Gluon saturation from DIS to AA collisions

IV – AA collisions : glasma instabilities

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General outline

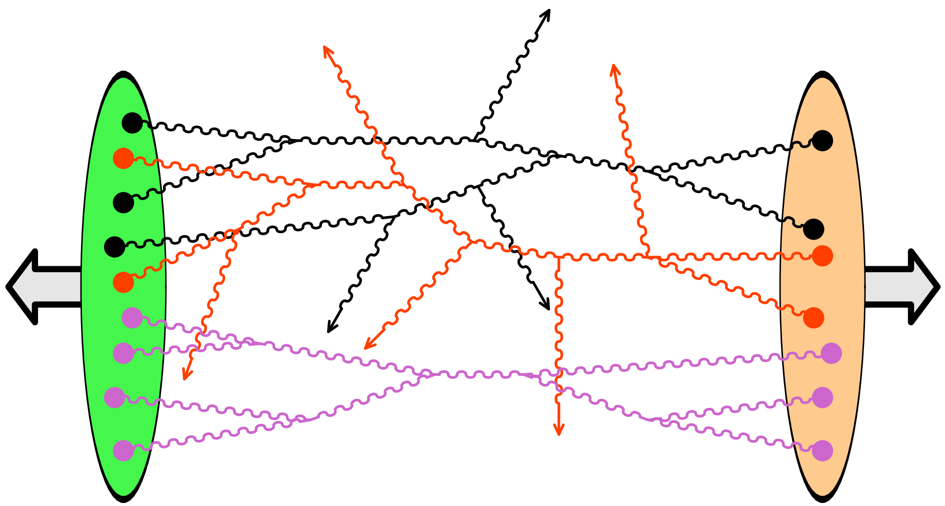
- Lecture I : Gluon saturation in DIS
- Lecture II : Proton-nucleus collisions
- Lecture III : AA collisions : gluon production
- Lecture IV : AA collisions : glasma instabilities

Lecture IV : AA : glasma instabilities

- Reminder : gluon production
- Glasma instabilities
- Unstable modes resummation
- Thermalization ?
- Possible link to the Weibel instability

Reminder: gluon production

Relevant graphs in the saturated regime



- Dilute regime : one parton in each projectile interact
- Dense regime : multiparton processes become crucial (+ pileup of many simultaneous scatterings)

Gluon spectrum at LO

Krasnitz, Nara, Venugopalan (1999 – 2001), Lappi (2003)

- Expansion in g^2 :

$$\frac{dN}{d^3\vec{\boldsymbol{p}}} = \frac{1}{g^2}\left[c_0 + c_1\, g^2 + c_2\, g^4 + \cdots\right]$$

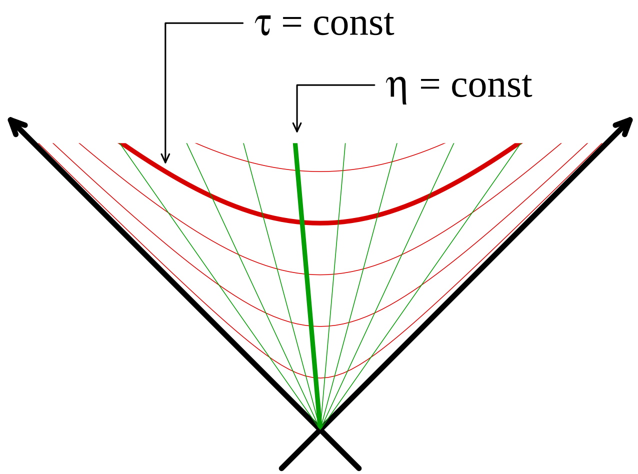
- The gluon spectrum at LO is given by :

$$\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\rm LO} \equiv \frac{c_0}{g^2} \propto \int_{x,y} e^{ip\cdot(x-y)} \;\cdots\; \mathcal{A}_\mu(x)\mathcal{A}_\nu(y)$$

- $\mathcal{A}_\mu(x)$ is the retarded solution of Yang-Mills equations :

$$\begin{cases} \left[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}\right] = J^\nu \\ \lim\limits_{t\to-\infty} \mathcal{A}^\mu(t,\vec{\boldsymbol{x}}) = 0 \end{cases}$$

Boost invariance



- Initial values at $\tau = 0^+$: the initial fields $\mathcal{A}_{\rm in}$ do not depend on the rapidity η

  $\triangleright$ they remain independent of η at all times (invariance under boosts in the z direction)

  $\triangleright$ numerical resolution performed in $1 + 2$ dimensions

Factorization at small x

- Naive loop expansion :

$$\frac{dN}{d^3\vec{\boldsymbol{p}}} = \frac{1}{g^2}\left[c_0 + c_1\, g^2 + c_2\, g^4 + \cdots\right]$$

- Problem : $c_{1,2,\dots}$ contain logarithms of $1/x_{1,2}$:

$$\begin{aligned} c_1 &= \qquad\qquad\qquad\qquad\quad c_{10} + c_{11}\,\ln\left(\frac{1}{x_{1,2}}\right) \\ c_2 &= c_{20} + c_{21}\,\ln\left(\frac{1}{x_{1,2}}\right) + \underbrace{c_{22}\,\ln^2\left(\frac{1}{x_{1,2}}\right)}_{\text{Leading Log terms}} \end{aligned}$$

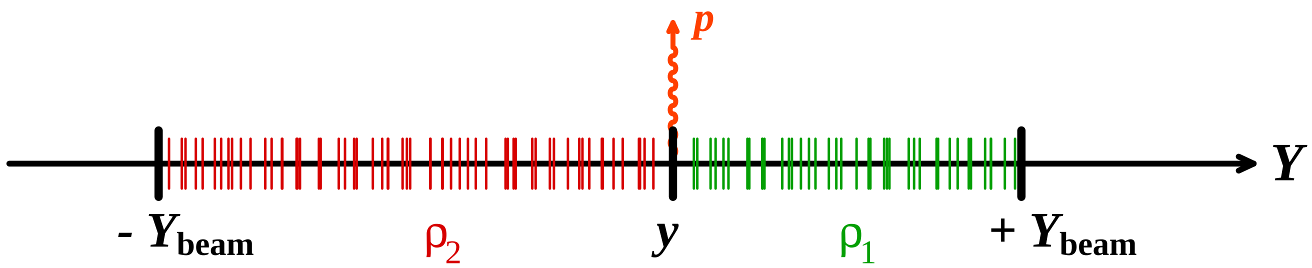
- At small $x_{1,2}$, these logs are large, and we would like to resum all the terms that have as many logs as powers of g^2

Factorization at small x

- For the single gluon spectrum in AA collisions, one would like to establish a formula such as :

$$\left\langle \frac{dN}{d^3\vec{\boldsymbol{p}}} \right\rangle \underset{\text{LLog}}{=} \int \left[D\rho_1\, D\rho_2\right] W_{Y_{\text{beam}}-y}[\rho_1]\, W_{y+Y_{\text{beam}}}[\rho_2]\; \frac{dN}{d^3\vec{\boldsymbol{p}}}\bigg|_{\text{LO}}$$

with $\dfrac{\partial}{\partial Y} W_Y = \mathcal{H}\, W$



  - All the leading logs of $1/x_{1,2}$ are absorbed in the $W's$
  - The $W's$ obey the JIMWLK evolution equation

NLO corrections

- The NLO corrections can be written as :

$$\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\rm NLO} = \Big[\frac{1}{2}\int\limits_{\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}\in{\rm LC}} \mathcal{G}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}})\,\mathbb{T}_{\boldsymbol{u}}\mathbb{T}_{\boldsymbol{v}} + \int\limits_{\vec{\boldsymbol{u}}\in{\rm LC}} \boldsymbol{\beta}(\vec{\boldsymbol{u}})\,\mathbb{T}_{\boldsymbol{u}}\Big]\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\rm LO}$$

- The operator $\mathbb{T}_{\boldsymbol{u}}$ is the generator of shifts of the initial value of the fields on the light-cone :

$$\mathcal{F}[\mathcal{A}_{\rm initial}+a] \equiv \exp\Big[\int_{\vec{\boldsymbol{u}}\in{\rm LC}} a(u)\cdot\mathbb{T}_{\boldsymbol{u}}\Big]\;\mathcal{F}[\mathcal{A}_{\rm initial}]$$

