effects

Outline

- **Theory without SSB:**

  Thermodynamics of the **Scalar Yukawa Theory**

  - **Mass and RG running effects**

- **Theory with SSB:**

  Chiral Phase Transition within the **Linear σ Model**

  - **Effects from interaction and fluctuations beyond mean-field**
  - **Logarithmic corrections: renormalization scale dependence**

- **Conclusions and Perspectives**

Thermodynamics of the cold and dense Yukawa Theory

$$\mathcal{L}_{\mathrm{int}} = \sum_{\alpha=1}^{N_F} g\,\overline{\psi}_\alpha \psi_\alpha \phi$$

Fermions interacting via scalar bosonic mediators ⇒ simplest renormalizable coupling.

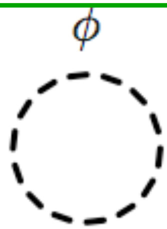
[Important in both effective and fundamental (Higgs) descriptions]

- **We calculate perturbatively the thermodynamic potential including:**
  - Interaction up to 2-loop order, without resummation (to this order, results are valid also for self-interacting bosons);
  - Fermions and bosons both massive;
  - Renormalization in the $\overline{\mathrm{MS}}$ scheme;
  - Influence of the complete RG flow (coupling and masses).
- **General analytical results.** **[LFP & Fraga (2008)]**

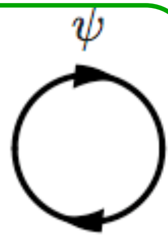
The thermodynamic potential

$$\Omega_Y \equiv -\frac{1}{\beta V} \ln Z_Y$$

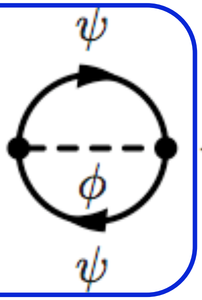
$$= -\frac{1}{\beta V}$$



$$+ \frac{1}{\beta V} N_F$$



$$+ \frac{1}{2}\frac{1}{\beta V} N_F$$



$$+$$

$$+ \text{[diagrams with counterterms]} + O(3\ loops),$$

Quantum Free Gas

Cancelation of UV divergences

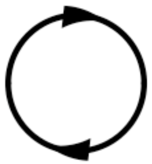
Exchange term
(2 massive fields)

Free Gas $(\mu \neq 0\,,\, T \to 0)$

ϕ

$$= V \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left[-\frac{1}{2}\,\beta\omega - \ln\left(1 - e^{-\beta\omega}\right) \right] \xrightarrow[\text{no ZPT}]{\beta \to \infty} 0$$

ψ



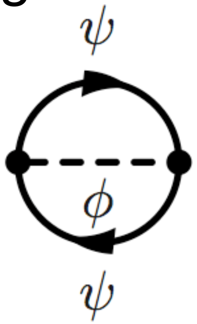
$$= -2V \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left[\beta E_{\mathbf{p}} + \ln\left(1 + e^{-\beta(E_{\mathbf{p}} - \mu)}\right) + \ln\left(1 + e^{-\beta(E_{\mathbf{p}} + \mu)}\right) \right]$$

$$\xrightarrow[\text{no ZPT}]{\beta \to \infty} -\beta V \,\frac{1}{24\pi^2} \left[2\,\mu\, p_f^3 - 3\, m^2\, u \right] \qquad u \equiv \mu p_f - m^2 \log\left(\frac{\mu + p_f}{m}\right)$$

Zero Point Terms (ZPT): in this case, they are purely vacuum contributions, which can be absorbed through a redefinition of the zero of Ω_Y.

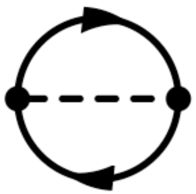
2-loop order: exchange diagram

Using in-medium Feynman rules:



$$= \beta V\, g^2 \sum\!\!\!\!\!\!\int_{P_1,P_2,K} \mathrm{Tr}\left[\frac{(2\pi)^3\beta\,\delta^{(4)}(K-P_1+P_2)}{(\not{P}_1-m)(m_\phi^2-K^2)(\not{P}_2-m)}\right]$$

After solving the 3 Matsubara Sums, in the cold and dense regime:



MEDIUM $(\sim n_f^2)$ $\quad (\sim n_f)$ $\quad$ VACUUM

$$\xrightarrow{\beta\to\infty} \boxed{-\beta V\, g^2\, J_1} + \boxed{\lim_{\beta\to\infty} \mathbf{L}_f} + \boxed{\mathcal{V}}$$

MEDIUM: UV finite

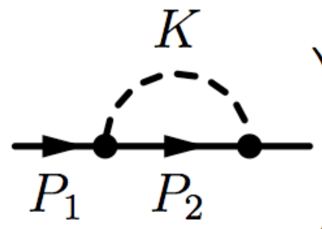
MEDIUM + VACUUM: UV divergent and medium-dependent ⇒ renormalization

VACUUM: UV divergent ZPT ⇒ reabsorption/subtraction

Renormalization:

- The divergence present in the medium-dependent term has its origin in a vacuum subdiagram: the fermionic self-energy.

$$\mathbf{L}_f = -2\,\beta V \left\{ \sum\!\!\!\!\!\!\!\int_P (-1)\,\mathrm{Tr}\left[\frac{1}{\not P - m} \left(i \;\; \right)^{\mathrm{VAC}}_{\mathrm{AMP\ M.S.}} \right] \right\}_{\mathrm{MATT}}$$



