Gluon saturation and the
Color Glass Condensate

Part I

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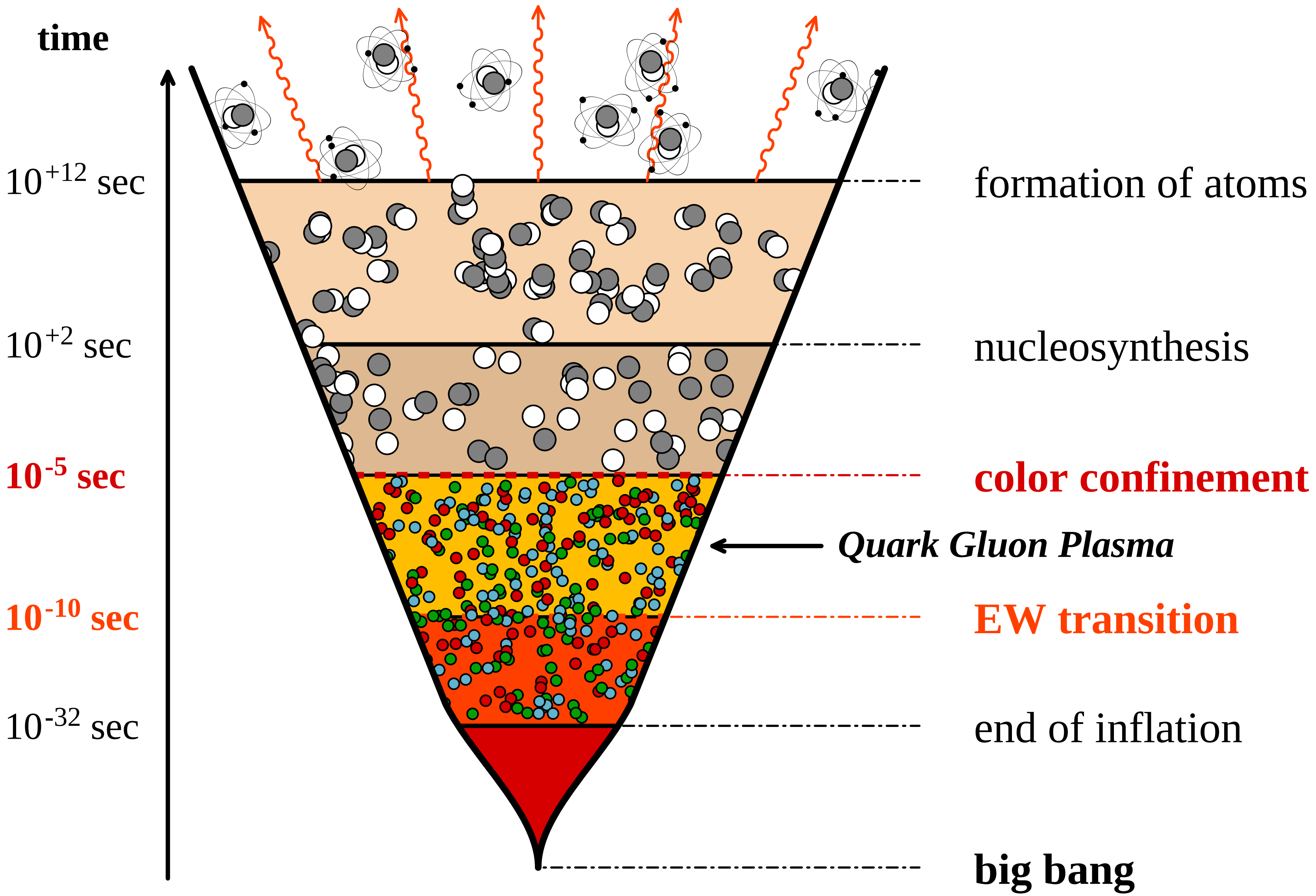
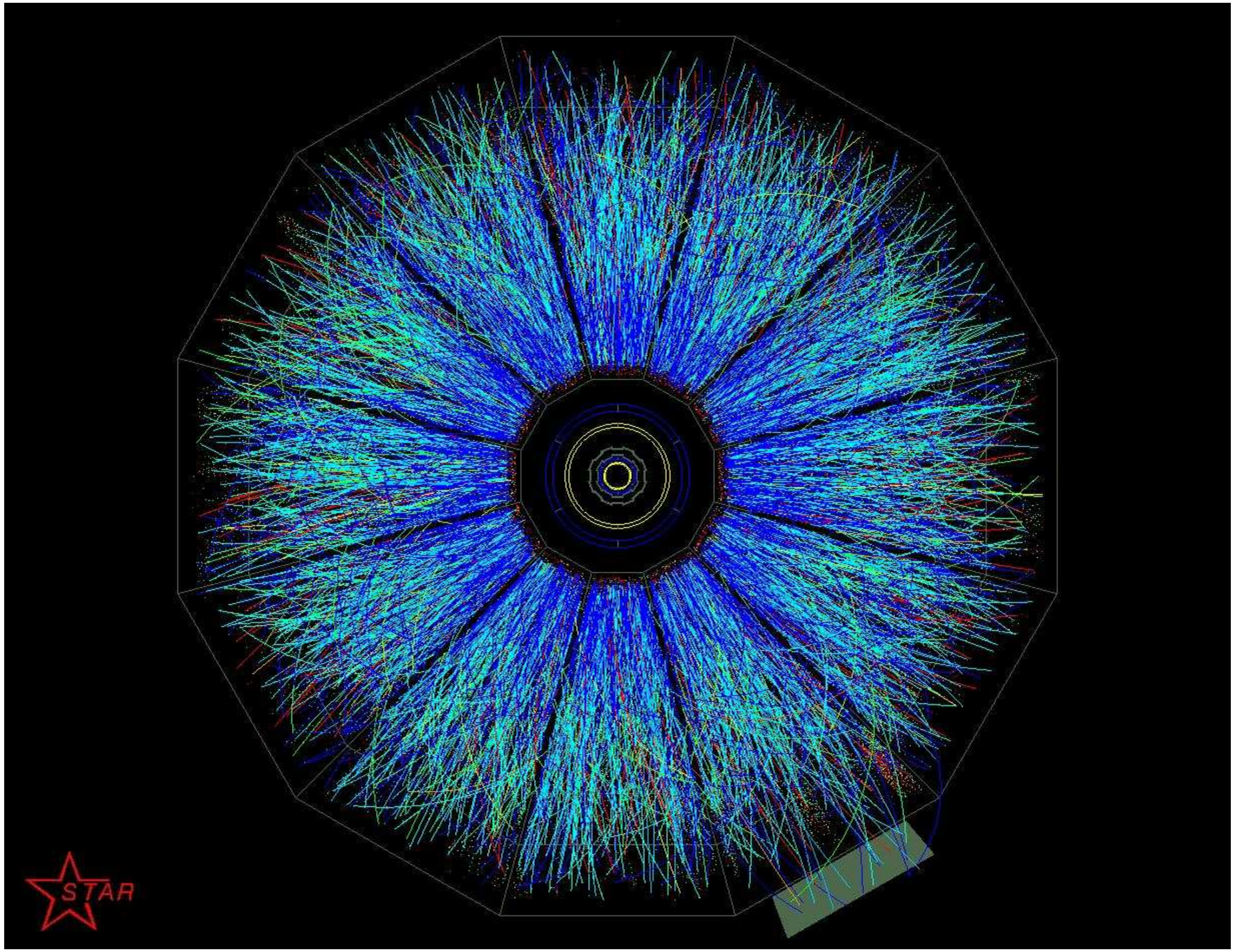
Bienvenu aux Houches !

The Big Bang (courtesy of François Gelis)

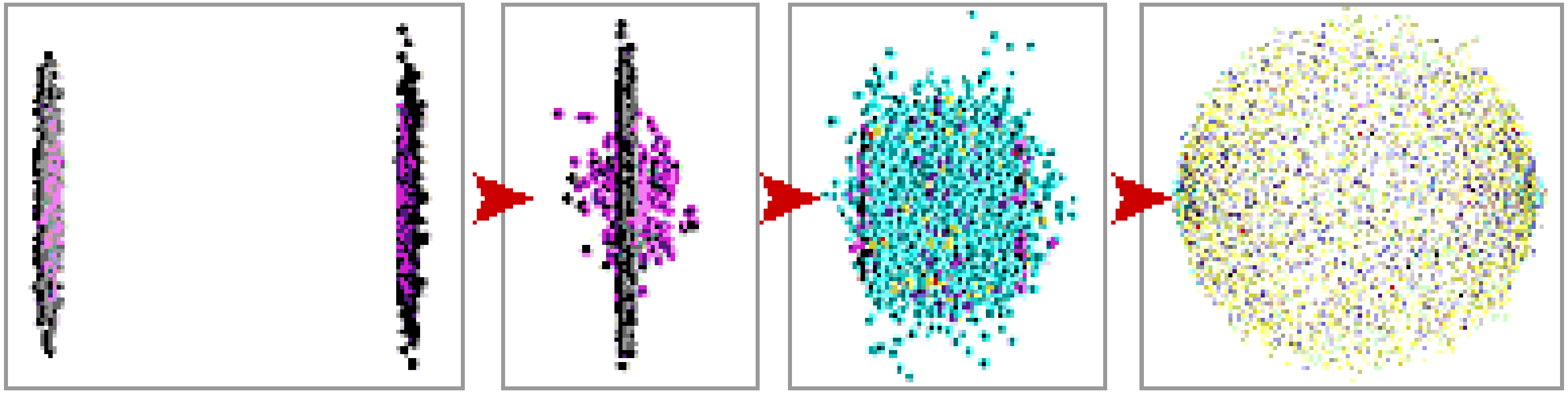


The Little Bang



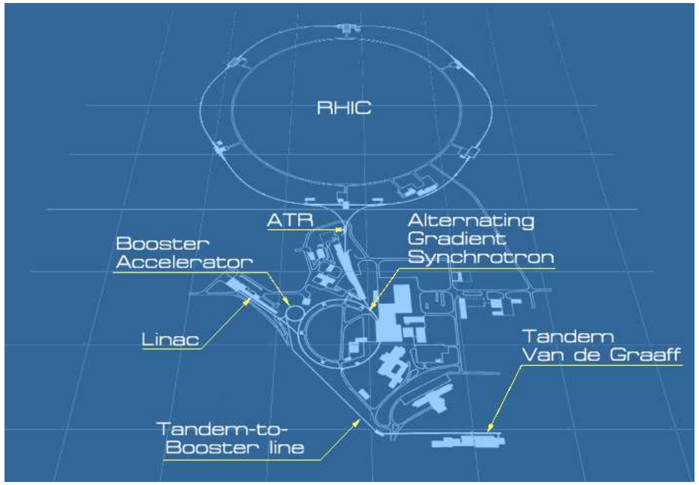
▷ Au–Au collision at RHIC (by STAR Coll.)
(cf. lectures by Thomas Ullrich)

The Little Bang



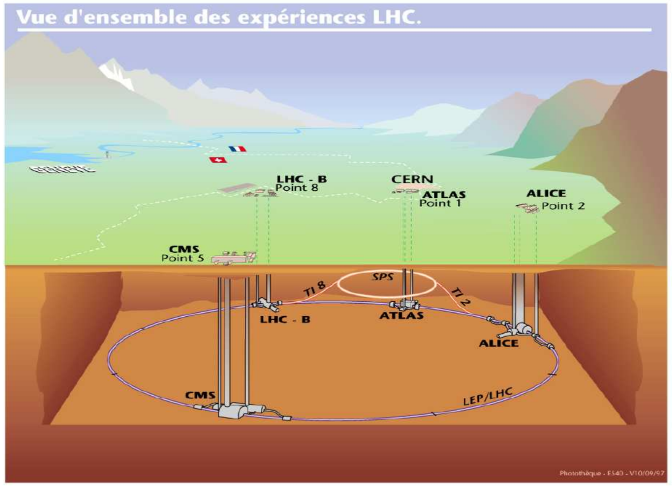
▷ This School should drive you through all these stages !

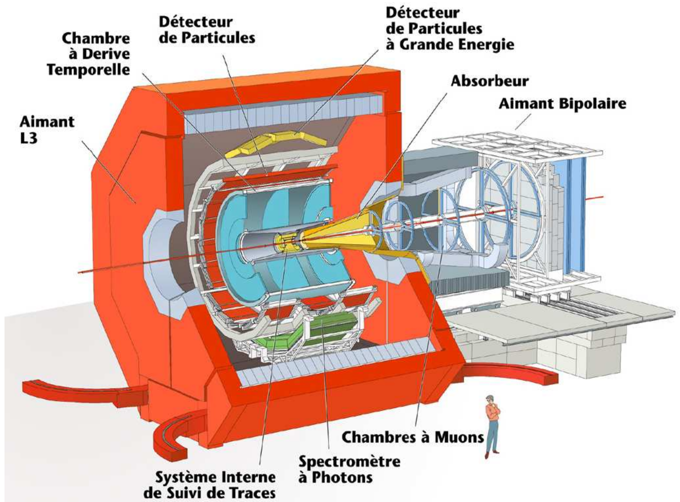
What have we learned from RHIC ? (since 2000)



