Gluon saturation and the

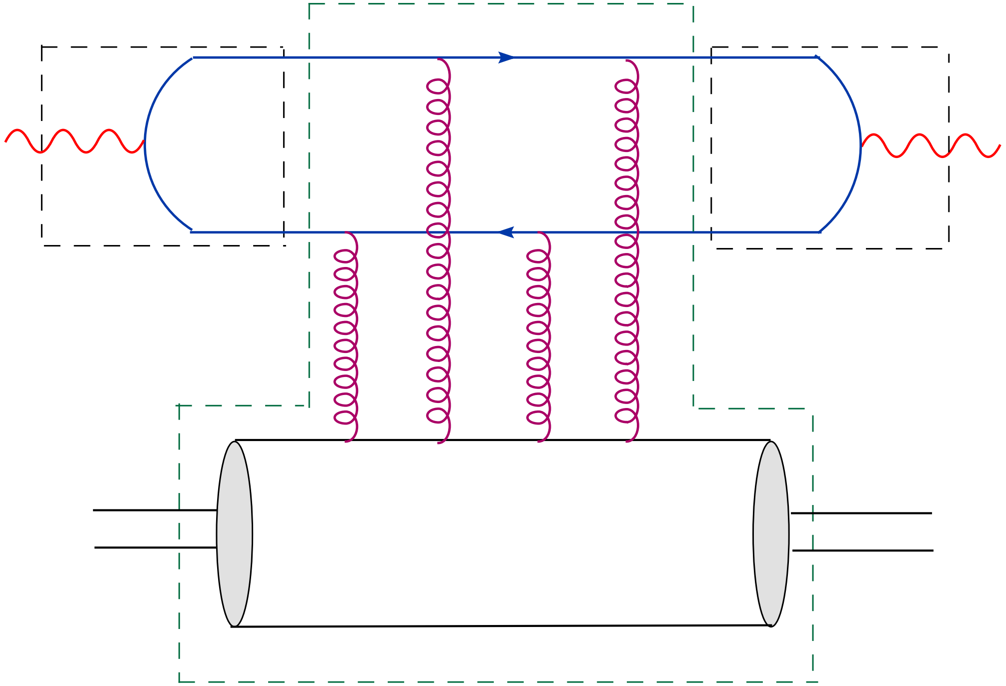
Color Glass Condensate

Part III

Edmond Iancu
SPhT Saclay & CNRS

Reminder : Dipole factorization for DIS

$$\sigma_{\gamma^* p}(x,Q^2) = \int_0^1 \mathrm{d}z \int \mathrm{d}^2\boldsymbol{r}\ |\Psi_\gamma(z,\boldsymbol{r};Q^2)|^2\ \sigma_{\mathrm{dipole}}(x,\boldsymbol{r})$$



$$\sigma_{\mathrm{dipole}}(x,r)\ =\ 2\int \mathrm{d}^2\boldsymbol{b}\ T(x,\boldsymbol{r},\boldsymbol{b})$$

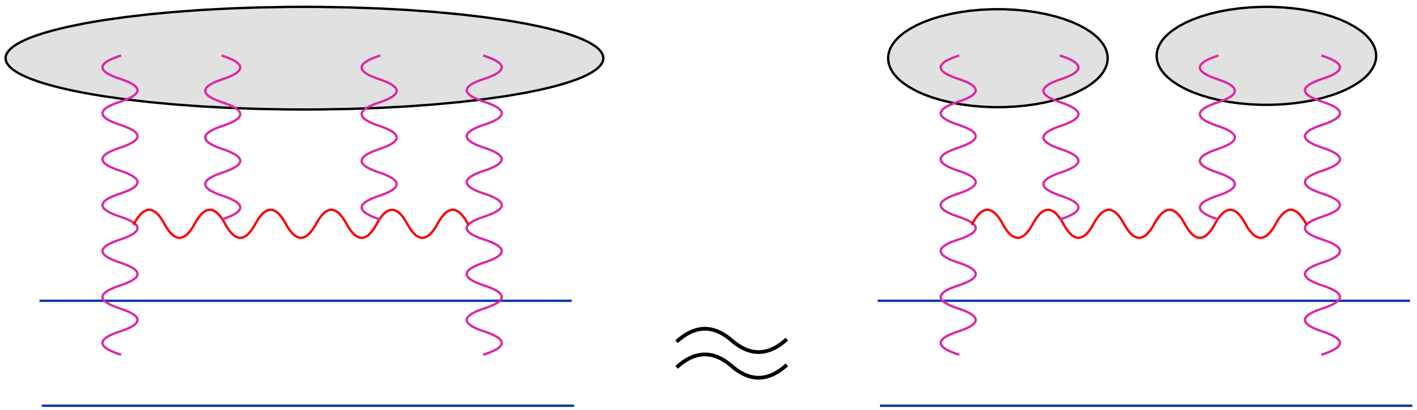
- Unitarity bound on the dipole amplitude: $T(x,\boldsymbol{r},\boldsymbol{b})\ \leq\ 1$

Reminder : Balitsky–Kovchegov equation

$$\frac{\partial}{\partial Y}\langle T(\boldsymbol{x},\boldsymbol{y})\rangle_Y = \frac{\bar{\alpha}_s}{2\pi}\int_{\boldsymbol{z}} \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2} \Big\langle \underbrace{-T(\boldsymbol{x},\boldsymbol{y})+T(\boldsymbol{x},\boldsymbol{z})+T(\boldsymbol{z},\boldsymbol{y})}_{\text{BFKL (linear)}} - \underbrace{T(\boldsymbol{x},\boldsymbol{z})T(\boldsymbol{z},\boldsymbol{y})}_{\text{non–linear}}\Big\rangle_Y$$

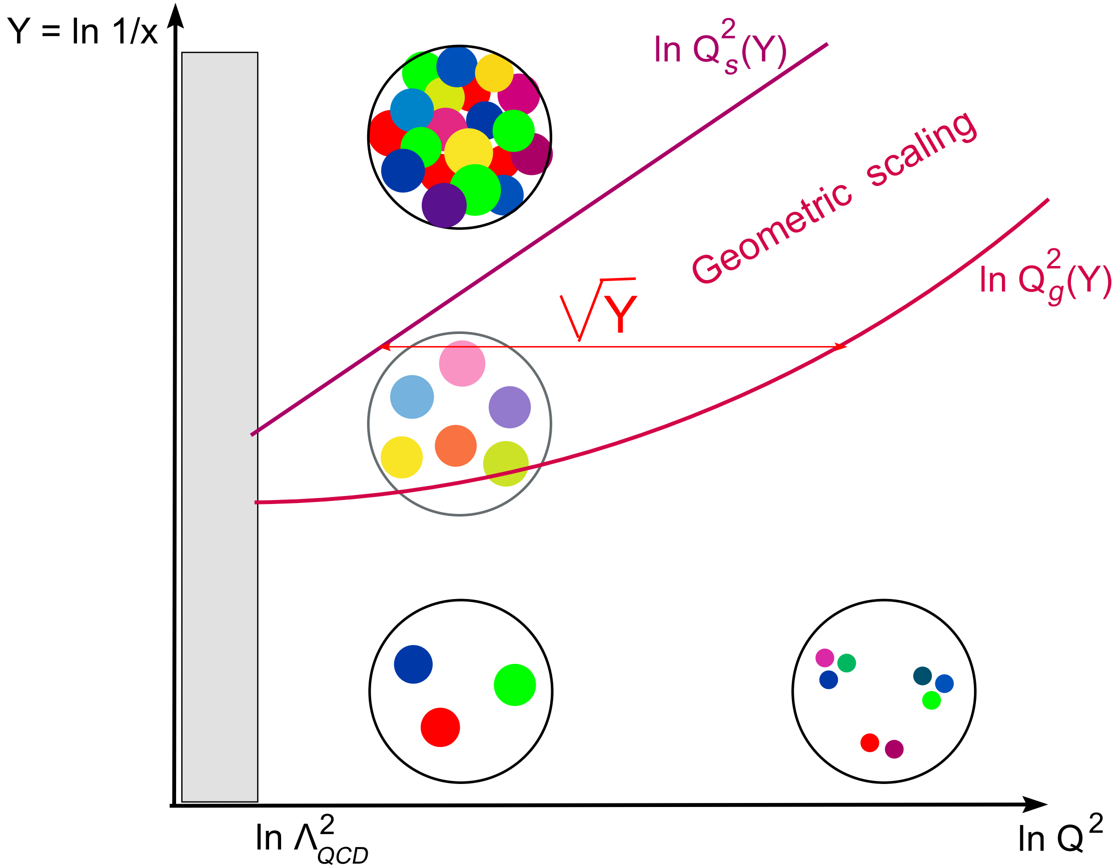
- Mean field approximation assuming factorization:

$$\langle T(\boldsymbol{x},\boldsymbol{z})T(\boldsymbol{z},\boldsymbol{y})\rangle_Y \approx \langle T(\boldsymbol{x},\boldsymbol{z})\rangle_Y \langle T(\boldsymbol{z},\boldsymbol{y})\rangle_Y$$



- Neglects correlations in the target wavefunction.

Reminder : Saturation & Geometric scaling

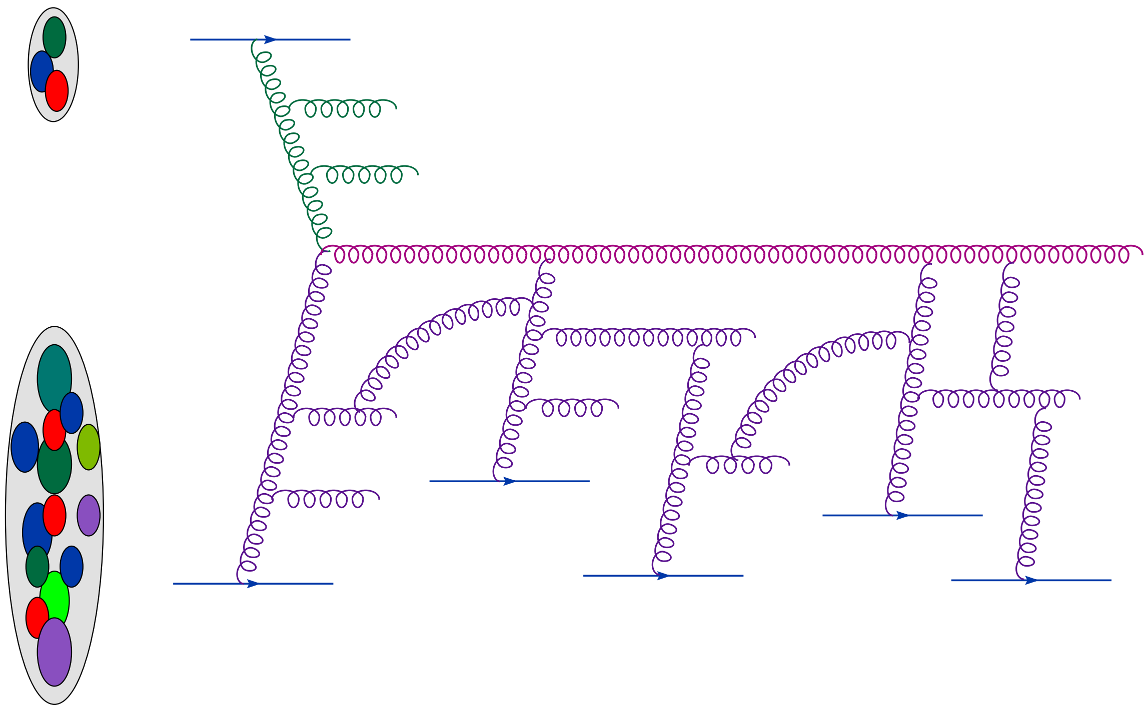


- Saturation line: $\ln Q_s^2(Y) = \lambda Y$ with $\lambda \simeq 0.3$ (NLO)
- Geometric scaling above saturation (in the 'dilute regime')

$$T(r,Y) \simeq \mathrm{e}^{-\gamma_s(\rho-\rho_s)} = (r^2 Q_s^2)^{\gamma_s}$$

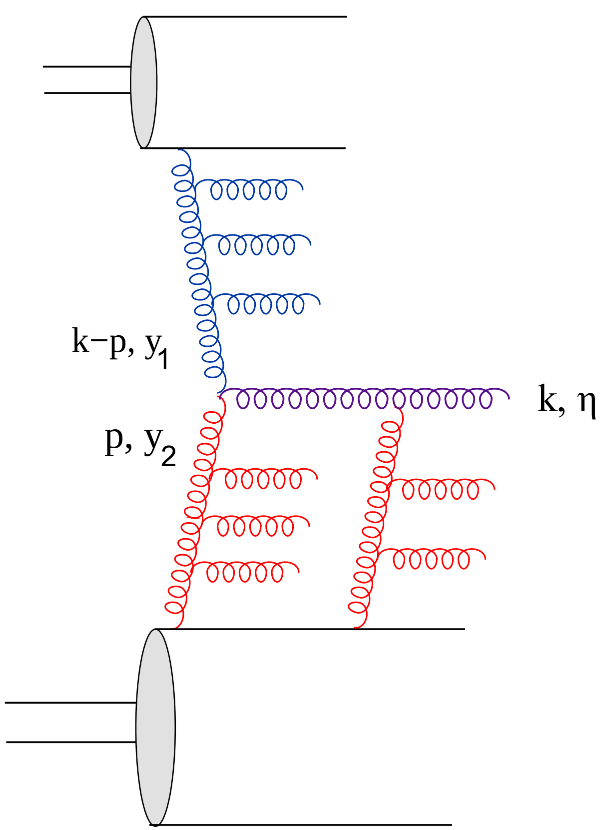
Gluon production in pp or pA collisions

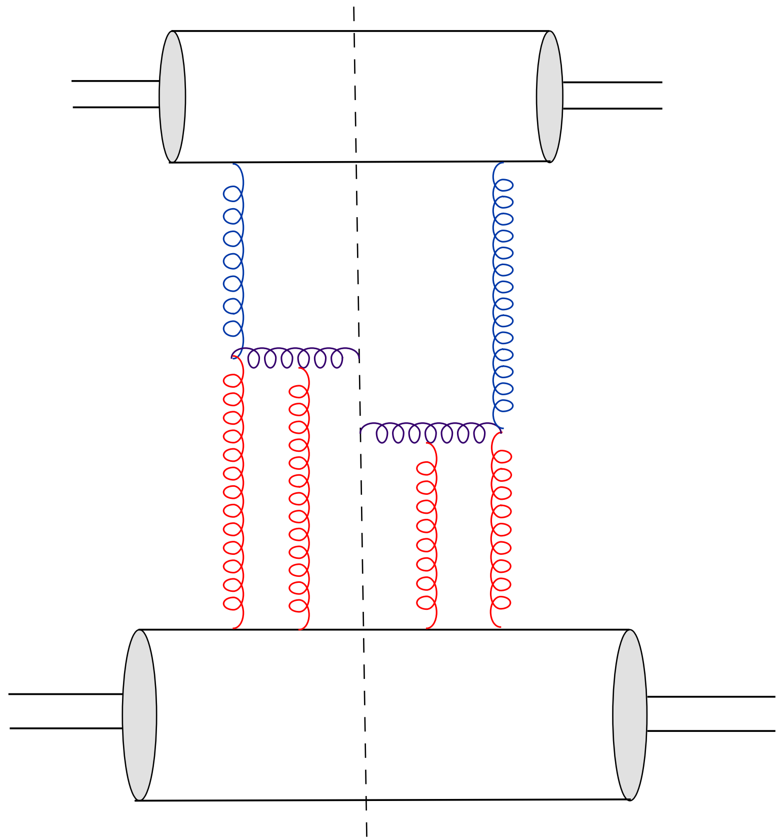
- ‘Dense–dilute’ scattering
  - pA collisions (RHIC, LHC)
  - pp collisions at ‘forward rapidity’ (LHC)
- Only one parton from the dilute projectile gets involved



- A probe of the gluon distribution inside the dense target !

