Nucleation and spinodal decomposition in the hadronization of QGP

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Introduction

- The Quark-Gluon Plasma may be found in Heavy-Ion collisions or in the early Universe (very high energy densities).
- In both scenarios: expanding hot “fireball” with deconfined matter that cools down below a certain T_c and suffers a phase transition (hadronization).
- The order of the hadronization phase transition is not yet a closed issue:

  * Lattice indicates a crossover.

  * Good results in hydrodynamics with 1st order transition.

Motivation

- Possible mechanisms for the dynamics of a 1st order phase conversion: bubble nucleation or spinodal decomposition.
- Our goal is to find clues leading to the predominant mechanism of hadronization in each of two (rather different) scenarios: in heavy-ion collisions and in the early Universe.
- Previous studies in this line: EoS based on the Bag Model and also resorted to the thin-wall approximation.
- Our analysis relies mainly on a mixed Lattice/Hadron Gas equation of state.

Outline

- Scenario description
- Physical assumptions
- Results for the nucleation rate
- Spinodal decomposition vs. nucleation
- Conclusions

Physical Scenarios

- We think of our phase conversion evolution as composed of three stages:

→ The (infinite) space is filled with hot QGP ($T>T_c$) and cools down due to expansion.

→ As the temperature reaches $T<T_c$ the QGP may hadronize via nucleation of hadron bubbles.

→ The temperature continues to decrease and eventually reaches T_{sp}, when any not yet hadronized supercooled QGP undergoes hadronization via spinodal decomposition.

Nucleation dynamics

[Csernai and Kapusta, 1992]

- Main assumptions:

→ Local thermal quasi-equilibrium.

→ Uniform expansion + conservation of entropy:

$$\frac{da(t)}{dt} = H(t)a(t)$$ and $$d(sa^3) = 0$$

→ Temperature drops due to expansion: supercooling

$$\Delta[T(t)] = \frac{3c_s^2}{t_h}t$$ where $$\Delta(T) := \frac{T_c - T}{T_c}$$

Nucleation dynamics

- A crucial quantity: nucleation rate of critical bubbles per unit time, per unit volume:

$$\Gamma(T) = \Gamma_0(T) \exp\left(-\frac{\Delta F}{T}\right)$$

[Langer, 1969]

- Coarse-grained free energy functional

$$F[e, \mathbf{M}] = \int d^3r \left[\frac{\mathbf{M}^2(\mathbf{r})}{2[e(\mathbf{r}) + p(\mathbf{r})]} + K\left[\nabla e(\mathbf{r})\right]^2 + f[e(\mathbf{r})]\right]$$

[Csernai and Kapusta, 1992]

$$F[e,\mathbf{M}] = \int d^3r \left[\frac{\mathbf{M}^2(\mathbf{r})}{2[e(\mathbf{r})+p(\mathbf{r})]} + K\left[\nabla e(\mathbf{r})\right]^2 + f[e(\mathbf{r})] \right]$$

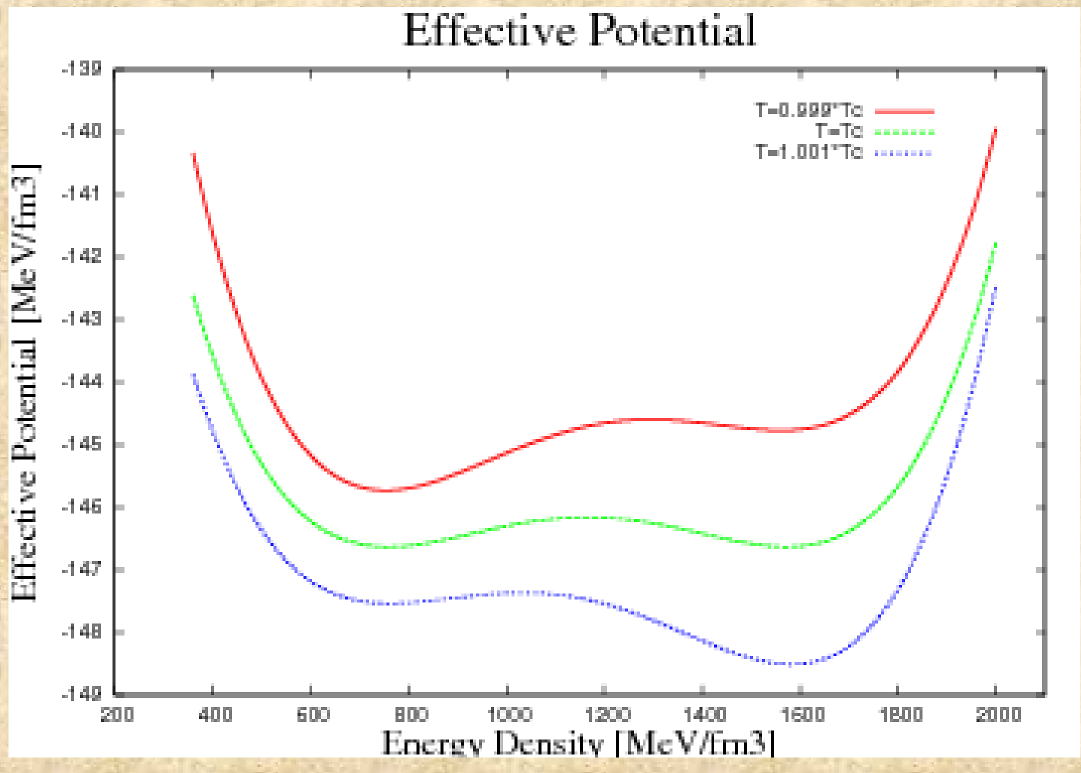
- *f(e)*: effective potential that can be derived from a Lagrangian or…
- … can be parametrized as, e.g., a 4th degree polynomial whose parameters are fixed by an adequate equation of state:

$$f(e) = \sum_{m=0}^{4} a_m (e - e_0)^m$$

$$a_m,\ e_0 = \text{functions of } T,\ T_c,\ e(T) \text{ and } p(T)$$

Effective potential

- The function *f(e)* must have two minima separated by a barrier in order to be a 1st order phase transition:



Equations of State

- Effective potential *f(e)* depends on the EoS.
- So far, most studies of nucleation apply Bag Model-inspired equations of state.
- We use a “mixed EoS”:

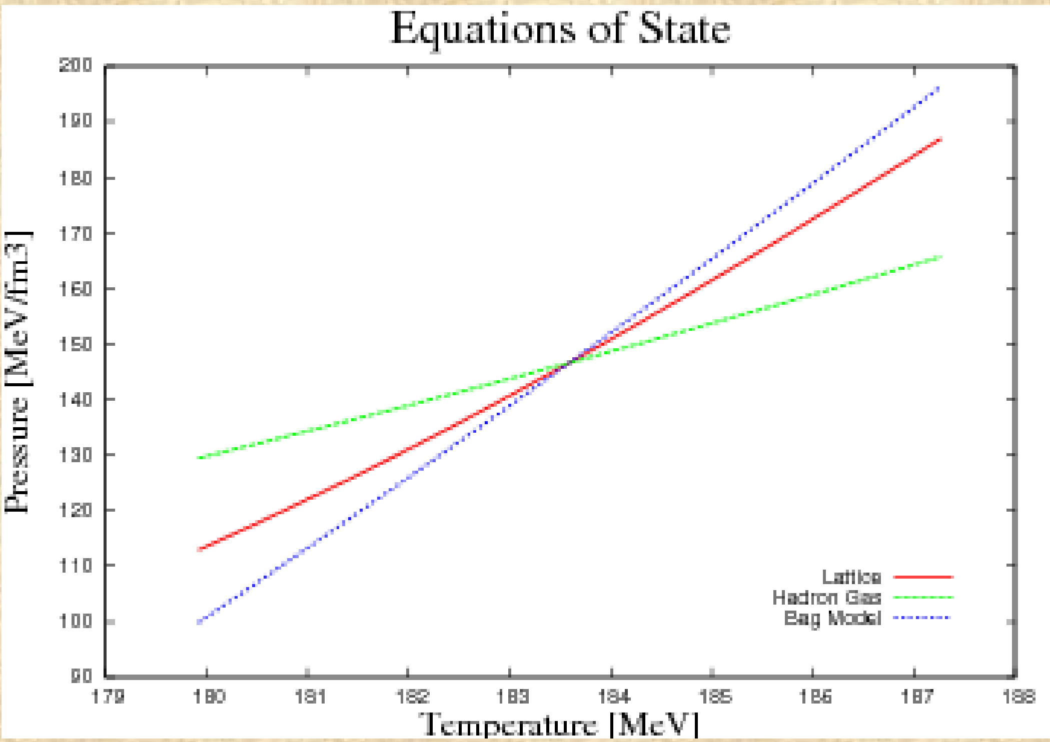
→ Lattice (N_f=2+1) results for the QGP phase

[Cheng et al., Phys.Rev.D77:014511,2008]

→ Hadron gas EoS (with 250 non-interacting massive hadronic resonances) for the confined one.