▷ cf. lectures by T. Ullrich

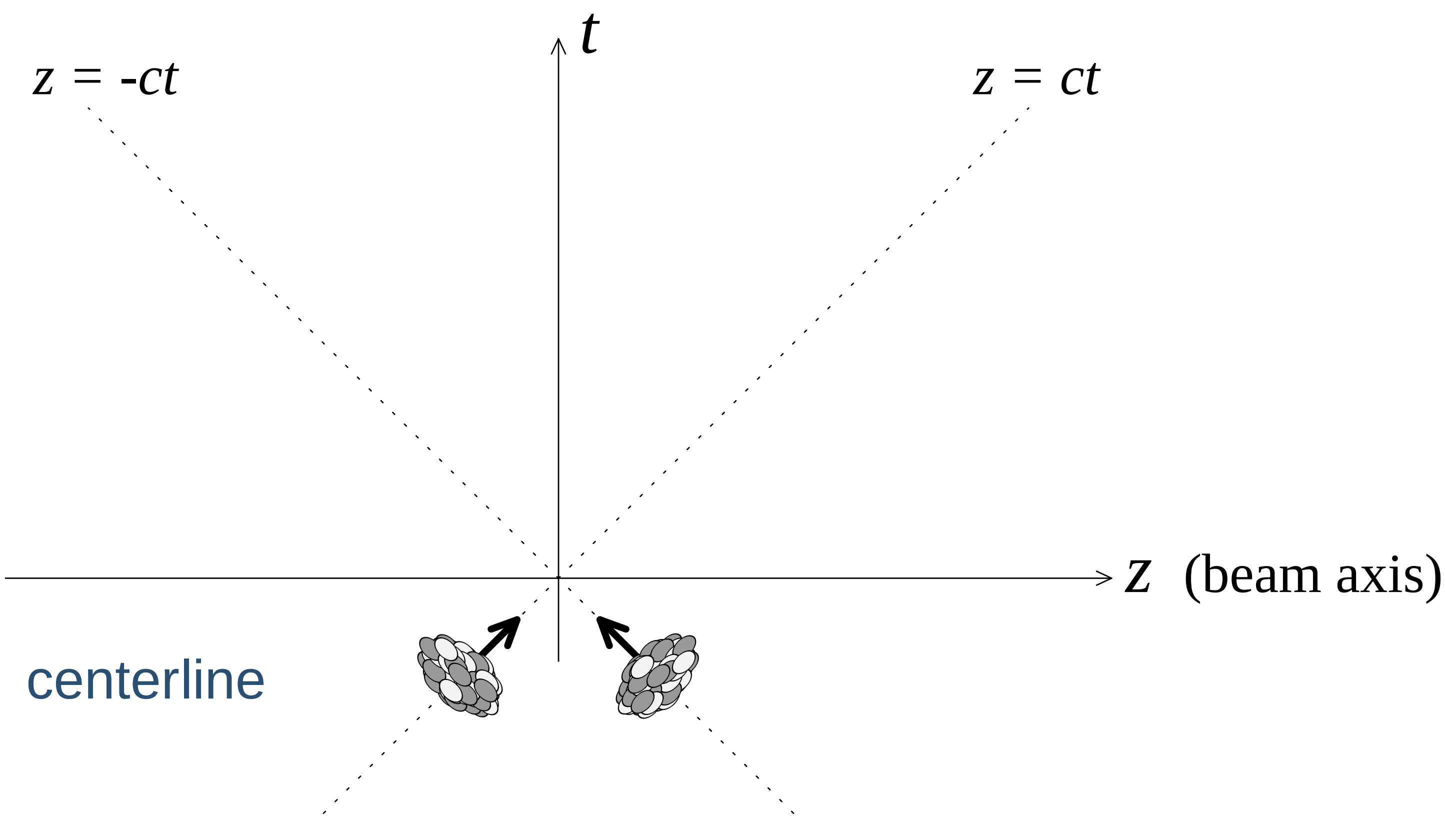
What can we measure at LHC ?





▷ cf. lectures by D. d'Enterria

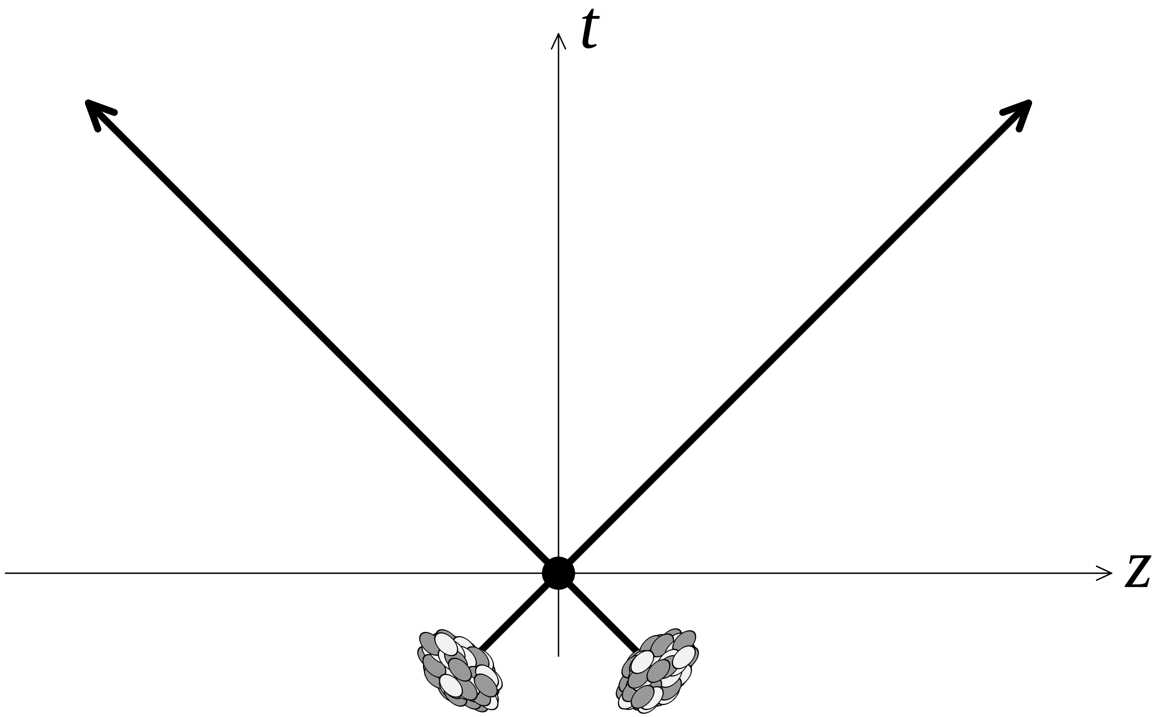
Stages of a heavy–ion collision (by François Gelis)



centerline

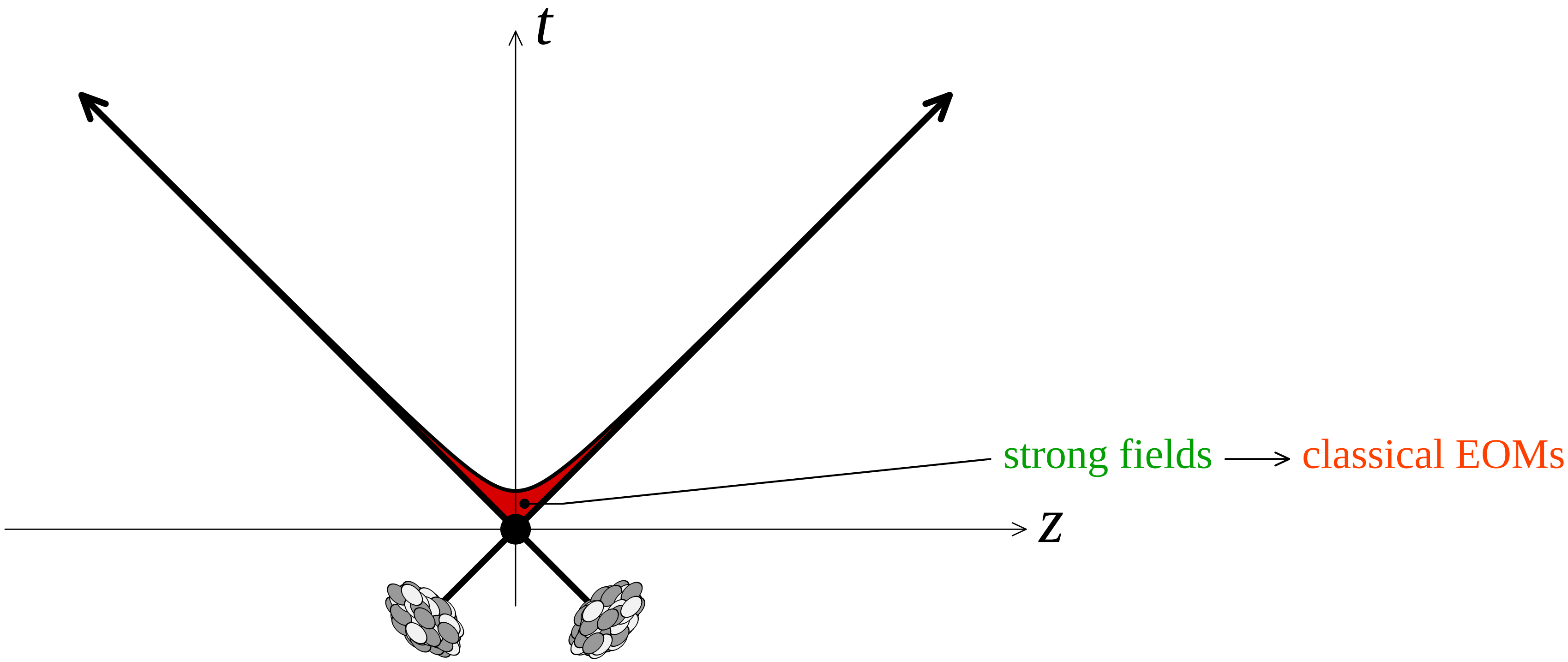
- $\tau < 0$ (before the collision)
  - the problem of 'initial conditions'
  - parton picture, gluon saturation, CGC
- This course (E.I.), but also
  Y. Dokshitzer, F. Gelis, R. Venugopalan

Stages of a heavy–ion collision (by François Gelis)



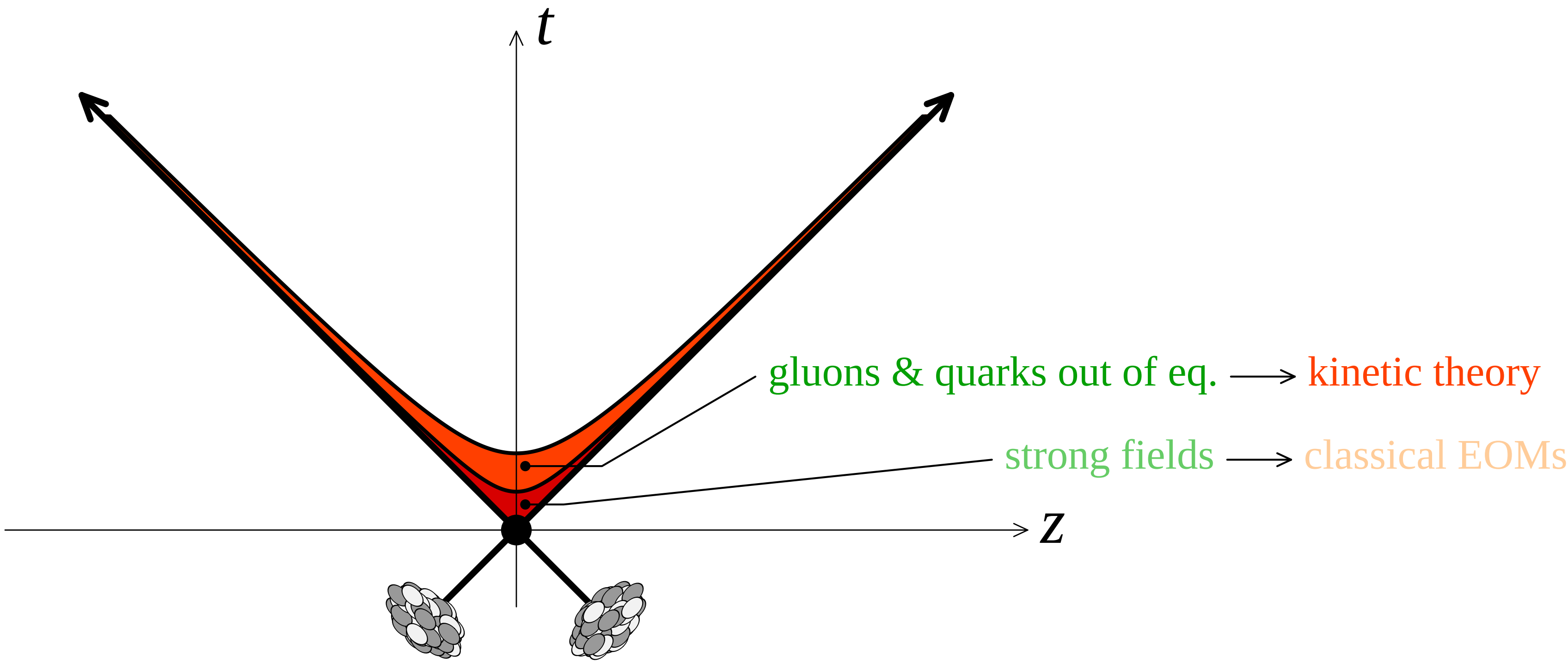
- $\tau \sim 0$ fm/c : Production of hard particles :
  - jets, direct photons, heavy quarks
- calculable with perturbative QCD (leading twist)
- cf. lectures by F. Gelis, U. Wiedemann

Stages of a heavy–ion collision (by François Gelis)



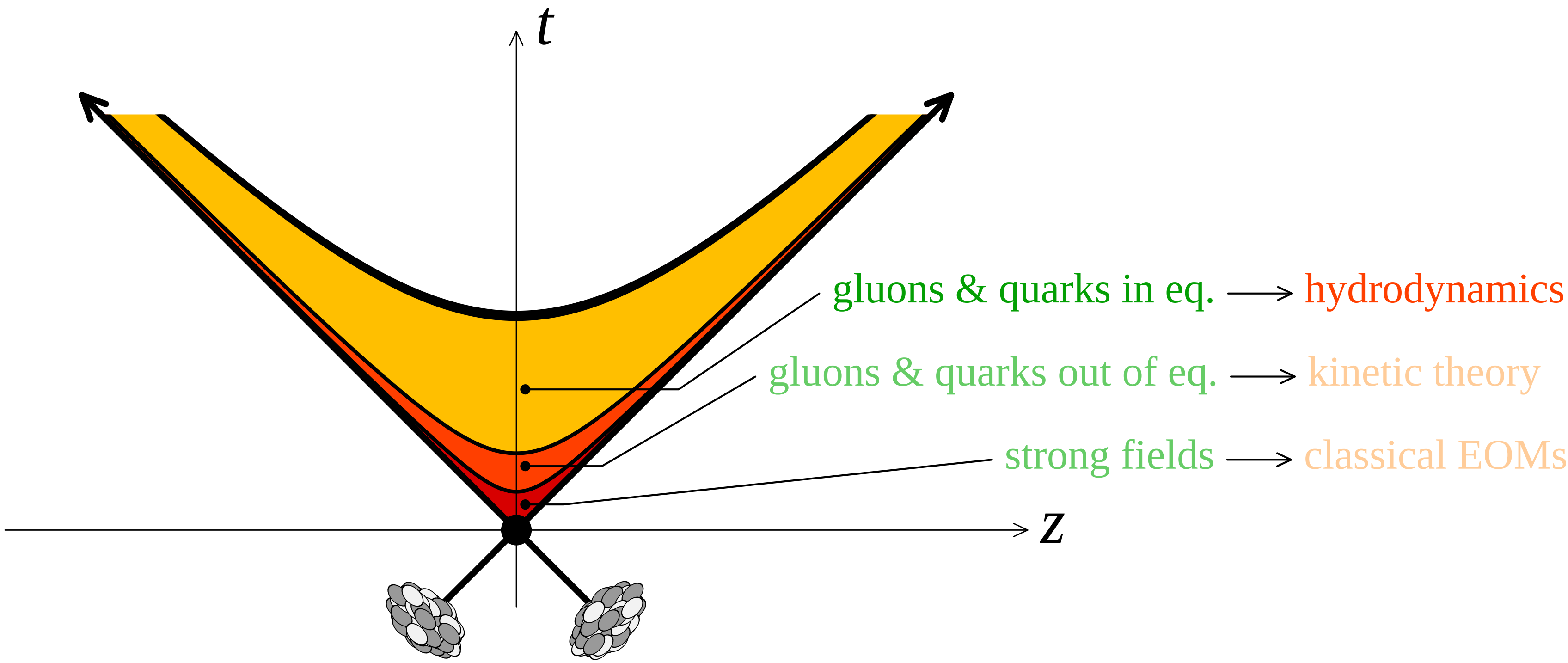
- $\tau \sim 0.2$ fm/c : Production of semi-hard particles
  - relatively small momentum : $p_\perp \lesssim 2$–3 GeV
  - make up for most of the multiplicity
  - sensitive to the physics of saturation (higher twist)
- cf. lectures by F. Gelis, R. Venugopalan