Forward gluon production (cf. lecture by F. Gelis)



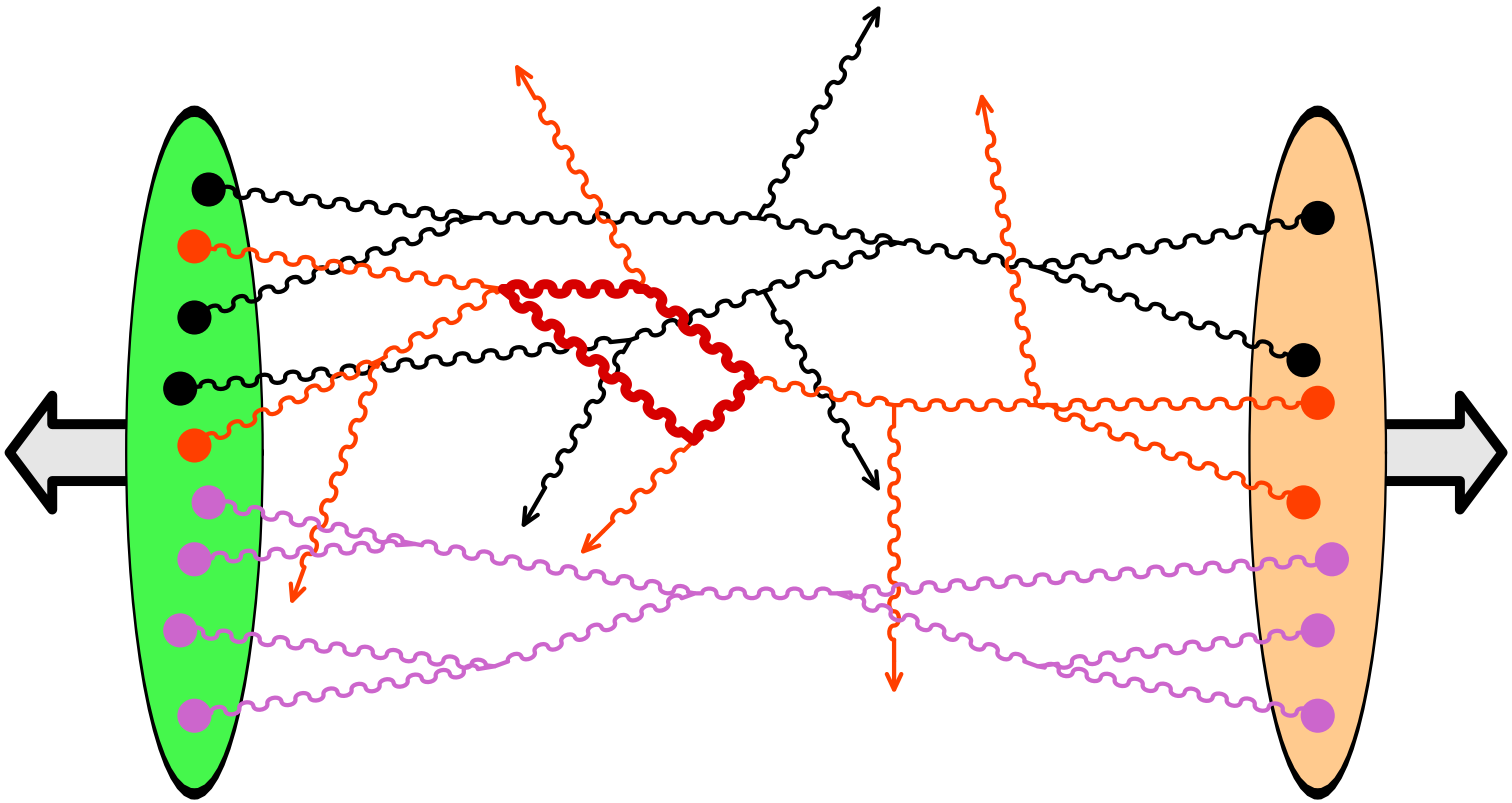


- Effective gluon–gluon dipole in the cross–section

$$\frac{\mathrm{d}\sigma^{pp\to JX}}{\mathrm{d}\eta\,\mathrm{d}^2k_\perp} \sim \frac{1}{k_\perp^2}\, xG_p(x_1, k_\perp^2) \int \mathrm{d}^2\boldsymbol{r}\ \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\ \nabla_r^2\sigma_{(gg)}(\boldsymbol{r}, x_2)$$

- $x_{1,2} = (k_\perp/\sqrt{s})\mathrm{e}^{\pm\eta}$

What about nucleus–nucleus collisions ?

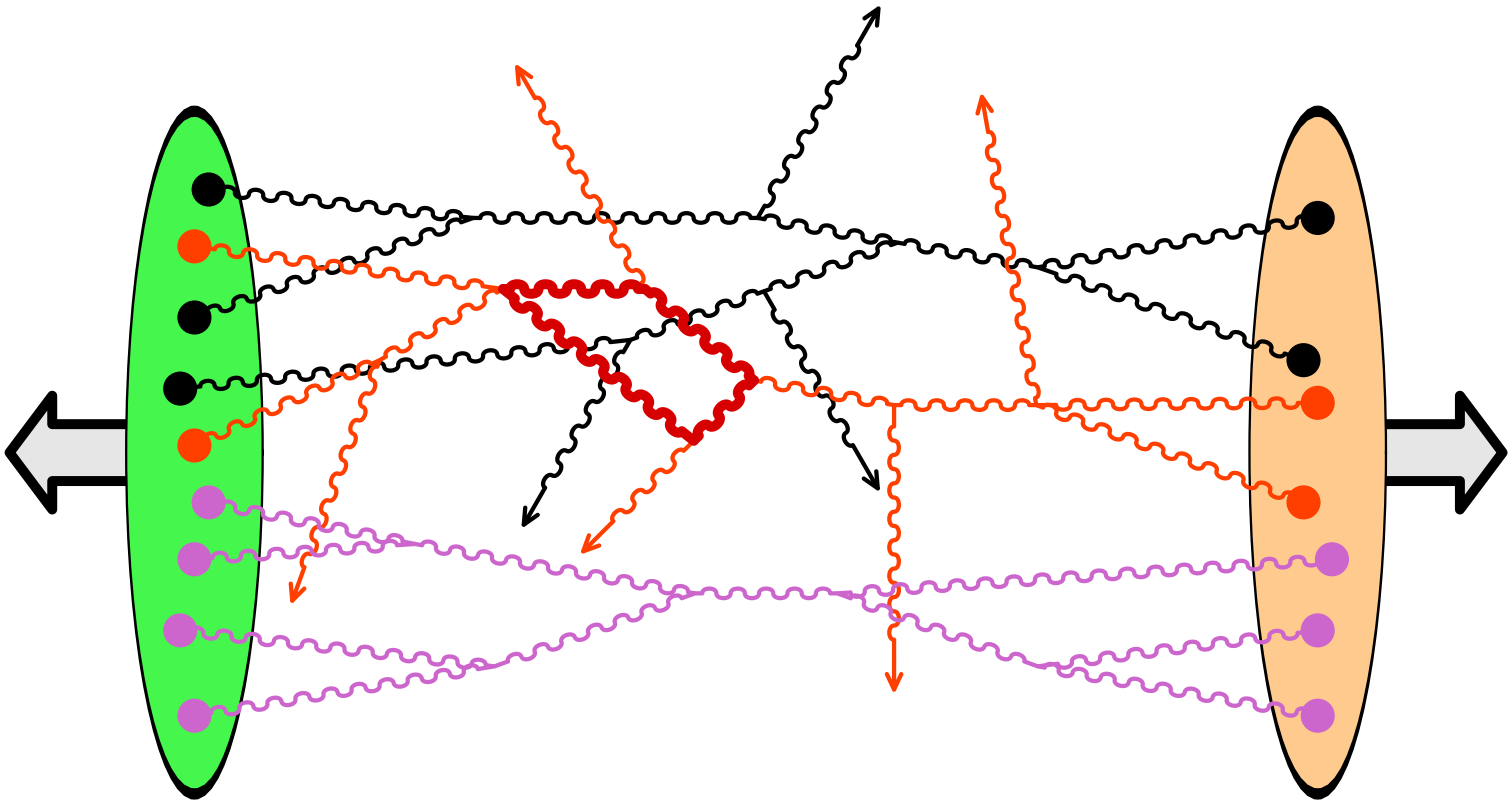


- ‘Dense–dense scattering’ : multi–particle interactions probe higher gluon correlations in the nuclear wavefunctions

  $\Longrightarrow$ a more complete description of the gluon distribution

- The CGC formalism

What about nucleus–nucleus collisions ?

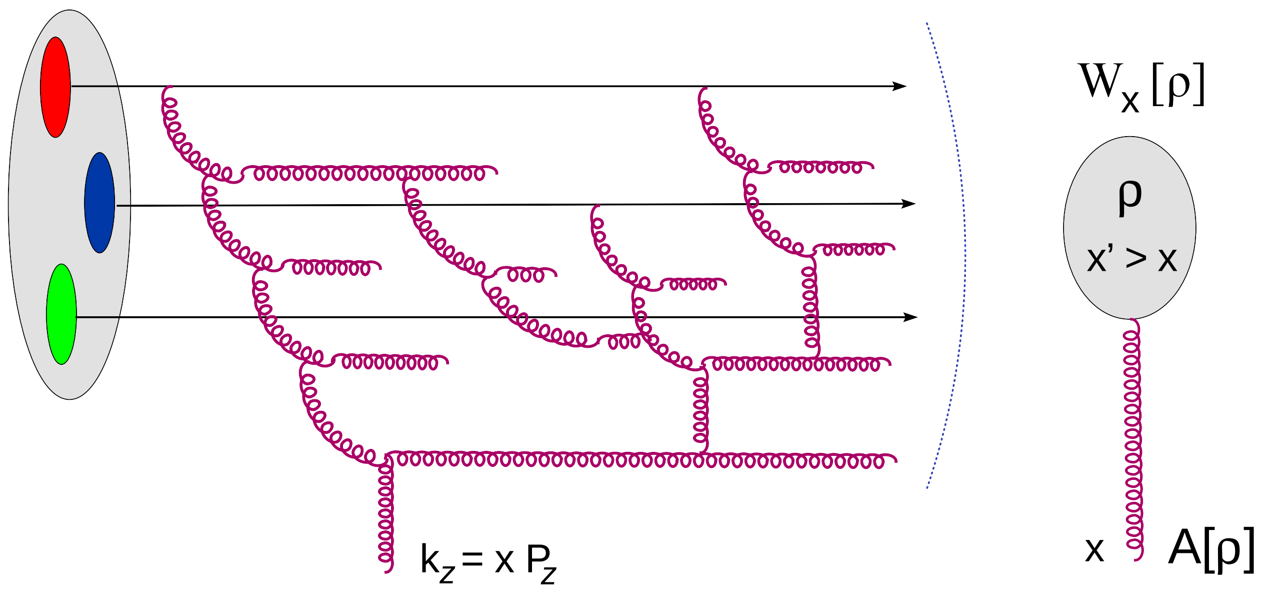


- complicated evolution of the wavefunction (JIMWLK)
- additional ‘complications’ due to final state interactions
- factorization is far from being obvious
- Factorization recently proven for the inclusive parton production (see lectures by F. Gelis and R. Venugopalan)

The Color Glass Condensate

(McLerran, Venugopalan, 1994; E.I., Leonidov, McLerran, 2000)

- Effective theory for the small–x gluons at/near saturation
- Small–x gluons: Classical color fields radiated by fast color sources ($x' \gg x$) 'frozen' in some random configuration

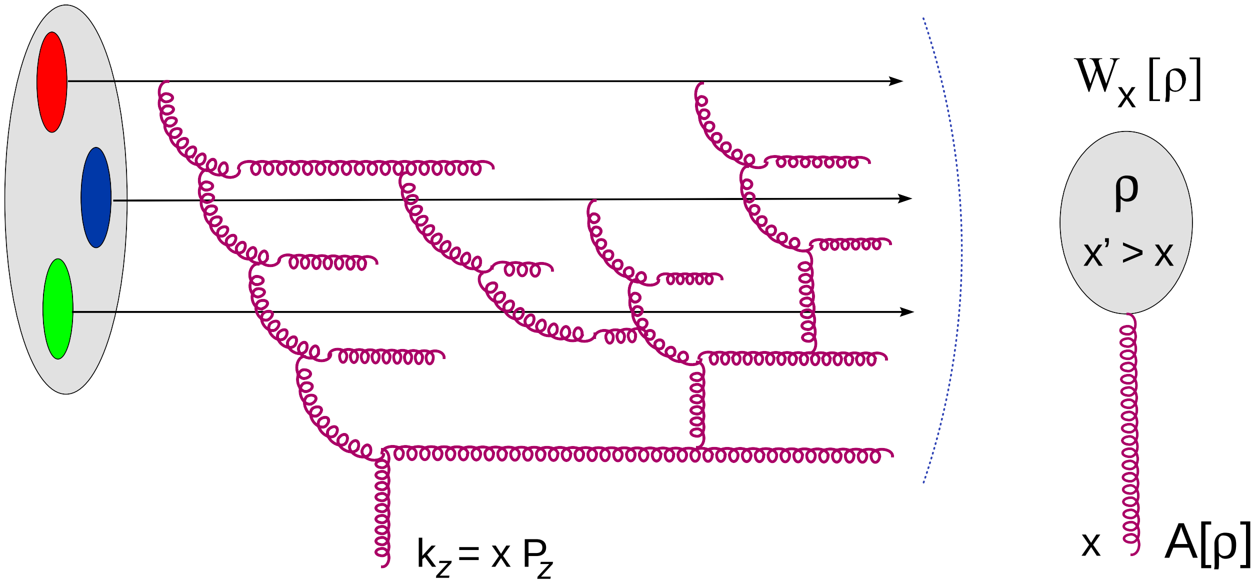


- Quantum modes with $x' > x$ (or $k^+ > \Lambda^+ = xP^+$) have been 'integrated out' and replaced with a random color charge distribution with density ρ^a and probability distribution $W_x[\rho]$

The Color Glass Condensate

(McLerran, Venugopalan, 1994; E.I., Leonidov, McLerran, 2000)

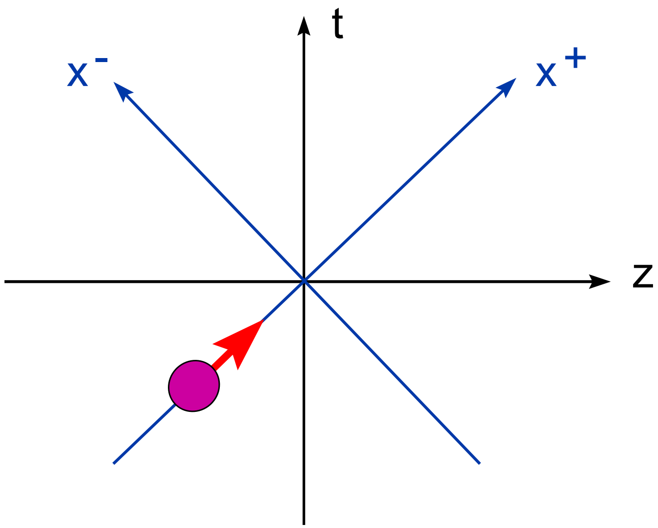
- Effective theory for the small–x gluons at/near saturation
- Small–x gluons: Classical color fields radiated by fast color sources ($x' \gg x$) 'frozen' in some random configuration

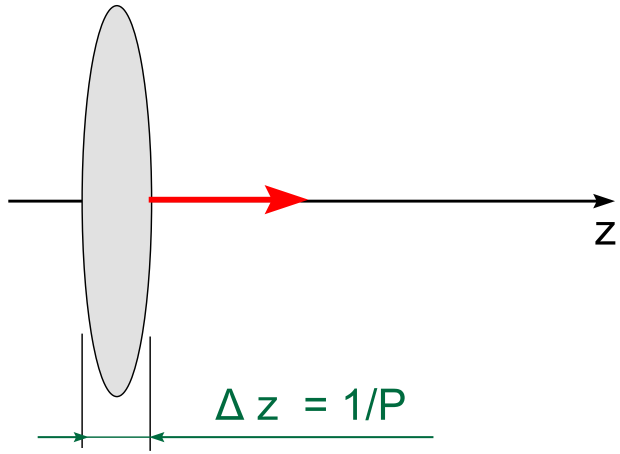


- Classical field equations (Yang–Mills) for the field $A^{\mu}_{a}[\rho]$
- Probability distribution for the charge density at x : $W_x[\rho]$
- Renormalization group equation for $W_x[\rho]$: JIMWLK