- It can be used to express fluctuations in terms of their initial value :

$$a^\mu(x) = \underbrace{\Big[\int_{\vec{\boldsymbol{u}}\in{\rm LC}} a(u)\cdot\mathbb{T}_{\boldsymbol{u}}\Big]}_{\text{initial condition}}\;\mathcal{A}^\mu(x)$$

NLO corrections

- The 2-point function $\mathcal{G}^{\mu\nu}$ can be written as

$$\mathcal{G}(x,y) = \int \frac{d^3\vec{\boldsymbol{k}}}{(2\pi)^3 2E_{\boldsymbol{k}}}\; a_{-\boldsymbol{k}}(x)\; a_{+\boldsymbol{k}}(y)$$

with
$$\begin{cases} \dfrac{\delta^2 \mathcal{S}_{YM}}{\delta \mathcal{A}^2} \cdot a_{\pm\boldsymbol{k}} = 0 \\ \lim\limits_{t\to-\infty} a_{\pm\boldsymbol{k}}(t,\vec{\boldsymbol{x}}) = \boldsymbol{\epsilon}(\boldsymbol{k})\; e^{\pm i k\cdot x} \end{cases}$$

- The equation of motion for β^μ reads

$$\frac{\delta^2 \mathcal{S}_{YM}}{\delta \mathcal{A}^2} \cdot \beta = \underbrace{\frac{\partial^3 \mathcal{S}_{YM}(\mathcal{A})}{\partial \mathcal{A}^3}}_{\text{3-gluon vertex in the background A}} \underbrace{\frac{1}{2}\int \frac{d^3\vec{\boldsymbol{k}}}{(2\pi)^3 2E_{\boldsymbol{k}}}\; a_{-\boldsymbol{k}}(x)\; a_{+\boldsymbol{k}}(x)}_{\text{value of the loop}}$$

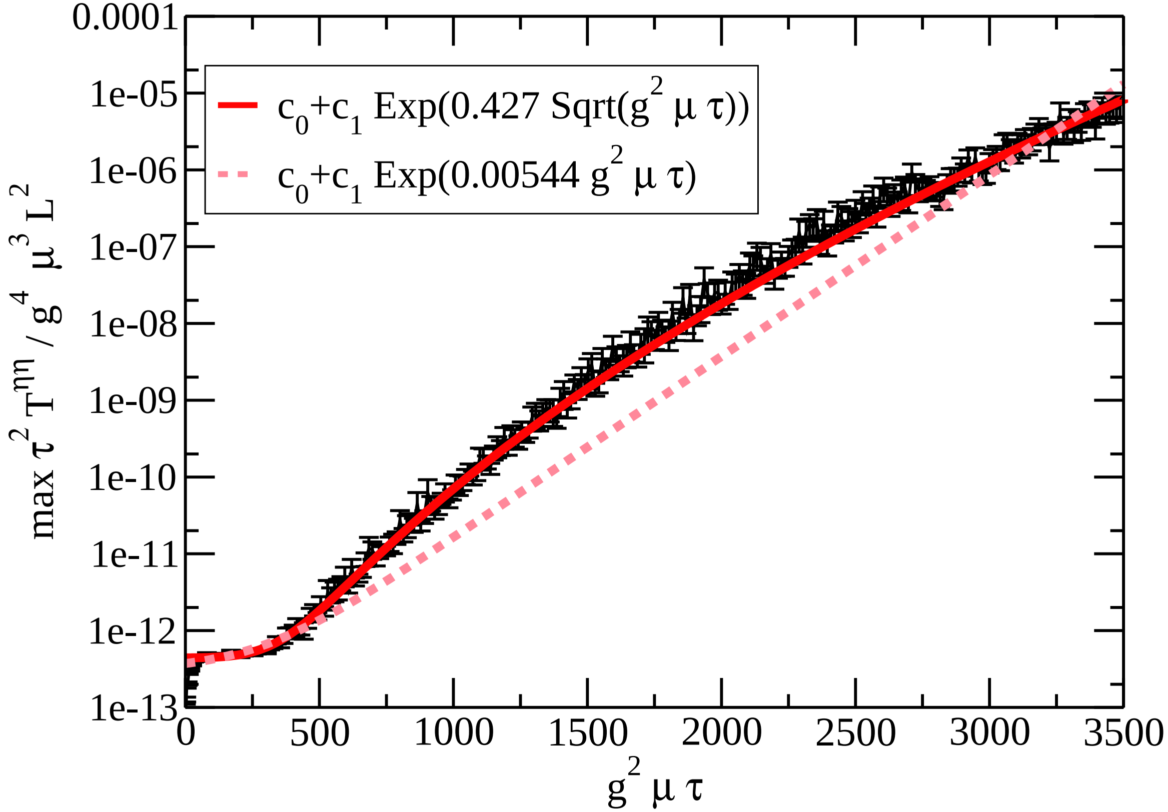
$$\lim_{t\to-\infty} \beta(t,\vec{\boldsymbol{x}}) = 0$$

Glasma instabilities

Numerical results

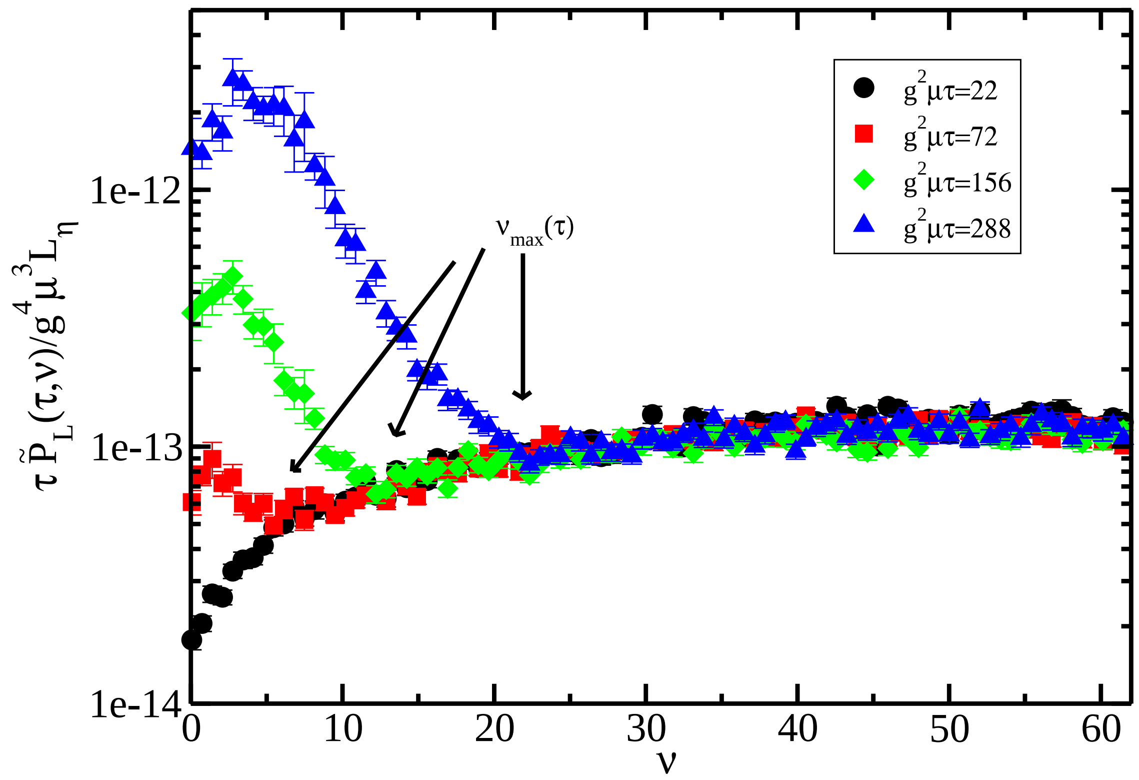
Romatschke, Venugopalan (2005)

- Rapidity dependent perturbations to the classical fields grow like $\exp(\sqrt{\mu\tau})$ until the non-linearities become important :



Numerical results

- Fastest growing modes (ν = Fourier conjugate of η) :



▷ the zero mode grows slower than the others

Unstable modes

- This numerical analysis tells us that the small field fluctuation equation of motion,

$$\frac{\delta^2 \mathcal{S}_{YM}}{\delta \mathcal{A}^2} \cdot a = 0 \; ,$$

has runaway solutions if the initial condition depends on η :

$$a(\tau, \eta, \vec{\boldsymbol{x}}_\perp) \underset{\tau \to \infty}{\sim} e^{\sqrt{\mu\tau}}$$