Stages of a heavy–ion collision (by François Gelis)



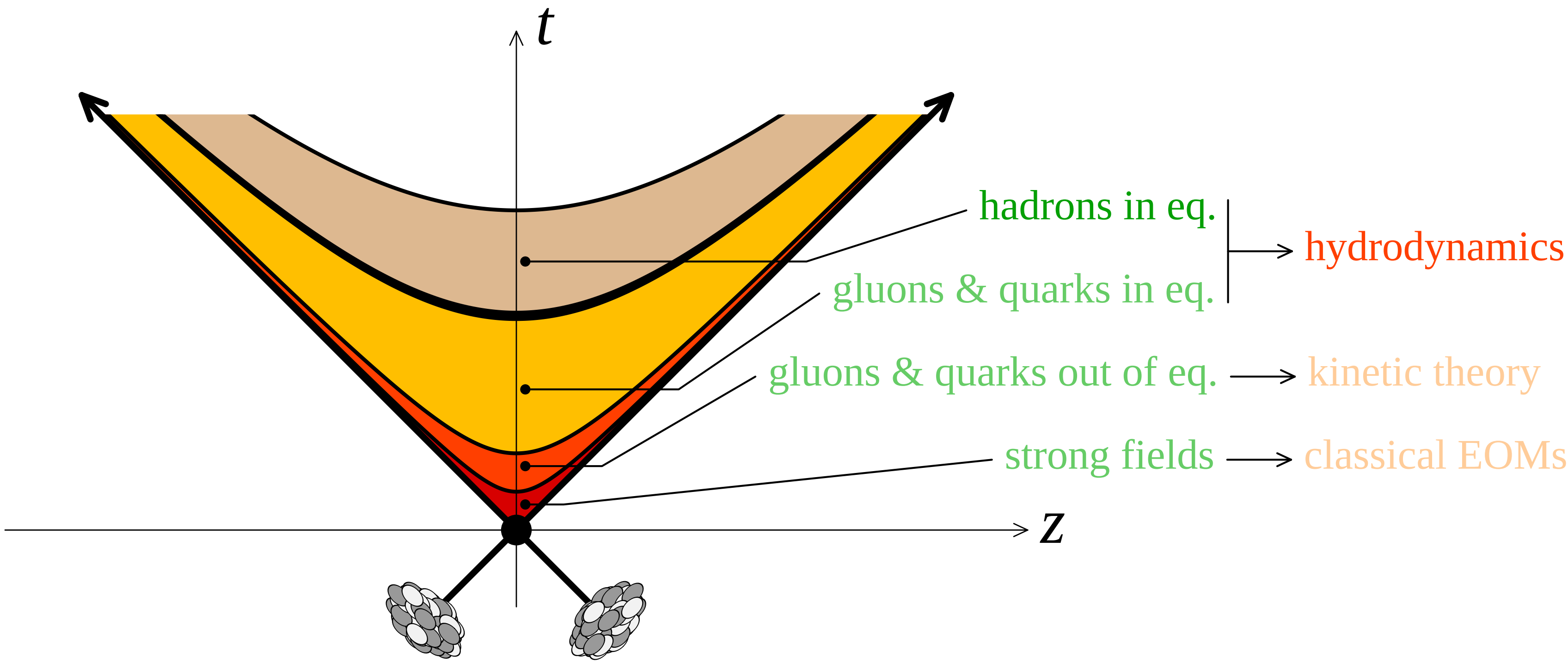
- $\tau \sim 1$–2 fm/c : Thermalization (?)
  - experiments suggest a fast thermalization
  - but this is still not well understood from QCD
- cf. lectures by D. Bödeker, A. Rebhan, R. Venugopalan A. Starinets

Stages of a heavy–ion collision (by François Gelis)



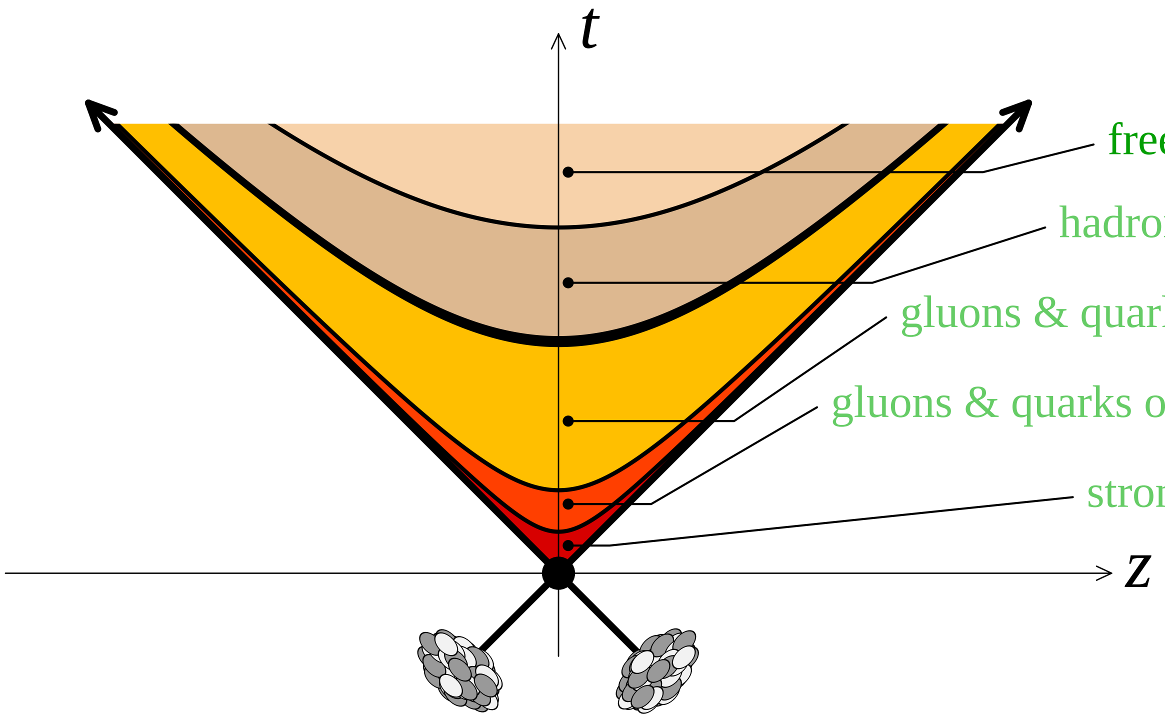
- $2 \leq \tau \lesssim 10$ fm/c : Quark Gluon Plasma
  - some features suggest a weakly–coupled system
  - some others don't
- cf. lectures by J.-P. Blaizot, J.-Y. Ollitrault, A. Starinets

Stages of a heavy–ion collision (by François Gelis)



- $10 \lesssim \tau \lesssim 20$ fm/c : Hot hadron gas
- cf. lectures by
  Y. Dokshitzer, J.-Y. Ollitrault, S. Pratt, U. Wiedemann

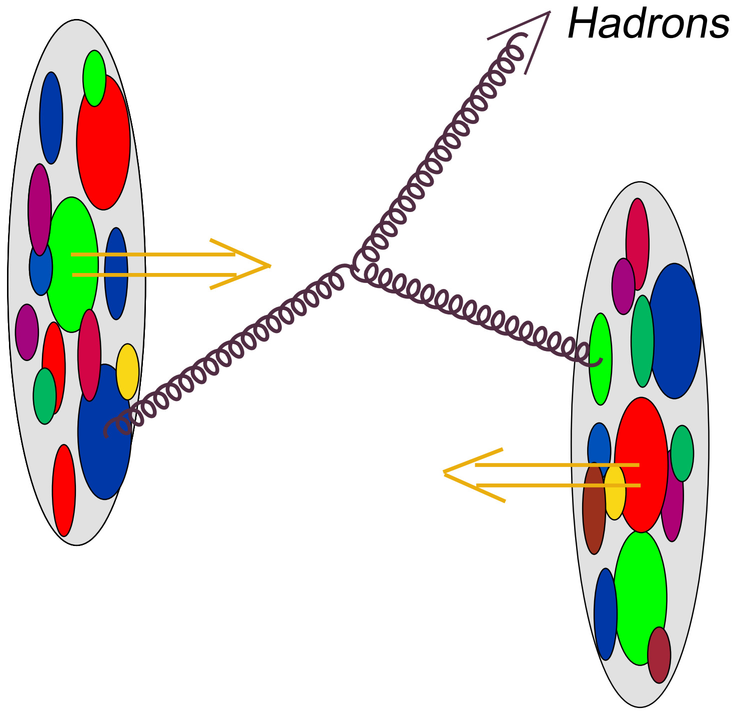
Stages of a heavy–ion collision (by François Gelis)



- $\tau \longrightarrow +\infty$
- Chemical freeze-out :
  density too small to have inelastic interactions
- Kinetic freeze-out :
  no more elastic interactions
- cf. lecture by S. Pratt

The ‘initial conditions’ problem

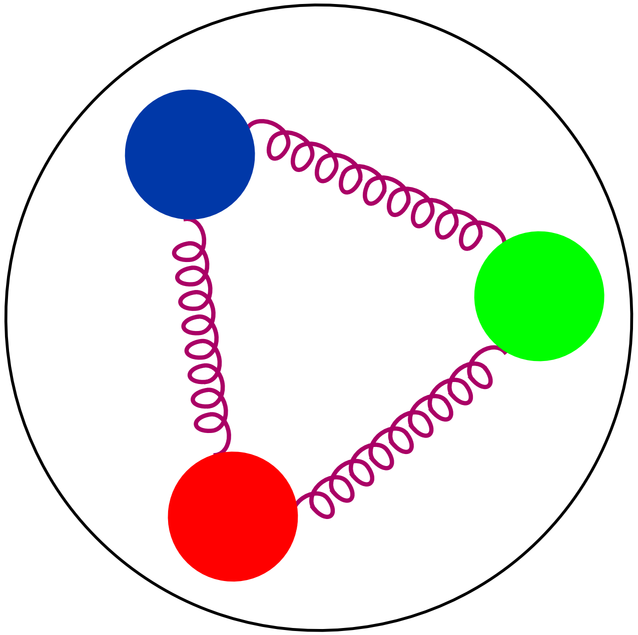
- Proton–proton, or nucleus–nucleus, collisions at high energy



- What are the high energy hadrons made of ?

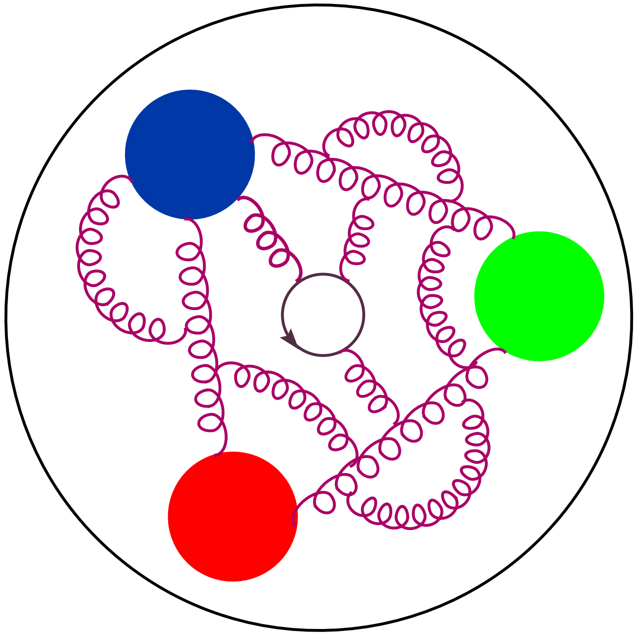
A hadron at rest

- A proton at rest, or at low energy $(P \sim M)$



A hadron at rest

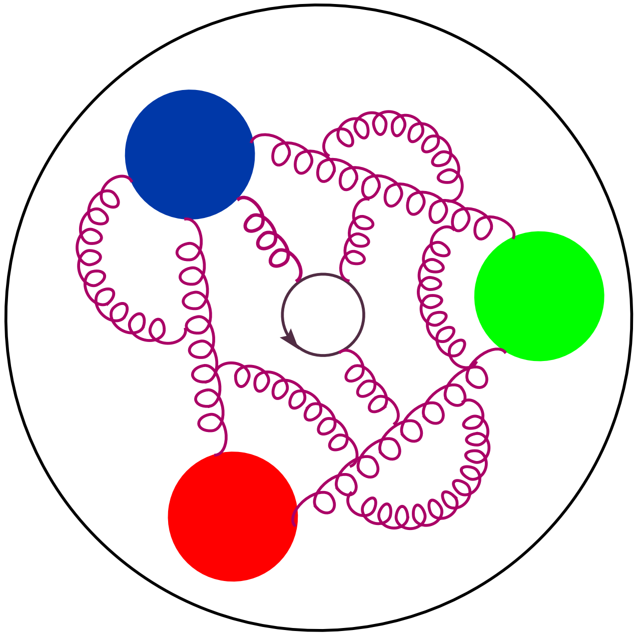
- A proton at rest, or at low energy ($P \sim M$)



- ... is a very complicated object ! (fully non–perturbative)

A hadron at rest

- A proton at rest, or at low energy $(P \sim M)$

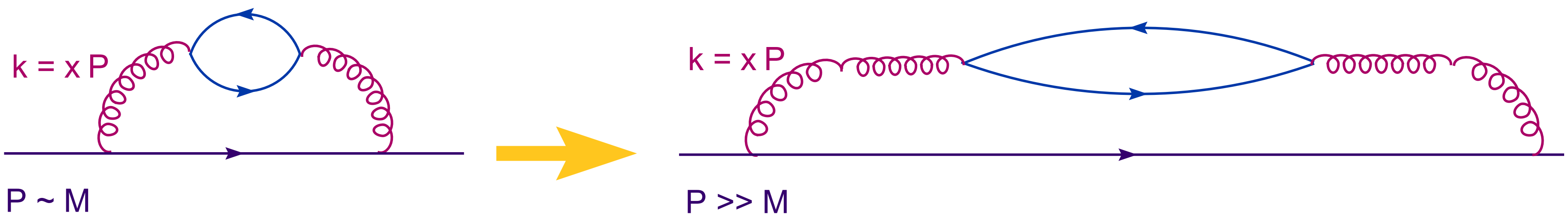


- The typical excitations have
  - energies, momenta, virtualities of order $\Lambda_{\rm QCD}$
  - lifetimes of order $1/\Lambda_{\rm QCD}$

... and the same is true for the vacuum excitations !

