Dissipative Dynamics of Deconfinement Transition

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Ecole de Physique des Houches 01/04/2008

Summary

- Deconfinement Transition
- Polyakov Loops
- Effective models for the Polyakov Loops
  - Order parameter obtained from the eigenvalues of the P.L.
  - Expectation value of the trace of the Polyakov Loop
- Langevin Evolution
- Results for pure gauge
- Perspectives

Deconfinement transition

Ultrarelativistic heavy ion collisions

Extreme conditions of pressure and temperature

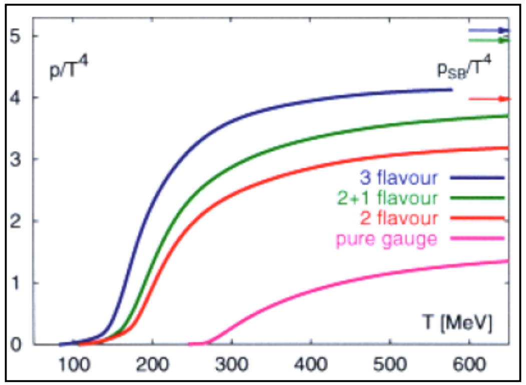
Lorentz contraction

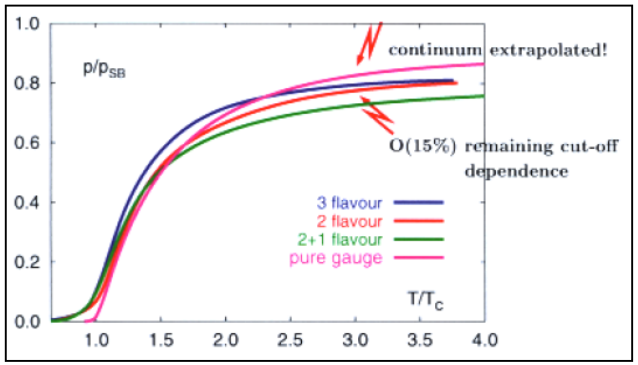
Quark gluon plasma

Deconfined state with collective effects

Because of flavor independence pure gauge results provide important results for QCD.

F. Karsch (QM 2001)





The Polyakov loop

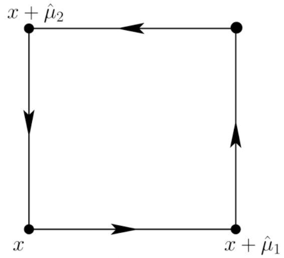
Covariant derivative:

$$n^\mu D_\mu \psi = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \left[\psi(x + \epsilon n) - U(x + \epsilon n, x)\psi(x) \right]$$

where

$$U(x + \epsilon n, x) = \mathbf{1} + ig\epsilon n^\mu A_\mu^a t^a + O(\epsilon^2)$$

On the lattice:



the link:

$$U^\mu(\vec{x}) \equiv e^{i\vec{\tau}\cdot\vec{A}^\mu(\vec{x})}$$

the path:

$$\tilde{W}[C] = \prod_{\vec{x}, \hat{\mu} \in C} U^{\hat{\mu}}(\vec{x})$$

Paths parallel to the temporal axis:

$$L(x) = \prod_{n=1}^{N_t} (U_{x+n\hat{t},0})$$

Continuum →

$$L(x) = Pexp\left[i \int_0^\beta dt \; \vec{\tau} \cdot \vec{A}^0(\vec{x}, t) \right]$$

To make it gauge invariant:

$$l(\vec{x}) = trL(\vec{x})$$

The Polyakov loop is an order parameter in systems with Z(N) symmetry, indicating when the symmetry is broken:

$$l_0 = 0, \quad T < T_c \quad ; \quad l_0 > 0, \quad T > T_c$$

Usually it is associated to the free energy of a probe quark through the relation:

$$\langle L \rangle = exp\left(-F_{teste}/T \right)$$

Z(N) symmetry in the deconfinement transition

QCD Lagrangean: $$\mathcal{L} = \frac{1}{2} G_{\mu\nu}^2 + \bar{q}\gamma_\mu D_\mu q$$