(see also : Fujii, Itakura (2008); Iwazaki (2008))

- Note : the square root is due to the longitudinal expansion

(Rebhan, Romatschke (2006))

Unstable modes

- This is a problem in loop corrections to $dN/d^3\vec{\boldsymbol{p}}$, because

$$\mathbb{T}_{\boldsymbol{u}}\mathcal{A}(x) \sim \frac{\delta\mathcal{A}(x)}{\delta\mathcal{A}_{\rm initial}(u)} \underset{\tau\to\infty}{\sim} e^{\sqrt{\mu\tau}} \qquad (\mu \sim Q_s)$$

- If we do not resum these unstable fluctuations, the CGC approach will break down at a time $\tau \sim \mu^{-1}\ln^2(1/g)$

- Power counting :
  - Naively : $\mathcal{A} \sim g^{-1}$, $\mathcal{A}_{\rm initial} \sim g^{-1}$, $\mathbb{T}_{\boldsymbol{u}}\mathcal{A}(x) \sim 1$
  - In reality : $\mathbb{T}_{\boldsymbol{u}}\mathcal{A}(x) \sim e^{\sqrt{\mu\tau}}$

- Note : the term $[\beta(u)\mathbb{T}_{\boldsymbol{u}}]\mathcal{A}(x)$ is not subject to this instability, because β is rapidity independent (1-point function in a boost invariant background)

  $\triangleright$ the unstable fluctuations come via terms with at least second derivatives, such as $[\mathcal{G}(u,v)\mathbb{T}_{\boldsymbol{u}}\mathbb{T}_{\boldsymbol{v}}]\mathcal{A}(x)$

Power counting

- So far, we have assumed that :

$$\frac{dN}{d^3\vec{\boldsymbol{p}}} = \frac{1}{g^2}\left[c_0 + c_1\, g^2 + c_2\, g^4 + \cdots\right]$$

$$c_1 = c_{10} + c_{11}\,\ln\left(\frac{1}{x_{1,2}}\right)$$

$$c_2 = c_{20} + c_{21}\,\ln\left(\frac{1}{x_{1,2}}\right) + c_{22}\,\ln^2\left(\frac{1}{x_{1,2}}\right)$$

with all the c_{np} coefficients are of order one

- We have resummed the terms that match a $\ln(1/x)$ to each g^2, and we have shown that all these leading log terms can be absorbed in the evolved distribution of sources $W[\rho_{1,2}]$

Power counting

- Because of the instabilities, we should write instead :

$$c_1 = c_{100} + c_{110} \ln\left(\frac{1}{x_{1,2}}\right) + \underline{c_{101}\, e^{2\sqrt{\mu\tau}}}$$

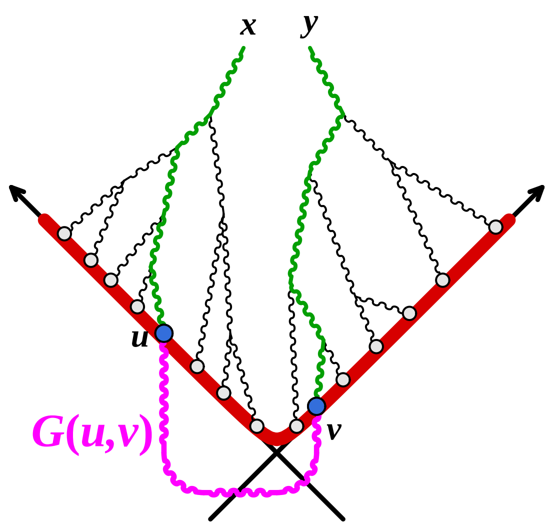
$$c_2 = c_{200} + c_{210} \ln\left(\frac{1}{x_{1,2}}\right) + c_{201}\, e^{2\sqrt{\mu\tau}} + c_{220} \ln^2\left(\frac{1}{x_{1,2}}\right) + \underline{c_{211} \ln\left(\frac{1}{x_{1,2}}\right) e^{2\sqrt{\mu\tau}}} + \underline{c_{202}\, e^{4\sqrt{\mu\tau}}}$$

- Note : because the logs of $1/x$ come from the zero η-modes, while the unstable terms come from the non-zero modes, there are only terms c_{npq} with $p + q \leq n$
- Resummation of leading logs : keep all the c_{nn0} terms
- Improved resummation : keep all the c_{npn-p} terms

  (these are the terms for which each g^2 is compensated by a large log(1/x) or a factor $e^{2\sqrt{\mu\tau}}$)

Power counting

- The instabilities are triggered by the 2-point function :



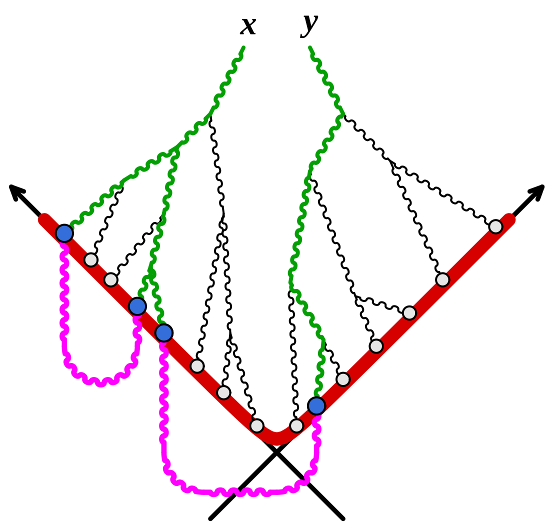
- Power counting : $\mathcal{G} \sim \mathcal{O}(1)$, $\bullet \sim \mathcal{O}(g\ e^{\sqrt{\mu\tau}})$

- This 1-loop term is of order $g^2\ e^{2\sqrt{\mu\tau}}$ relative to the LO contribution to the gluon spectrum

Power counting

- At n-loop order, one must pick the terms that have the fastest growth in time

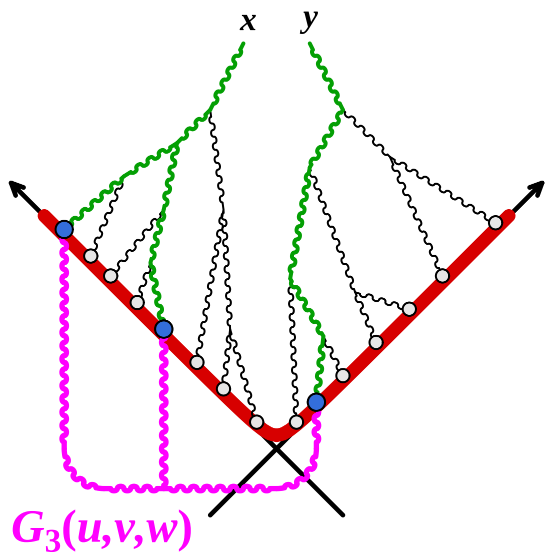
  ▷ one must maximize the number of locations where the initial field is perturbed on the light-cone, while minimizing the powers of α_s



- This 2-loop term is of order $g^4 \, e^{4\sqrt{\mu\tau}}$ relative to the LO contribution to the gluon spectrum

Power counting

- Non-Gaussian correlations are suppressed :



- Power counting : $\mathcal{G}_3 \sim \mathcal{O}(g)$, $\bullet \sim \mathcal{O}(g\, e^{\sqrt{\mu\tau}})$
- This 2-loop term is of order $g^4\, e^{3\sqrt{\mu\tau}}$ relative to the LO contribution to the gluon spectrum $\triangleright$ subleading

Unstable modes resummation

Resummation

- 1-loop contributions :

$$\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\rm NLO} = \Bigg[\ln\left(\frac{\Lambda^+}{p^+}\right)\,\mathcal{H}_1 + \ln\left(\frac{\Lambda^-}{p^-}\right)\,\mathcal{H}_2 + \frac{1}{2}\int_{\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}\in{\rm LC}} \mathcal{G}_{\nu\not=0}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}})\,\mathbb{T}_{\boldsymbol{u}}\mathbb{T}_{\boldsymbol{v}}\Bigg]\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\rm LO}$$

  - $\mathcal{G}_{\nu\not=0}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}})$ does not contain the zero η-mode
  - This formula does not make sense beyond $\tau_{\rm max}$