A hadron at high energy

- 'Infinite momentum frame' : $P \gg M \implies P^\mu \simeq (P, 0, 0, P)$
- A virtual excitation with given momentum fraction $x = k/P$:



- Its lifetime is amplified by Lorentz time dilation :

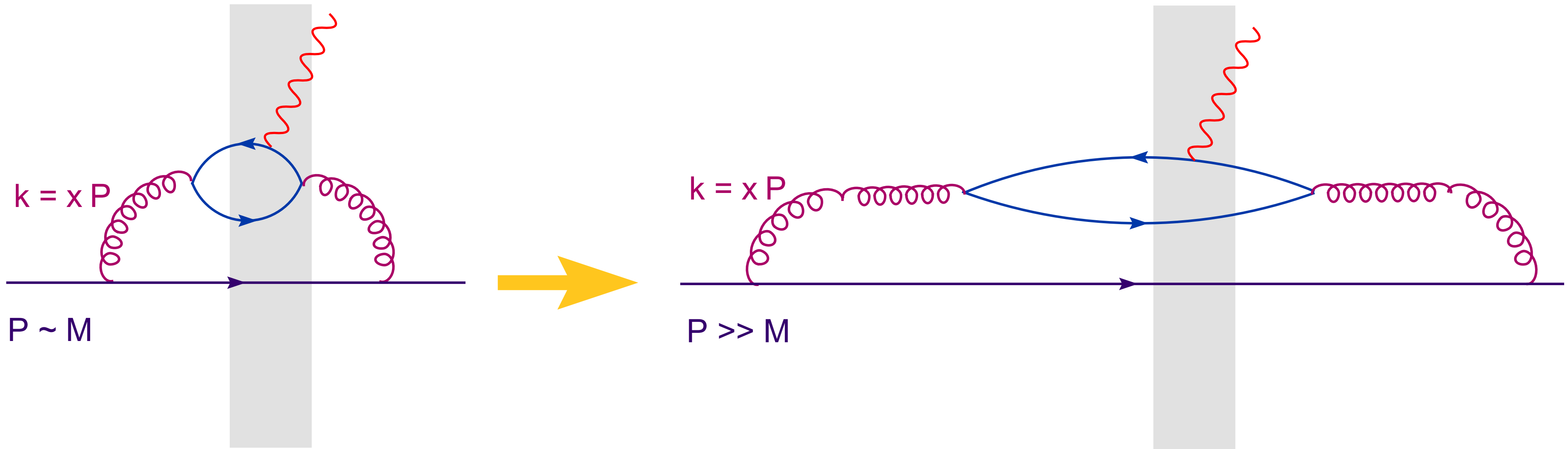
$$\Delta t_{\rm IMF} \sim \gamma\, \Delta t_{\rm RF} \sim \frac{xP}{\Lambda^2} \gg \frac{1}{\Lambda} \qquad \left(\gamma \equiv \frac{xP}{\Lambda} \gg 1\right)$$

$\Longrightarrow$ A nearly on–shell excitation with 4–momentum

$k^\mu \approx (xP, k_\perp, xP)$ and $xP \gg k_\perp$: a 'parton'

High energy interactions

- ‘Partons’ : virtual excitations which live much longer than the vacuum excitations or the interaction time



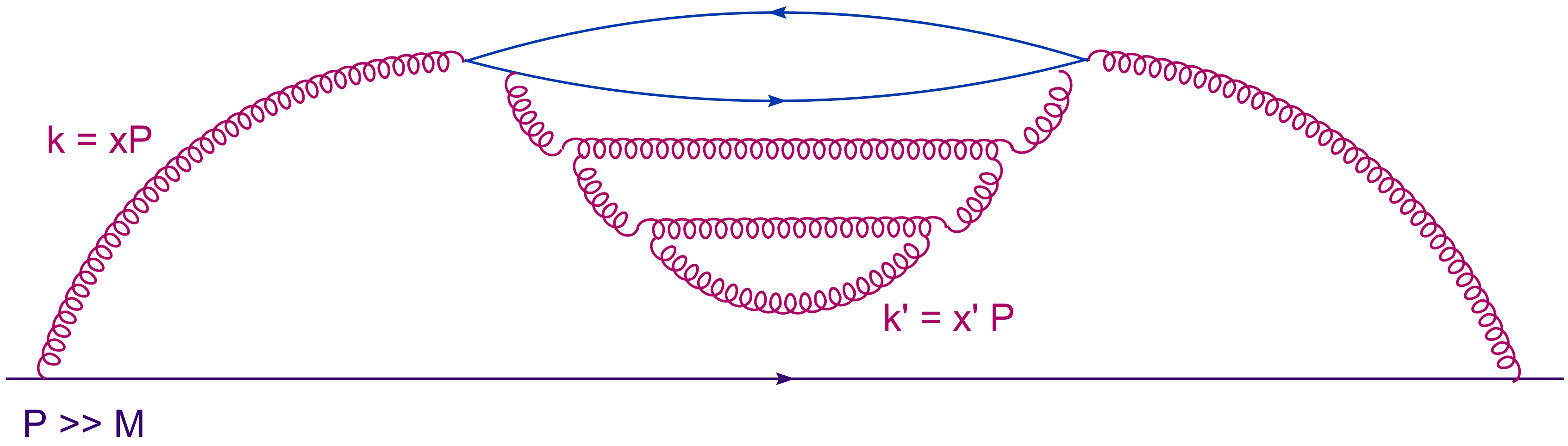
- Scattering with some external projectile
- The excitation is ‘frozen’ over the time of scattering

$\Longrightarrow$ Parton factorization

$$\sigma = \text{parton distribution functions} \otimes \text{partonic cross} - \text{sections}$$

High energy evolution

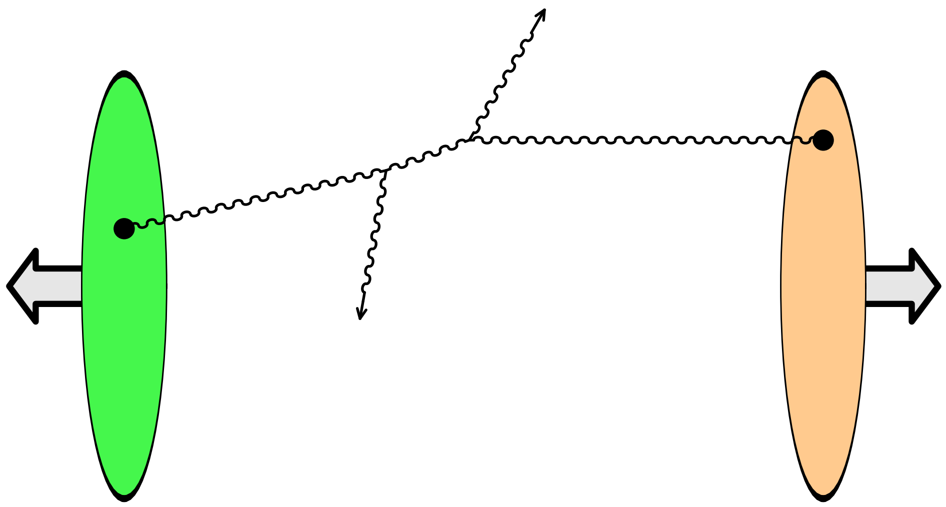
- The 'frozen' excitations act as color sources for the emission of new gluons, with lower longitudinal momenta $(x' < x)$



- Strategy: use perturbative QCD to
  - compute the evolution of the parton distributions
  - justify factorization
  - and compute partonic cross–sections

Particle production

- Justified so long as we are interested in very hard ‘jets’ (sufficiently large transverse momenta)

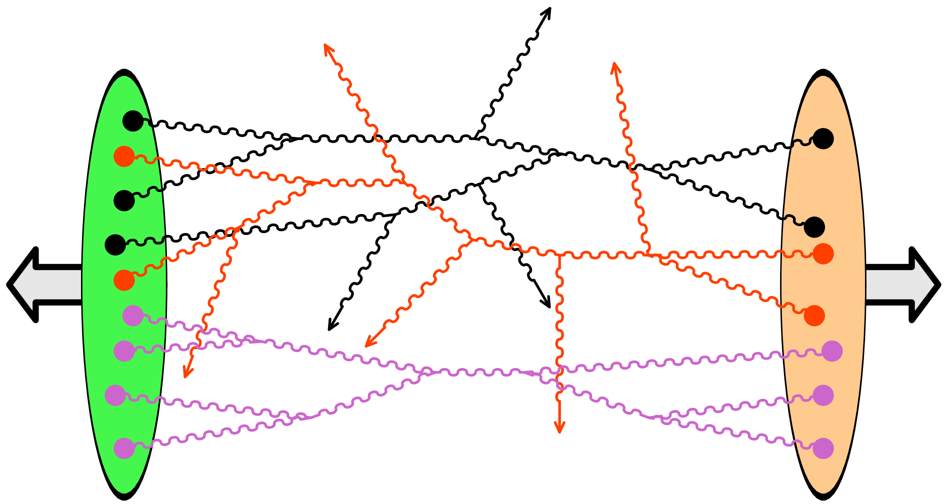


- Hard partons ‘see’ a dilute regime (probe short distances)

  $\Longrightarrow$ Only one particle (‘parton’) from each nucleus participate in the collision

Particle production

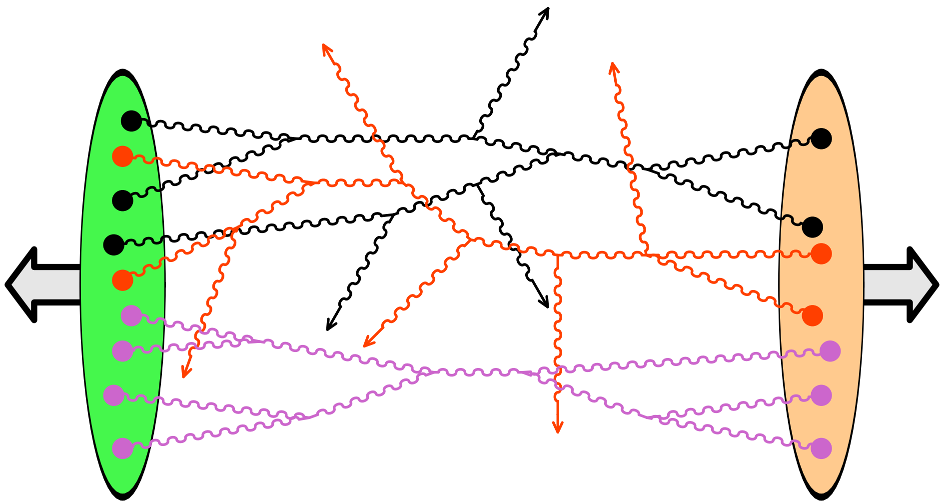
- But for softer jets, multiple interactions are essential.



- On a ‘soft’ resolution scale, the hadron look dense
  - multiparton processes inside the hadron wavefunction
  - pileup of many partonic scatterings in every AA collision

Particle production

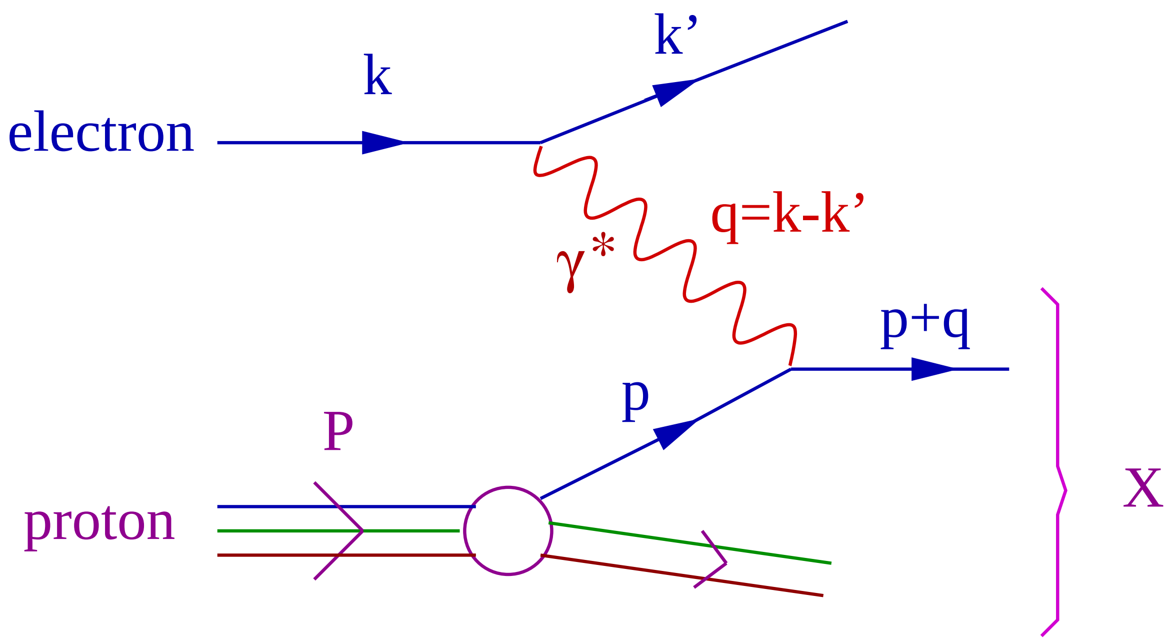
- But for softer jets, multiple interactions are essential.