Light Cone notations & Kinematics

- The hadron moves in the positive z direction, with $v \simeq c = 1$





- Longitudinal momentum $P \gg M \Longrightarrow P^\mu = (E \approx P, 0, 0, P)$

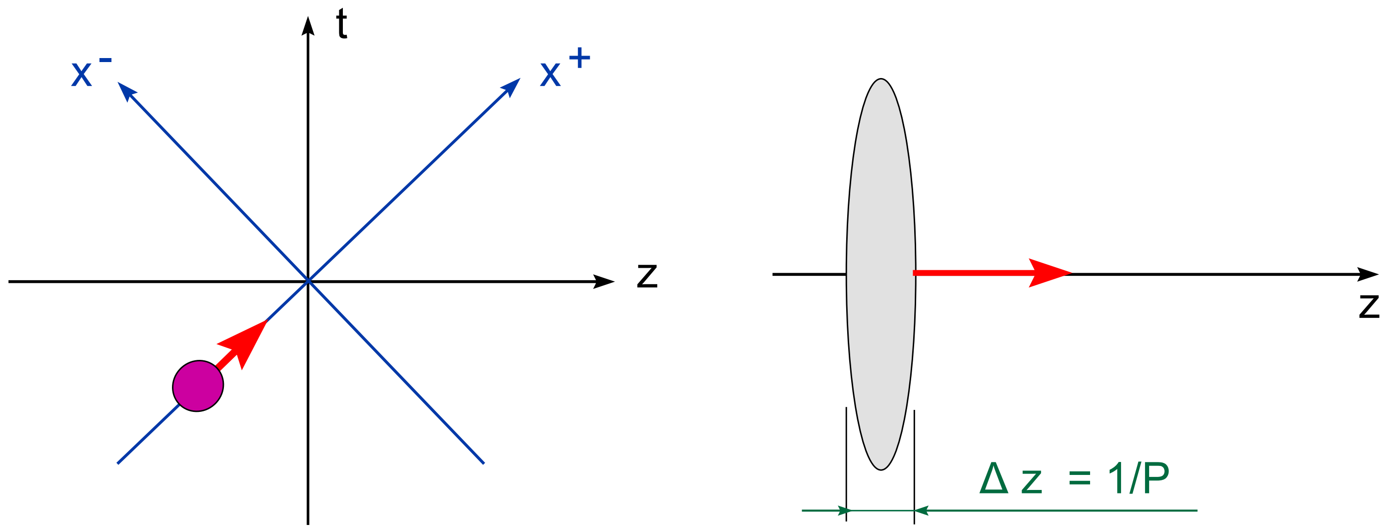
$$P^+ \equiv \frac{1}{\sqrt{2}}(E+P) \simeq \sqrt{2}P\,, \qquad P^- \equiv \frac{1}{\sqrt{2}}(E-P) \simeq 0$$

- A classical particle: $z \simeq t$ or $x^- \simeq 0$ with

$$\underbrace{x^+ \equiv \frac{1}{\sqrt{2}}(t+z) \simeq \sqrt{2}t\,,}_{\text{LC time}} \qquad \underbrace{x^- \equiv \frac{1}{\sqrt{2}}(t-z) \simeq 0}_{\text{LC longitudinal coord.}}$$

Light Cone notations & Kinematics

- The hadron moves in the positive z direction, with $v \simeq c = 1$



- Longitudinal momentum $P \gg M \Longrightarrow P^\mu = (E \approx P, 0, 0, P)$

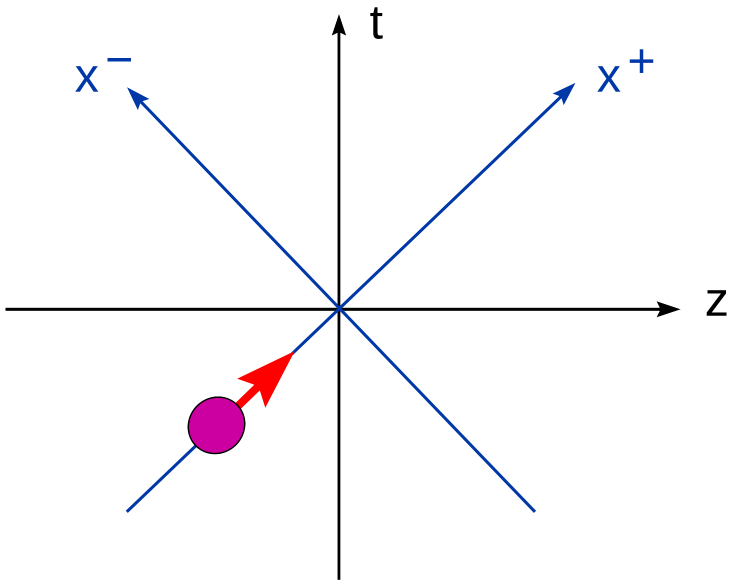
$$P^+ \equiv \frac{1}{\sqrt{2}}(E+P) \simeq \sqrt{2}P\,, \quad P^- \equiv \frac{1}{\sqrt{2}}(E-P) \simeq 0$$

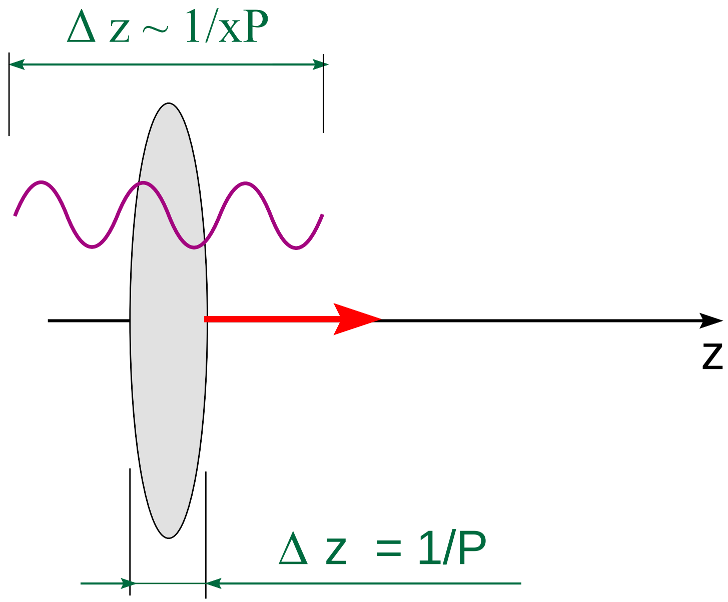
- Even for the quantum system, the wavefunction is strongly localized near $x^- = 0$ ("pancake")

$$\Delta x^- \sim \frac{1}{P^+} \sim \frac{1}{\gamma M} \ll \frac{1}{M}$$

Light Cone notations & Kinematics

- The hadron moves in the positive z direction, with $v \simeq c = 1$





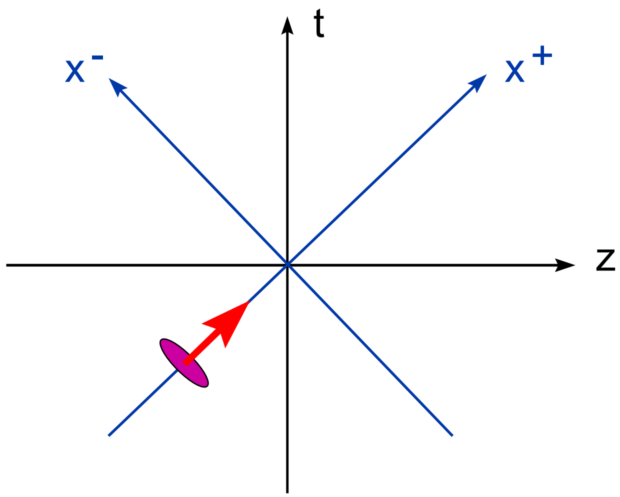
- Longitudinal momentum $P \gg M \Longrightarrow P^\mu = (E \approx P, 0, 0, P)$

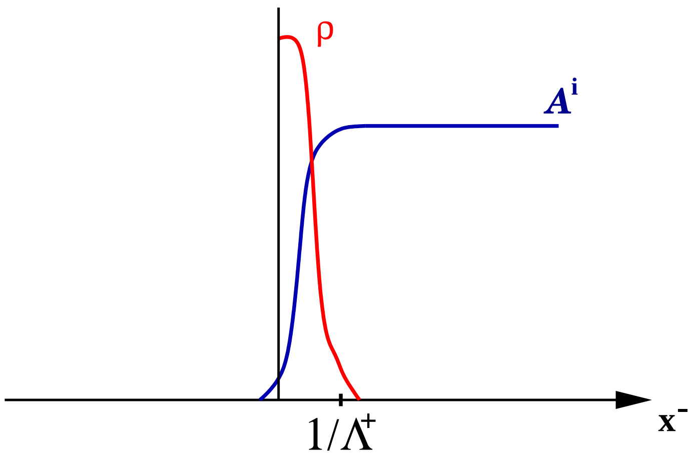
$$P^+ \equiv \frac{1}{\sqrt{2}}(E+P) \simeq \sqrt{2}P\,, \quad P^- \equiv \frac{1}{\sqrt{2}}(E-P) \simeq 0$$

- Small–x gluons are, however, more delocalized

$$\Delta x^- \sim \frac{1}{xP^+} \ll \frac{1}{P^+}$$

The Yang–Mills equations





- The ‘color source’ : a current in the ‘plus’ direction

$$\left(D_\nu F^{\nu\mu}\right)_a(x) \;=\; \delta^{\mu+}\rho_a(x^-,x_\perp)$$

- The source ρ_a is
  - independent of the LC time x^+ (Lorentz time dilation)
  - localized near $x^-=0$ (i.e., $z=t$) within a distance

$$\Delta x^- \;\sim\; 1/\Lambda^+ \quad \text{with} \quad \Lambda^+ \;=\; \mathrm{xP}^+$$

Classical solution (cf. lecture by F. Gelis)

- Only one independent field d.o.f. (independent of x^+)
- Coulomb gauge : $\nabla^i A^i_a = 0 \Longrightarrow A^i_a = 0, \quad i = 1, 2$

$$-\nabla^2_\perp A^+_a(x^-, x_\perp) = \rho_a(x^-, x_\perp)$$

- Exercise: Show that the solution is of the form

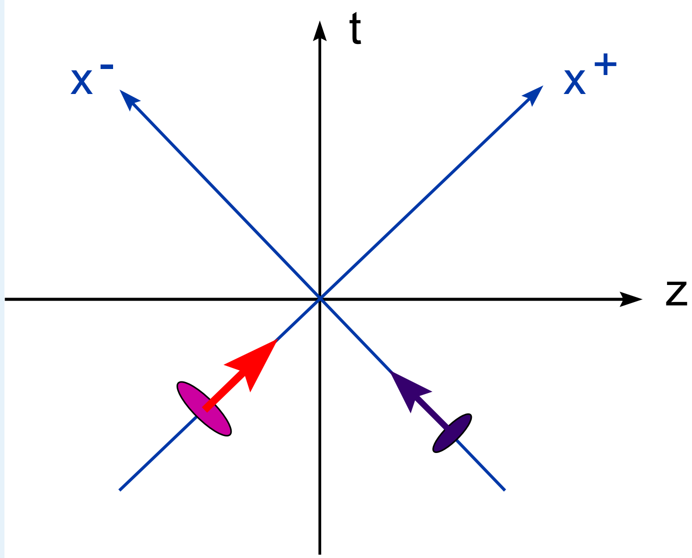
$$A^+_a(x^-, \boldsymbol{x}) = \int \mathrm{d}^2\boldsymbol{y}\, \frac{1}{4\pi} \ln \frac{1}{(\boldsymbol{x}-\boldsymbol{y})^2\mu^2}\, \rho_a(x^-, \boldsymbol{y})$$

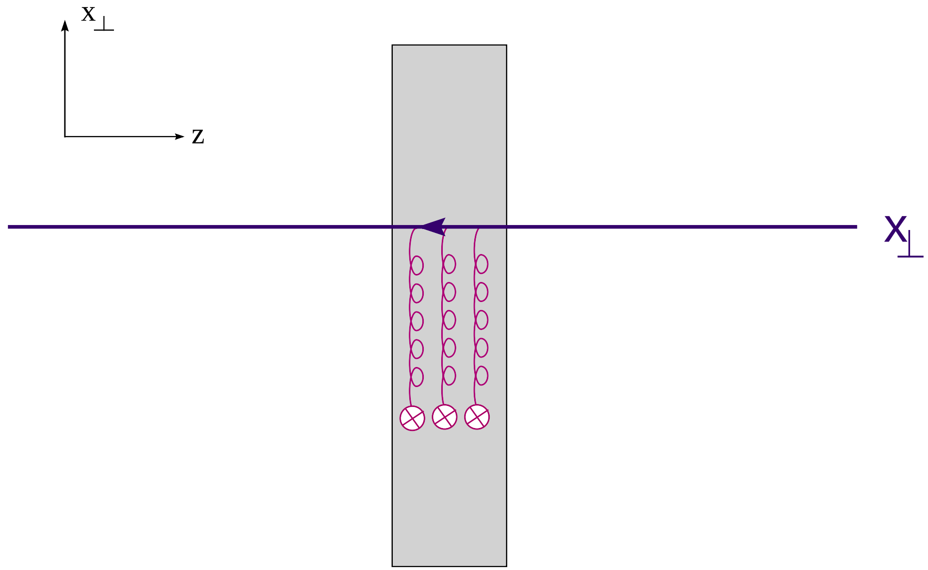
  NB : Localized near $x^- = 0$, so like the color charge itself.

  ‘Weiszäcker–Williams color field’

- One can easily trade ρ^a for A^+_a (e.g., $W_x[\rho] \to W_x[A^+]$)
- Where are the non–linear effects ?!

Eikonal Approximation





- Right moving target (CGC) + Left moving projectile ($q,\ \bar{q},\ g$...)
- Field equations in the background field:

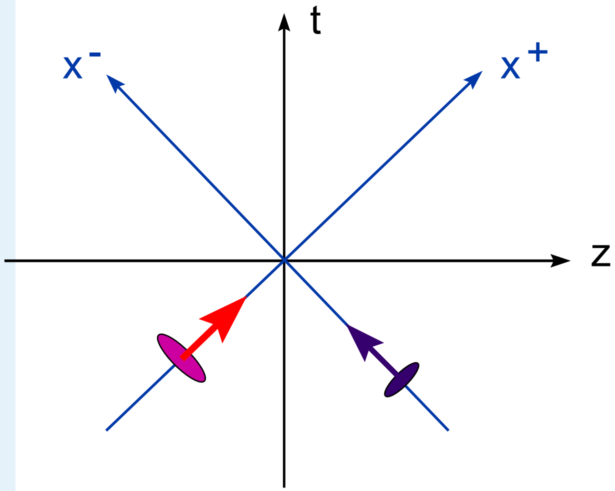
$$\gamma_\mu D^\mu \psi(x) = 0 \qquad (D^\mu = \partial^\mu - igT^a A_a^\mu)$$

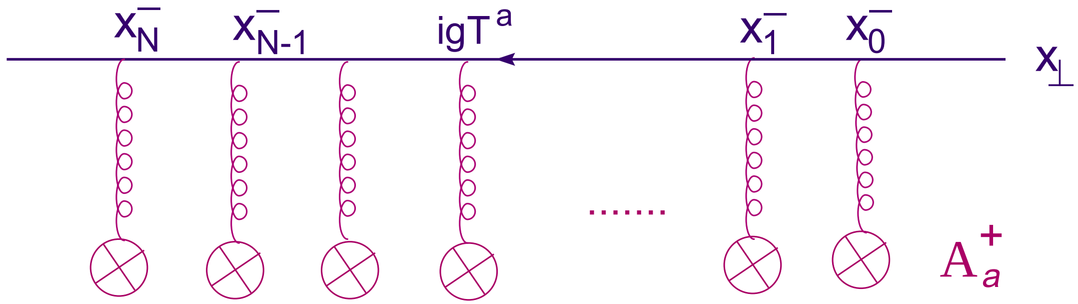
- $D^+ \sim 1/\Delta x^-$ is much larger than $D_\perp$

$\Longrightarrow\ D^+ S(x^-, \boldsymbol{x}_\perp) \approx 0$

$\Longrightarrow$ straight line trajectory: $\boldsymbol{x}_\perp =$ const.