- Assume that the resummation of these terms to all orders can be written as :

$$\sum_{n=0}^{\infty}\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{{\rm N}^n{\rm LO}} = \mathcal{U}_1\,\mathcal{U}_2\;\;F[\mathbb{T}_{\boldsymbol{v}}]\;\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\rm LO}$$

where $\mathcal{U}_{1,2}$ are evolution operators for the JIMWLK Hamiltonians (factorization is due to the non-mixing of the various divergences)

Resumming the unstable terms

- Introduce the “Laplace transform” of $F[\mathbb{T}_{\boldsymbol{u}}]$,

$$F[\mathbb{T}_{\boldsymbol{u}}] \equiv \int \big[Da(\vec{\boldsymbol{u}})\big] \; F[a(\vec{\boldsymbol{u}})] \; \underbrace{\exp \int_{\rm LC} a(\vec{\boldsymbol{u}}) \cdot \mathbb{T}_{\boldsymbol{u}}}$$

translation operator for the initial classical field

- The effect of $F[\mathbb{T}_{\boldsymbol{u}}]$ is :

$$F[\mathbb{T}_{\boldsymbol{v}}] \quad \frac{dN}{d^3\vec{\boldsymbol{p}}}\bigg|_{\rm LO} = \int \big[Da(\vec{\boldsymbol{u}})\big] \; F[a(\vec{\boldsymbol{u}})] \quad \frac{dN}{d^3\vec{\boldsymbol{p}}}[\mathcal{A}+a]\bigg|_{\rm LO}$$

▷ resumming the unstable modes amounts to add a fluctuating field to the initial value of the classical field on the light-cone, with a distribution $F[a(\vec{\boldsymbol{u}})]$

Resumming the unstable terms

- Summing both the large logs of $1/x_{1,2}$ and the unstable terms, we get :

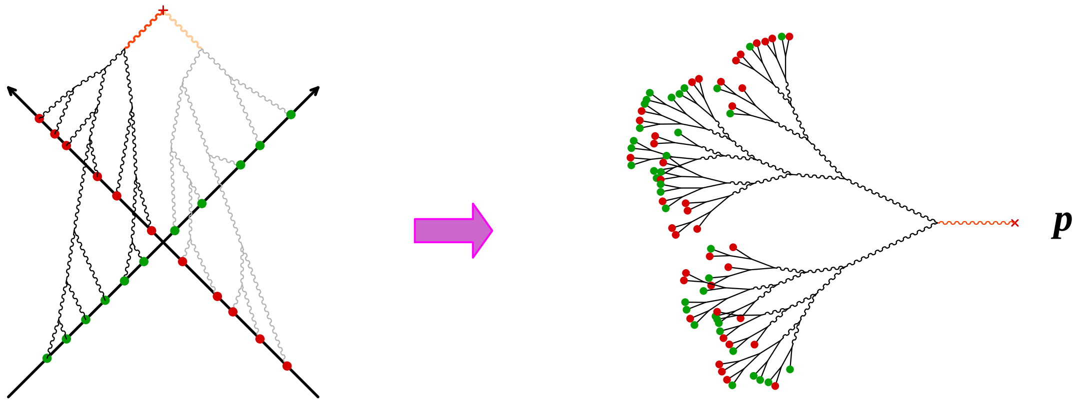
$$\left\langle \frac{dN}{d^3\vec{\boldsymbol{p}}} \right\rangle \underset{\substack{\text{improved}\\ \text{resummation}}}{=} \int [D\rho_1\, D\rho_2]\; W_{Y_1}[\rho_1]\, W_{Y_2}[\rho_2] \times \int [Da(\vec{\boldsymbol{u}})]\; F[a(\vec{\boldsymbol{u}})]\; \left.\frac{dN}{d^3\vec{\boldsymbol{p}}}[\mathcal{A}+a]\right|_{\text{LO}}$$

- Note : after this resummation, the instabilities do not lead to divergences when $\tau \to +\infty$

  (the quantum fluctuations are now absorbed in the initial condition of the non-linear YM equations – instead of being treated in the linear approximation)

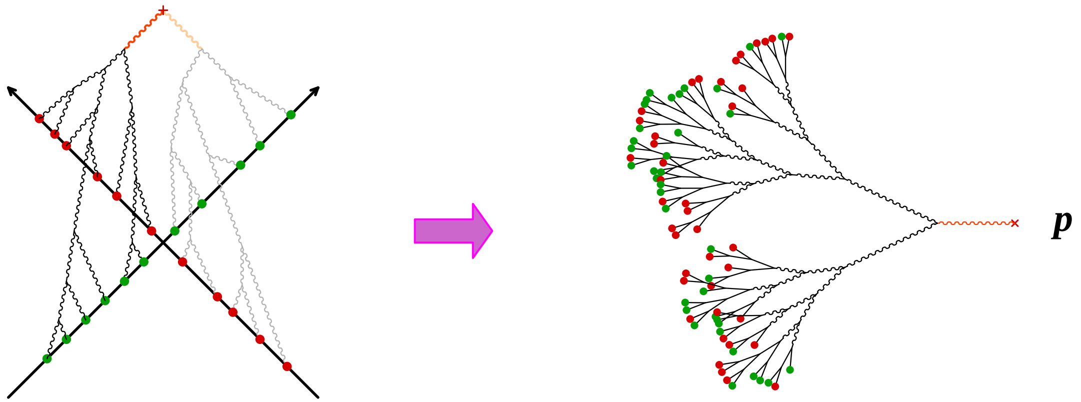
Instabilities and gluon splitting

- Tree level :

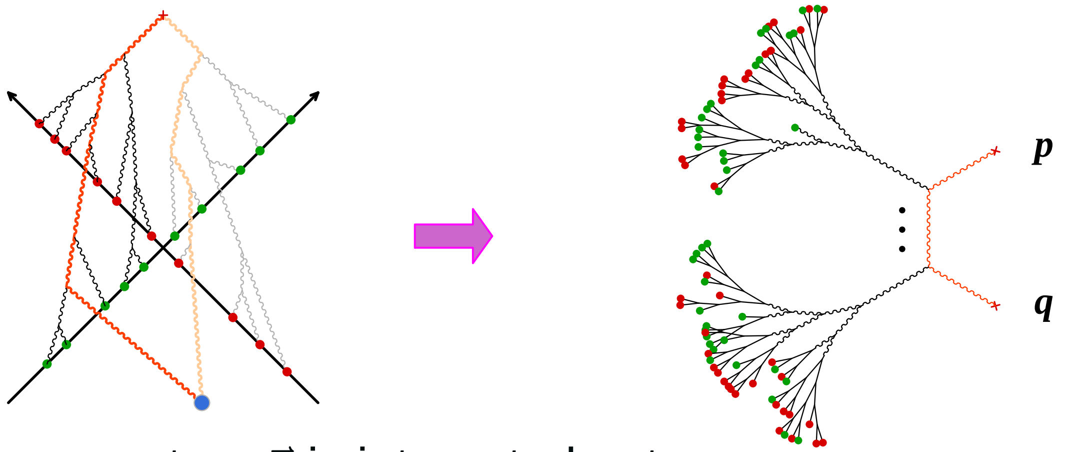


Instabilities and gluon splitting

- Tree level :



- One loop ▷ gluon pairs :



▷ The momentum $\vec{q}$ is integrated out

▷ If $\alpha_s^{-1} \lesssim |y_{\boldsymbol{p}} - y_{\boldsymbol{q}}|$, the correction is absorbed in $W[\rho_{1,2}]$

▷ If $|y_{\boldsymbol{p}} - y_{\boldsymbol{q}}| \lesssim \alpha_s^{-1}$: gluon splitting in the final state

Initial Gaussian fluctuations

- Some numerical results have been obtained with a toy model for the distribution of initial fluctuations

- With some approximations, one can obtain a spectrum of Gaussian fluctuations characterized by :

$$\big\langle a_i(\eta,\vec{\boldsymbol{x}}_\perp)\,a_j(\eta',\vec{\boldsymbol{x}}'_\perp)\big\rangle =$$

$$= \frac{1}{\tau\sqrt{-(\partial_\eta/\tau)^2-\boldsymbol{\partial}_\perp^2}}\left[\delta_{ij}+\frac{\partial_i\partial_j}{(\partial_\eta/\tau)^2}\right]\delta(\eta-\eta')\,\delta(\vec{\boldsymbol{x}}_\perp-\vec{\boldsymbol{x}}'_\perp)$$

