Shock wave propagation in causal dissipative hydrodynamics

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Motivation

Hydrodynamics for Heavy Ions

- good description for mid and low p_T data with ideal hydro
- implies *Thermalization* and *Equation of State*?
- or just *conservations* and *collective motion*?
- jet induced Mach cone as a genuine hydrodynamical manifestation
- information on the EoS from angular correlations
- AdS/CFT indicates lower viscosity bound
- jet propagation introduces high inhomogeneities
- entropy production
- extends validity of hydrodynamics

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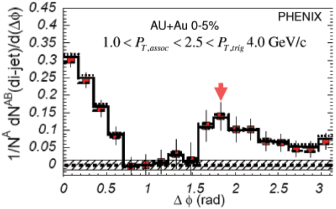
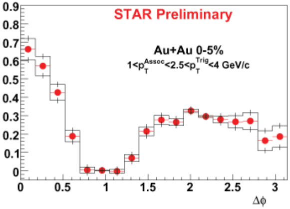
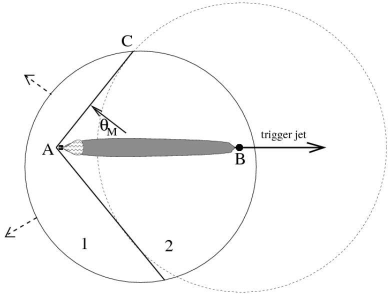
Jets and Mach Cone

Jet data





Mach cone Scenario



- fast ($c_s^2 < v \sim 1$) parton deposits its energy in the medium.
- assume fast thermalization of energy deposition and hydrodynamical evolution

Hydrodynamics

- General equations for non-conformal and non-isotropic fluid in Landau Frame $\varepsilon u^\nu = u_\mu T^{\mu\nu}$

$$\partial_\mu\big((\varepsilon + P + \mathit{\Pi})u^\mu u^\nu - g^{\mu\nu}(P + \mathit{\Pi})\big) + \partial_\mu(\mathcal{P}^{(\mu\nu)}_{\sigma\rho}\pi^{\sigma\rho}) = 0$$

$$\partial_\mu(nu^\mu) + \partial \cdot q = 0$$

$\mathit{\Pi}$ bulk, π shear and q heat flux
$u_\mu \mathit{\Delta}^{\mu\nu} = 0$ and $g_{\mu\nu}\mathcal{P}^{\mu\nu}_{\sigma\rho} = 0$

- Entropy equation

$$T\partial_\mu(su^\mu - \alpha q^\mu) = -\mathit{\Pi}\partial \cdot u + \mathcal{P}^{\mu\nu}_{\sigma\rho}\pi^{\sigma\rho}\partial_\mu u_\nu - T\mathit{\Delta}^{\mu\nu}q_\mu\partial_\nu\alpha$$

≥ 0 production!

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$$\geq 0 \qquad \text{production!}$$

Viscosity and Causality

$$\begin{pmatrix} \Pi & q_\mu & \pi^{\sigma\rho} \end{pmatrix} \begin{pmatrix} -\partial \cdot u \\ -\Delta^{\mu\nu} T \partial_\nu \alpha \\ \mathcal{P}^{(\mu\nu)}_{\sigma\rho} \partial_\mu u_\nu \end{pmatrix} \geq 0, \qquad \sum_A \mathcal{F}_A X_A \mathcal{J}_A \geq 0$$

- Landau-Lifshitz: algebraically quadratic

$$\mathcal{F} = \xi X \mathcal{J}$$

leads to non-causality [Hiscock and Lindblom '83]

- Israel-Stewart ['79]: second order thermodynamics

$$\mathrm{d}_\tau \mathcal{F} = X \Big(-\sum_B \beta_{AB} \mathcal{F}_B + \xi \mathcal{J} \Big)$$

cross terms, up to 19 coefficients
causality is possible! (constraints on coefficients)