Wilson lines





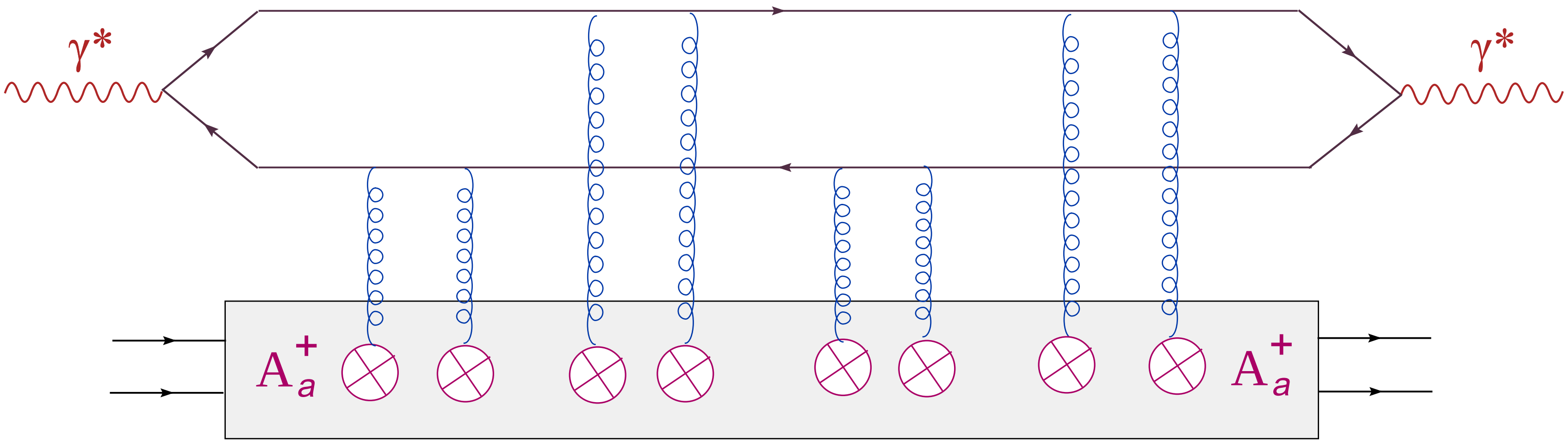
$$D^+ S(x^-, \boldsymbol{x}_\perp) \equiv \left(\frac{\partial}{\partial x^-} - igT^a A_a^+ \right) S(x^-, \boldsymbol{x}_\perp) = 0$$

$$\Longrightarrow \quad S(x^-, \boldsymbol{x}_\perp) = \mathrm{P} \exp\left(\mathrm{i}g \int_{-\infty}^{x^-} \mathrm{d}y^- A_a^+(x^-, \boldsymbol{x}_\perp) T^a \right) \equiv V(x^-, \boldsymbol{x}_\perp)$$

- Path–ordered exponential : color rotation, non–linear in A^+

$$V(x^-) = \mathrm{e}^{\mathrm{i}g\epsilon A^+(x_N^-)}\, \mathrm{e}^{\mathrm{i}g\epsilon A^+(x_{N-1}^-)} \ldots \mathrm{e}^{\mathrm{i}g\epsilon A^+(x_0^-)}$$

Dipole scattering off the CGC

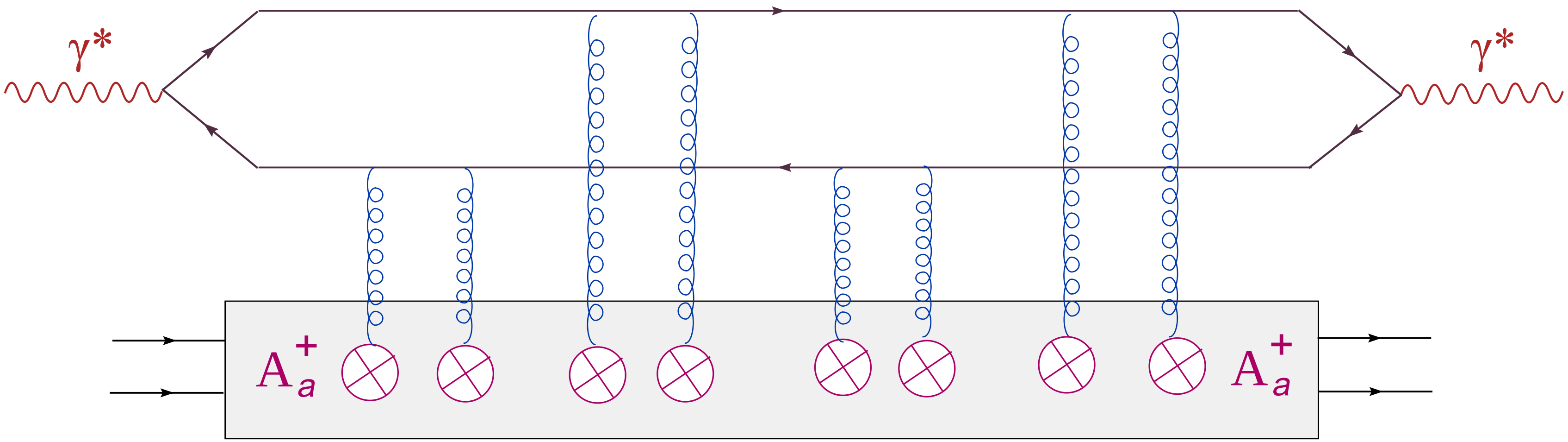


$$S(\boldsymbol{x},\boldsymbol{y})[A^+] = \frac{1}{N_c}\mathrm{tr}\big(V(\boldsymbol{x})\,V^\dagger(\boldsymbol{y})\big)$$

$$V(\boldsymbol{x}) \equiv \mathrm{Pexp}\Big(\mathrm{i}g\int \mathrm{d}x^- A_a^+(x^-,\boldsymbol{x})t^a\Big) \quad \in \mathrm{SU}(N_c)$$

- Color trace: the dipole is color neutral
- Color transparency: when $\boldsymbol{x}\to\boldsymbol{y}$, $S\to 1$
- Unitarity manifest: $|S|\leq 1$ (multiple scattering)

The weight function $W_Y[\rho]$



- $S(\boldsymbol{x},\boldsymbol{y})[A^+]$ = the 'event-by-event' S–matrix : valid for a given configuration of the color sources (fields) in the target
- The physical amplitude: average over all configurations average over ρ with the weight function $W_Y[A^+]$:

$$\langle S(\boldsymbol{x},\boldsymbol{y})\rangle_Y = \int \mathrm{D}[A^+]\ W_Y[A^+]\ S(\boldsymbol{x},\boldsymbol{y})[A^+]$$

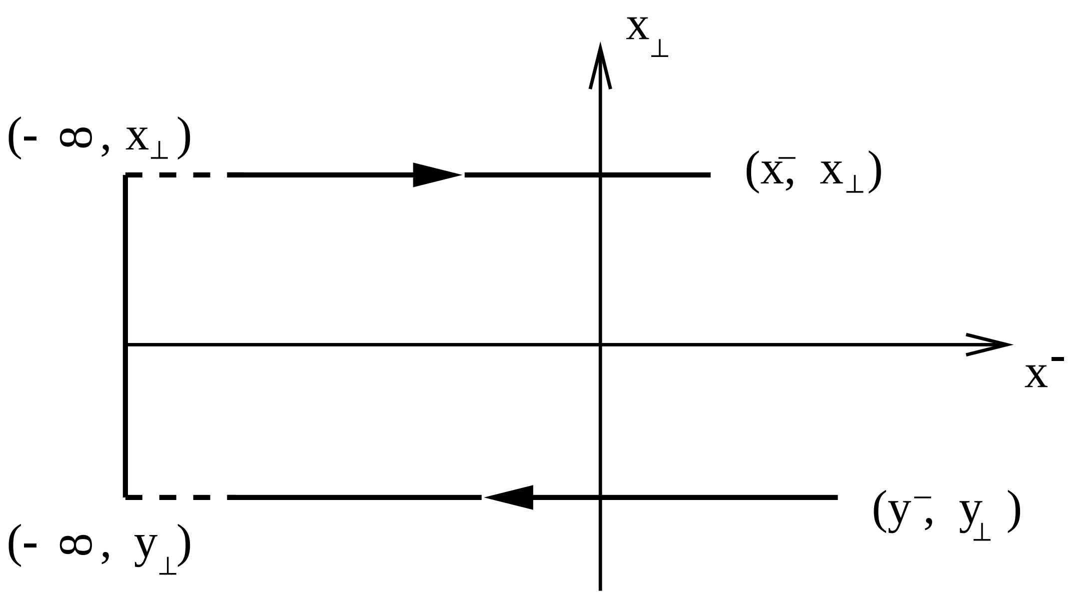
- Computing $W_Y[A^+]$: the main issue in the CGC formalism

The (unintegrated) gluon distribution

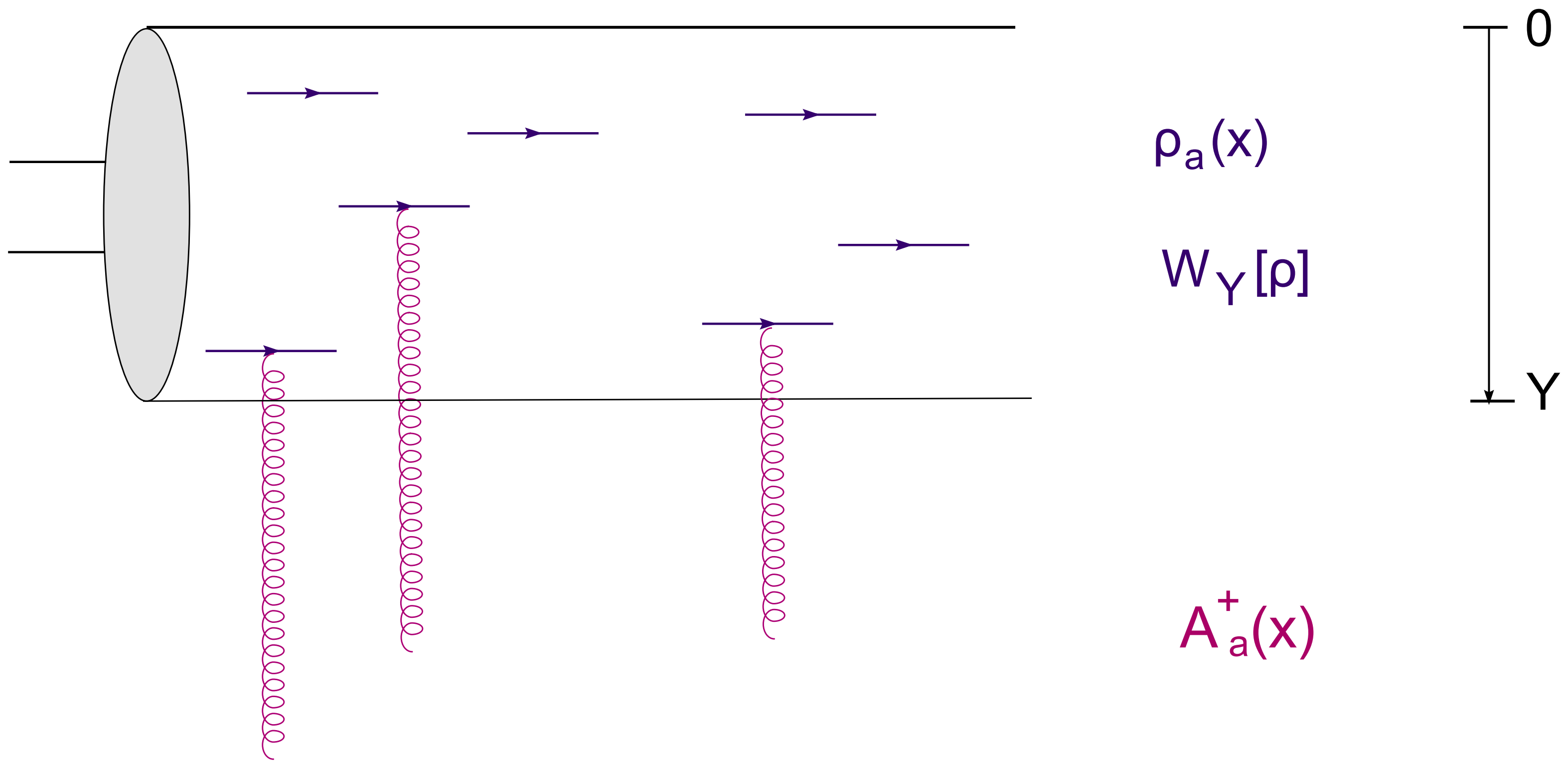
$$n(Y, k_\perp, b_\perp) \equiv \frac{(2\pi)^3}{2(N_c^2-1)} \frac{\mathrm{d}N}{\mathrm{d}Y\,\mathrm{d}^2k_\perp\,\mathrm{d}^2b_\perp}$$

- What is ‘gluon number’ ?! (a priori, gauge–dependent)
- A Fourier transform of the gauge–invariant 2–point function

$$\left\langle \mathrm{Tr}\,\left\{E^i(x)\,W_\gamma(x,y)\,E^i(y)\,W_\gamma(y,x)\right\}\right\rangle_Y, \quad W_\gamma(x,y) = V(\boldsymbol{x})V^\dagger(\boldsymbol{y})$$