- On a ‘soft’ resolution scale, the hadron look dense
  - multiparton processes inside the hadron wavefunction
  - pileup of many partonic scatterings in every AA collision
- The simplest framework to address all that :
  deep inelastic electron–hadron scattering

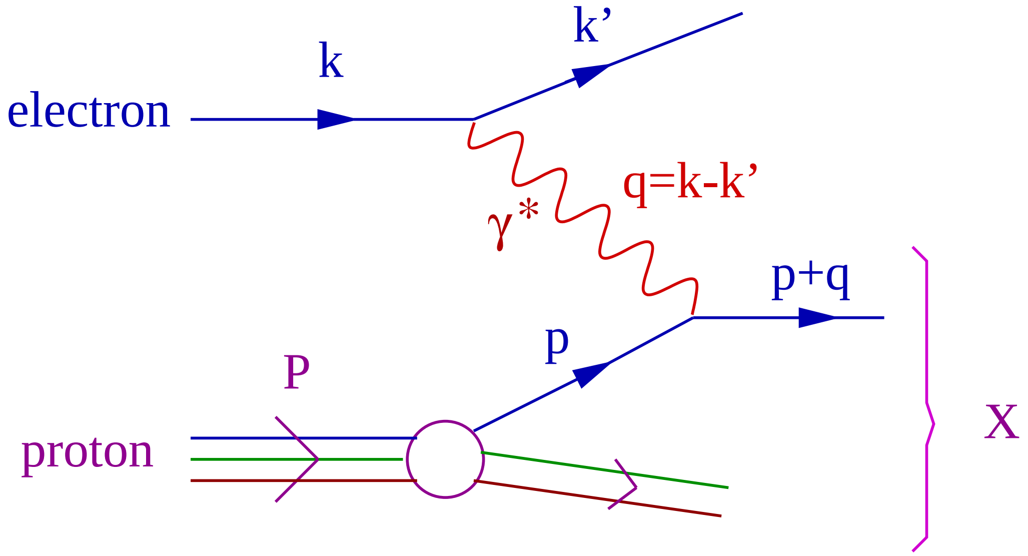
Deep Inelastic Scattering

electron (k) + proton (P) $\longrightarrow$ electron (k') + X (P_X)



- 'Inclusive cross–section' : One allows for all the possible final states X of the hadronic system
- One measures the momentum k'^{μ} of the deflected lepton
  $\Longrightarrow$ One deduces the transferred momentum $q^{\mu} = k^{\mu} - k'^{\mu}$

Deep Inelastic Scattering: Kinematics

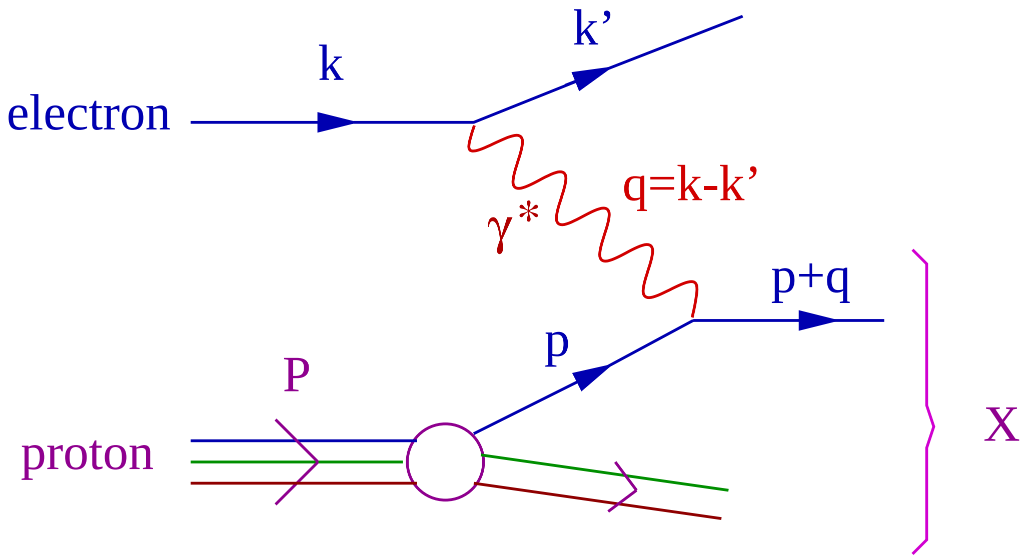


- Useful kinematical invariants (2 independent ones) :
  - The virtuality of the exchanged photon: $Q^2 \equiv -q^2 \geq 0$

    Exercice: use $k^2 = (k-q)^2 = m_e^2$ *to deduce that* $q^2 < 0$
  - The invariant energy of the photon–proton system:

    $$s \equiv (q+P)^2 = 2P \cdot q + M^2 - Q^2$$
  - The Bjorken–x variable: $$x \equiv \frac{Q^2}{2P \cdot q}$$

Deep Inelastic Scattering: Small–x



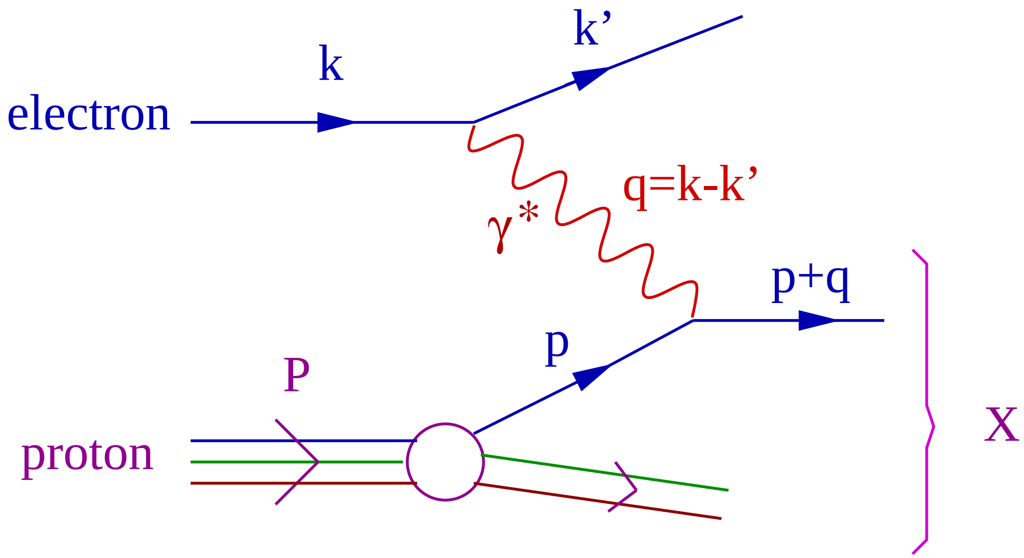
- "Deeply Inelastic Scattering" : $Q^2 \gg M^2$

$$0 < x \equiv \frac{Q^2}{2P \cdot q} \simeq \frac{Q^2}{s + Q^2} < 1$$

- "Small–x regime" $\longleftrightarrow$ high–energy : $s \gg Q^2$

$$x \simeq \frac{Q^2}{s} \ll 1$$

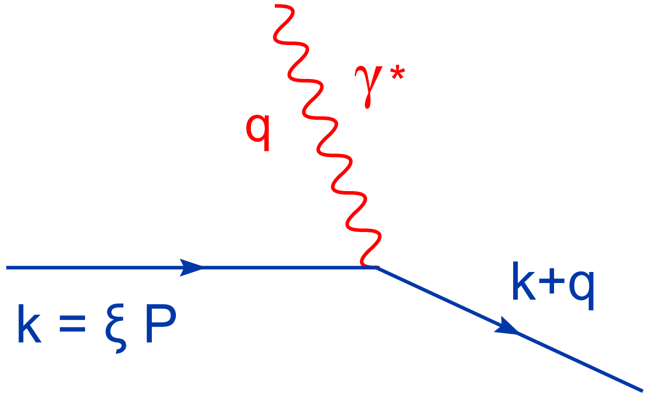
Resolution scales in DIS



- Direct physical interpretation for Q^2 and x :
  - the virtual photon resolution in transverse space ...
  - and, respectively, longitudinal momentum.
- Virtual photon absorbed by a quark excitation of the proton
  - with transverse size $\Delta x_\perp \sim 1/Q$
  - and longitudinal momentum $p_z = xP$

Partons in DIS

- The absorption of the virtual photon in the proton IMF :



$$P^\mu = (P, 0, 0, P)$$
$$k^\mu \approx (\xi P, k_\perp, \xi P)$$

$$(k+q)^2 \approx 0 \quad \Longrightarrow \quad -Q^2 + 2\xi P\cdot q \approx 0$$
$$\Longrightarrow \quad \xi = \frac{Q^2}{2P\cdot q} = x$$

- Uncertainty principle $\Longrightarrow$ longitudinal resolution $\lambda \sim 1/xP$
- At high energy, the hadron looks thicker then its naive Lorentz contracted width:

$$\Delta z_{\rm class} \sim \frac{R}{\gamma} \sim \frac{RM}{P} \sim \frac{1}{P} \ll \frac{1}{xP}$$

Partons lifetime

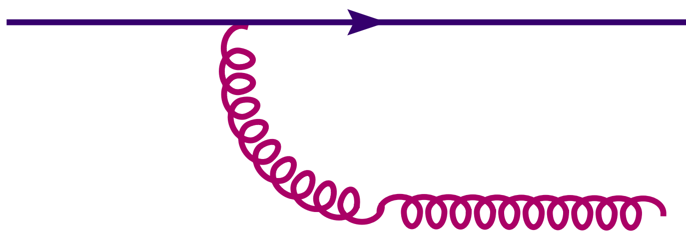
Motivation

Partons in DIS

- DIS
- Resolution
- Partons in DIS
- F2
- Distributions
- Partons at HERA

Gluon evolution

- Partons have a finite lifetime, as they can radiate

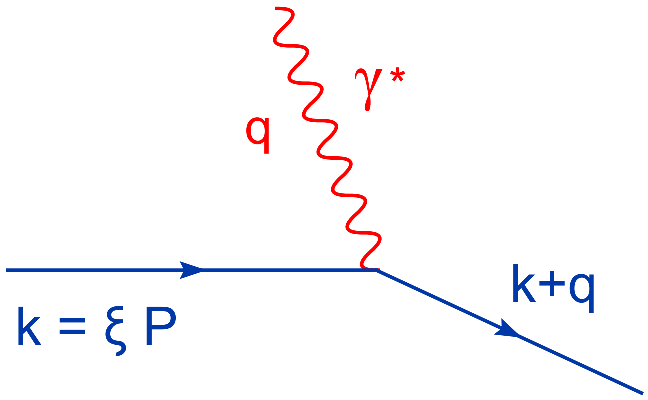


$$\begin{aligned}\Delta E &= -P + \sqrt{x^2P^2 + k_\perp^2} + \sqrt{(1-x)^2P^2 + k_\perp^2} \\ &\approx -P + xP + \frac{k_\perp^2}{2xP} + (1-x)P + \frac{k_\perp^2}{2(1-x)P} \\ &\approx \frac{k_\perp^2}{2x(1-x)P}\end{aligned}$$

- By the uncertainty principle (for small $x \ll 1$)

$$\Delta t \sim \frac{1}{\Delta E} = \frac{2xP}{k_\perp^2} = \frac{2k_z}{k_\perp^2}$$ (Lorentz time dilation)

- The collision time is similarly evaluated :



$$\Delta t_{\rm col} \sim \frac{2xP}{Q^2}$$

- The parton lifetime should be larger than the collision time

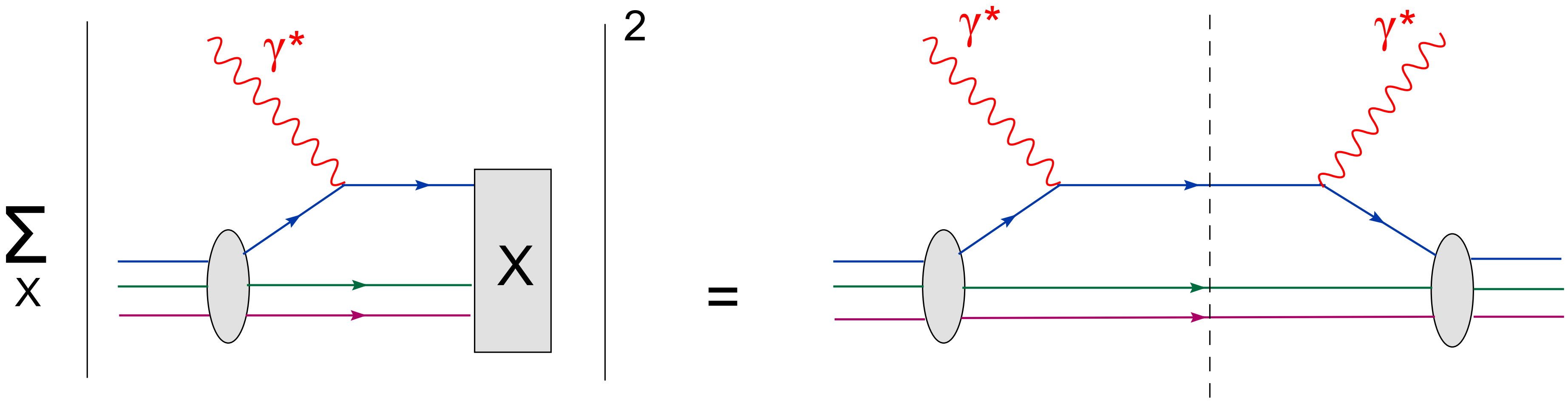
$$\Delta t_{\rm part} \sim \frac{2xP}{k_\perp^2} \;>\; \Delta t_{\rm col} \sim \frac{2xP}{Q^2}$$

$\Longrightarrow$ The photon 'sees' all the partons having $k_\perp^2 < Q^2$

- By the uncertainty principle, such partons are localized within an area $\Delta\Sigma \sim 1/Q^2$ in the transverse plane

The proton structure function

- Differential cross section for virtual photon absorbtion :



$$\sigma_{\gamma^* p}(x, Q^2) = \frac{4\pi^2 \alpha_{\rm em}}{Q^2} F_2(x, Q^2)$$

- $F_2(x, Q^2) =$ the 'proton structure function' $= F_T + F_L$

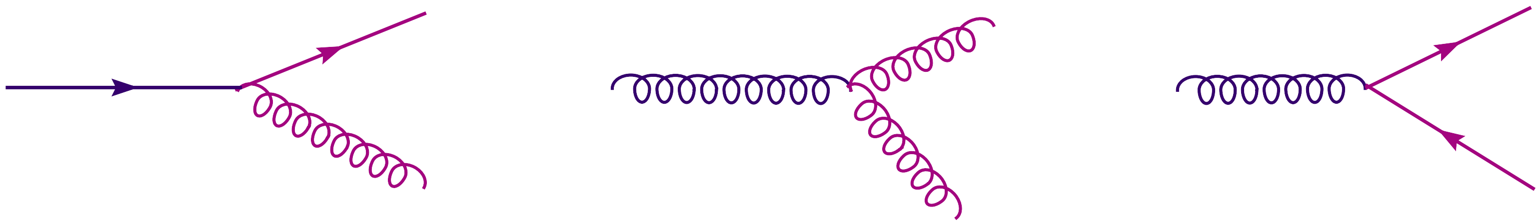
If 'proton' = a single, point–like, quark $\Rightarrow \hat{F}_2(x, Q^2) = \delta(x - 1)$