Renorm. $\overline{\mathrm{MS}}$ →

$$\lim_{\beta\to\infty} \mathbf{L}_f^{\mathrm{REN}} = -2\,\beta V \frac{g^2}{64\pi^4}\, m^2\, u\, [2\,\alpha_1]$$

$$\alpha_1 = -4\frac{m_\phi}{m}\left(1-\frac{m_\phi^2}{4m^2}\right)^{\frac{3}{2}} \left\{ \tan^{-1}\left[\sqrt{\frac{1}{\frac{4m^2}{m_\phi^2}-1}}\right] + \tan^{-1}\left[\frac{\frac{1}{2}-\frac{m_\phi^2}{4m^2}}{\sqrt{\frac{m_\phi^2}{4m^2}}\sqrt{1-\frac{m_\phi^2}{4m^2}}}\right]\right\}$$
$$+\frac{7}{2}-\frac{m_\phi^2}{2m^2}-\frac{3}{2}\log\left(\frac{m^2}{\Lambda^2}\right)+\frac{m_\phi^2}{m^2}\left(\frac{3}{2}-\frac{m_\phi^2}{4m^2}\right)\log\left(\frac{m^2}{m_\phi^2}\right),$$

Purely medium term ⇒ The integral J_1:

$$J_1 = \int \frac{d^3\vec{p}_1 d^3\vec{p}_2}{(2\pi)^6} \frac{\theta(\mu - E_1)\theta(\mu - E_2)}{2E_1E_2} \left\{ \frac{4m^2 - m_\phi^2}{(E_1 - E_2)^2 - |\vec{p}_1 - \vec{p}_2|^2 - m_\phi^2} - 1 \right\}$$

In general, the inclusion of finite masses brings about much complication to the analytical calculation.

However, mass effects are relevant

⇒ in the QCD phase diagram;

⇒ in the comparison with lattice and experiments, increasingly precise **[Lattice07, Boels *et al* (2007)]**;

⇒ when there are effects that alter the masses (e.g. condensates); ...

Here we calculate *analytically* the integral J_1, and the thermodynamic potential, for *arbitrary* values of m, m_ϕ and μ.

Result for the 2-loop thermodynamic potential

$$\Omega_Y = -N_F \frac{1}{24\pi^2}\left[2\,\mu p_f^3 - 3m^2\,u\right] - \frac{1}{2}N_F\,g^2\left[J_1 + \frac{1}{16\pi^4}m^2 u\alpha_1\right]$$

$$u \equiv \mu p_f - m^2 \log\left(\frac{\mu+p_f}{m}\right) \qquad J_1 = \frac{1}{(2\pi)^4}\left\{2m^2\left(1-\frac{m_\phi^2}{4m^2}\right)\mathcal{I}_I - \frac{1}{2}\,u^2\right\}$$

$$\mathcal{I}_I = \mu^2\frac{m_\phi^2}{2m^2}\log\left[\frac{m_\phi^2}{4p_f^2+m_\phi^2}\right] + \left(1-\frac{m_\phi^2}{2m^2}\right)\frac{u^2-\mu^2\,p_f^2}{m^2} + \\ + \frac{m_\phi}{m}\sqrt{1-\frac{m_\phi^2}{4m^2}}\,(\mu\,p_f+u)\,D_{\tan} + p_f^2 - 2m^2\,\overline{\mathcal{K}}_{log}\left(\frac{p_f}{\mu+m},\frac{m_\phi^2}{4m^2}\right)$$

$$D_{\tan} \equiv \tan^{-1}\left(\frac{-p_f m_\phi}{2m(\mu+m)\sqrt{1-\frac{m_\phi^2}{4m^2}}}\right) + \tan^{-1}\left(\frac{p_f}{m_\phi\sqrt{1-\frac{m_\phi^2}{4m^2}}}\left[2-\frac{m_\phi^2}{2m(\mu+m)}\right]\right)$$

$$\overline{\mathcal{K}}_{\log}(\bar{x},z) = \sqrt{z}\,\Delta(\bar{x},z) + \frac{1}{2}\sqrt{z(z-1)}\,\mathcal{C}_{\mathrm{Li}}(\bar{x},z)$$

[LFP & Fraga (2008)]

Result for the 2-loop thermodynamic potential (cont.)

$$\overline{\mathcal{K}}_{\log}(\bar{x}, z) = \sqrt{z}\, \Delta(\bar{x}, z) + \frac{1}{2}\sqrt{z(z-1)}\, \mathcal{C}_{\mathrm{Li}}(\bar{x}, z)$$

$$\mathcal{C}_{\mathrm{Li}}(\bar{x}, z) = \Bigg[\mathbf{Li}_2\left(1-\left(z-\sqrt{z(z-1)}\right)(1-\bar{x})\right) - \mathbf{Li}_2\left(1-\left(z+\sqrt{z(z-1)}\right)(1-\bar{x})\right) +$$

$$+\mathbf{Li}_2\left(\frac{1-\bar{x}}{1+\bar{x}}\left[1-\left(z+\sqrt{z(z-1)}\right)(1-\bar{x})\right]\right) - \mathbf{Li}_2\left(\frac{1-\bar{x}}{1+\bar{x}}\left[1-\left(z-\sqrt{z(z-1)}\right)(1-\bar{x})\right]\right)\Bigg] + \Big[\bar{x} \mapsto -\bar{x}\Big]$$

$$\Delta(\bar{x}, z) = \begin{cases} \Delta_<(\bar{x}, z) & \text{, if } z < 1 \\ \Delta_>(\bar{x}, z) & \text{, if } z > 1 \end{cases}$$