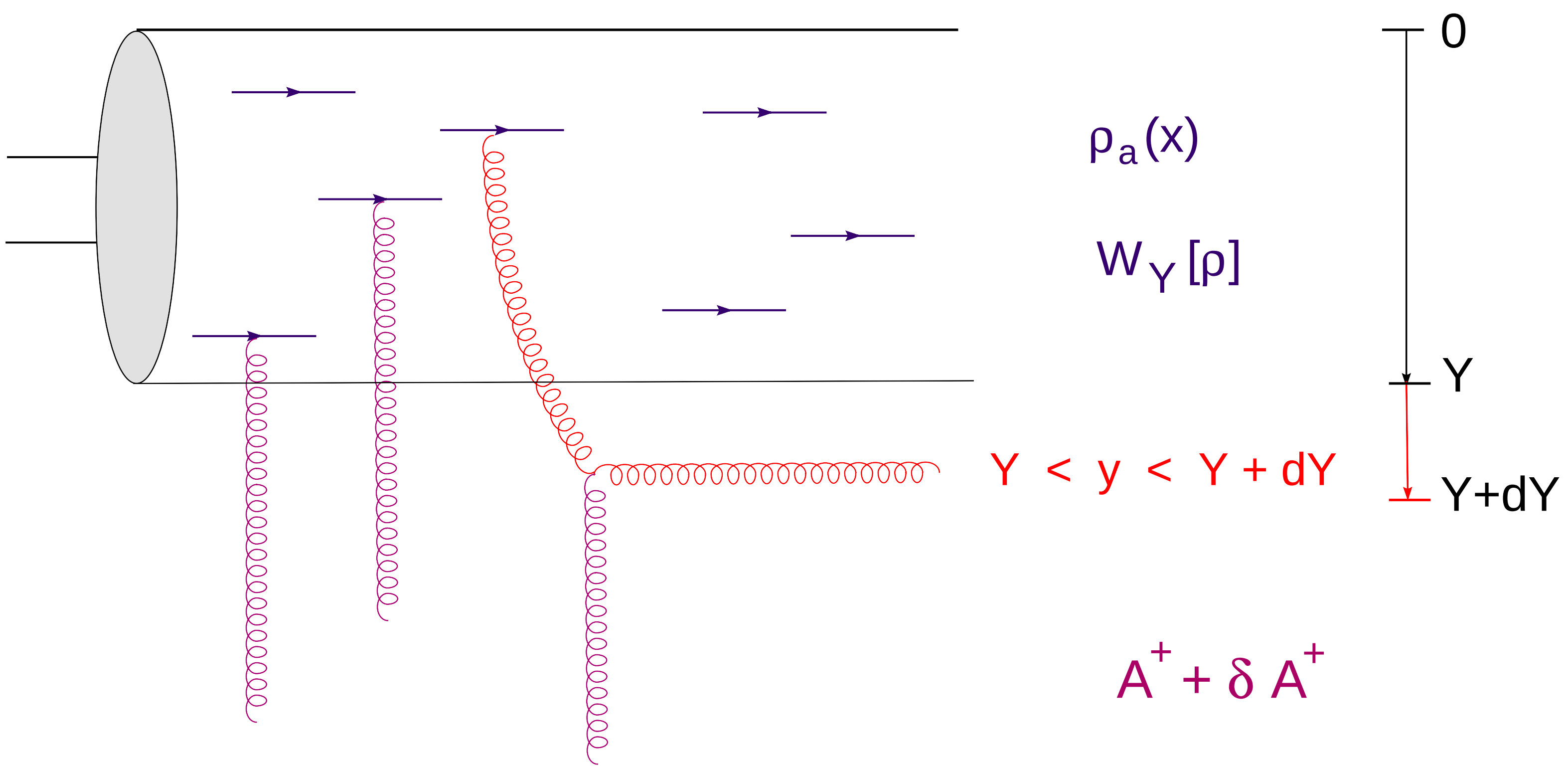


Renormalization group at small x



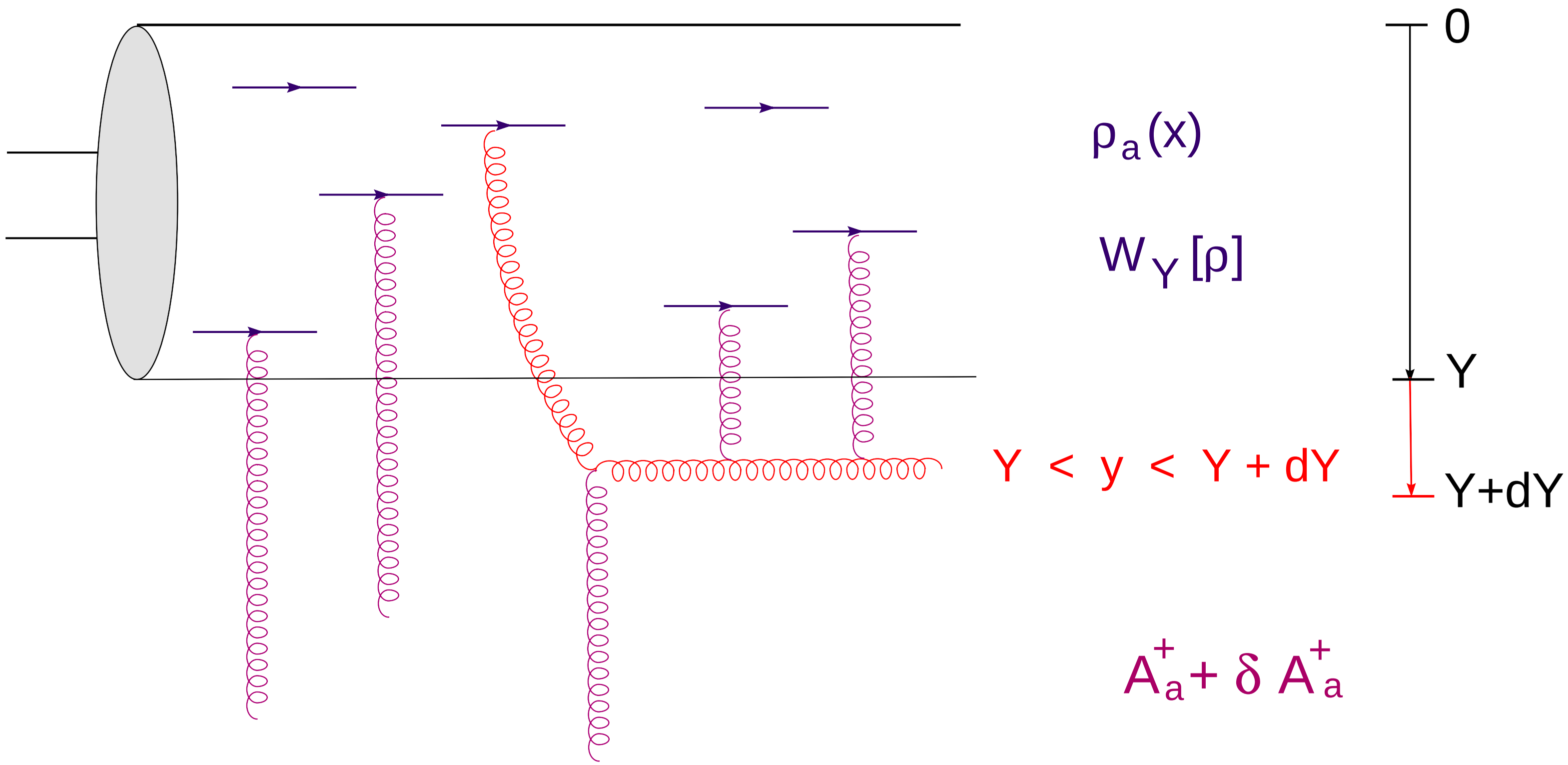
- Rapidity $Y = \ln\frac{1}{x}$: The relevant information about the gluon distribution has been included in the weight function $W_Y[\rho]$
- Alternatively: $W_Y[A^+]$ (a simple change of variables)

Renormalization group at small x



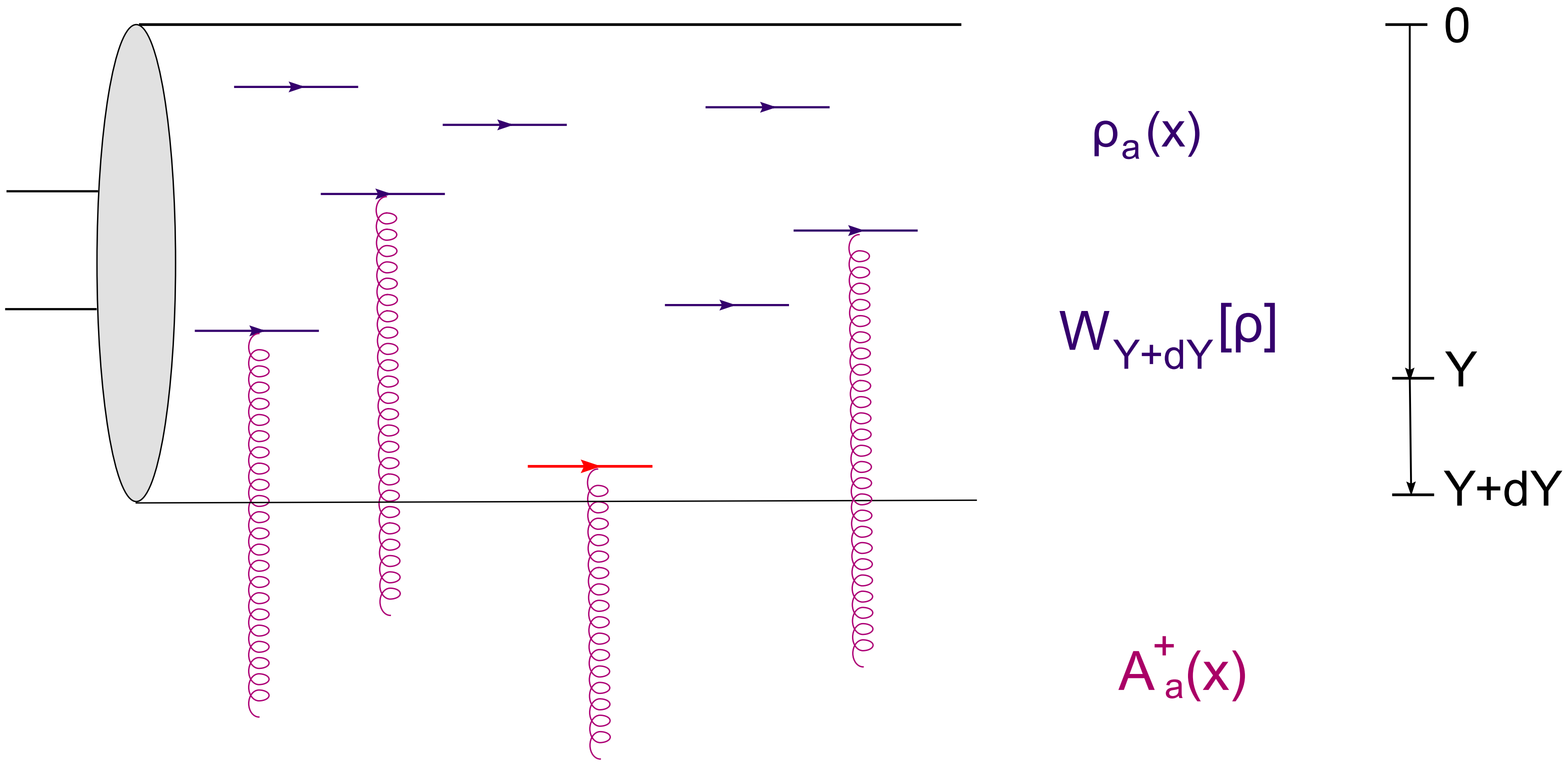
- **Rapidity $Y + \mathrm{d}Y$:** One addition ‘color source’ (gluon) is being radiated, from one of the color sources at Y
- **Low density/energy :** The new gluon is incoherently produced from any of the previous sources: $\delta\rho \propto \rho$

Renormalization group at small x



- **High density/energy :** The new gluon can rescatter off the color field produced by other sources: $\delta\rho =$ non–linear in ρ
- **Strategy:** Absorb the change in ρ and in the correlations into a change of the weight function: $W_Y[\rho] \longrightarrow W_{Y+\mathrm{d}Y}[\rho]$

Renormalization group at small x



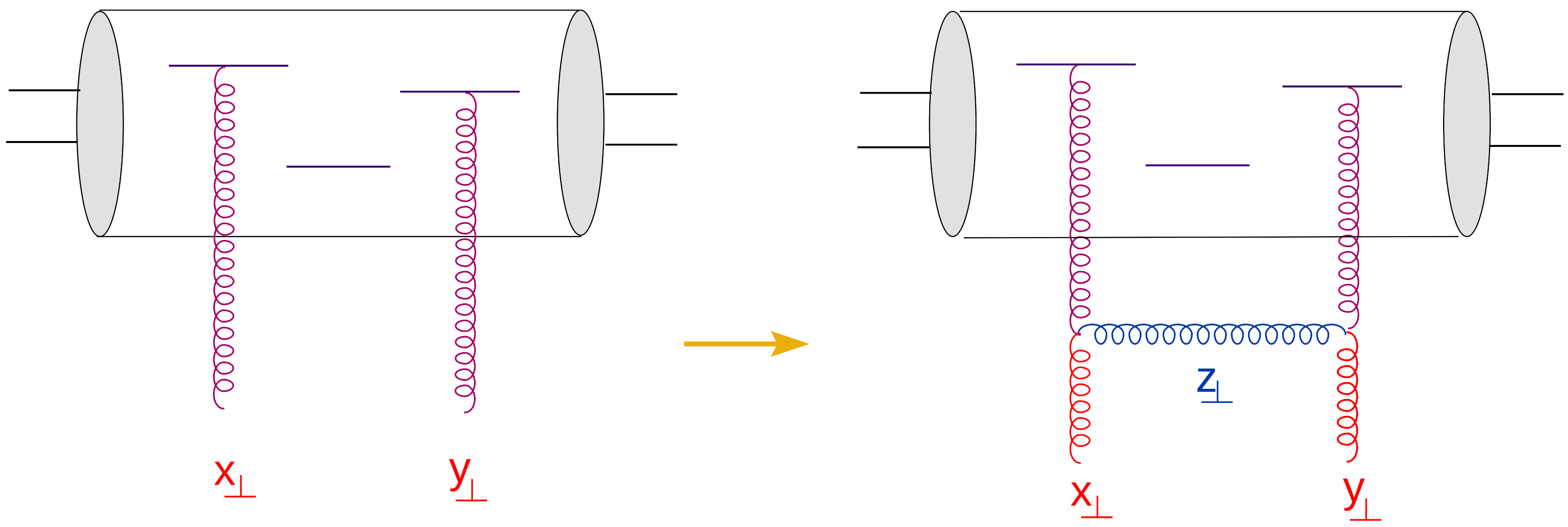
- Evolution equation for $W_Y[\rho]$ ('JIMWLK')

$$\frac{\partial W_Y[\rho]}{\partial Y} = H_{\mathrm{JIMWLK}}\left[\rho, \frac{\delta}{\delta\rho}\right] W_Y[\rho]$$

(Jalilian-Marian, E.I., McLerran, Weigert, Leonidov, and Kovner, 97–00)

Low density: BFKL Hamiltonian

- $W_Y[\rho]$ = a probability distribution $\Longrightarrow$ 'cut' diagrams : (amplitude $\times$ complex conjugate amplitude)
- Weak fields/low density: one must recover BFKL evolution

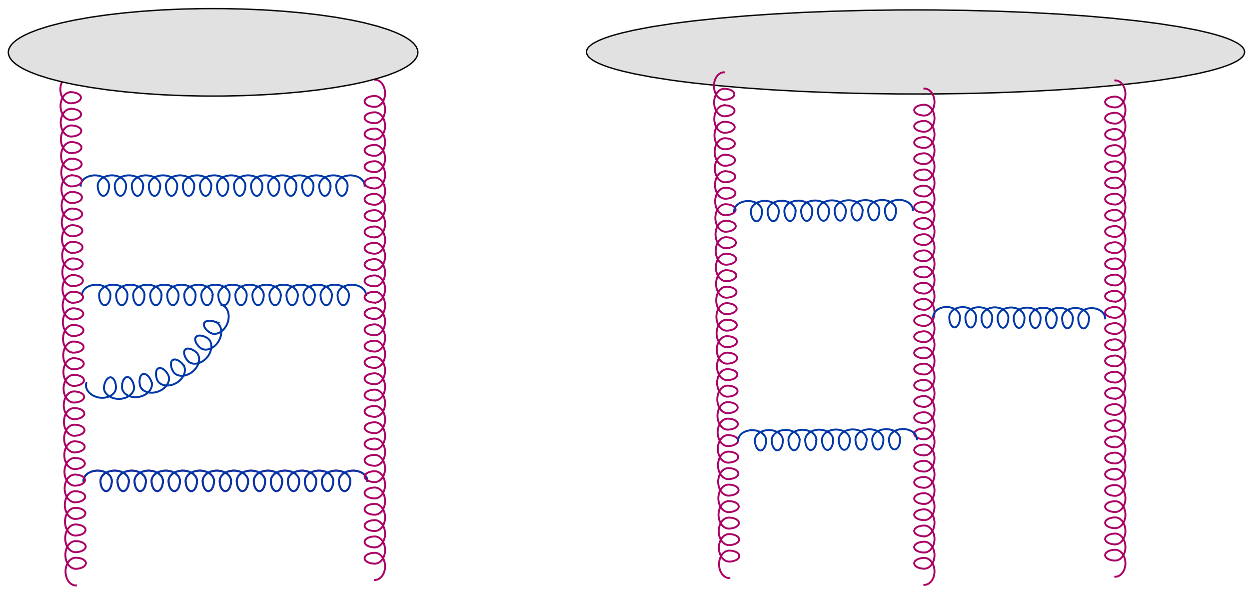


$$H_{\rm JIMWLK} \approx \frac{1}{2}\,(\mathcal{K}_{\rm BFKL}\,A\,A)\,\frac{\delta}{\delta A}\,\frac{\delta}{\delta A} \equiv H_{\rm BFKL}$$

BFKL as color glass evolution

- The weak–field (BFKL) limit of the JIMWLK Hamiltonian

$$\frac{\partial W_Y[A]}{\partial Y} \approx \frac{1}{2}\mathcal{K}_{\rm BFKL}\left(\frac{\delta}{\delta A}\,A\,A\,\frac{\delta}{\delta A}\right)W_Y \equiv H_{\rm BFKL}\,W_Y[A]$$