Parton distributions

- The quark distribution function :

$$F_2(x,Q^2) \;=\; \sum_f e_f^2 \left[x q_f(x,Q^2) + x\bar{q}_f(x,Q^2)\right]$$

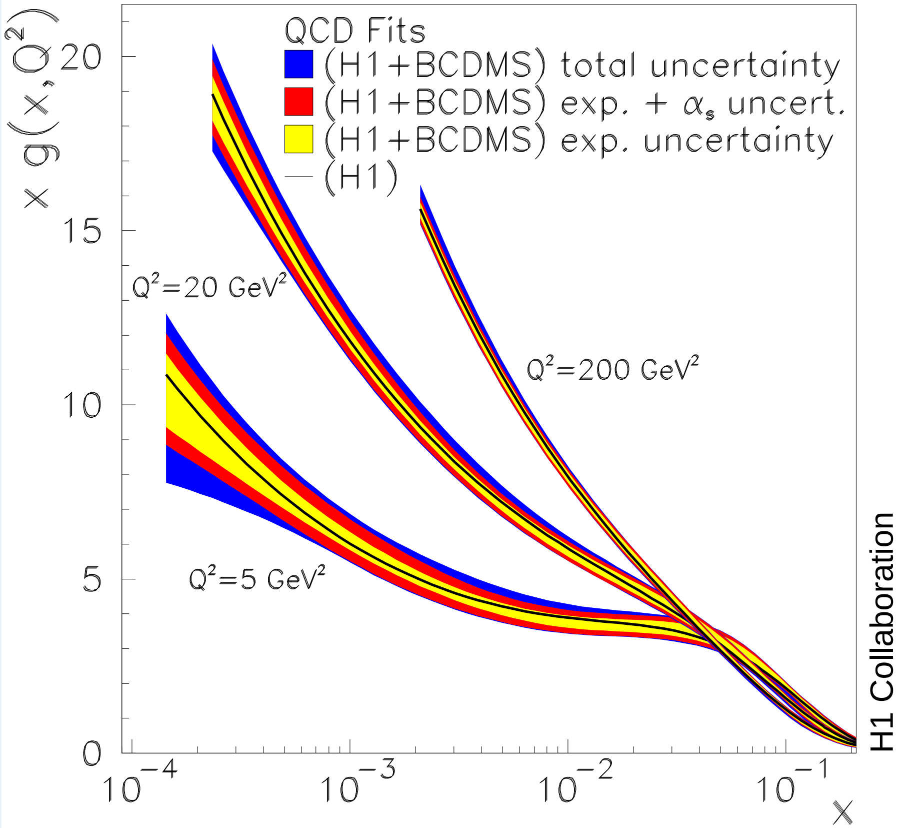
- $q_f(x,Q^2)\mathrm{d}x$: number of quarks of flavor f
  - with momentum fraction between x and $x+\mathrm{d}x$
  - localized in the transverse space within an area $1/Q^2$
- Parton evolution (DGLAP) $\Longrightarrow$ Gluon distribution $xG(x,Q^2)$

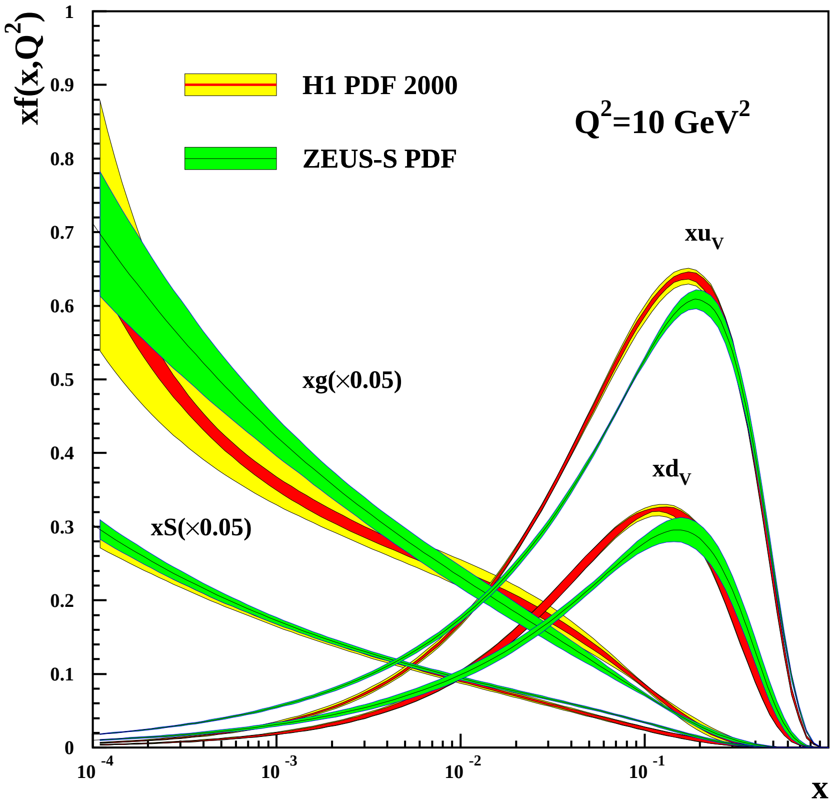


$$\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2} \;\simeq\; \frac{\alpha_s}{3\pi}\Big(\sum_f e_f^2\Big)\, xG(x,Q^2)$$

Partons at HERA

▷ The gluon distribution rises very fast at small x ! $(\sim 1/x^{0.3})$

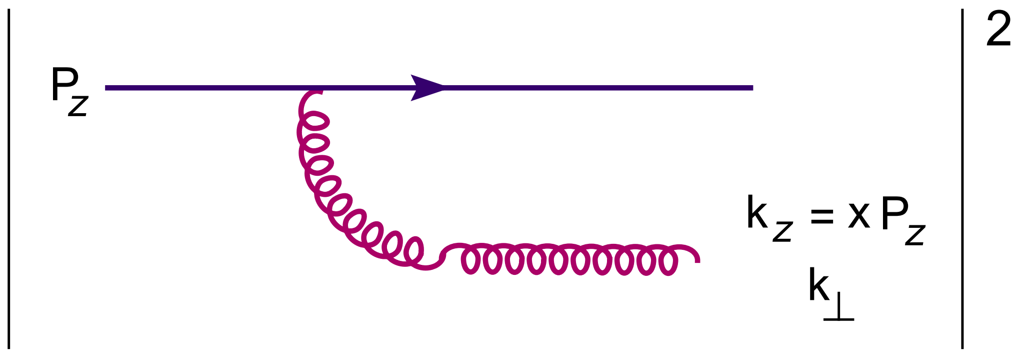




▷ The ‘sea’ quark distribution rises as well (driven by the gluons)

Bremsstrahlung $(q \to qg)$

- The 'infrared sensitivity' of bremsstrahlung favors the emission of 'soft' (= small–x) gluons



$$dP_{\rm Brem} \simeq \frac{\alpha_s C_F}{\pi^2}\frac{d^2k_\perp}{k_\perp^2}\frac{dk_z}{k_z} \propto \alpha_s \frac{dx}{x} \equiv \alpha_s\, dY$$

$$Y \equiv \ln\frac{1}{x} \sim \ln s \implies dY = \frac{dx}{x} : \quad \text{“rapidity”}$$

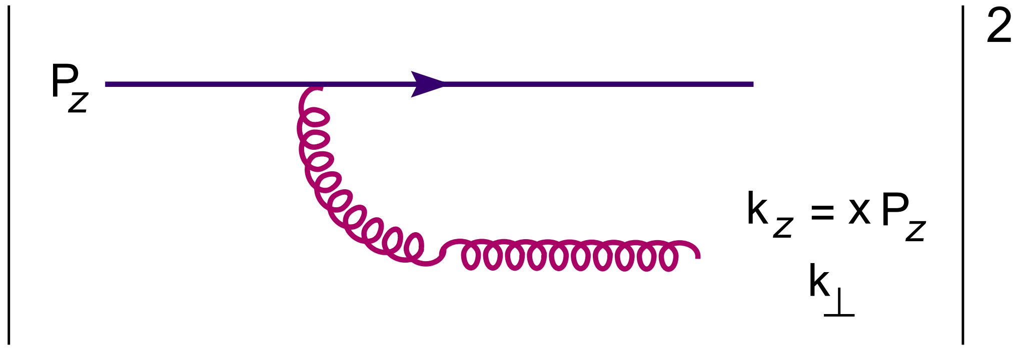
- A probability of $\mathcal{O}(\alpha_s)$ to emit one gluon per unit rapidity.

The gluon distribution of a single quark

- $N(x, k_\perp) = \#$ of gluons with longitudinal momentum fraction x and transverse momentum $k_\perp$ radiated by a quark.

“unintegrated gluon distribution” : $$xG(x, Q^2) = \int^{Q^2} \mathrm{d}\, k_\perp^2\, N(x, k_\perp)$$

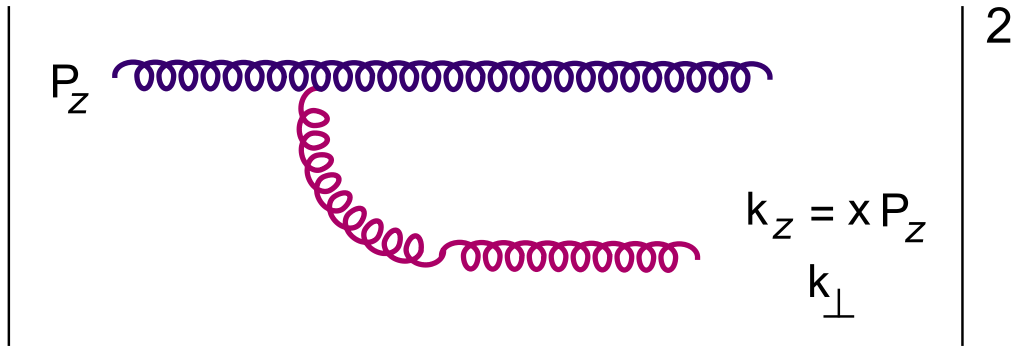
- Lowest order in α_s :



$$N(x, k_\perp) = x\, \frac{\mathrm{d}P_{\mathrm{Brem}}}{\mathrm{d}x\, \mathrm{d}k_\perp^2} \simeq \frac{\alpha_s C_F}{\pi}\, \frac{1}{k_\perp^2}$$

Bremsstrahlung $(g \to gg)$

- A 'soft' gluon can radiate an even softer one !

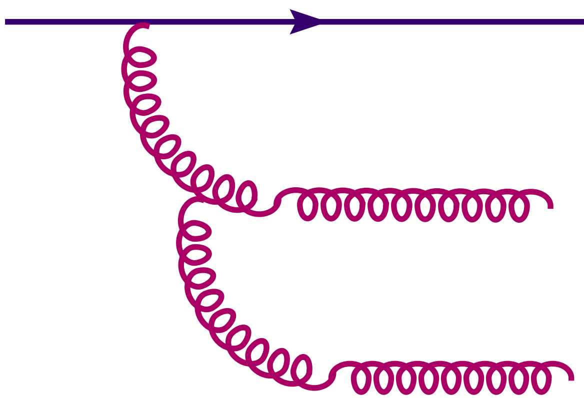


$$\mathrm{d}P_{\mathrm{Brem}} \simeq \frac{\alpha_s N_c}{\pi^2} \frac{\mathrm{d}^2 k_\perp}{k_\perp^2} \frac{\mathrm{d}k_z}{k_z} \propto \alpha_s \frac{\mathrm{d}x}{x} \equiv \alpha_s \, \mathrm{d}Y$$

- Radiative corrections due to soft gluon emission are potentially important !

BFKL evolution (qualitatively)

- $\mathcal{O}(\alpha_s \ln(1/x))$: First order correction ("leading logarithmic approximation")
- One intermediate gluon with x_1 in the range $x \ll x_1 \ll 1$



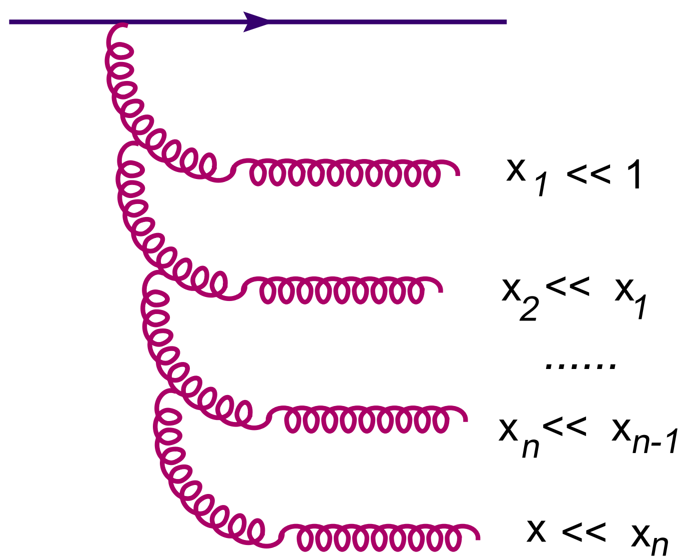
- A contribution of relative order

$$\mathcal{P}(1) \propto \alpha_s \int_x^1 \frac{dx_1}{x_1} = \alpha_s \ln \frac{1}{x}$$

BFKL evolution (qualitatively)