[T. Kodama, Private Communication]

Equations of State

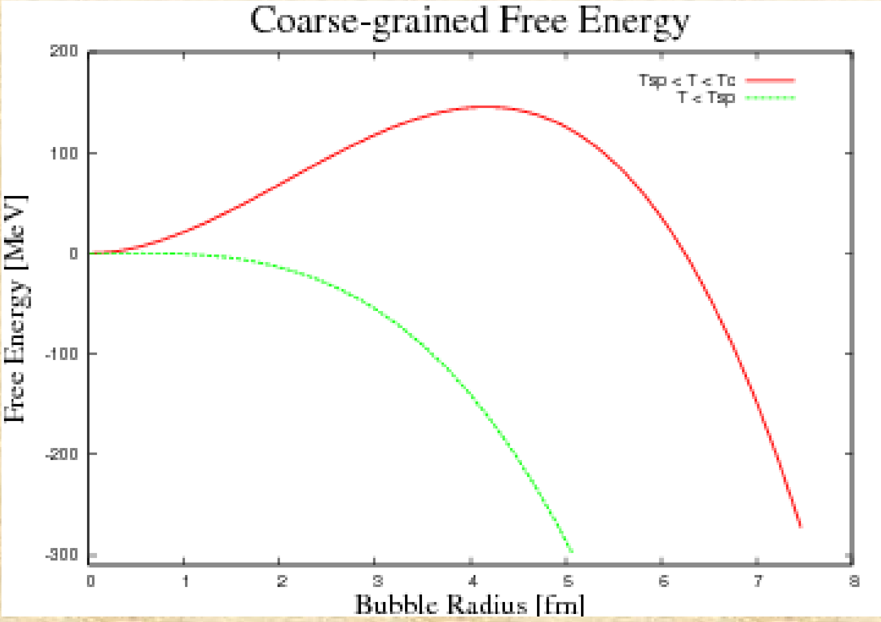


- Mixed EoS predicts weaker 1st order phase transition than Bag model

→ Different conversion dynamics?

Free energy of a bubble

- "Bubble": a sphere of hadronic matter.
- Coarse-grained free energy of a bubble of radius:
- Bubbles (coherent thermal fluctuations) with $R<R_c$ shrink and bubbles with $R>R_c$ grow.



- Maximum at $R=R_c$ defines a critical bubble.

Energy density profiles

- Critical bubbles are static and unstable solutions of

$$\left.\frac{\delta F[e]}{\delta e(\mathbf{r})}\right|_{\mathrm{M}=0} = 0$$

- Spherical symmetry leads to

$$-K\left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}\right]e(r) + f_0'' \frac{(e-e_0)(e-e_q)(e-e_h)}{(e_h-e_0)(e_q-e_0)} = 0$$

[using 4^{th} degree *f(e)*]

$$-K\left[\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}\right]e(r)+f_0''\frac{(e-e_0)(e-e_q)(e-e_h)}{(e_h-e_0)(e_q-e_0)}=0$$

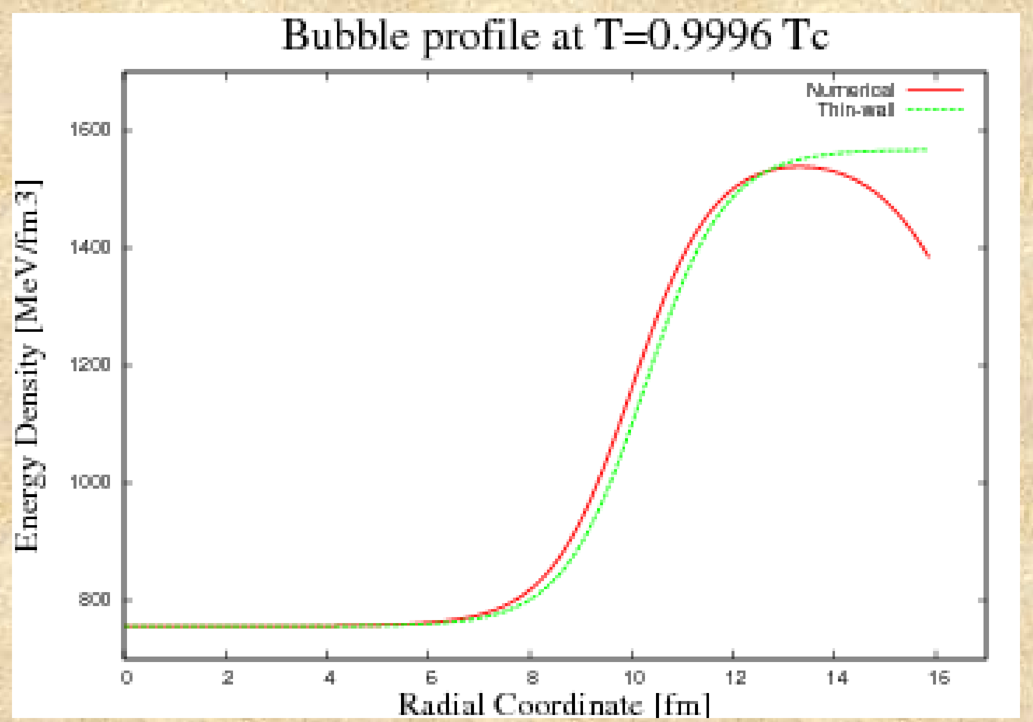
- Formally equivalent to the 1-d motion of a classical particle under a potential energy *-f(e)*, plus a dissipative force.