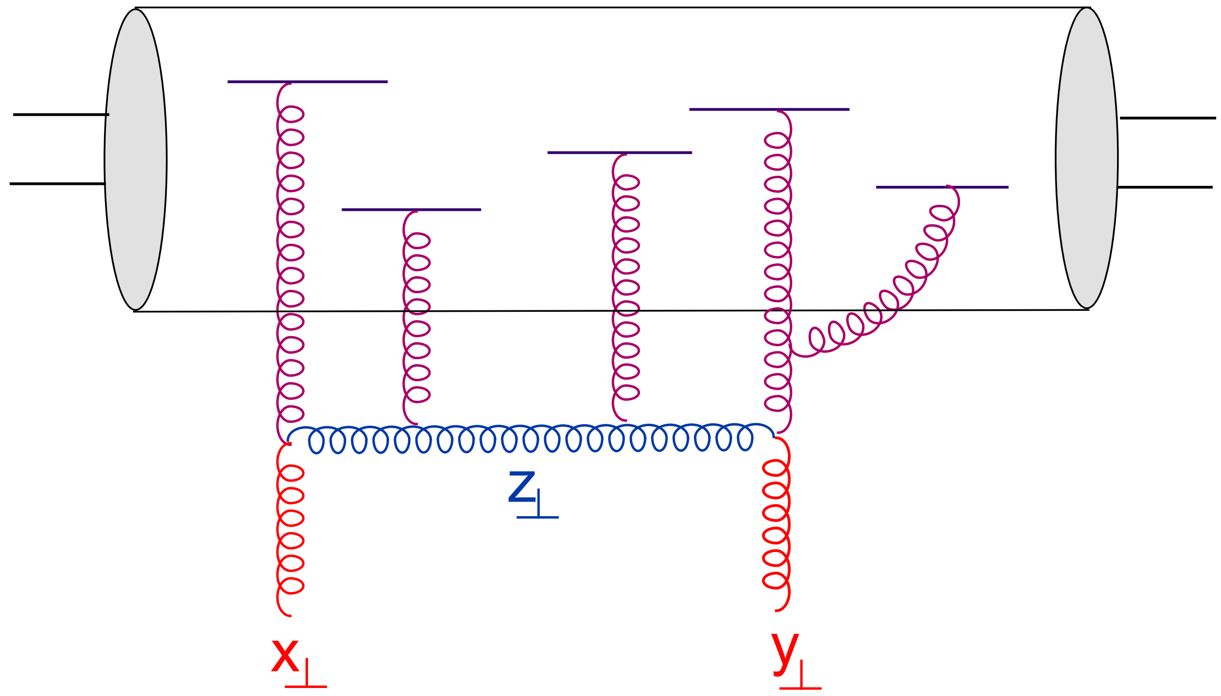


- Each step in the evolution : $2 \rightarrow 2$ gluon vertex

  Insert a 'BFKL exchange' in between each pair of fields

- All gluon correlations rise exponentially with Y

The general case: JIMWLK Hamiltonian



- Strong fields $gA^+ \sim \mathcal{O}(1)$: The quantum gluon rescatters of the background field in the eikonal approximation

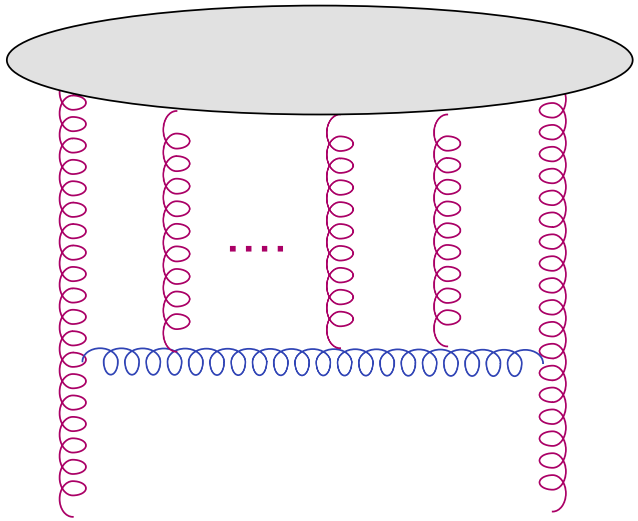
$$H_{\mathrm{JIMWLK}} = \frac{1}{2}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} \frac{\delta}{\delta A^a(\boldsymbol{x}_\perp)}\,\chi_{ab}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)[V,V^\dagger]\,\frac{\delta}{\delta A^b(\boldsymbol{y}_\perp)}$$

- V and $V^\dagger$: Wilson lines in the adjoint representation.

Coherent emission and saturation

$$\chi_{ab}(\boldsymbol{x},\boldsymbol{y}) = \int_{\boldsymbol{z}} \underbrace{\frac{z^i - x^i}{(\boldsymbol{z}-\boldsymbol{x})^2}\,\frac{z^i - y^i}{(\boldsymbol{z}-\boldsymbol{y})^2}}_{\text{Lipatov vertex}} \underbrace{\left(1 - V^\dagger(\boldsymbol{x})V(\boldsymbol{z})\right)_{ac}\left(1 - V^\dagger(\boldsymbol{z})V(\boldsymbol{y})\right)_{cb}}_{\to (A(\boldsymbol{x})-A(\boldsymbol{z}))(A(\boldsymbol{z})-A(\boldsymbol{y}))}$$

- $\chi[A]$: the gluon emission rate in the background field
- $n \rightarrow 2$ gluon vertices with arbitrary n (non–linear evolution)



- ‘Gluon recombination’ — more correctly, coherent emission

Coherent emission and saturation

$$\chi_{ab}(\boldsymbol{x},\boldsymbol{y}) = \int_{\boldsymbol{z}} \underbrace{\frac{z^i - x^i}{(\boldsymbol{z}-\boldsymbol{x})^2}\,\frac{z^i - y^i}{(\boldsymbol{z}-\boldsymbol{y})^2}}_{\text{Lipatov vertex}} \underbrace{\big(1 - V^\dagger(\boldsymbol{x})V(\boldsymbol{z})\big)_{ac}\big(1 - V^\dagger(\boldsymbol{z})V(\boldsymbol{y})\big)_{cb}}_{\to (A(\boldsymbol{x})-A(\boldsymbol{z}))(A(\boldsymbol{z})-A(\boldsymbol{y}))}$$

- The new gluon is coherently produced out of the preexisting color sources
- Strong fields $A \sim 1/g$
  - the Wilson lines $V \sim \exp(igA^+)$ rapidly oscillate
  - the bilinear $V^\dagger(\boldsymbol{z})V(\boldsymbol{x})$ self–averages to zero
  - the emission rate saturates at a field–independent value
- However strong the field is, there will be only one additional gluon emitted per unit rapidity

  $\Longrightarrow$ the gluon density rises linearly in $Y \sim \ln s$, rather than exponentially (as in the low density/weak field regime)

Coherent emission and saturation

$$\chi_{ab}(\boldsymbol{x},\boldsymbol{y}) = \int_{\boldsymbol{z}} \frac{z^i - x^i}{(\boldsymbol{z}-\boldsymbol{x})^2}\,\frac{z^i - y^i}{(\boldsymbol{z}-\boldsymbol{y})^2}\,\big(1 - V^\dagger(\boldsymbol{x})V(\boldsymbol{z})\big)_{ac}\,\big(1 - V^\dagger(\boldsymbol{z})V(\boldsymbol{y})\big)_{cb}$$

- This is the mechanism for gluon saturation !
- This only happens on sufficiently large distances, i.e., for sufficiently small transverse momenta for the emitted gluons.
- Remember: $V^\dagger(\boldsymbol{z})V(\boldsymbol{x})$ = the S–matrix for a color dipole

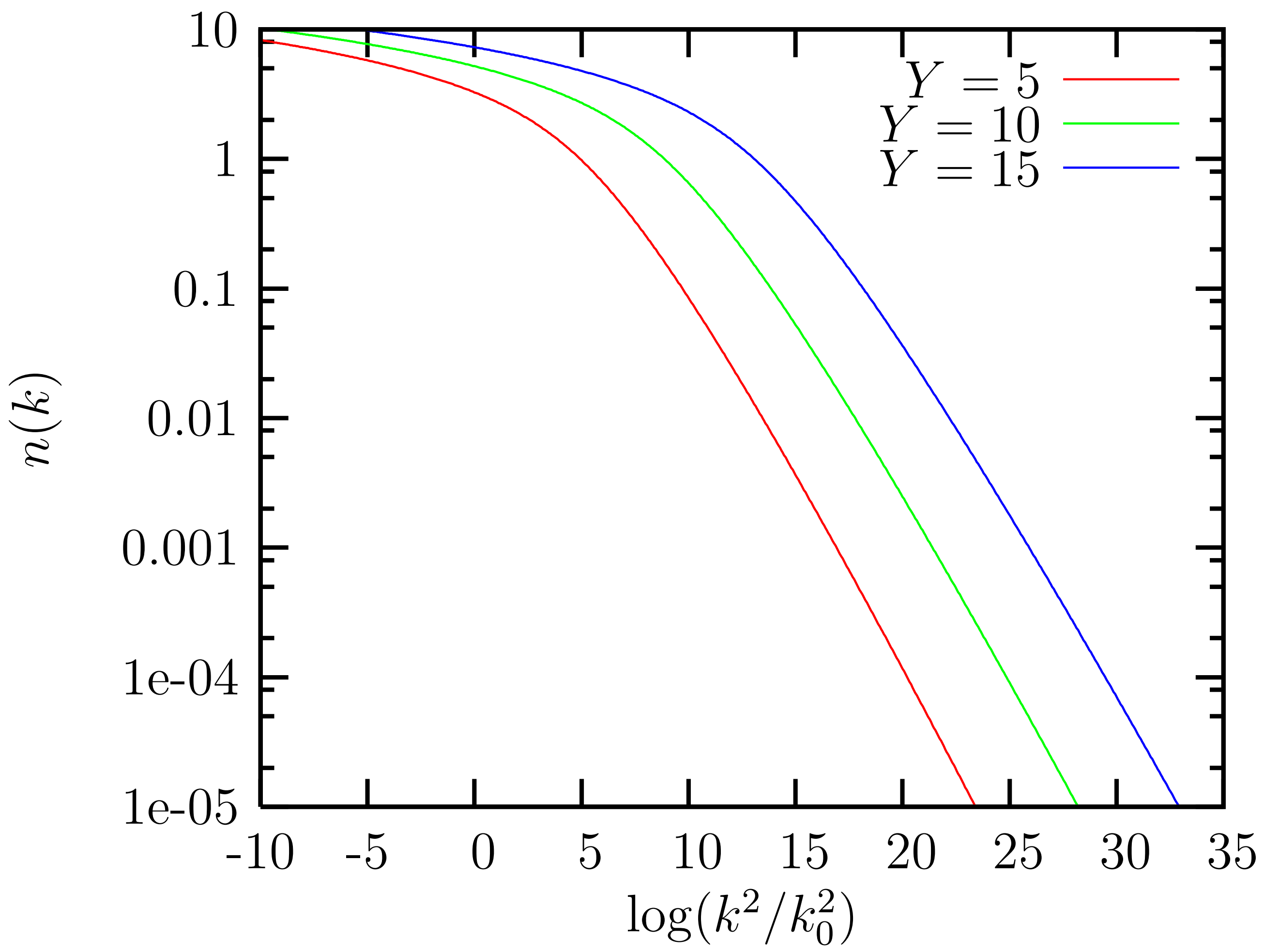
  $V^\dagger(\boldsymbol{z})V(\boldsymbol{x}) \to 0$ over distances $|\boldsymbol{z}-\boldsymbol{x}| \gtrsim 1/Q_s(Y)$
- For a given Y, saturation occurs only for gluons with transverse momenta $k_\perp \lesssim Q_s(Y)$

$$\chi(\boldsymbol{x},\boldsymbol{y}) \approx \ln\big((\boldsymbol{x}-\boldsymbol{y})^2 Q_s^2(Y)\big) \Longrightarrow n(k_\perp, Y) \sim \ln \frac{Q_s^2(Y)}{k_\perp^2} \propto Y$$

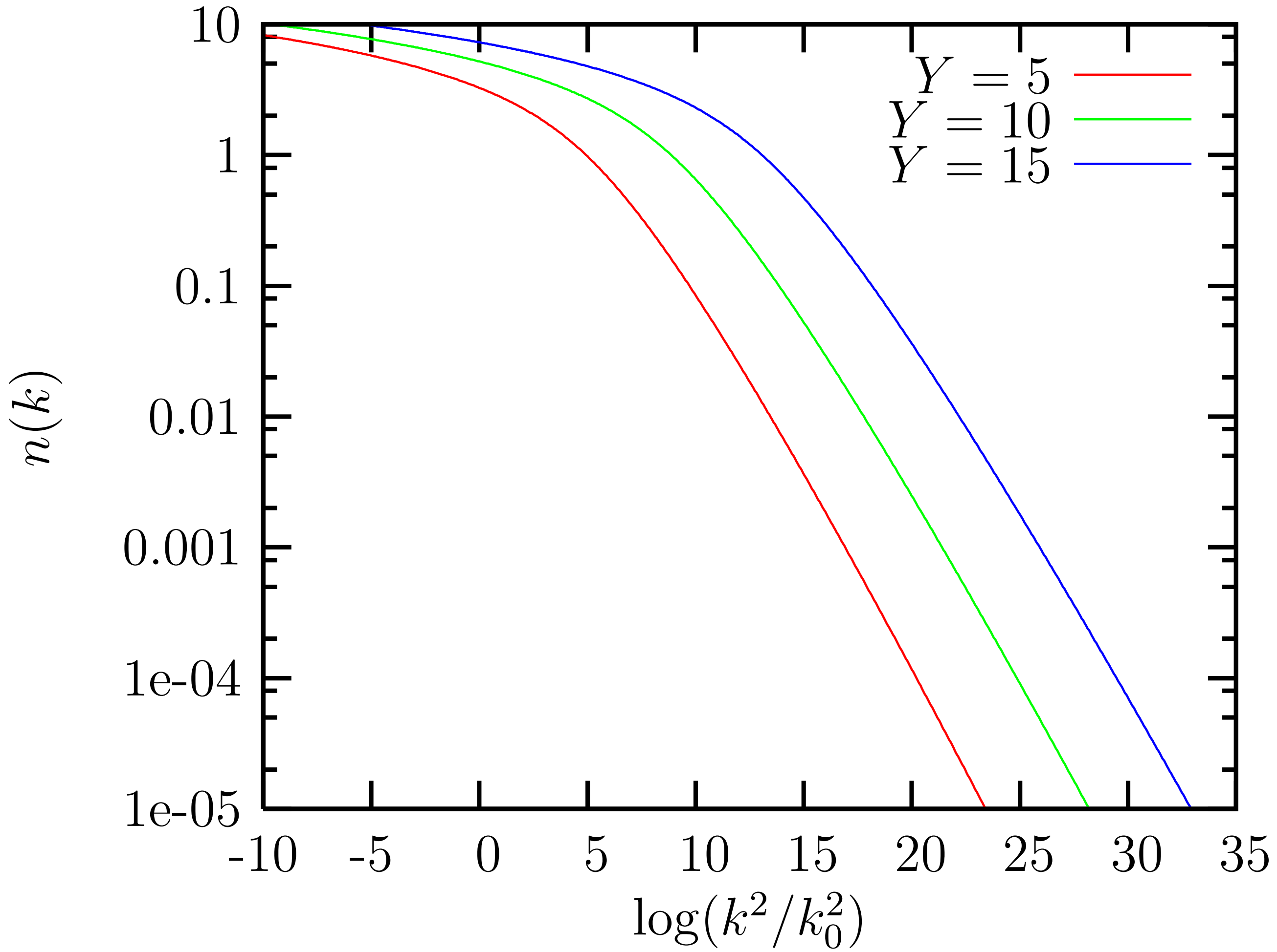
- The connection saturation $\leftrightarrow$ dipole unitarity is now manifest

The gluon occupation number

$$n(Y, k_\perp, b_\perp) \equiv \frac{(2\pi)^3}{2(N_c^2-1)} \frac{\mathrm{d}N}{\mathrm{d}Y\,\mathrm{d}^2k_\perp\,\mathrm{d}^2b_\perp}$$