Gluons are periodic on imaginary time τ and quarks must be antiperiodic:

$$A_\mu(\vec{x},\beta) = +A_\mu(\vec{x},0), \quad q(\vec{x},\beta) = -q(\vec{x},0)$$

Considering transformations periodic under the center of the group, the fields transform like:

Adjoint representation → $$A^\Omega(\vec{x},\beta) = \Omega_c^\dagger A_\mu(\vec{x},\beta)\Omega_c = +A_\mu(\vec{x},0),$$

Fundamental representation → $$q^\Omega(\vec{x},\beta) = \Omega_c^\dagger q(\vec{x},\beta) = e^{-i\phi} q(\vec{x},\beta) \neq -q(\vec{x},0)$$

Quarks break the Z(N) symmetry!

Effective models for the Polyakov loop

Model 1: Order parameter obtained from the eigenvalues of the Polyakov loop [P. Meisinger, T. Miller, M. Ogilvie (2002)]

Diagonalizing the Polyakov loop, the eigenvalues can be seen as degrees of freedom: $P_{jk} = exp(i\theta_j)\delta_{jk}$

Once: $e^{-F/T} = \langle Tr_f P(\vec{x}) \rangle$

When P=0, the free energy is finite , indicating confinement.

Any perturbative calculation yields to: $f_{pert} = T^4 F(p, g(T))$

The leading order behaviour is independent of g(T) and gives the blackbody behaviour expected in high temperatures.

Performing the perturbative calculation one reaches:

$$f_{pert}(\theta) = -\frac{1}{\beta} \sum_{j,\ k=1}^{N} 2\left(1 - \frac{1}{N}\delta_{jk}\right) \times \int \frac{d^3k}{(2\pi)^3} \sum_{n=1}^{\infty} \frac{1}{N}\ e^{-n\beta\omega_k + in\Delta\theta_{jk}}$$

Where: $\omega_k \equiv |\vec{k}|$

- Purely perturbative result predicts a gas of confined gluons for any temperature, and there is no indication that high order corrections modify this result.
- It is necessary to insert a scale in a way that the high T behaviour is preserved.

Inserting a mass scale: $\omega_k = \sqrt{k^2 + M^2}$

$$f_{pert}(\theta) = -T^4 * f_1 + T^2 M^2 * f2$$

High temperature → Quartic term dominates → Min. in $\Delta\theta_{jk} = 0$

Low temperature → Quadratic term dominates → Max. in $\Delta\theta_{jk} = 0$

Parametrizing the diagonal matrix on the following way

$$diag[exp(i\phi_{N/2}, ..., i\phi_1, -i\phi_{-1}, ..., -i\phi_{-N/2})]$$

We find the minima in: $$\phi_j = \frac{2\pi j}{N}$$

In the SU(2) case we have two minima and the free energy is given by:

$$f = -\frac{\pi^2 T^4}{15} + \frac{4T^4}{3\pi^2}\phi^2(\phi-\pi)^2 + \frac{M^2T^2}{4} + \frac{M^2T^2}{\pi^2}\phi(\phi-\pi)$$

To evidence the Z(2) symmetry we define a new variable $\psi = \pi/2 - \phi$ and the free energy becomes:

$$f = -\frac{\pi^2 T^4}{15} + \frac{T^4}{12\pi^2}(\pi^2 - 4\psi^2)^2 + \frac{M^2T^4}{4} - \frac{M^2T^2}{4\pi^2}(\pi^2 - 4\psi^2)$$

The constant M can be determined through the value of the critical temperature T_d

$$M = \pi(2/3)^{1/2}T_d \approx 2.5651T_d$$

Following a similar way for the case of SU(3) we get for the free energy:

$$f = -T^4\frac{8\pi^2}{45} + \frac{T^4}{6\pi^2}[8\phi^2(\phi-\pi)^2 + \phi^2(\phi-2\pi)^2] + \frac{2T^2M^2}{3} + \frac{T^2M^2}{2\pi^2}[2\phi(\phi-\pi) + \phi(\phi-2\pi)].$$

Again it is possible change variables to make the symmetry more evident:

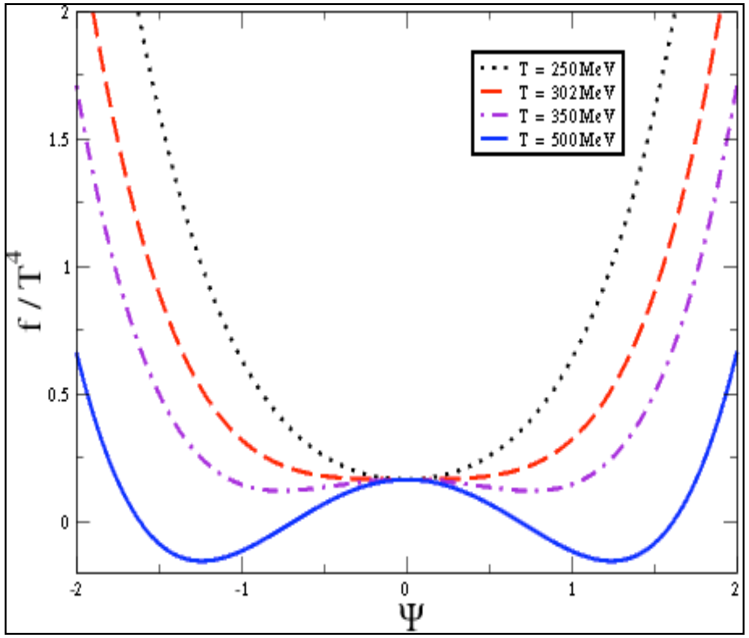
$$\psi = 2\pi/3 - \phi$$

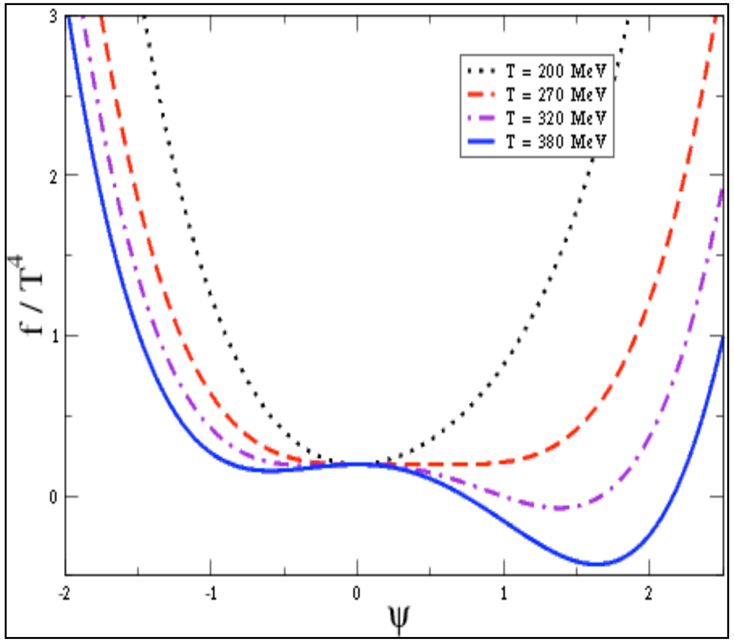
$$f = \frac{8\pi^2}{405}T^4 + \left(\frac{3}{2\pi^2}T^2M^2 - \frac{2}{3}T^4\right)\psi^2 - \frac{2}{3\pi}T^4\psi^3 + \frac{3}{2\pi^2}T^4\psi^4$$

with

$$M = \frac{20\pi}{9}\frac{1}{\sqrt{10}}\,T_d \approx 2.2077\,T_d$$

Free energy for SU(2) and SU(3)





SU(2) free energy: as the temperature increases two degenerate minima appear.

SU(3) free energy: immediately above the critical temperature there is a barrier that quickly disappear.