- $\mathcal{O}((\alpha_s Y)^n)$: n intermediate gluons strongly ordered in x

$$x \ll x_n \ll x_{n-1} \cdots \ll x_2 \ll x_1 \ll 1$$

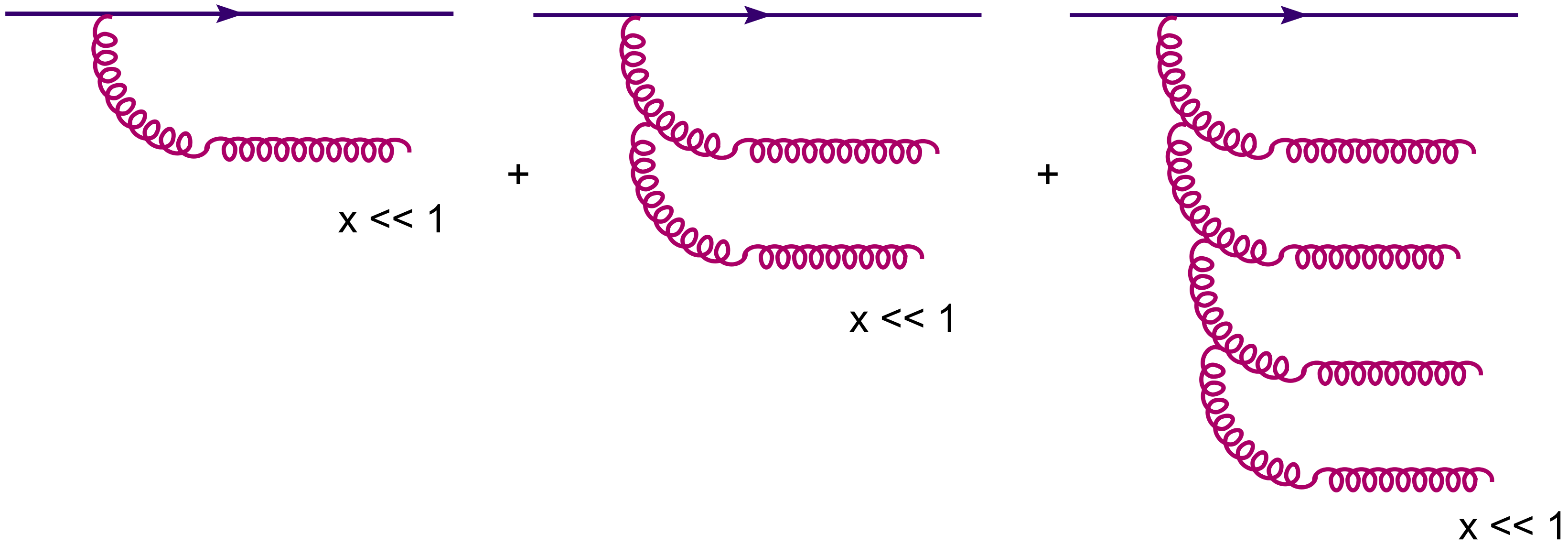


$$\mathcal{P}(n) \propto \frac{1}{n!}\left(\alpha_s \ln \frac{1}{x}\right)^n$$

BFKL evolution (qualitatively)

- The sum of all contributions exponentiate:

$$\sum_n \frac{1}{n!}\left(\alpha_s \ln\frac{1}{x}\right)^n \sim \; e^{\omega\alpha_s Y} \quad \text{with} \quad Y \equiv \ln\frac{1}{x}$$



$$N(Y, k_\perp) \approx \frac{\alpha_s C_F}{\pi}\,\frac{1}{k_\perp^2}\,e^{\omega\alpha_s Y}$$

- "BFKL resummation" *(Balitsky, Fadin, Kuraev, Lipatov, 78)*

BFKL evolution (qualitatively)

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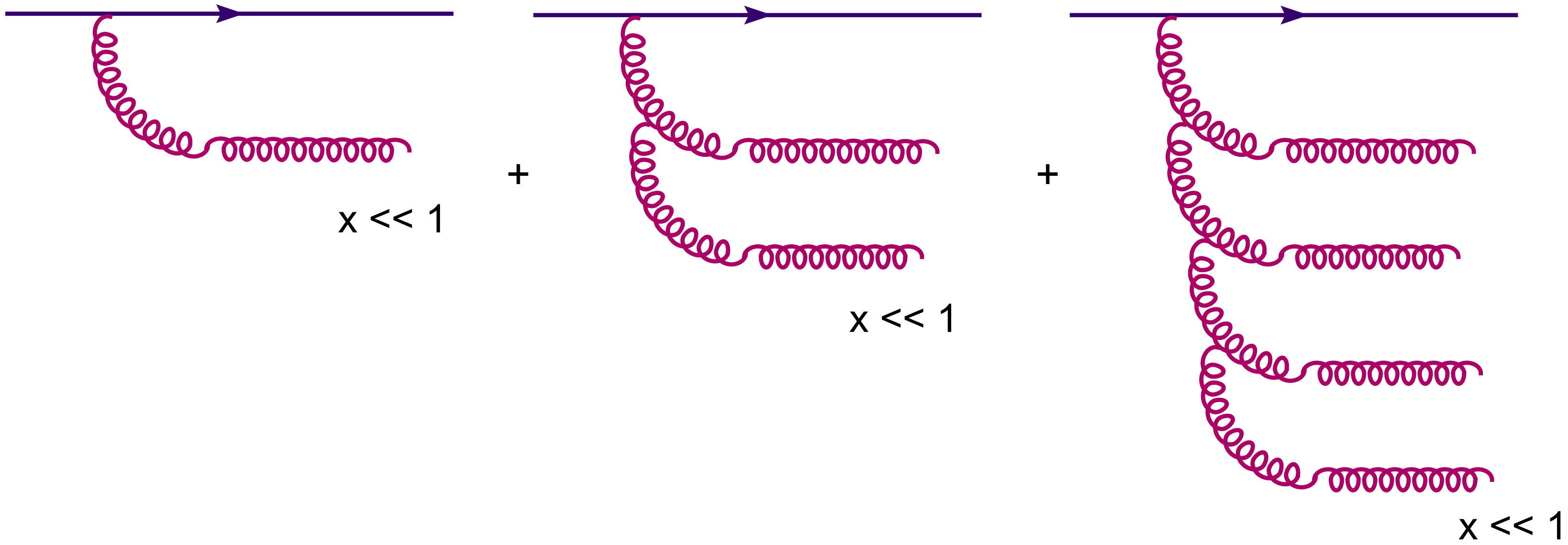
$$\sum_n \frac{1}{n!}\left(\alpha_s \ln\frac{1}{x}\right)^n \sim e^{\omega\alpha_s Y} \quad \text{with} \quad Y \equiv \ln\frac{1}{x}$$



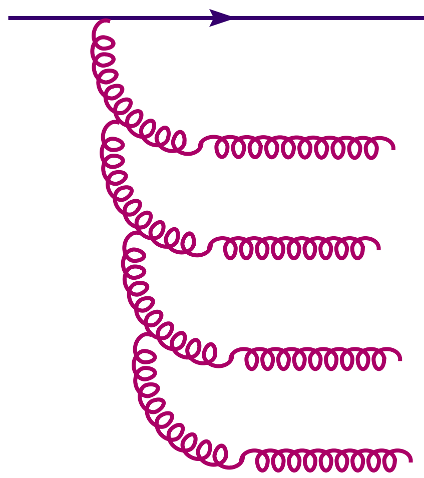
$$N(Y, k_\perp) \approx \frac{\alpha_s C_F}{\pi}\left(\frac{1}{k_\perp^2}\right)^\gamma e^{\omega\alpha_s Y} \ldots$$

- "BFKL resummation" *(Balitsky, Fadin, Kuraev, Lipatov, 78)*

BFKL evolution (qualitatively)

- Remember: The lifetime of the virtual gluon is proportional to its longitudinal momentum

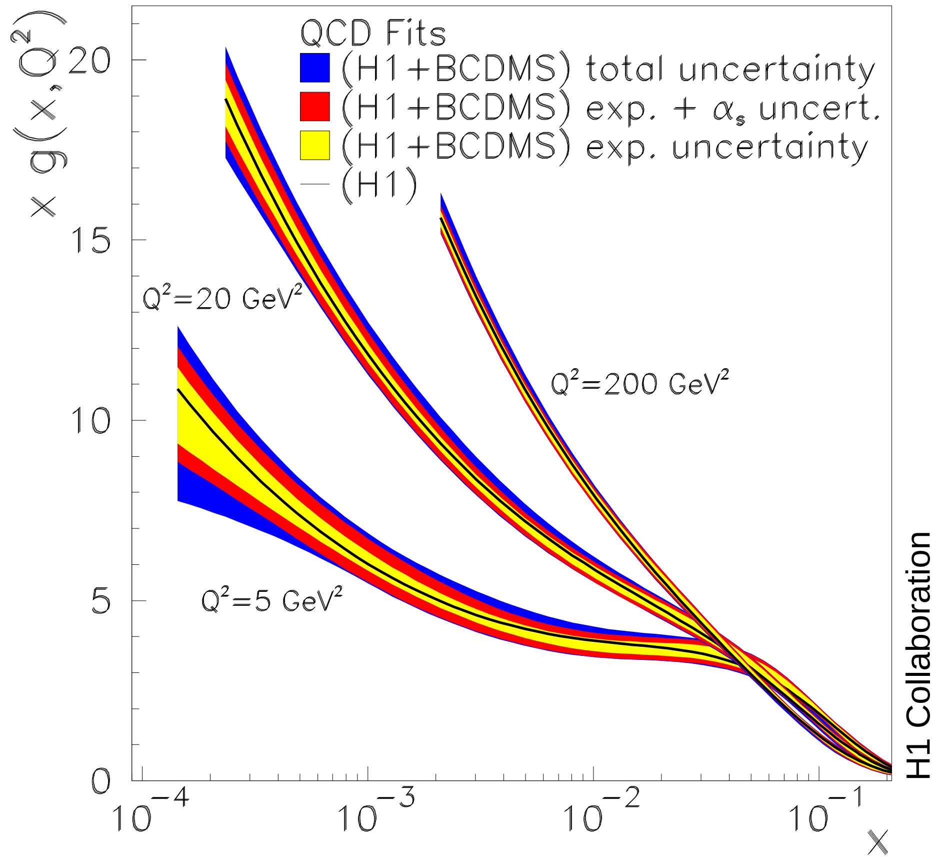
$$\Delta t \sim \frac{\hbar}{\Delta E} = \frac{2k_z}{k_\perp^2} = \frac{2xP}{k_\perp^2}$$



- The smaller x, the shorter the lifetime $\Longrightarrow$ coherence !
- The ‘upper’ (large–x) part of the cascade is ‘frozen’ : “glass”

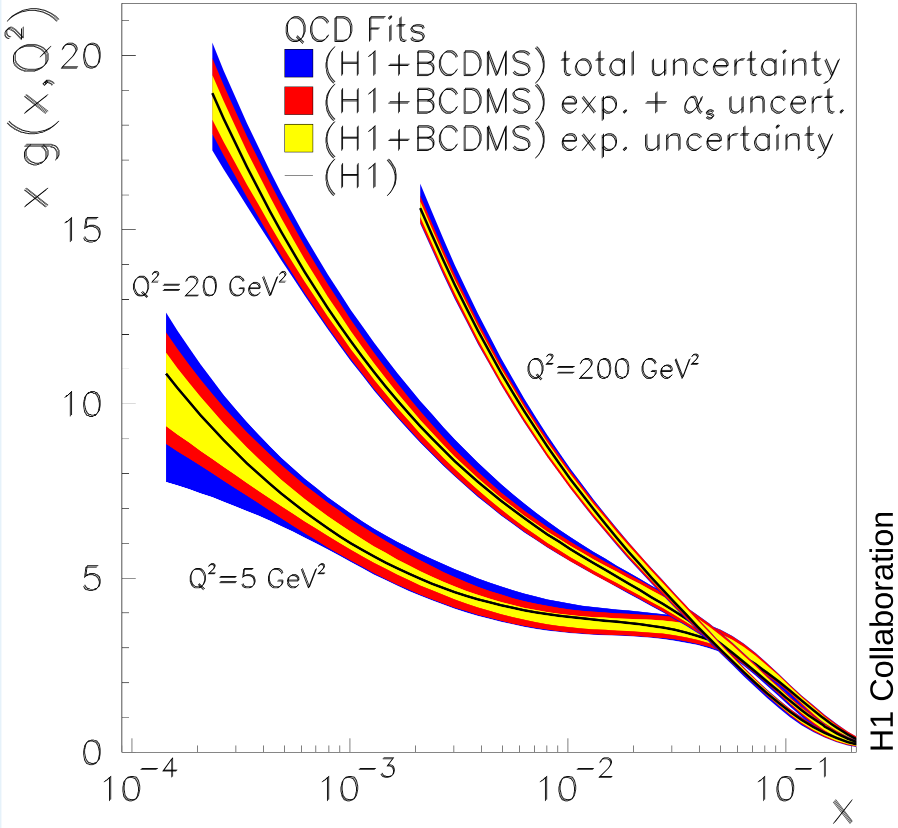
Gluons at HERA ... revisited

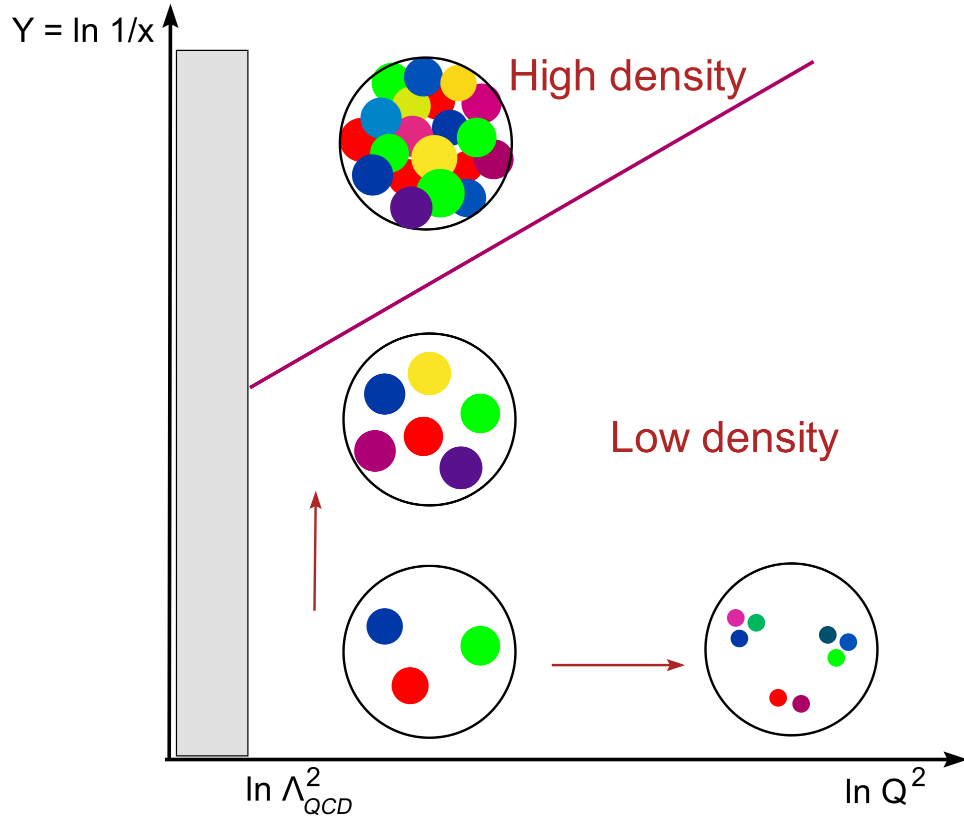
The gluon distribution rises with both Q^2 and $1/x$ ($\sim 1/x^{\lambda}$, $\lambda \sim 0.3$)



$xG(x,Q^2) \approx$ # of gluons with transverse size $\Delta x_\perp \sim 1/Q$ and $k_z = xP$