$$\Delta_<(\bar{x}, z) = \sqrt{1-z}\Bigg\{\log(1-\bar{x})\left[\tan^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{z}}\right) - \tan^{-1}\left(\frac{\sqrt{z(1-z)}}{z+\left(\frac{1+\bar{x}^2}{2\bar{x}}-1\right)^{-1}}\right)\right] +$$

$$+\log(1+\bar{x})\left[\tan^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{z}}\right) - \tan^{-1}\left(\frac{\sqrt{z(1-z)}}{z+\left(\frac{1+\bar{x}^2}{2\bar{x}}+1\right)^{-1}}\right) - \pi\left[1-\theta\left(mz\frac{(1+\bar{x})^2}{1-\bar{x}^2} - \frac{2m\bar{x}}{1-\bar{x}^2}\right)\right]\right]\Bigg\}$$

$$\Delta_>(\bar{x}, z) = \frac{1}{2}\sqrt{z-1}\Bigg\{\log\left(\frac{1-\bar{x}^2}{(1+\bar{x})^2}\right)\log\left(\frac{|\sqrt{z}(1+\bar{x}^2) - 2\bar{x}\sqrt{z-1}|}{|\sqrt{z}(1+\bar{x}^2) + 2\bar{x}\sqrt{z-1}|}\right) + \pi^2\Bigg[-\theta[z(1-\bar{x}^2)-1] -$$

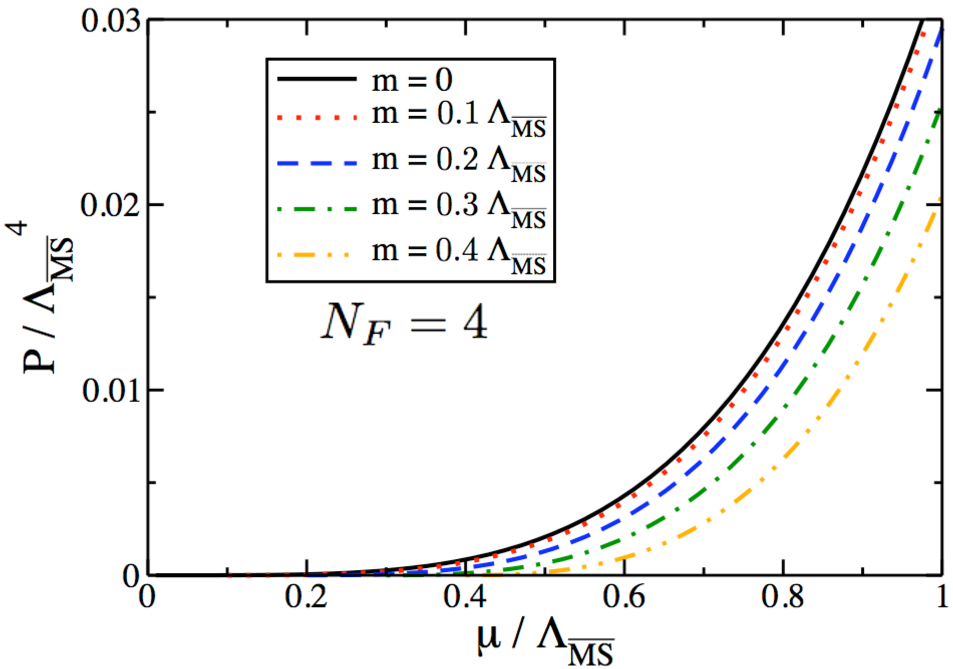
$$-\theta\left(1+\frac{\bar{x}}{(1-\bar{x}^2)\sqrt{z(z-1)}}[2-z(1-\bar{x}^2)]\right) + \theta\left(1-\frac{\bar{x}}{(1-\bar{x}^2)\sqrt{z(z-1)}}[2-z(1-\bar{x}^2)]\right) + 1\Bigg]\Bigg\}$$

Pressure of the cold and dense Yukawa Theory ($m_\phi = 0$)

For $m_\phi = 0$, the pressure $P = -\Omega_Y$ assumes the much simpler form:

$$\Omega_Y = -N_F \frac{1}{24\pi^2}\left[2\,\mu p_f^3 - 3m^2\,u\right] - N_F \frac{g^2}{64\pi^4}\left\{3\,u^2 - 4\,p_f^4 + m^2\,u\left[7 - 3\log\left(\frac{m^2}{\Lambda^2}\right)\right]\right\}$$

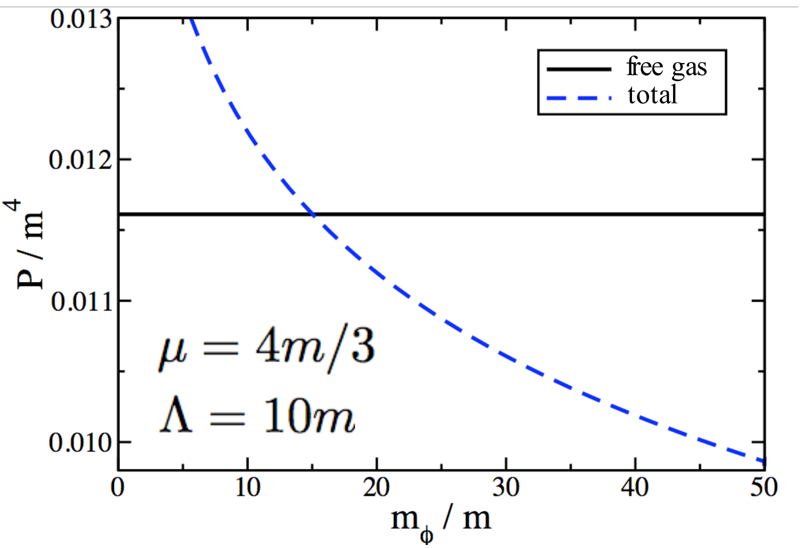
⇒ For massless fermions, the dependence on the scale Λ arises only at higher orders.