Model 2: expectation value of the Polyakov loop as the order parameter

[R. D. Pisarski (2001)]

The Polyakov loop under a transformation of the Z(N) group behaves as:

$$l_n \to e^{ni\phi} l_n$$

Considering the mass terms in an effective Lagrangean:

$$\mathcal{L}^{eff} = m_1^2 |l_1|^2 + m_2^2 |l_2|^2 + ...$$

The simplest possibility is:

$$m_1^2 < 0 \quad , \quad T > T_c \quad , \quad m_1^2 > 0 \quad , \quad T < T_c ,$$

with the superior orders loops mass always positive

$$m_2^2 > 0 \quad , \quad m_3^2 > 0 ...$$

From the global symmetry Z(N), to have a gauge invariant potential:

[B. Svetitsky and L. Yaffe (1982)]

$$V = \left(-\frac{b_2}{2}|l|^2 - \frac{b_3}{3}[l^3 + (l^*)^3] + \frac{1}{4}(|l|^2)^2 \right) b_4 T^4$$

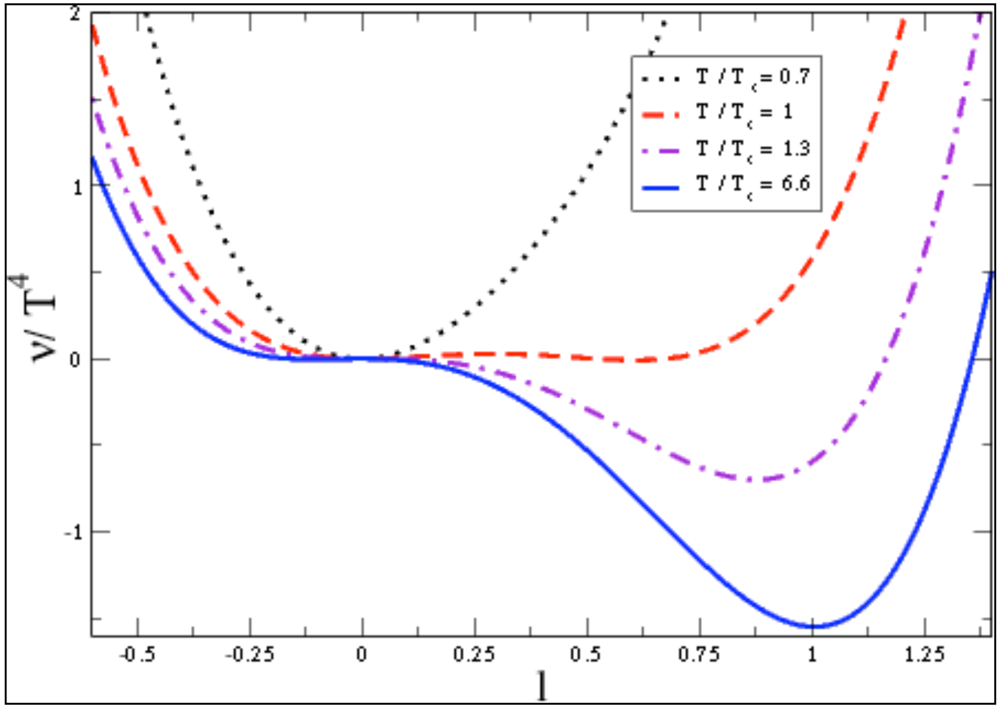
these coefficients can be obtained from lattice data:

$$b_2(T) = (1 - 1.11/x)(1 + 0.265/x)^2(1 + 0.300/x)^3 - 0.487$$

$$b_3 = 2 \qquad b_4 \approx 0.6061$$

with: $$x = \frac{T}{T_d}$$

SU(3) potential obtained from the second model:



Section of the potential on the real axis. On the complex plane at any temperature above Td the vacuum is N-degenerated. Above the critical temperature there is the barrier, in accordance with the previous model, that disappears when the temperature increases.

Langevin evolution

Langevin equation → Random force and dissipation

A Langevin equation can be derived in classical mechanics, quantum mechanics or field theory.

Langevin equation derived for the $\lambda\varphi^4$ model using real time treatment, finite temperature and calculating up to 2-loops.

- Dominant contribution in the noise is multiplicative
- Colored noise
- Dissipation term proportional to the field squared.