Gluons at HERA ... revisited

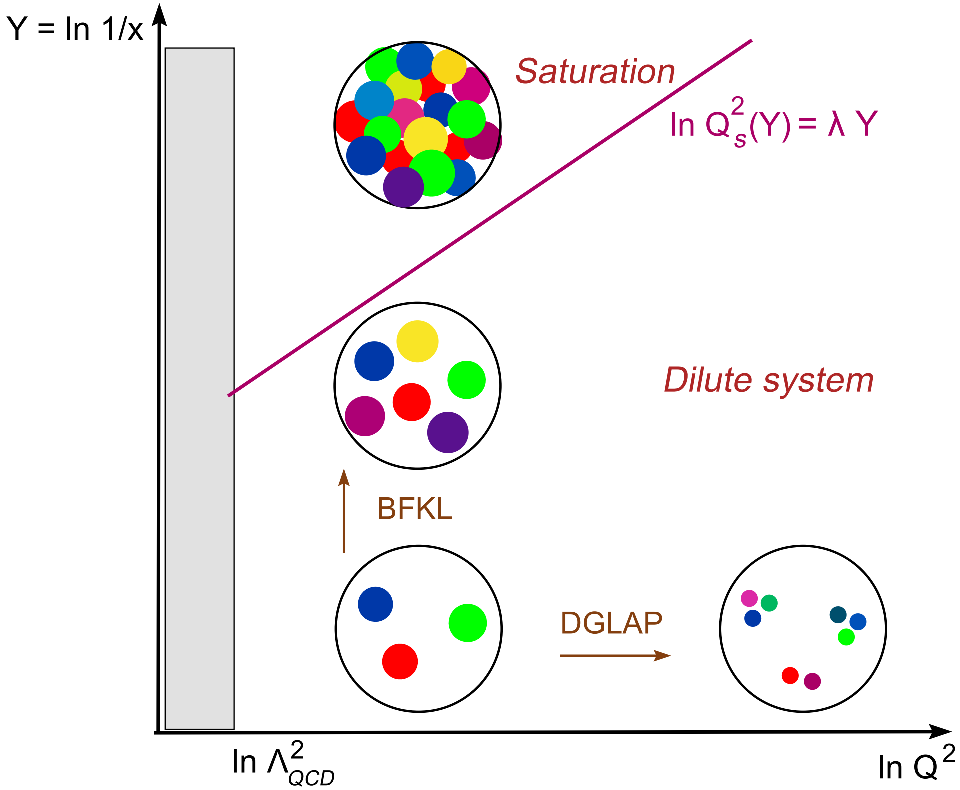




▷ High–Q^2 evolution (DGLAP) : The parton density is decreasing

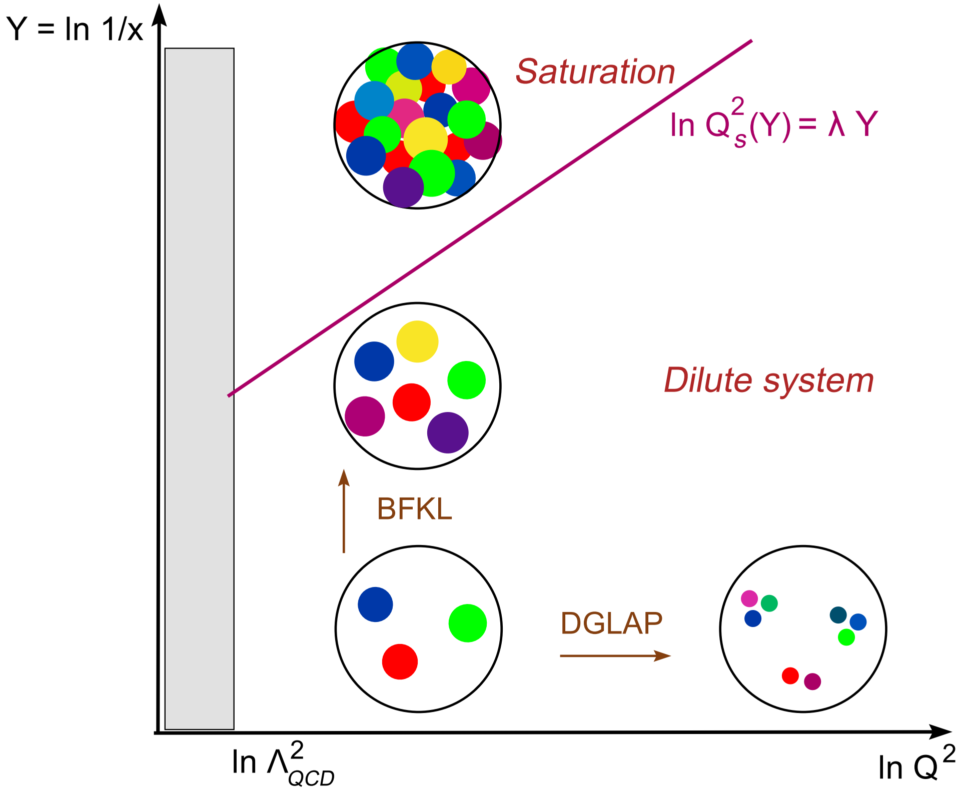
▷ High–energy evolution : An evolution towards increasing density !

Gluon saturation (qualitatively)



- **Linear evolution (DGLAP, BFKL):** After being emitted, partons in the wavefunction do not know about each other.
- Natural for DGLAP: an evolution towards diluteness
- Eventually wrong for BFKL: gluons overlap with each other

Gluon saturation (qualitatively)



- Criterion: large gluon occupation number $n \sim 1/\alpha_s$

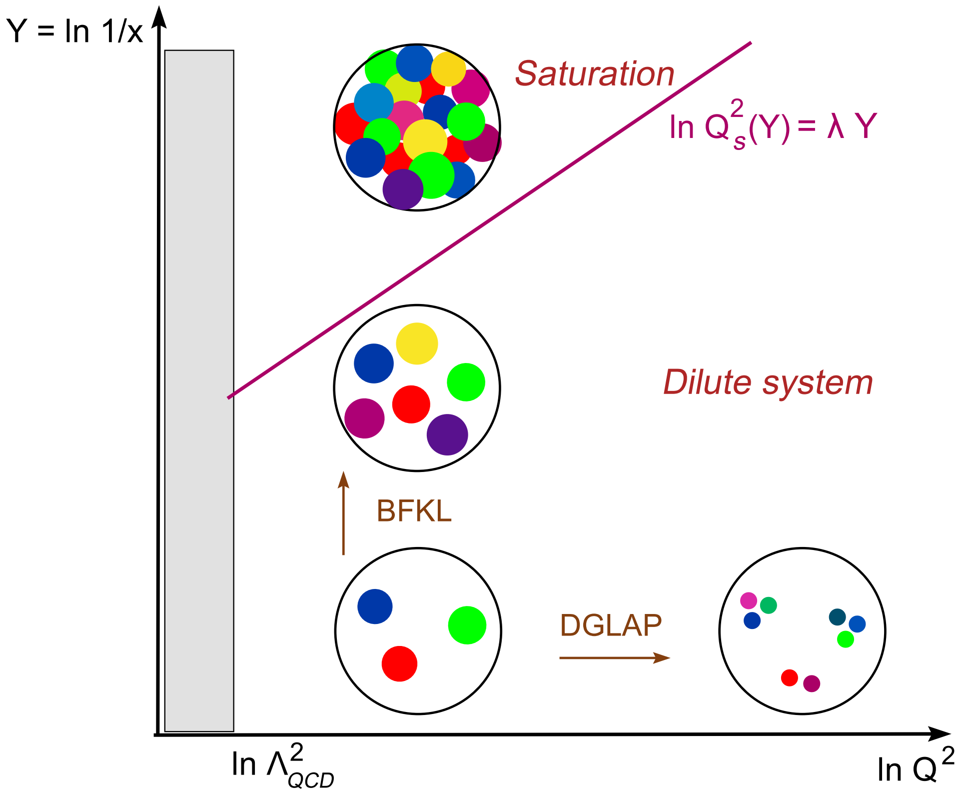
$$n(x, b_\perp, k_\perp) \equiv \frac{N(x, k_\perp)}{\pi R^2} \approx \frac{\pi}{Q^2} \times \frac{xG(x, Q^2)}{\pi R^2}$$

- The gluons must be numerous enough (small x) and large enough (low Q^2) to strongly overlap with each other.

Gluon saturation (qualitatively)

- Critical line : Saturation momentum $Q_s(x)$

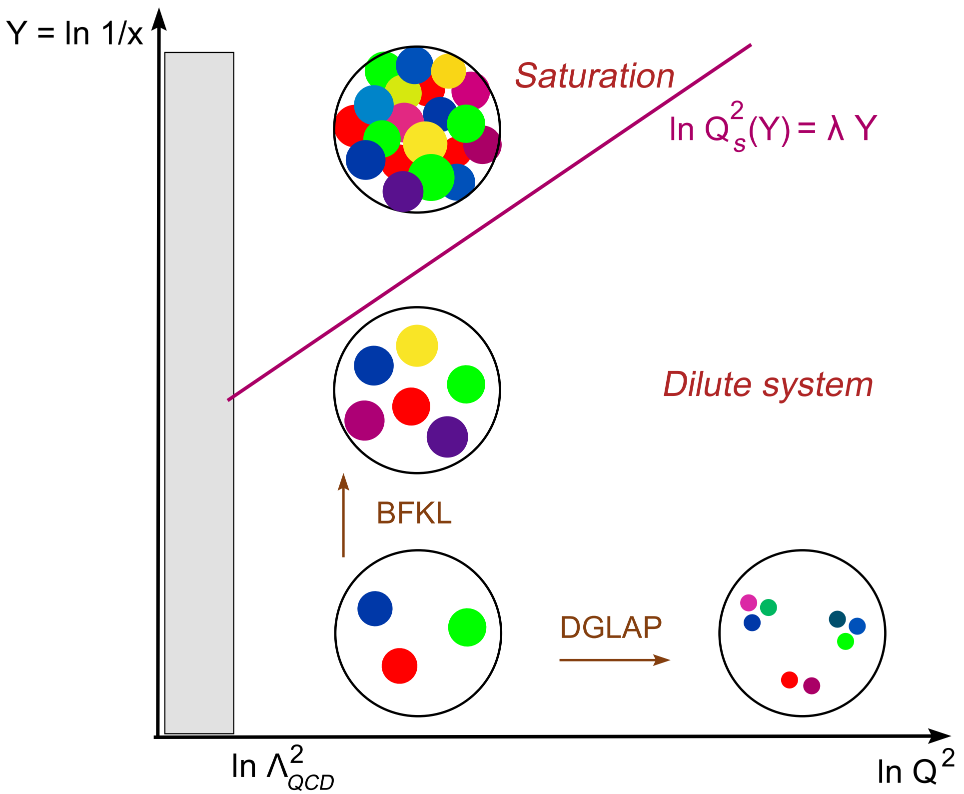
$$Q_s^2(x) \simeq \alpha_s \frac{xG(x, Q_s^2)}{\pi R^2} \sim x^{-\lambda} \equiv e^{\lambda Y}$$



Gluon saturation (qualitatively)

- Large nucleus : $A \gg 1 \implies xG_A(x,Q^2) \propto A$

$$Q_s^2(x,A) \simeq \alpha_s \frac{xG_A(x,Q_s^2)}{\pi R_A^2} \sim x^{-\lambda} A^{1/3}$$



The Color Glass Condensate

(McLerran, Venugopalan, 1994; E.I., Leonidov, McLerran, 2000)

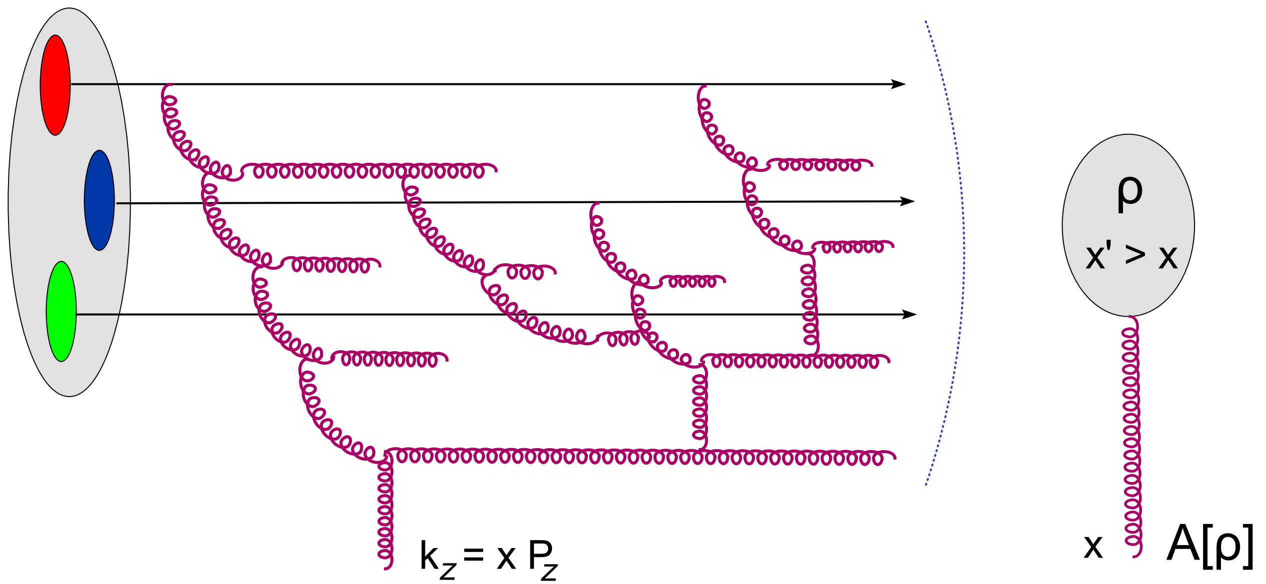
- The theory of saturation within perturbative QCD
- An effective theory for the small–x gluons in the high–density environment characteristic of saturation
- Large occupation numbers ($n \sim 1/\alpha_s$) : 'condensate'
  - The gluons can be described as classical color fields
- Separation of scales (longitudinal momentum/time) : 'glass'
  - The smaller x, the shorter the lifetime of the gluon

$$\Delta t \sim \frac{\hbar}{\Delta E} = \frac{2xp}{k_\perp^2}$$

  - The gluons with $x' \gg x$ are 'frozen' over the typical time scale for the dynamics at x

The Color Glass Condensate

- Small–x gluons: Classical color fields radiated by fast color sources ($x' \gg x$) 'frozen' in some random configuration



- Classical field equations (Yang–Mills) for the field $A_a^\mu[\rho]$
- Probability distribution for the charge density at Y : $W_Y[\rho]$
- Renormalization group equation for $W_Y[\rho]$: JIMWLK