→ Significant mass effects

Possible large mass effects within effective models of cold and dense matter.

Pressure of the cold and dense Yukawa Theory $(m_\phi \neq 0)$

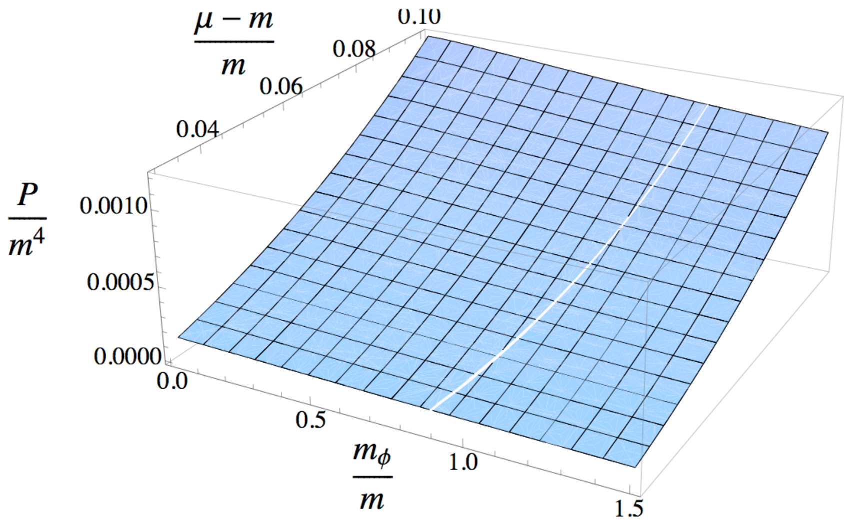


➔ Cold and dense case: quantum Fermi pressure

➔ Interaction mediated by massive boson generates fermionic quasi-particles with effective masses:

$$\begin{cases} \text{lower, for} & m_\phi/m \lesssim 15 \\ \text{higher, for} & m_\phi/m \gtrsim 15 \end{cases}$$

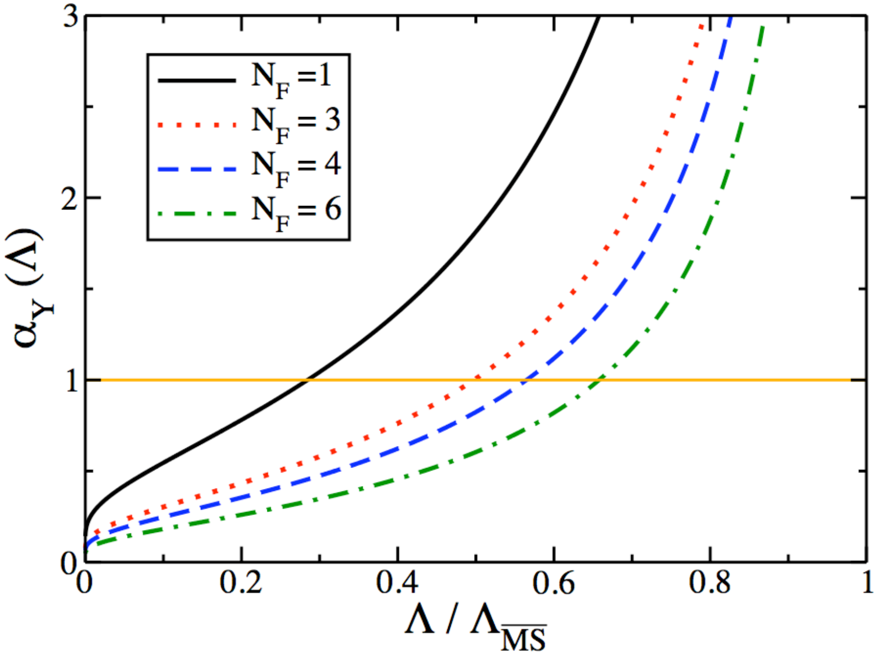
The effect is intensified as the density increases. ⬅



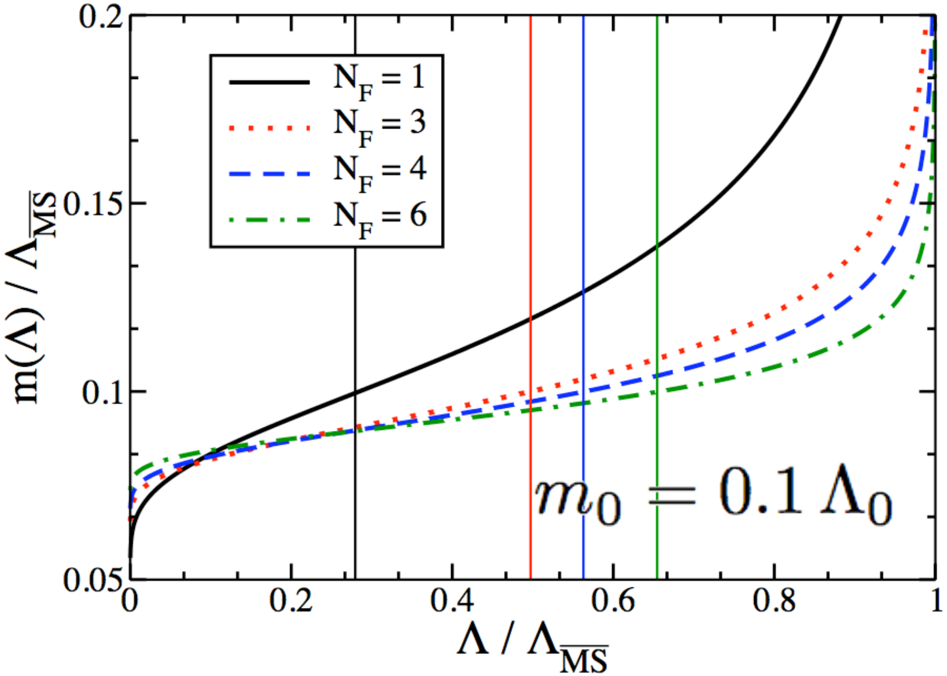
Strong quantitative dependence on the renormalization scale Λ.