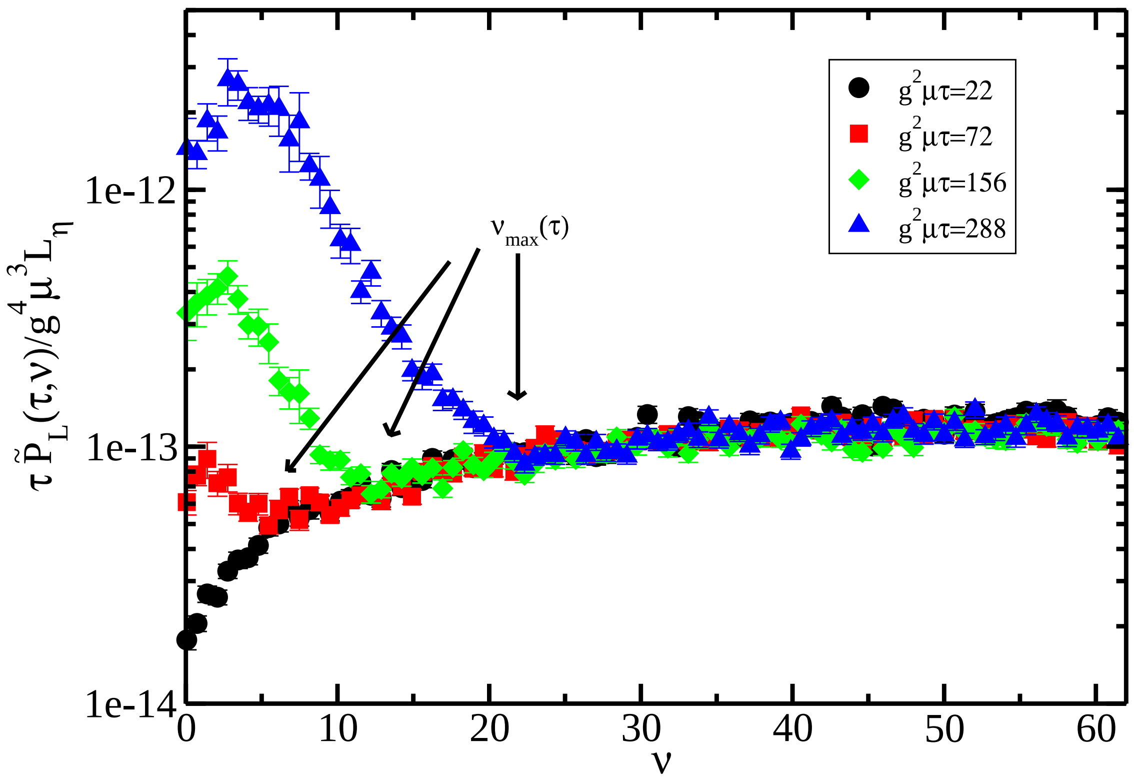
(Fukushima, FG, McLerran (2006))

- Problem: loop corrections have UV divergences...
  - Cutoff the fluctuation spectrum at $k \le \Lambda$
  - Renormalize the classical action, with counterterms computed in cutoff regularization
  - Multiply by the overall renormalization factor for the operator of interest

Thermalization ?

Numerical results

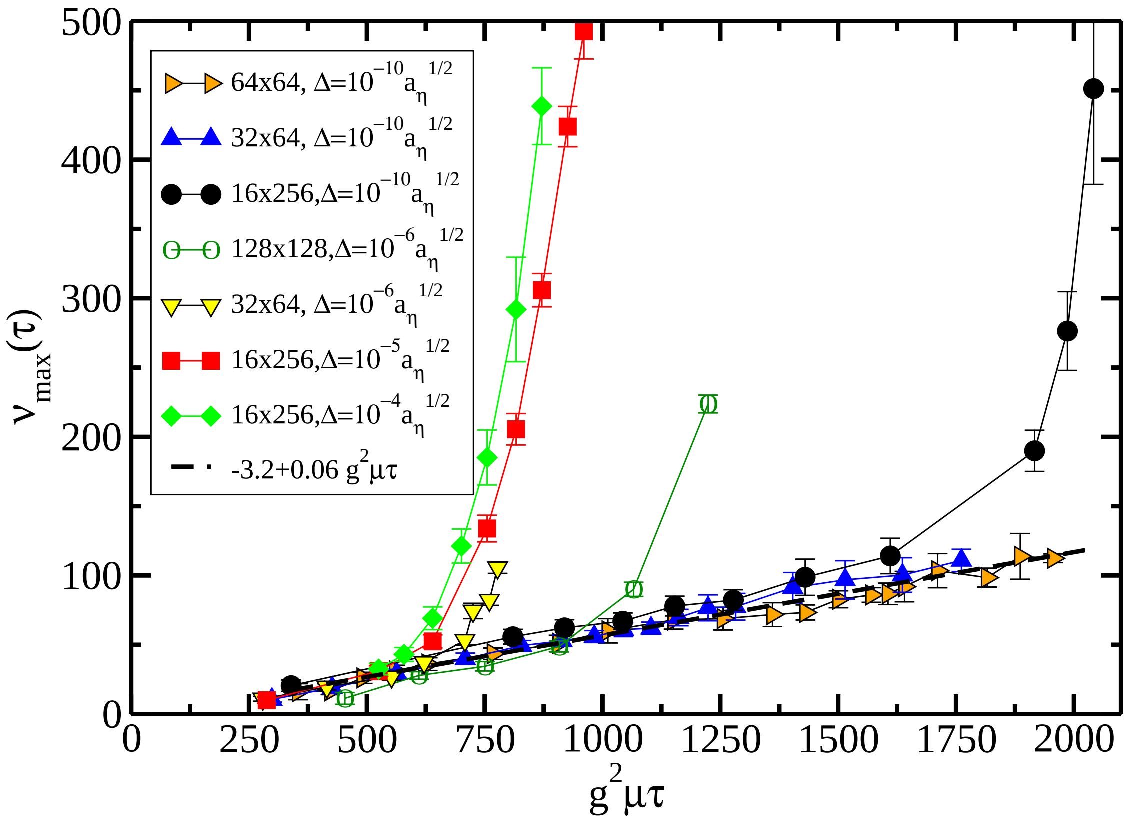
- Fastest growing modes ($\nu =$ Fourier conjugate of η) :



▷ the zero mode grows slower than the others

Numerical results

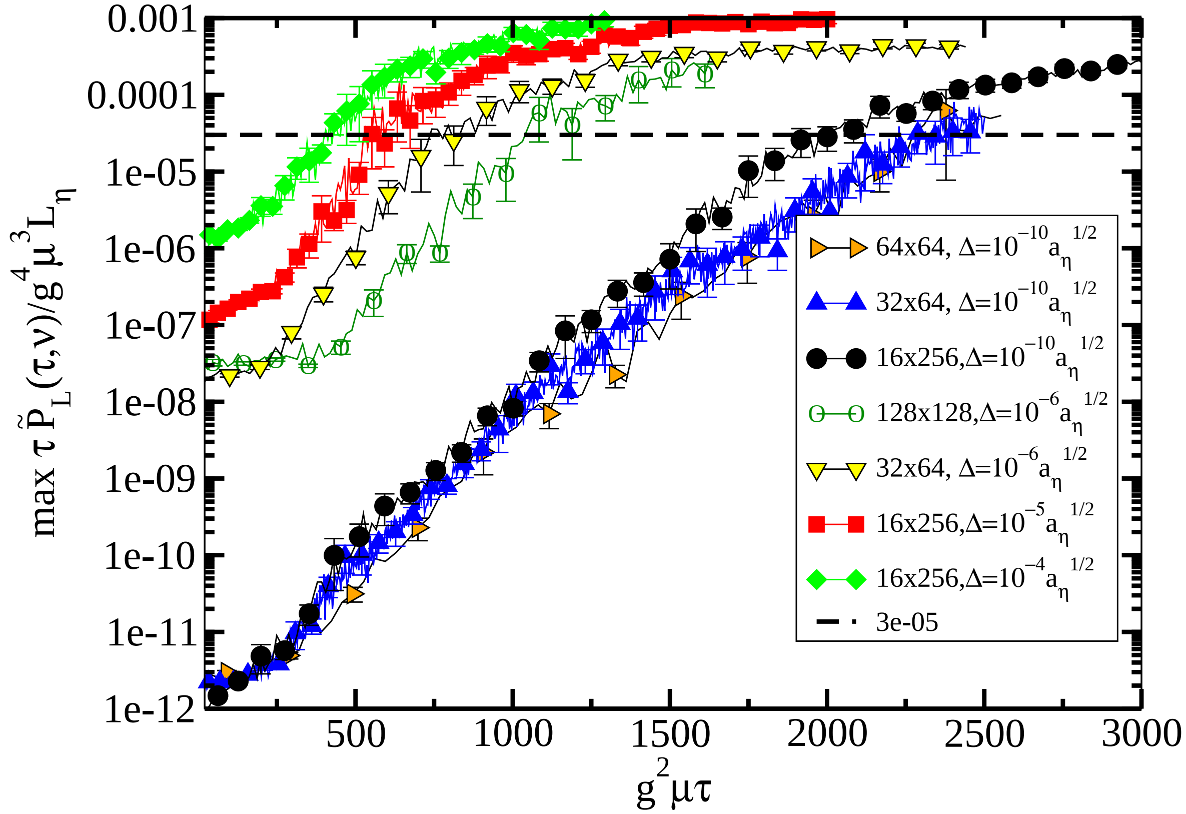
- Maximal growing mode as a function of time :