Out of equilibrium field theory

Let us consider a $\lambda\varphi^4$ theory with Lagrangean:

$$\mathcal{L}[\phi] = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$$

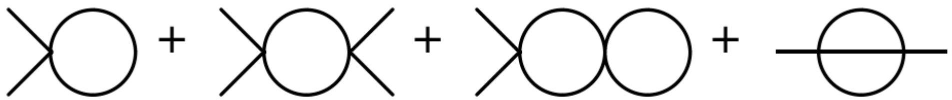
The functional generator is:

$$Z[J] = \int_C D\phi exp\{iS[\phi, J]\}$$

Considering the contributions up to 2-loops and order λ^2, diagramatically, we have:

[M. Gleiser e R. O Ramos (1994)]

$$\Gamma[\varphi] = S[\varphi] +$$



$$+ \mathcal{O}(\lambda^3)$$

where $\Gamma[\varphi]$ is the effective action and $S[\varphi]$ is the classical action.

- Real time formalism
- A change of variables
- One associates imaginary terms (rapid oscilation) to gaussian fluctuation fields

Minimizing the action with respect to the field:

$$\left.\frac{\delta S_{eff}[\varphi_\Delta, \varphi_c, \xi_1, \xi_2]}{\delta \varphi_\Delta}\right|_{\varphi_\Delta=0} = 0$$

and considering the dressed propagators we reach:

$$\left[\Box + m_T^2\right] \varphi_c(\vec{x},t) + \frac{\lambda_T}{3!}\varphi_c^3(\vec{x},t) + \eta_1 \varphi_c^2(\vec{x},t)\dot{\varphi}_c(\vec{x},t) = \varphi_c(\vec{x},t)\xi_1(\vec{x},t).$$

If we take into account interactions with other fields the solution would be more general:

$$\Box\phi + \left[\Gamma_1 + \Gamma_2\,\phi^2\right]\frac{\partial \phi}{\partial t} + V'(\phi) = \xi_1(\vec{x},t) + \xi_2(\vec{x},t)\,\phi.$$

Lattice counterterms

K. Farakos et al (1994}

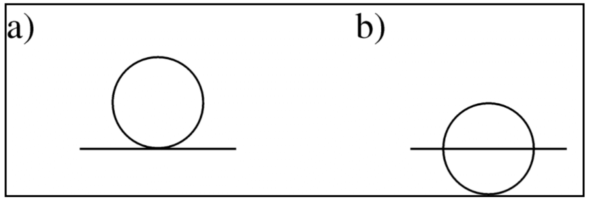
L. M. A. Bettencourt (2001}

For a $\lambda\varphi^4$ theory the ultraviolet divergences can be eliminated adding mass counterterms in the action:

$$\delta S = -\int d^3x(\delta m^2 \phi^\dagger \phi + \frac{1}{2}\delta m_D^2 A_0^a A_0^a)$$

$$\delta m^2 = f_1 \Sigma_{tad} + f_2 \Sigma_{sun}$$

Σ_{tad} and Σ_{sun} are proportional to the diagrams a) tadpole and b) sunset:



These diagrams are calculated in the lattice and one obtains:

$$\delta m^2 = 6\lambda T \left(\frac{0.252731}{a}\right) - \frac{12\lambda^2 T^2}{16\pi^2}\left(log\frac{6}{a\mu_3} + 0.09\right)$$

Pure gauge results

Considering a system characterized by the following free energy:

$$F(\psi, T) = \int d^3x \left[\frac{B}{2} (\nabla\psi)^2 + V_{eff}(\psi, T) \right]$$

The Langevin equation will be:

$$B\left(\frac{\partial^2\psi}{\partial t^2} - \nabla^2\psi\right) + \Gamma \frac{\partial\psi}{\partial t} + V'_{eff}(\psi) = \xi$$

The factor B plays a role of superficial tension and comes from the perturbative calculation. [T. Bhattacharya, A. Gocksch, C. P. Korthals-Altes, R. D. Pisarski (1992)]

Γ is obtained through Monte Carlo simulation (relation between Monte Carlo time and real time obtained comparing with experimental data)

We get the appropriated counterterms comparing our potential with the potential $\lambda\varphi^4$ and identifying m and λ:

$$\delta m^2 = -0.75\frac{\lambda T}{a} + \frac{6\lambda^2 T^2}{16\pi^2} log\left(\frac{6}{am} + 0.09\right)$$

Lattice → 64^3 sites with periodic boundary conditions.