RG flow in the scalar Yukawa Theory

Coupling: $\alpha_Y = g^2/4\pi$



Fermion mass:



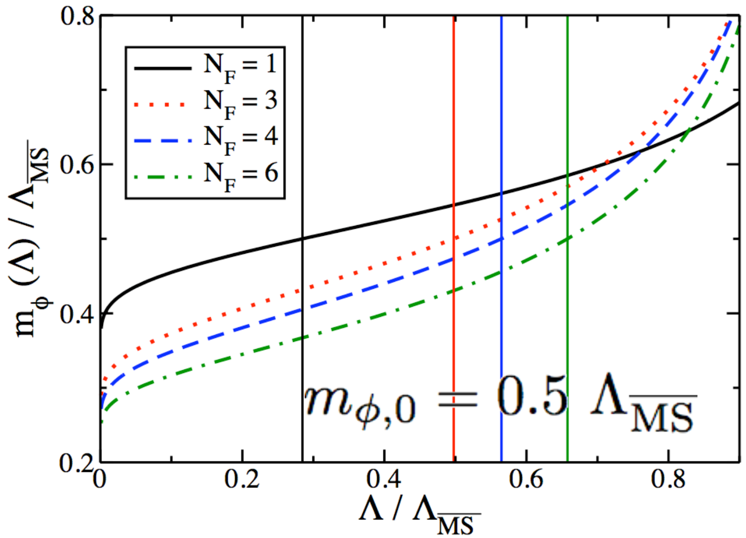
In the large N_F limit:

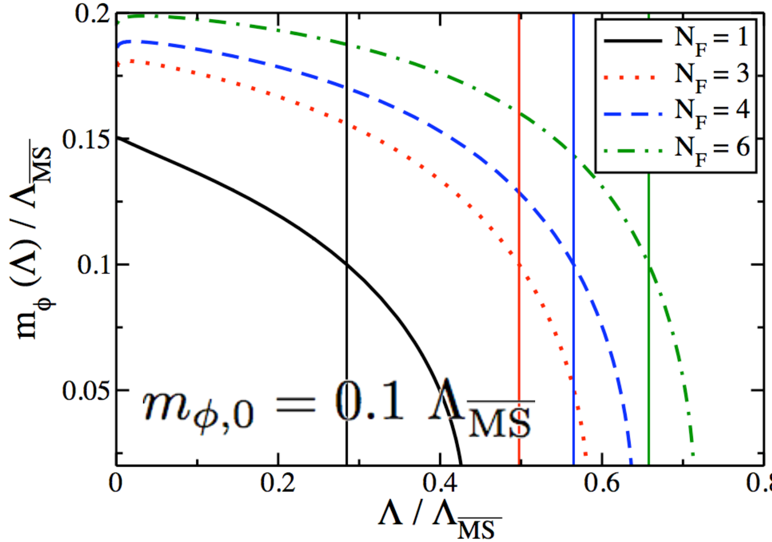
- **Larger perturbative domain**
- **The fermion mass tends to become scale-invariant**

Mediator mass RG flow

$$m_\phi^2(\Lambda) = m^2(\Lambda)\left\{\frac{12N_F}{2N_F+3} + \mathcal{C}\left[\log\left(\frac{\Lambda}{\Lambda_{\overline{MS}}}\right)\right]^{-\frac{2N_F-3}{2N_F+3}}\right\}$$

$$\mathcal{C} \;=\; \left[\log\left(\frac{\Lambda_0}{\Lambda_{\overline{MS}}}\right)\right]^{\frac{2N_F-3}{2N_F+3}}\left\{\frac{m_{\phi,0}^2}{m_0^2} - \frac{12N_F}{2N_F-3}\right\} \quad ; \; m_0 = 0.1\ \Lambda_{\overline{MS}}$$