▷ eventually, explosion of the hard modes

Numerical results

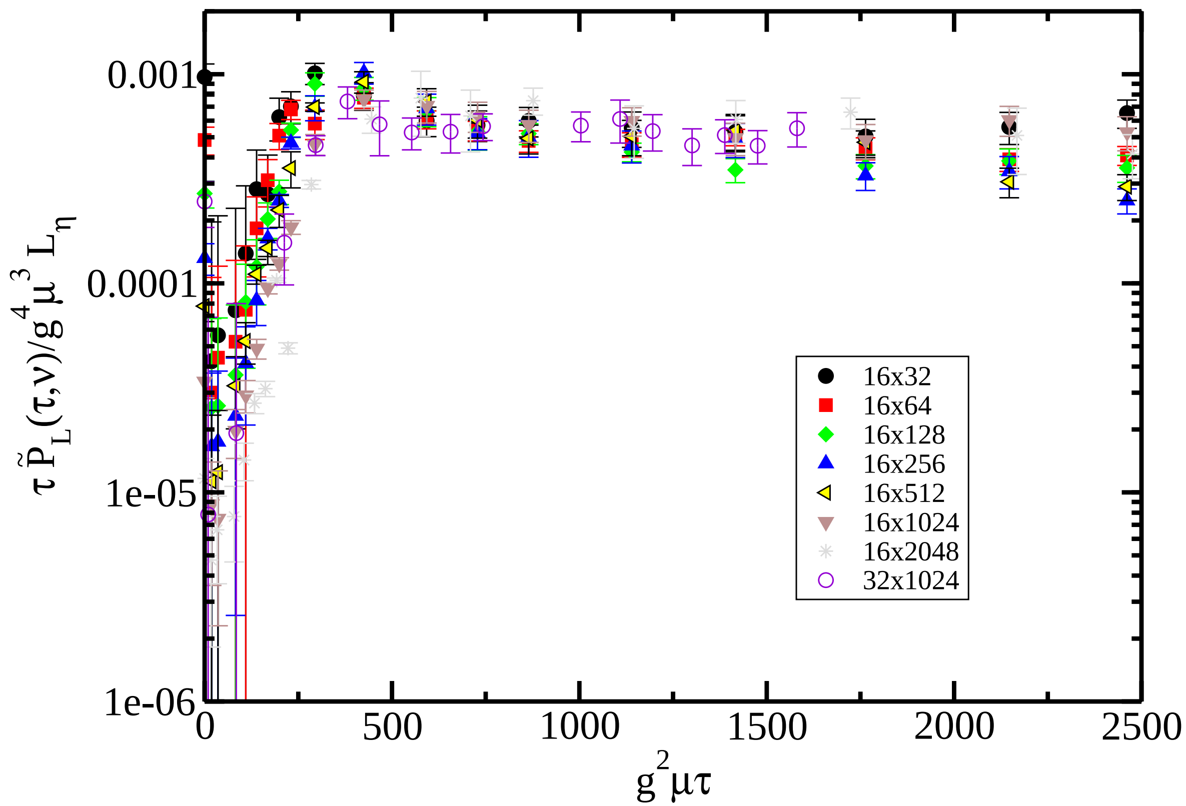
- Maximal amplitude as a function of time (weak anisotropy) :



▷ the UV explosion occurs when this amplitude reaches some fixed value

Numerical results

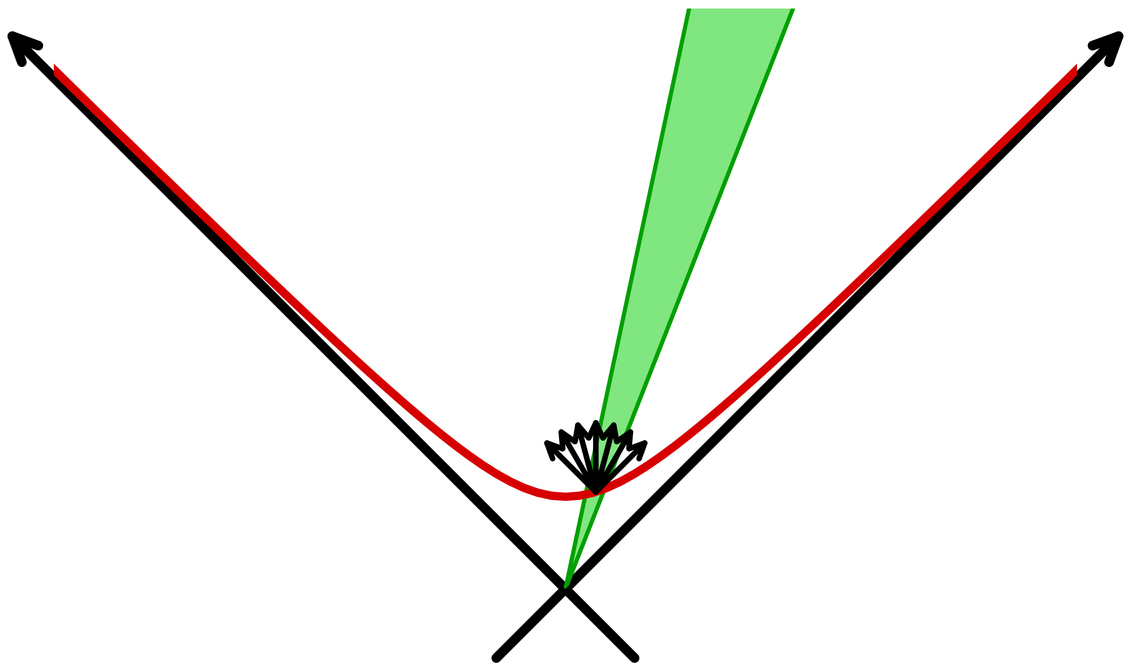
- Maximal amplitude as a function of time (larger anisotropy) :



▷ the longitudinal pressure grows faster for a larger initial anisotropy

Longitudinal expansion

- If nothing else happened, the distribution of produced particles would quickly become very anisotropic :