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Viscosity and Causality

- Baryon-free $n = 0$ implies $\kappa = 0$ ($q^\mu = 0$) (Landau frame)
- Isotropic $T^{\mu\nu}$ implies $\eta = 0$ ($\pi^{\mu\nu} = 0$)
- Then

$$u^\mu \mathrm{d}_\tau(\omega + \Pi) + (\omega + \Pi)(u^\mu \partial \cdot u + \mathrm{d}_\tau u^\mu) = \partial^\mu (P + \Pi)$$

$$T(\mathrm{d}_\tau s + s\partial \cdot u) = -\Pi \partial \cdot u$$

and

$$\tau_{\mathrm{R}} \mathrm{d}_\tau \Pi = -\Pi - \zeta \partial \cdot u$$

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Viscous Coefficients

- First principles calculations:
  - pQCD for shear $\eta/\tau_{\rm R} = \frac{2}{3}P = \frac{1}{6}(\varepsilon + P)$ [Baier, Romatschke & Wiedemann '06]
  - AdS/CFT for shear $\eta/s \geq 1/4\pi \approx 0.08$
- Hints from a better look on the equations: solvability depends on

$$\tilde{\omega} = \varepsilon + P + \Pi \neq 0, \qquad \left(\left(c_{\rm s}^2 + \frac{1}{\tilde{\omega}}\frac{\zeta}{\tau_{\rm R}}\right) - 1\right)\mathbf{u}^2 \neq 1$$

  - for $\Pi \longrightarrow -\omega$:

$$\mathrm{d}_\tau \tilde{\omega} \longrightarrow -\frac{\zeta}{\tau_{\rm R}}\partial \cdot u + \frac{\omega}{\tau_{\rm R}} > 0$$

  $\zeta/\tau_{\rm R} = a\tilde{\omega}$ with $v^2 := a + c_{\rm s}^2 < 1$ and $1/\tau_{\rm R} = bT$ guarantees both inequalities (implies $\zeta = a\tilde{\omega}/(bT) =: a\tilde{s}/b$)
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Smoothed Particle Hydrodynamics

[Monaghan '92]

- mesh-free and dimension independent algorithm.
- for ideal evolution equations are derived from variational principle
- derivatives of analytic kernel function $W_h(r)$

$$(\gamma\sigma)_a = \sum_{\mathbf{r}_b \in \mathcal{S}^d_{2h}} \nu_b W_h(r_{ab}), \quad \nabla \cdot \mathbf{v}_a = \sum_{\mathbf{r}_b \in \mathcal{S}^d_{2h}} \nu_b \mathbf{v}_{ab} \cdot \hat{\mathbf{r}}_{ab} W'_h(r_{ab})$$

$$\mathrm{d}_t \left(\frac{\tilde{\omega}\mathbf{u}}{\sigma} \right)_a = \sum_{\mathbf{r}_b \in \mathcal{S}^d_{2h}} \frac{\nu_b}{(\gamma\sigma)_b} \left(\frac{(P+\mathit{\Pi})_b}{(\gamma\sigma)_b^2} + \frac{(P+\mathit{\Pi})_a}{(\gamma\sigma)_a^2} \right) W'_h(r_{ab}) \hat{\mathbf{r}}_{ab}$$

$$\partial \cdot u = \mathbf{v} \cdot \mathrm{d}_t \mathbf{u} + \nabla \cdot \mathbf{v}$$

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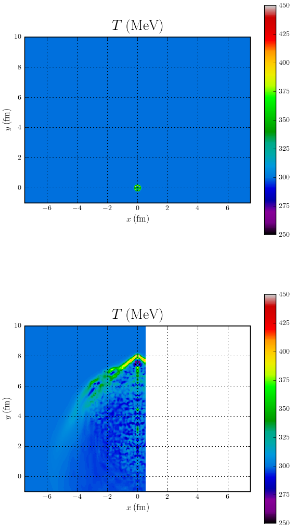
Initial Condition