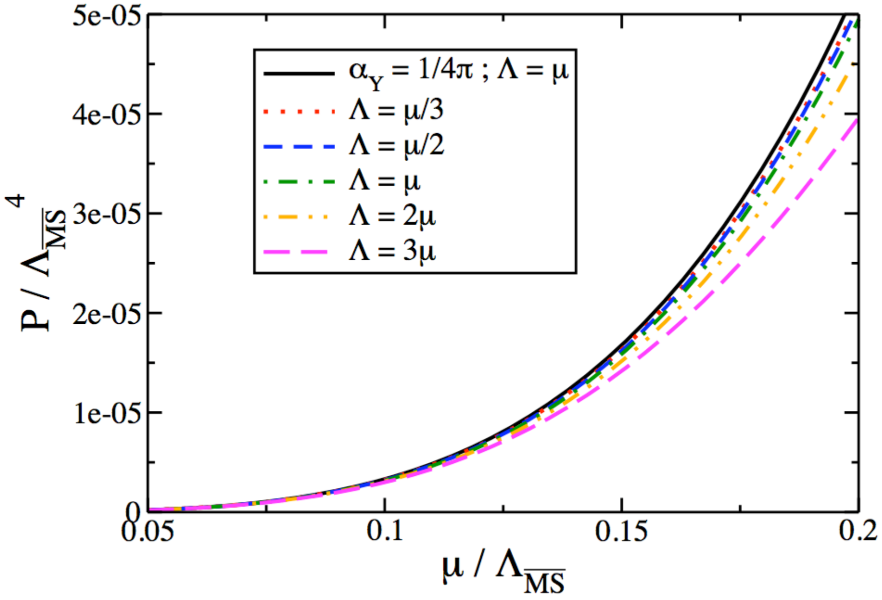


Competition between masses ⇒ 2 regimes of bosonic mass flow

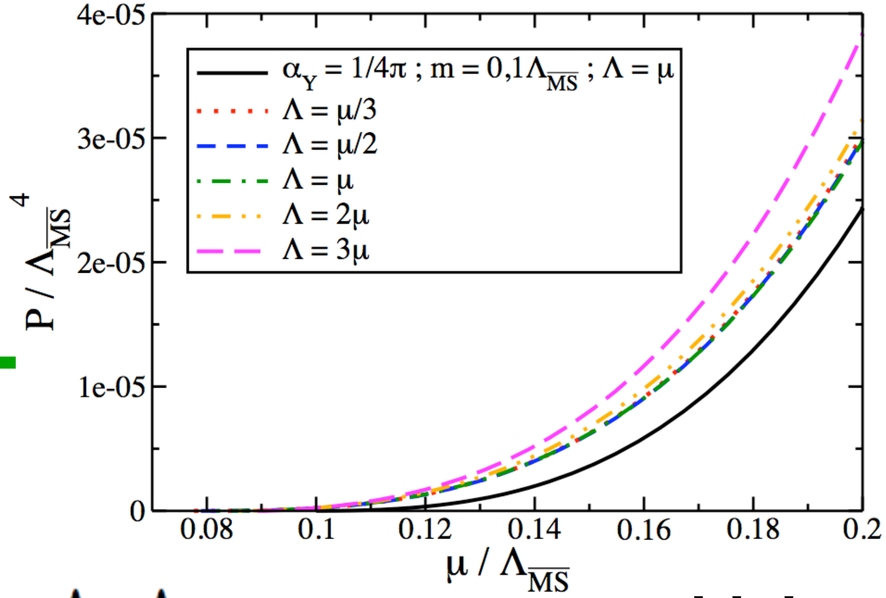
$$\frac{\partial\, m_\phi^2}{\partial \log\Lambda} = \alpha_Y \frac{N_F}{\pi}\left[m_\phi^2 - 6m^2\right]$$

Influence of the RG flow on the Thermodynamics

$$g\,,\,m\,,\,m_\phi \;\mapsto\; g(\Lambda)\,,\,m(\Lambda)\,,\,m_\phi(\Lambda)$$



- ➔ $m_\phi = 0$ and $m = 0$
- ➔ The running coupling decreases the pressure.



The inclusion of $m(\Lambda)$ reverts ⇐ the effect.

Freedom of choice of the scales $\Lambda, \Lambda_{\overline{MS}}, m_0, m_{\phi,0}$, which depend on the specificities of the physical system.

The Linear σ Model

$$
\begin{aligned}
\mathcal{L}_{\mathrm{L}\sigma\mathrm{M}} &= \overline{\psi}\left[i\partial\!\!\!/ - m_q\right]\psi + \frac{1}{2}(\partial_\mu\vec{\pi})\cdot(\partial^\mu\vec{\pi}) + \frac{1}{2}(\partial_\mu\sigma)(\partial^\mu\sigma) \\
&\quad - g\overline{\psi}\left[i\gamma^5\vec{\mathbf{t}}\cdot\vec{\pi} + \sigma\right]\psi - \mathcal{V}(\vec{\pi},\sigma)\,,
\end{aligned}
$$

quark-meson coupling: Yukawa-type

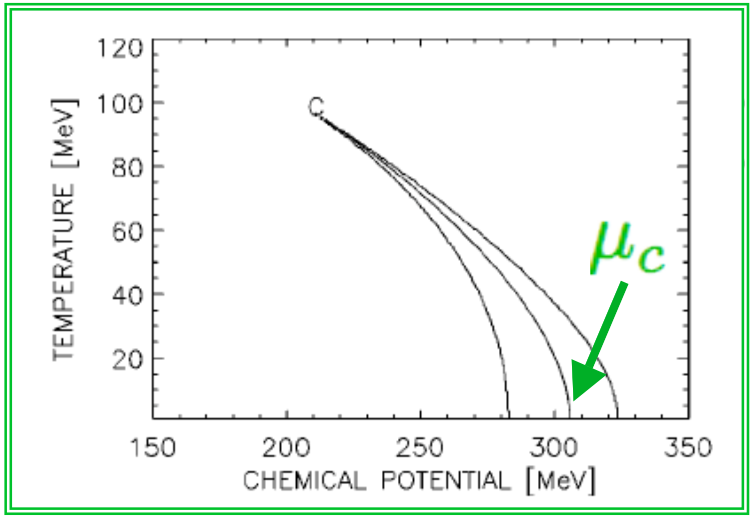
$$
\mathcal{V}(\vec{\pi},\sigma) \equiv \frac{\lambda^2}{4}[(\vec{\pi}^2+\sigma^2) - v^2]^2 - h\sigma
$$