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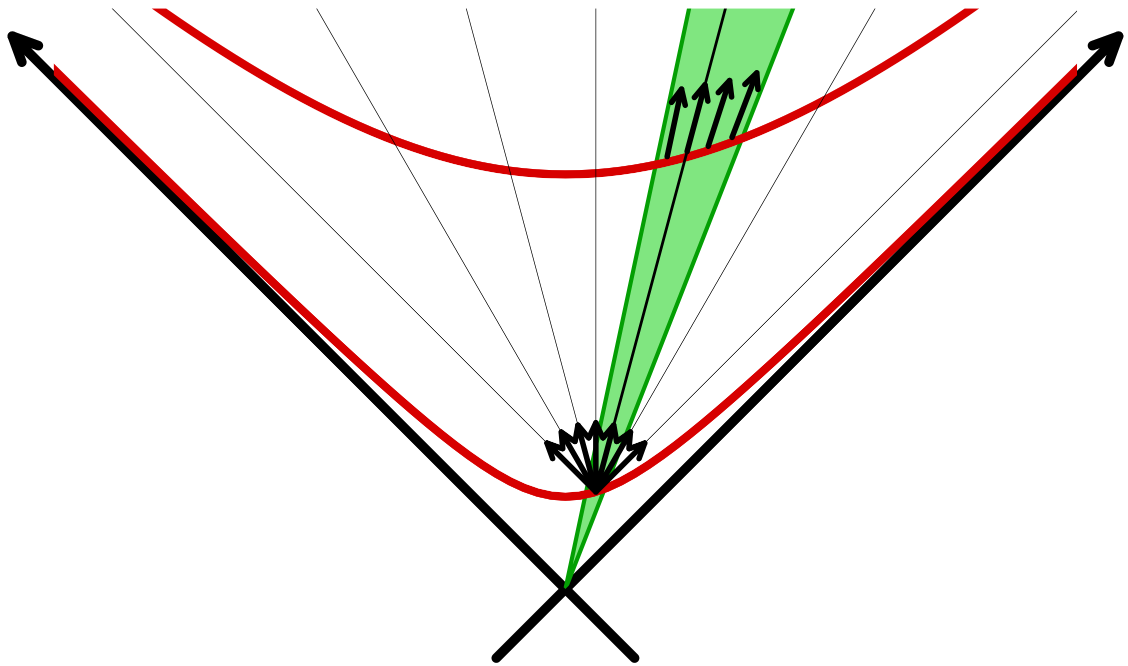
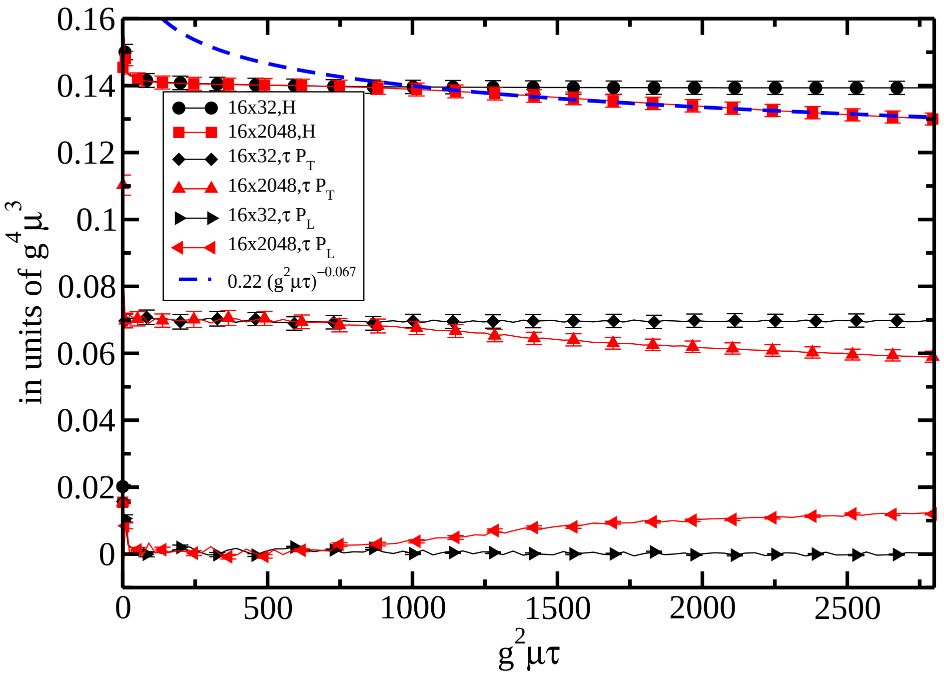


▷ if particles fly freely, only one longitudinal velocity can exist at a given η : $v_z = \tanh(\eta)$

▷ the longitudinal expansion of the system is the main obstacle to local isotropy

Glasma instability (expanding system)

- The Glasma instability seems to help fighting the expansion :



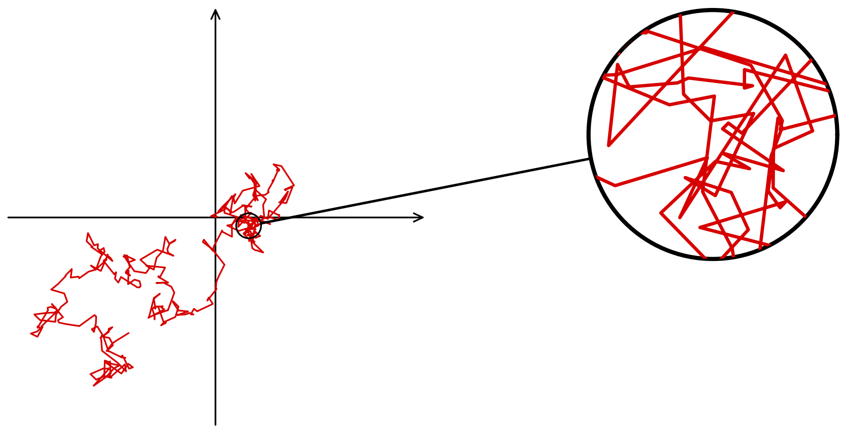
- The energy density drops slightly faster than τ^{-1} ($\tau^{-1.33}$ needed for local thermalization)
- This is for rather tiny initial fluctuations. In QCD, they are suppressed only by $\alpha_s \approx 0.3$

Uncertainty bound on η/s

- $\eta \sim \lambda\epsilon$ ($\lambda =$ mean free path, $\epsilon =$ energy density). Thus,

$$\frac{\eta}{s} \sim \lambda \underbrace{\frac{\epsilon}{s}}_{\text{energy per particle}}$$

- Heisenberg inequalities forbid the mean free path to be smaller than the De Broglie wavelength of the particles. Scatterings by an $\mathcal{O}(1)$ angle can occur only every λ_{Broglie} at most :



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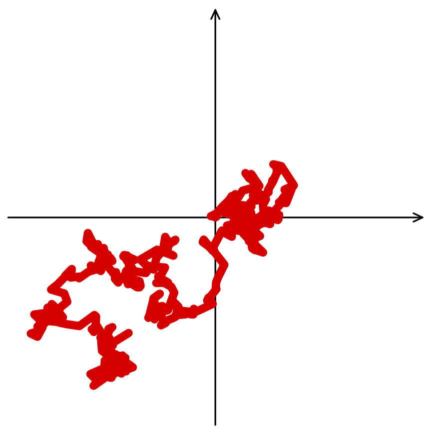

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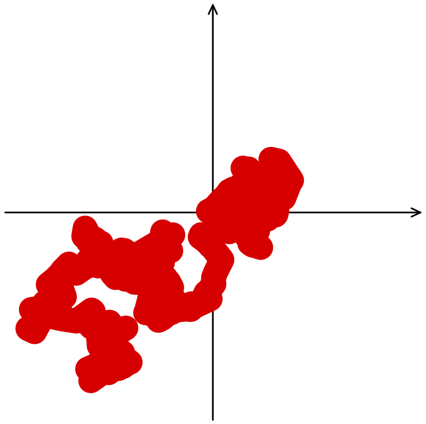


- Hence, $\dfrac{\eta}{s} \geq \mathcal{O}(1)$

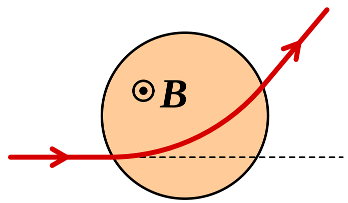
Anomalous transport

Asakawa, Bass, Muller (2006)

- Assume that $\alpha_s = \frac{g^2}{4\pi} \ll 1$
- Consider a domain of size Q_s^{-1}, in which the magnetic field is uniform and large, of order $B \sim Q_s^2/g$
- Let a particle of energy $E \sim Q_s$ go through this domain. The Lorenz force deflects its trajectory by an angle of order unity :