- Boundary conditions

$$\frac{de}{dr}(r=0)=0$$ and $$e(r\to\infty)=e_q(T)$$

Energy density profiles

- Numerical solution (undershoot/overshoot).
- Comparison between the numerical bubble profile and the thin-wall approximation.



How long does it take to reach $T_{spinodal}$?

- Nucleation rate (overestimate) ⟶ $$\Gamma(T) = T_c^4 \exp\left(-\frac{\Delta F}{T}\right)$$
- The rate Γ is not sufficient to determine the importance of nucleation.
- Criterion for importance of nucleation: how much of the (expanding) plasma is hadronized through bubble nucleation before the system's temperature reaches $T_{spinodal}$.
- We need a relation between time and temperature (or supercooling Δ).
- Neglecting reheating effects (release of latent heat),

$$\Delta[T(t)] = \frac{3c_s^2}{t_h} t$$

where

$$\Delta(T) := \frac{T_c - T}{T_c}$$

How long does it take to reach $T_{spinodal}$?

- At $T=T_{spinodal}$ the barrier of the effective potential *f(e)* disappears.
- Our EoS and *f(e)* lead to

$$T_{sp} = 0.996 T_c$$

- In terms of supercooling:

$$\Delta_{sp} = 0.004$$

- We define t_{sp} as the time the system takes to cool down to $T_{spinodal}$:

$$\Delta_{sp} = \frac{3c_s^2}{t_H} t_{sp}$$

Thin wall nucleation rate

- The thin-wall approximation is good only for small supercooling ($\Delta \ll \Delta_{sp}$).
- It also generally leads to an overestimate of the nucleation rate.
- We will use the thin wall approximation to overestimate the nucleation rate with our mixed EoS.

$$\Gamma_{thin}(T) = T_c^4 \exp\left[-\frac{A}{\Delta^2(T)}\right]$$

How much of space is nucleated?

- The fraction of space hadronized before $T_{spinodal}$ is reached is smaller than

$$\eta := \frac{4\pi}{3} t_{sp}^3 \int_0^{t_{sp}} dt \, \Gamma[T(t)]$$

- In the thin-wall approximation with $\Delta \ll 1$:

$$\eta = t_{sp}^3 \int_0^{t_{sp}} dt \, \Gamma[T(t)] \approx t_{sp}^3 \int_0^{t_{sp}} dt \, T_c^4 \exp\left(-\frac{A}{\Delta^2[T(t)]}\right)$$

where $A := \frac{16\pi}{3} \frac{\sigma^3}{\ell^2 T_c}$

How much of space is nucleated?

- The "mixed" EoS provides the critical temperature

$$T_c = 183\,\mathrm{MeV}$$

- And the latent heat

$$\ell = 814\,\mathrm{MeV}/\mathrm{fm}^3$$

- We take the surface tension from the lattice

$$\sigma = 2\,\mathrm{MeV}/\mathrm{fm}^2$$

[Beinlich, Karsch, Peikert, 1998]

How much of space is nucleated?

- Therefore, $A \approx 1.7 \times 10^{-5} \ll 1$
- However, as we really want an overestimate, let us take $A \cong 0$
- Finally, our overestimate for η is given by

$$\eta \approx 4(t_{sp}T_c)^4$$

How important is nucleation?

How important is nucleation in heavy-ion collisions?

A) Bag Model: $\eta_{Bag} \approx 15 \longrightarrow 1$

B) “Mixed” EoS: $\eta_{Mixed} \approx 0.1$

- An equation of state with a weaker phase transition makes nucleation less important in heavy ion collisions.

Nucleation in the early Universe

- The early Universe expands very slowly (as compared to typical HIC time scales).

- Its typical times are much longer than those in HIC

$$H_{Univ}^{-1} = 10^{19} H_{HIC}^{-1}$$

- We may readily see that

$$\eta_{Univ} \approx 10^{76} \eta_{HIC}$$

- Spinodal decomposition can almost surely be neglected, regardless of the equation of state.

Conclusions

- In heavy-ion collisions, nucleation is probably a secondary mechanism of phase conversion. This strongly suggests that spinodal decomposition should play a major role in a 1st order hadronization process.

- In the early universe, the phase conversion is completely dominated by bubble nucleation and spinodal decomposition may be neglected. This is a known result, although the huge values of η now give a quantitative indication of the strength of this result.

However...
... some improvements are surely needed:

- Reheating
- Finite-size
- Dynamical expansion
- Real-time phase transition dynamics