In each step we have done an average of the value of the loop in each site:

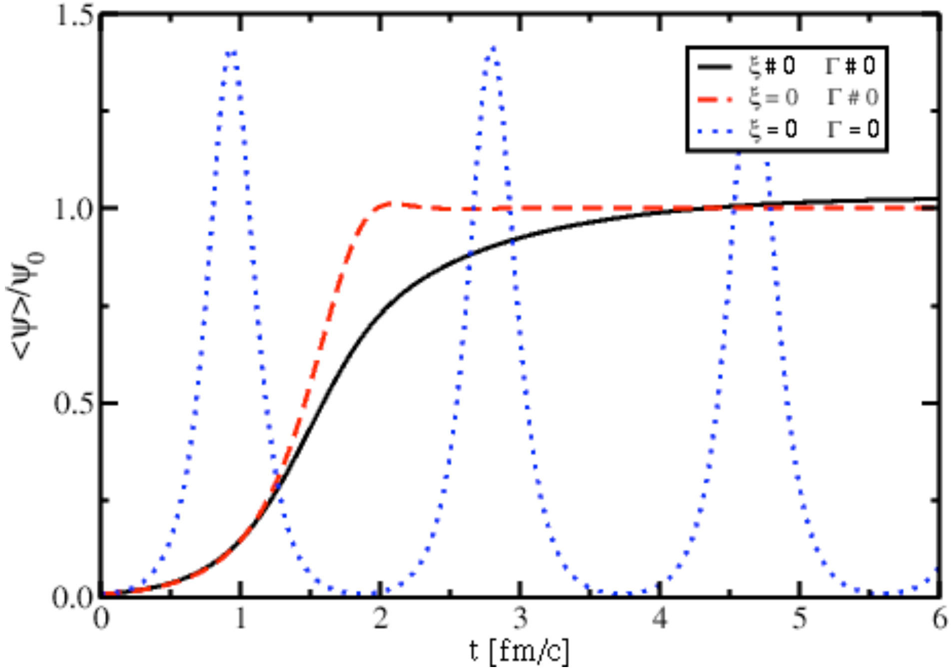
$$\langle\psi\rangle = \frac{1}{N^3}\sum_{ijk}\psi_{ijk}(t)$$

We took the average over many realizations with different initial configurations around $\psi \sim 0$, also taking different noise configurations .

Temporal discretization → Semi-implicit method/ leap-frog

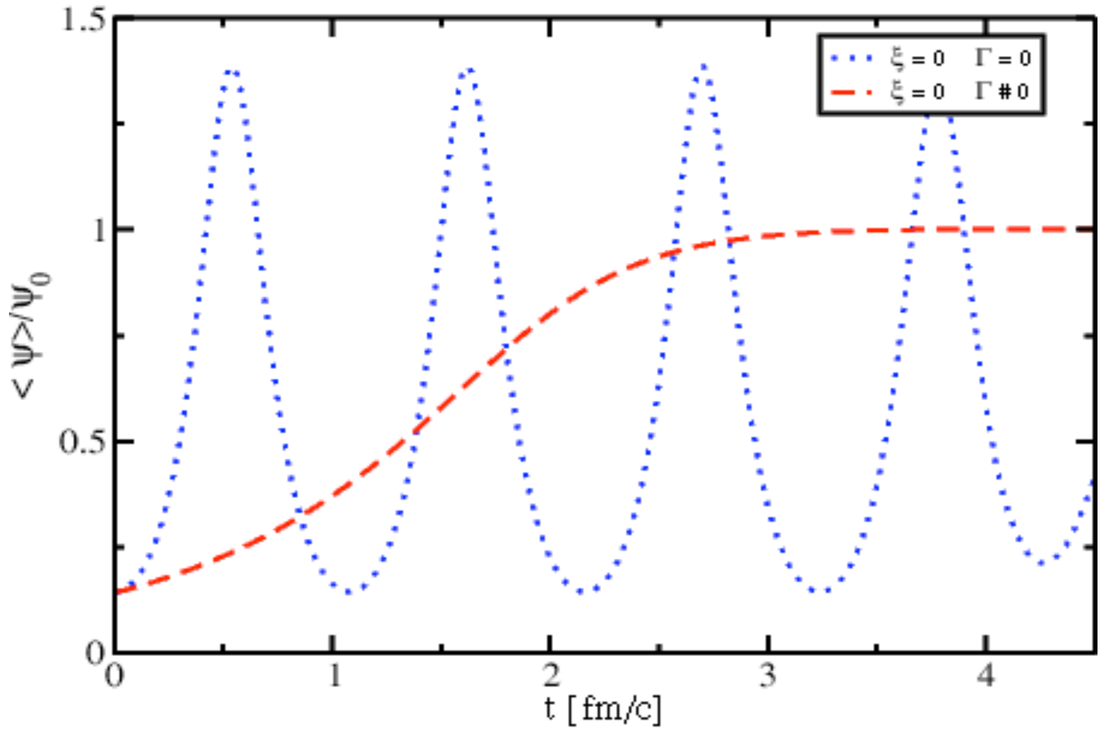
Spatial discretization → Finite differences – Fast Fourier transform/ Fourier collocation