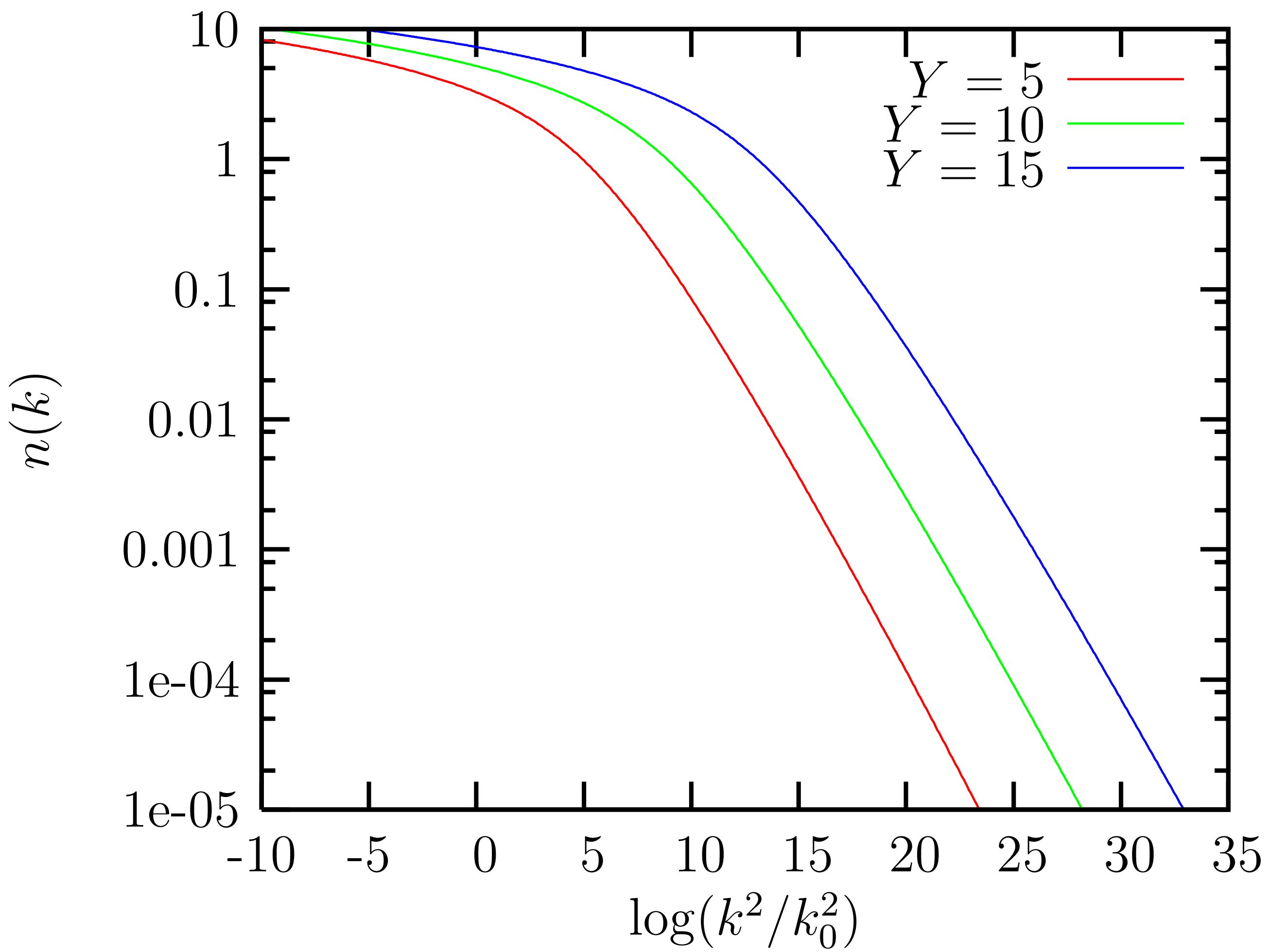


Gluon occupation number



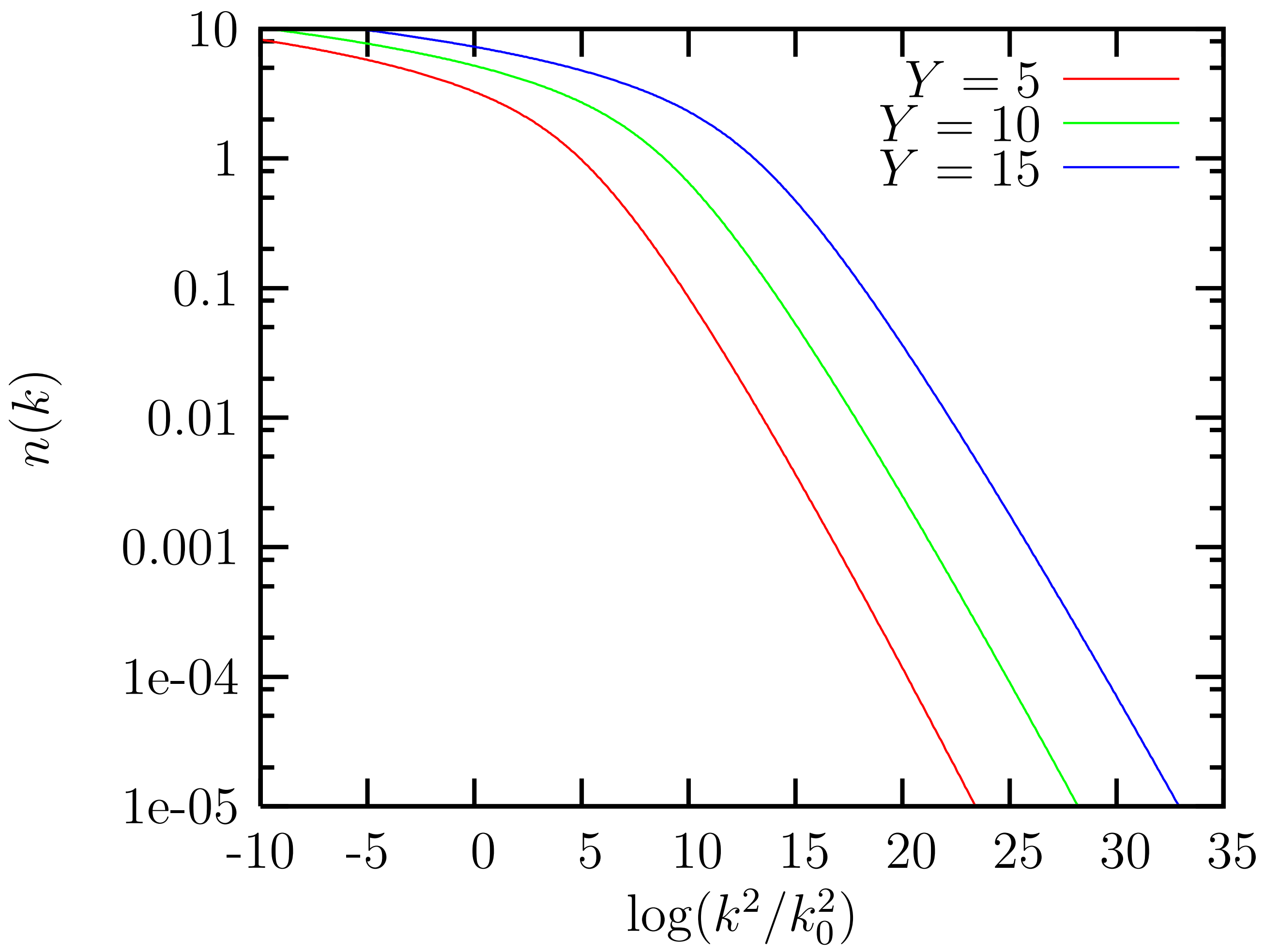
- For $k_\perp \gg Q_s(Y)$: $n(k, Y) \propto 1/k_\perp^2$ (bremsstrahlung)

Gluon occupation number



- For $k_\perp \gg Q_s(Y)$: $n(k,Y) \propto (Q_s^2/k_\perp^2)^{\gamma_s}$ (geometric scaling)

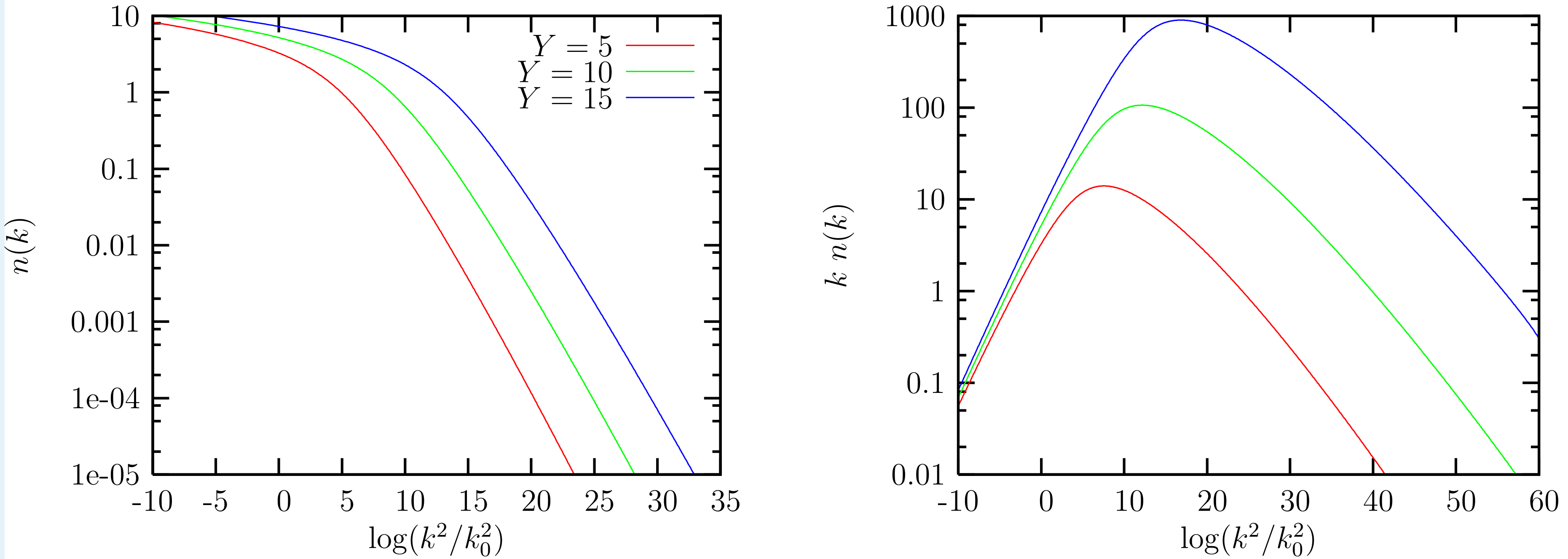
Gluon occupation number



For $k_\perp \lesssim Q_s(Y)$: $n(Y, k_\perp) \approx \dfrac{1}{\alpha_s N_c} \ln \dfrac{Q_s^2(Y)}{k_\perp^2} \sim \dfrac{1}{\bar{\alpha}_s}$

Gluon occupation number

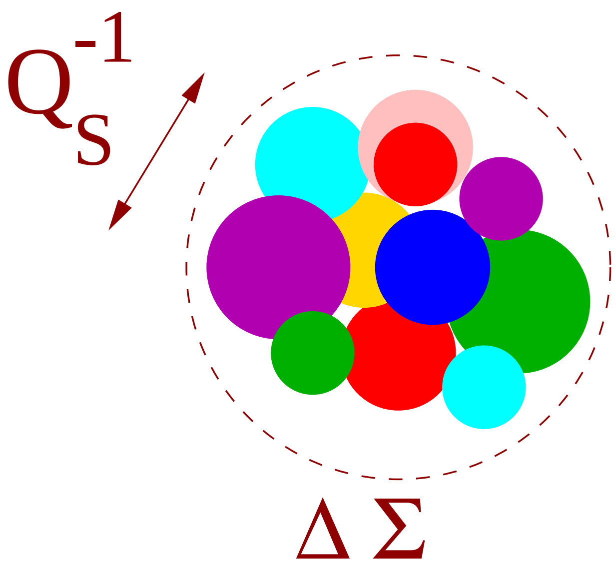
$$xG(x,Q^2) = \int \mathrm{d}^2b \int^Q \mathrm{d}k\, k\, n(k,Y)$$



- $Q_s(Y)$: the typical transverse momentum of the gluons
- High energy (Y) $\Longrightarrow$ large Q_s^2 $\Longrightarrow$ weak coupling !
- At sufficiently high energy, gluon saturation allows for a meaningful perturbation theory !

Color neutrality at saturation

- In a low energy hadron, color is screened by confinement: $r \sim \Lambda_{\rm QCD}^{-1} \sim 1\ {\rm fm}$
- At high energy, the densely packed gluons screen each other, in such a way that color neutralization occurs already at the perturbative scale $Q_s^{-1} \ll \Lambda_{\rm QCD}^{-1}$



- $Q_s(Y)$ is the "infrared cutoff" at high energy, and is hard !

Evolution of observables

- Recall: Observables are obtained by averaging over ρ, or A :

$$\langle \mathcal{O}[A] \rangle_Y = \int \mathrm{D}[A] \; W_Y[A] \; \mathcal{O}[A]$$

- Example: The dipole S–matrix: $S(\boldsymbol{x}, \boldsymbol{y}) = \frac{1}{N_c} \mathrm{tr}\big(V(\boldsymbol{x}) \, V^\dagger(\boldsymbol{y}) \big)$
- Differentiate w.r.t. Y, use JIMWLK, and integrate by parts:

$$\begin{aligned} \partial_Y \langle \mathcal{O}[A] \rangle_Y &= \int \mathrm{D}[A] \; (\partial_Y W_Y) \; \mathcal{O}[A] \\ &= \int \mathrm{D}[A] \; \frac{1}{2} \left(\frac{\delta}{\delta A_a} \chi_{ab} \frac{\delta W_Y}{\delta A_b} \right) \mathcal{O}[A] \\ &= \int \mathrm{D}[A] \, W_Y[A] \; \frac{1}{2} \frac{\delta}{\delta A_a} \chi_{ab} \frac{\delta}{\delta A_b} \; \mathcal{O}[A] \end{aligned}$$

- ... or, simply,

$$\partial_Y \langle \mathcal{O}[A] \rangle_Y = \langle H_{\mathrm{JIMWLK}} \; \mathcal{O}[A] \rangle_Y$$

Recovering Balitsky equations

- Exercise: Use JIMWLK to deduce the first Balitsky equation :

$$\frac{\partial}{\partial Y}\langle S(\boldsymbol{x},\boldsymbol{y})\rangle_Y = \frac{\bar{\alpha}_s}{2\pi}\int_{\boldsymbol{z}}\frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2}\Big\{-\langle S(\boldsymbol{x},\boldsymbol{y})\rangle_Y + \langle S(\boldsymbol{x},\boldsymbol{z})S(\boldsymbol{z},\boldsymbol{y})\rangle_Y\Big\}$$

- The action of the functional derivatives on Wilson lines

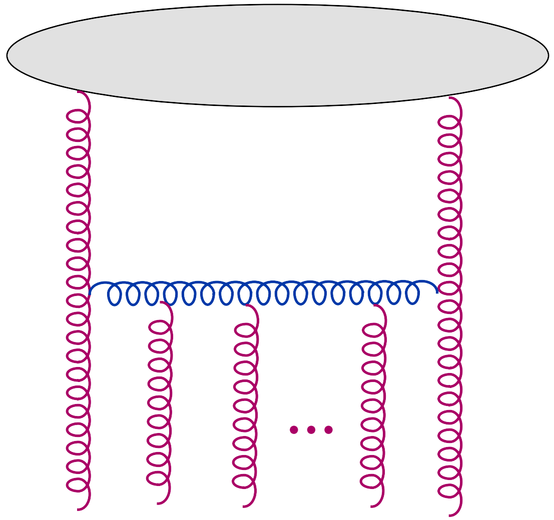
$$V(\boldsymbol{x}) \equiv \mathrm{Pexp}\Big(ig\int_{-\infty}^{\infty}\mathrm{d}x^- A_a^+(x^-,\boldsymbol{x})T^a\Big)$$

$$\frac{\delta}{\delta A_a^+(\boldsymbol{x})}V(\boldsymbol{x}) \equiv \frac{\delta}{\delta A_a^+(x^-\to\infty,\boldsymbol{x})}V(\boldsymbol{x}) = igT^a V(\boldsymbol{x})$$

- The derivative w.r.t. the field at $x^- = \infty$! ("Lie derivative")
- The CGC color source/field gets built in layers of x^-
- The higher equations in the Balitsky hierarchy can be similarly obtained.