2D box

$T_{\text{BG}} = 0.3\text{GeV}$

$T_{\text{jet}} = 0.4\text{GeV}$

$R_{\text{jet}} = 0.1\text{fm}$

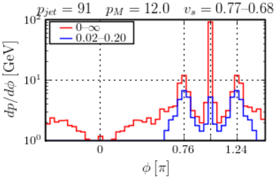
$\gamma = 10$

$p \approx 150\text{GeV}$

$\zeta/\tau_{\mathrm{R}} = \frac{1}{6}\tilde{\omega} \qquad v = \sqrt{0.5} \approx 0.71$

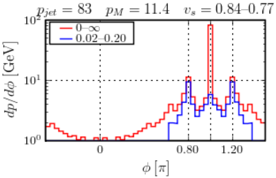
$$1/\tau_{\mathrm{R}} = 20T$$

$$\zeta/\tilde{s} = \frac{1}{120}$$



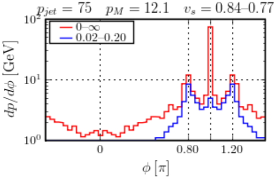
$$1/\tau_{\mathrm{R}} = 4T$$

$$\zeta/\tilde{s} = \frac{1}{24}$$



$$1/\tau_{\mathrm{R}} = T$$

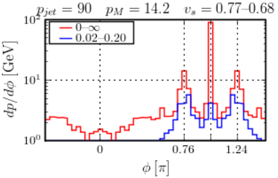
$$\zeta/\tilde{s} = \frac{1}{6}$$



$1/\tau_{\rm R} = 2T$

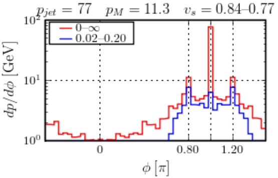
$$\zeta/\tau_{\rm R} = \frac{1}{15}\,\tilde{\omega} \qquad \zeta/\tilde{s} = \frac{1}{90}$$

$$v = \sqrt{\frac{6}{15}} \approx 0.63$$



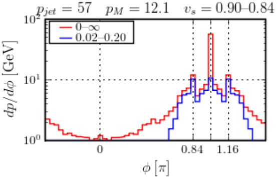
$$\zeta/\tau_{\rm R} = \frac{1}{6}\,\tilde{\omega} \qquad \zeta/\tilde{s} = \frac{1}{12}$$

$$v = \sqrt{\frac{1}{2}} \approx 0.71$$



$$\zeta/\tau_{\rm R} = \frac{1}{3}\,\tilde{\omega} \qquad \zeta/\tilde{s} = \frac{1}{6}$$

$$v = \sqrt{\frac{2}{3}} \approx 0.82$$



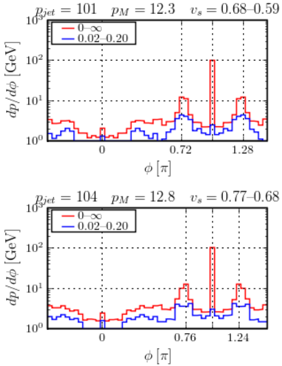
Ideal limit

$$\zeta/\tau_{\rm R} = \frac{1}{3}\,\tilde{\omega} \qquad \zeta/\tilde{s} = \frac{1}{6}$$

$$\frac{1}{\tau_{\rm R}} = 100T \quad v = \sqrt{\frac{201}{600}} \approx 0.58$$

$$\zeta/\tau_{\rm R} = 0 \qquad \zeta/\tilde{s} = 0$$

$$v = \sqrt{\tfrac{1}{3}} \approx 0.58$$



Conclusions and Remarks

- working n-dimensional hydrocode with causal bulk viscosity
- our choice of the coefficients seem to avoid the singular points $\tilde{\omega} = 0$ and $v > 1$

To do

- realistic values for p for RHIC
- compare with simulations with expansion (needs higher precision)
- profile at $T = T_{\rm f}$
- Cooper-Frye freeze-out and compare with experimental data
- 3D simulations (time consuming)