Model 1: SU(2) case



- Strong delay effects in the thermalization caused by the dissipation and noise.

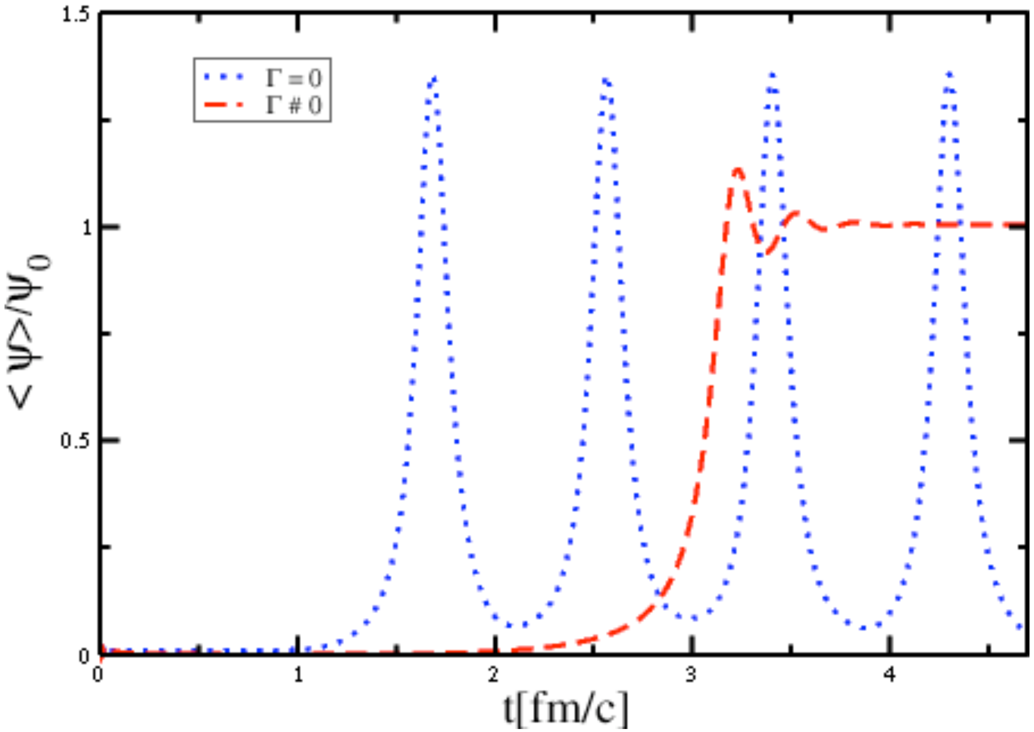
Model 1: SU(3) case



- Effects even stronger than in the case of chiral transition.
- The noise also must play an important role.

[E. S. Fraga, G. Krein (2005)]

Model 2: SU(3) case



- The noise also must play an important role.
- Dissipation effect in accordance with the obtained using the other model.

Conclusions

- The dissipation plays an important role and its effects must be taken into account.
- The effect in the deconfinement transition is even larger than in the chiral transition.
- The results can be compared to pure gauge results on the lattice.
- A more robust model can provide information possible to compare to experimental data.
- Perspectives:
    - Coupling to chiral fields.
    - Memory effects.
    - More robust effect models.
    - Inclusion of other effects that can change considerably the time scale of thermalization such as inhomogeneities, plasma expansion, finite size, etc.