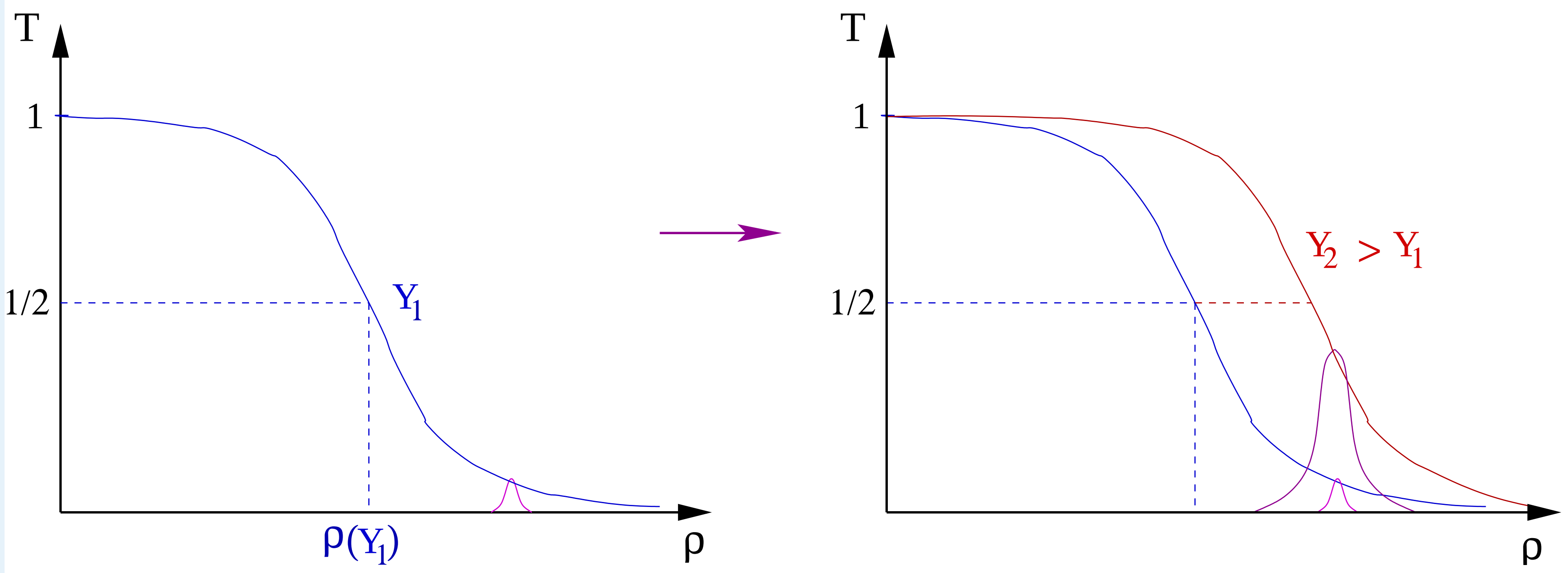
Gluon number fluctuations

- What about the reversed, missing, diagrams ?
- $2 \to n$ gluon vertices with $n > 2$



- Clearly, irrelevant at high density ! (higher powers of g without compensating factors of the strong field $A \sim 1/g$)
- Generate gluon number fluctuations at low density
- And so what ?! ... Higher–order effects anyway !

Reminder: Pulled front



- The front is driven by the linear (BFKL) growth in the tail of the amplitude at large $\rho \gg \rho_s(Y)$, where $T \ll 1$
  $\Longrightarrow$ potentially strong sensitivity to fluctuations !

- High–energy evolution in pQCD : a stochastic process in the same universality as the reaction–diffusion process $A \rightleftharpoons 2A$

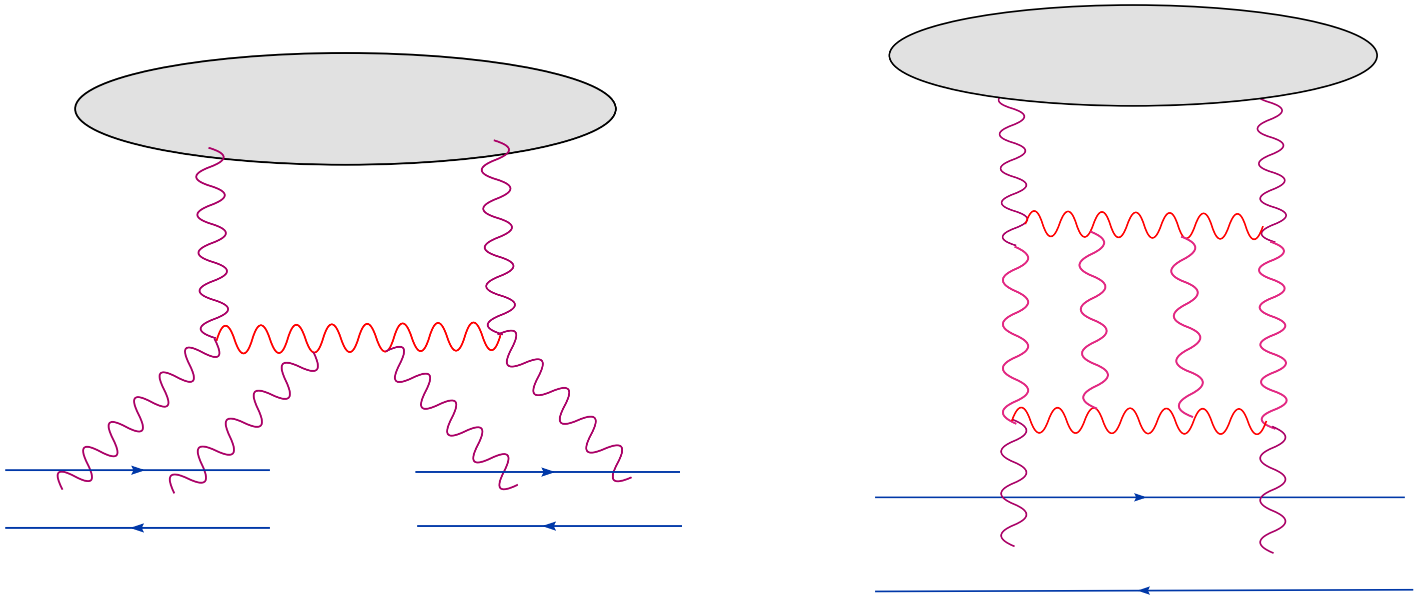
  (E.I., Mueller, Munier; E.I., Triantafyllopoulos, 2004)

The Pomeron loop equations

- Additional terms in the evolution equations (large N_c)

$$\frac{\partial \langle T \rangle}{\partial t} \simeq \langle T \rangle - \langle T\,T \rangle$$

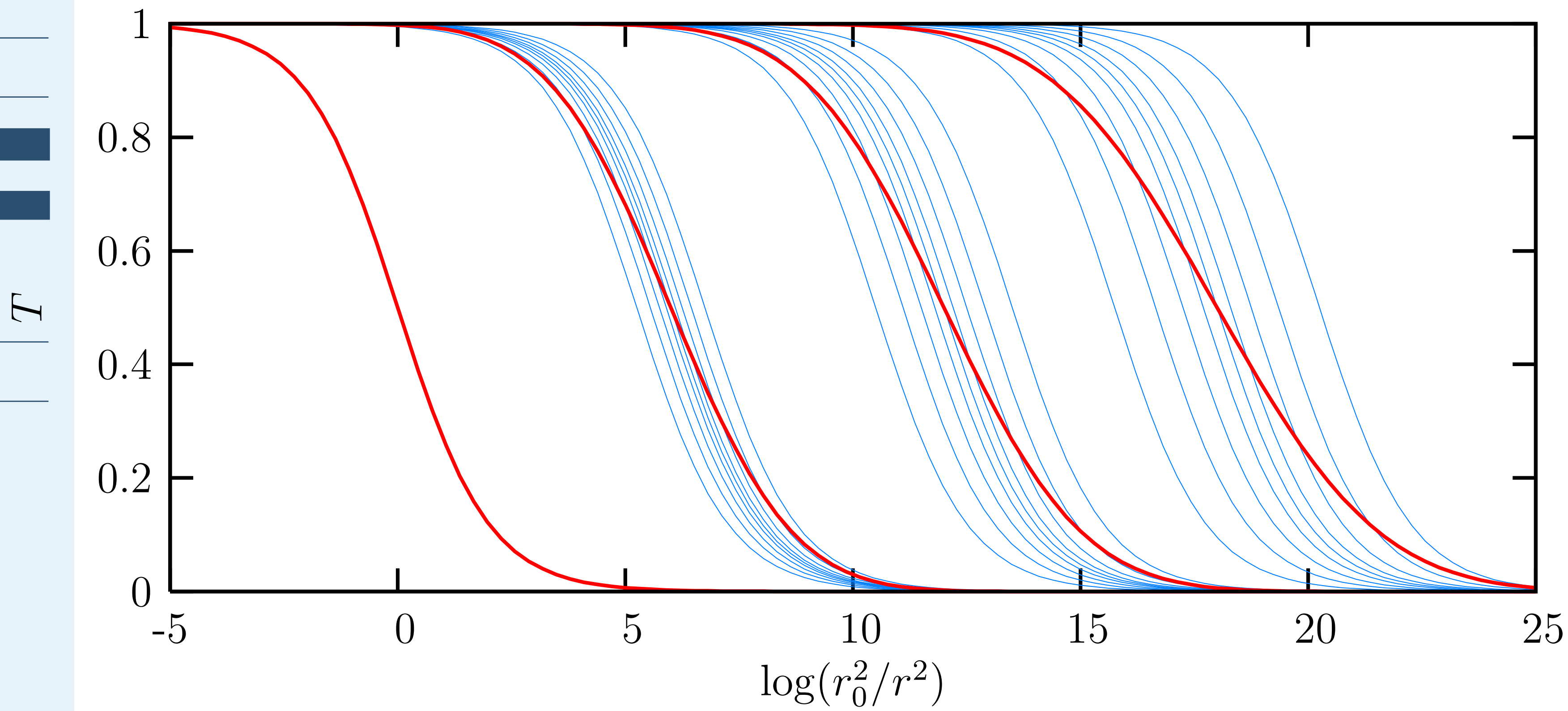
$$\frac{\partial \langle T\,T \rangle}{\partial t} \simeq 2\langle T\,T \rangle - 2\,\langle T\,T\,T \rangle + \alpha_s^2 \langle T \rangle$$



- The fluctuation term dominates when $\langle T \rangle \lesssim \alpha_s^2$ (dilute tail)

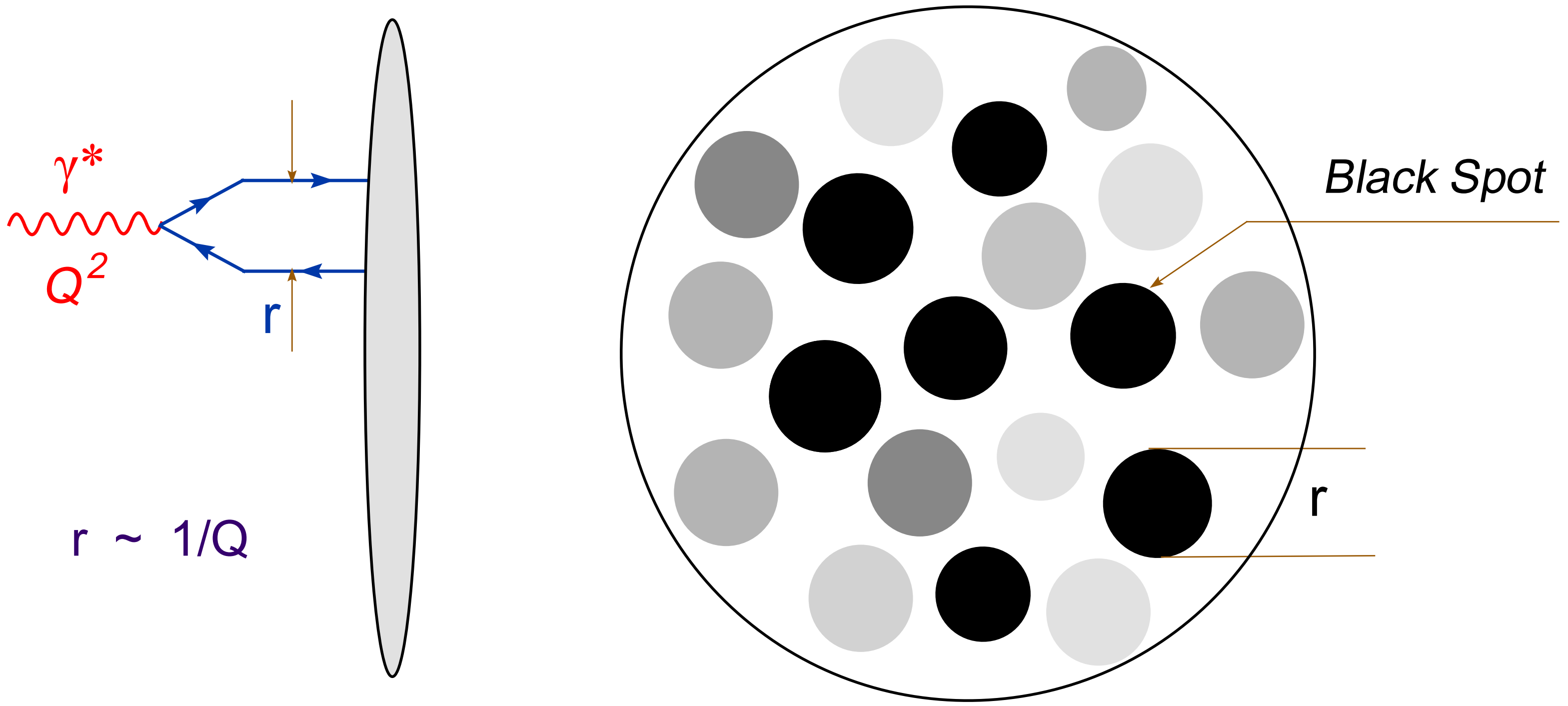
New phenomena due to fluctuations

- Front diffusion : $\Longrightarrow$ geometric scaling is progressively washed out and replaced by diffusive scaling



New phenomena due to fluctuations

- Highly inhomogeneous gluon distribution in impact parameter space: black spots



- ... All that mostly coming from the correspondence with statistical physics.

Running coupling effects

- Unfortunately, the fluctuations appear to be 'washed out' by the effects of the running of the coupling

  'washed out' = delayed up to very high energies

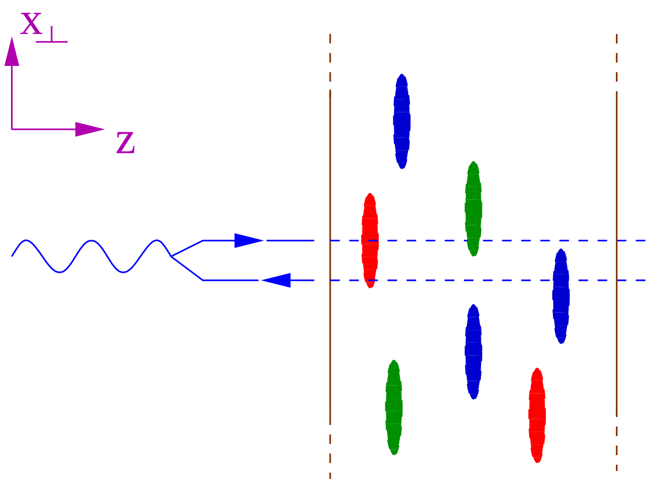
  $(Y \gtrsim 100 \Longrightarrow \sqrt{s} \gg M_{Planck})$

  (Dumitru, E. I., Portugal, Soyez, Triantafyllopoulos, 07)

- To be followed !

Prediction : Next week end

Initial condition: The MV model



- The gluon distribution of a large nucleus ($A \gg 1$)
  at not so large energy : $\alpha_s Y \ll 1$ (say, $x = 10^{-1} \div 10^{-2}$)
- Gluon (BFKL) evolution is negligible
  $\Longrightarrow$ the only 'color sources' : the $3A$ valence quarks

$$W_A[\rho] = \mathcal{N} \exp\left\{-\frac{1}{2}\int \mathrm{d}^2\boldsymbol{x}\,\frac{\rho_a(\boldsymbol{x})\rho_a(\boldsymbol{x})}{\mu_A}\right\}, \qquad \mu_A \propto A^{1/3}$$

- Reasonable initial condition ($Y = 0$) for the JIMWLK equation