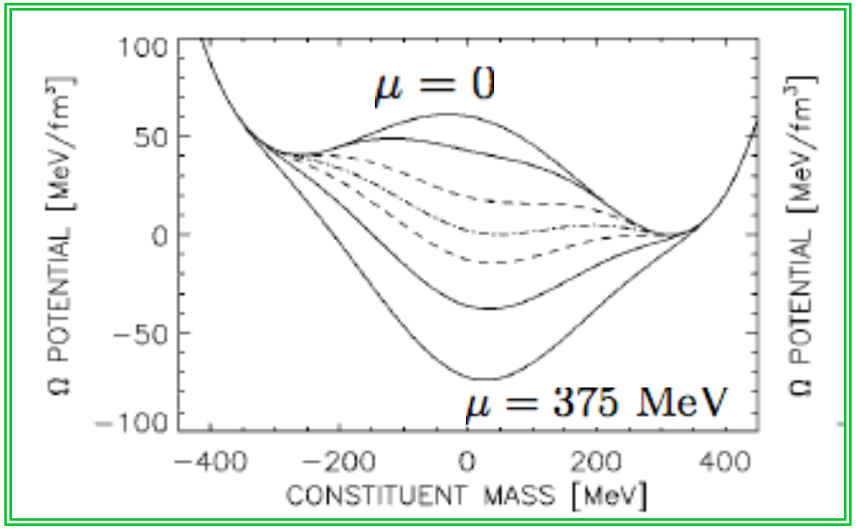
Explicit breaking $m_{\mathrm{quarks}} \neq 0$

- In QCD: chiral symmetry is approximate spontaneously broken symm.
- Spontaneous Symmetry Breaking: $\sigma = \langle\sigma\rangle(T,\mu) + \delta\sigma$
- Renormalizable:
  UV infinities are absorbed systematically, but a new energy scale, Λ , is introduced in the process.
- The parameters g , m_q , λ , v , h , Λ are fixed to reproduce (measured or simulated) properties of the QCD vacuum.

Some previous results within the Linear σ Model

- Mean-field calculation of the effective potential for finite T and μ
(No fluctuations nor logarithmic ZPT corrections)





[Scavenius *et al* (2001)]

- Beyond mean-field for $T > 0$ (With resummation)
  - ZPT + 2 loops + MSC Resummation **[Caldas, Mota & Nemes (2001)]**
  - CJT + Hartree Approx. **[Röder, Ruppert & Rischke (2003)]**
  - ZPT+ Averaged Sigma fluctuations **[Mócsy, Mishustin & Ellis (2004)]**
  - ...

Beyond mean-field approximation at finite density

- Cold and dense case
- **Study the σ direction (without pions)**

  Other condensates do not seem to affect much $\langle\sigma\rangle$

  [Röder, Ruppert & Rischke (2003)]
- **Including:**
  - **Bosonic and fermionic** fluctuations up to order of **1 and 2 *loops*, without resummation**;
  - **Zero PoinT logarithmic corrections** renormalized in the $\overline{\mathrm{MS}}$ scheme

    $\Rightarrow$ analysis of renormalization scale dependence
- **Investigate the consequences over the chiral transition in QCD at finite density**

[LFP & Fraga, in prep.]

The effective potential for the LσM without pions

$$V_{eff}(\langle\sigma\rangle) = \mathcal{V} + [\Omega_{\delta\sigma}]^{\text{ren}}$$

In this case, the **effective theory for the fluctuations** is given by:

- $N_f N_c$ species of massive constituent quarks ; $m = m_q + g\langle\sigma\rangle$
- Massive self-interacting scalar fluctuations ; $m_\sigma^2 = \lambda^2(3\langle\sigma\rangle^2 - v^2)$
- **Yukawa-type interaction**

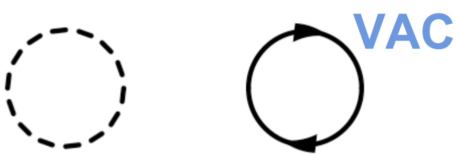
i.e., exactly the Yukawa theory with massive fermions and bosons whose thermodynamics we solved up to 2-loop order.

However: $m \mapsto m(\langle\sigma\rangle(\mu))$; $m_\phi \mapsto m_\sigma(\langle\sigma\rangle(\mu))$

⇒ **purely vacuum terms are now medium dependent** implicitly through the variation of the masses.

⇒ ZPT contributions previously subtracted need to be calculated here:

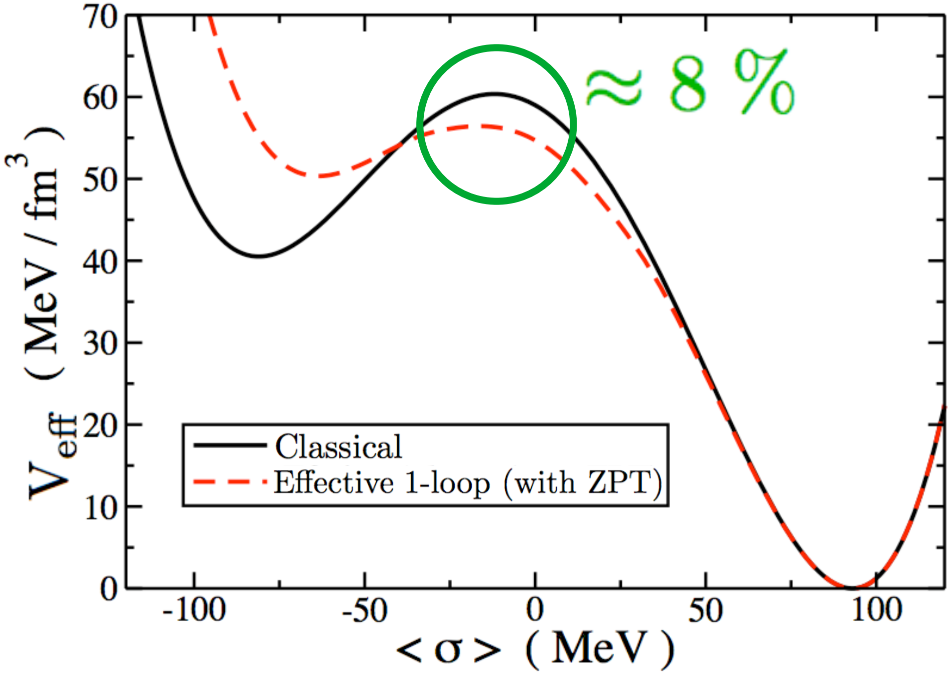
One *loop*:



Two *loops*:

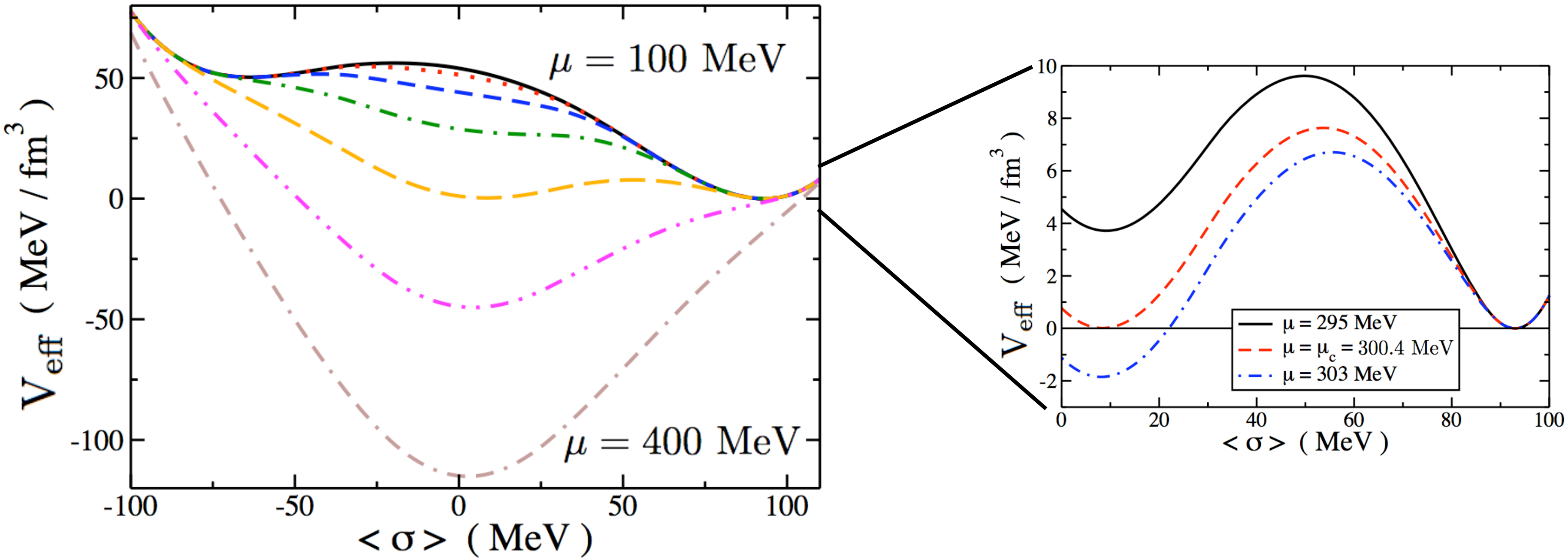
1-loop results ⇒ ZPT effects

- Up to 1-loop order, the logarithmic corrections modify considerably the **central region** of the vacuum effective potential ⇒ **possibly relevant for the dynamics of phase conversion.**



1-loop results ⇒ Chiral Transition

- The transition is qualitatively the same: 1st order.
- The critical chemical potential is slightly reduced (~2%)
  ⇒ Indication of weak dependence of the critical region on the renormalization scale



Conclusions

- We solved completely, obtaining ***general analytical results***, the thermodynamics of a scalar Yukawa Theory with:
  - **fermions and bosons both massive;**
  - **Interaction up to 2-loop order;**
  - **Complete RG flow.**

  ➡ Significant mass effects

- **Chiral transition at finite density** within the Linear σ Model, incorporating **zero-point logarithmic corrections:**

  ➡ **Results on the criticality were not significantly altered** $\Rightarrow$ **indication of a weak dependence of the critical region on the renormalization scale.**

  ➡ **The central region of the effective potential suffered modifications ~ 8%** $\Rightarrow$ **possible consequences for the phase conversion dynamics.**

Perspectives

- **<u>Work in progress:</u>**
  - **2-loop effective potential for the LσM:**

    Fully calculated, including ZPT;

    Still lacking parameter fixing to reproduce QCD vacuum
  - **RG flux implementation**
- **Obtain** $\mu_c(m_q)$
- **Comparison with lattice simulations of effective models**

  [Taurines, *priv. comm.*]
- **ZPT effects on the dynamics of phase conversion.**

  [Mizher, LFP & Fraga]