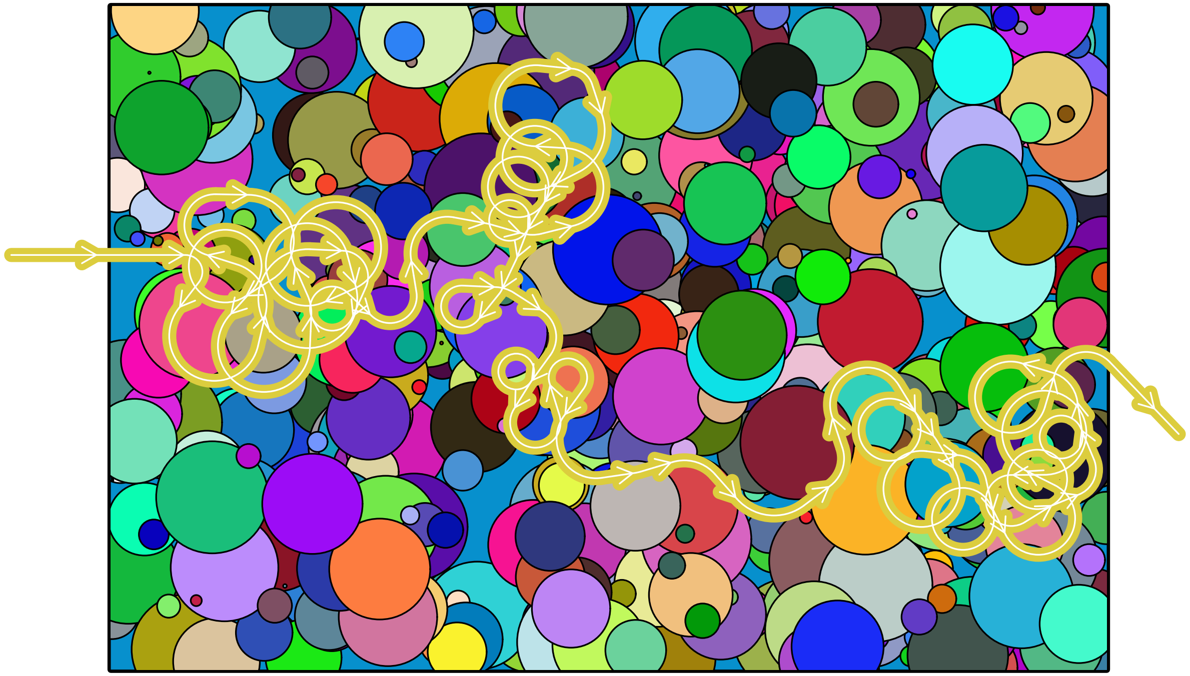
$$\frac{d\vec{\boldsymbol{p}}}{dt} = g\,\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}} \quad \Rightarrow \quad \dot{\theta} = \frac{gB}{E} \sim Q_s$$

time spent in the domain : $\delta\tau \sim Q_s^{-1}$



Anomalous transport

- Consider now a region filled with such domains, with random orientations for the magnetic field in each domain



▷ In such a medium, the mean free path of a particle of energy Q_s is of order Q_s^{-1}, i.e. as low as permitted by the uncertainty principle ▷ fast thermalization?

Link to Weibel instabilities

Weibel instabilities

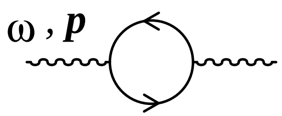
- Weibel (1959) : instability in anisotropic electron-ion plasmas
- Mrowczynski (1998-2003) : similar instabilities exist in QCD
- Romatschke-Strickland (2003-2004) : Weibel instability through the screening properties of the anisotropic QGP
- Arnold, Lenaghan, Moore, Yaffe (2005) : instabilities and thermalization
- Recent numerical investigations of these instabilities :
  Arnold, Moore, Yaffe
  Rebhan, Romatschke, Strickland
  Dumitru, Nara, Strickland
  Bödeker, Rummukainen
- Is there a relation between this instability, that occurs in anisotropic plasmas, and the Glasma instability discussed so far?

Dressed propagator (equilibrium)

- In order to assess how the medium affects the propagation of excitations, one must compute the gluon polarization tensor $\Pi^{\mu\nu}(x,y) \equiv \left\langle J^\mu(x) J^\nu(y) \right\rangle$

- Because one is after the long distance properties of the plasma, one also makes the approximation $|\vec{\boldsymbol{p}}| \ll |\vec{\boldsymbol{k}}|$ (Hard Thermal Loops : Braaten, Pisarski - 1990)

- For instance, the spatial part Π^{ij} of the polarization tensor reads :



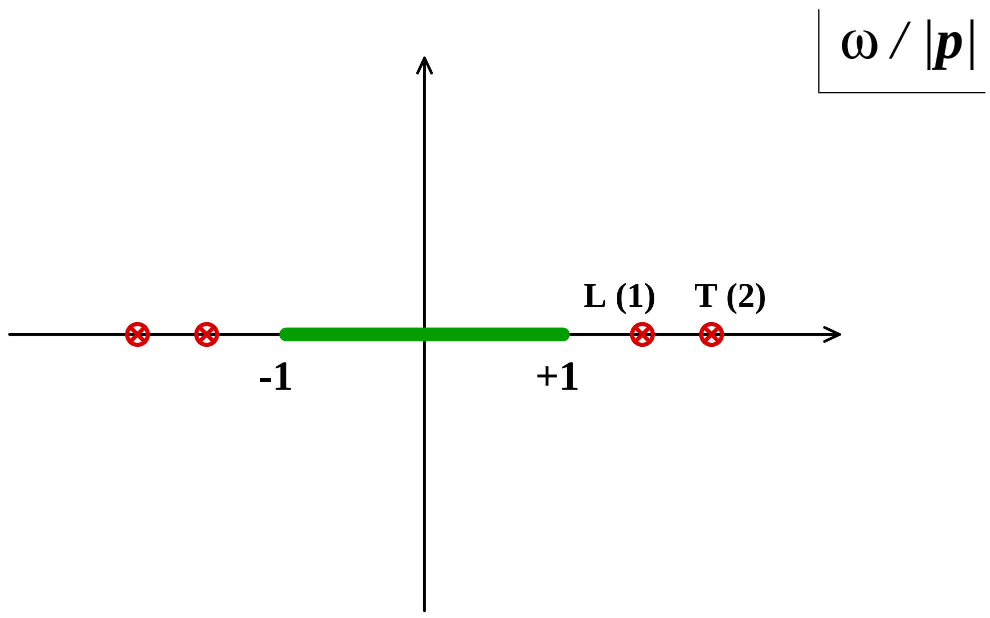
$$= -g^2 T \int \frac{d^3\vec{\boldsymbol{k}}}{(2\pi)^3}\, \widehat{\boldsymbol{v}}^i_{\boldsymbol{k}} \frac{\partial f(\vec{\boldsymbol{k}})}{\partial \boldsymbol{k}^l} \left[\delta^{jl} - \frac{\widehat{\boldsymbol{v}}^j_{\boldsymbol{k}} \widehat{\boldsymbol{v}}^l_{\boldsymbol{k}}}{\omega - \widehat{\boldsymbol{v}}_{\boldsymbol{k}} \cdot \vec{\boldsymbol{p}} + i\epsilon} \right]$$

($\widehat{\boldsymbol{v}}_{\boldsymbol{k}} \equiv \vec{\boldsymbol{k}}/|\vec{\boldsymbol{k}}|$)

  - It depends on the distribution $f(\vec{\boldsymbol{k}})$ of particles in the plasma

Singularities (equilibrium)

- In the complex plane of $\omega/|\vec{p}|$, the dressed propagator has poles (quasi-particles) and a cut (Landau damping) :



Debye screening (equilibrium)

- The Coulomb potential of a static charge reads :

$$V(\vec{\boldsymbol{r}}) = g \int \frac{d^3\vec{\boldsymbol{q}}}{(2\pi)^3} \; \frac{e^{i\vec{\boldsymbol{q}}\cdot\vec{\boldsymbol{r}}}}{\vec{\boldsymbol{q}}^2 + \Pi_L(0,\vec{\boldsymbol{q}})}$$

- In a plasma, $\Pi_L(0,\vec{\boldsymbol{q}}) = \frac{g^2T^2}{3} \equiv m_D^2$. The Fourier transform gives

$$V(\vec{\boldsymbol{r}}) = g \int \frac{d^3\vec{\boldsymbol{q}}}{(2\pi)^3} \; \frac{e^{i\vec{\boldsymbol{q}}\cdot\vec{\boldsymbol{r}}}}{\vec{\boldsymbol{q}}^2 + m_D^2} = \frac{g}{4\pi|\vec{\boldsymbol{r}}|} \; e^{-m_D|\vec{\boldsymbol{r}}|}$$

$\triangleright$ the potential is unmodified at $r \ll 1/m_D$, but exponentially suppressed at large distance

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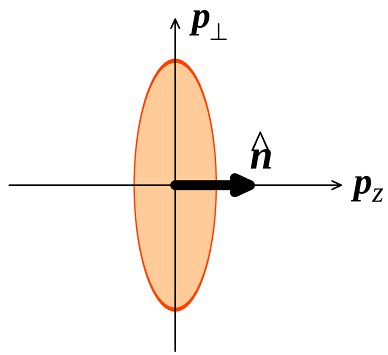
Medium effects (anisotropic)

- Most of the previous analysis can be carried through in the case of a plasma with an anisotropic distribution of particles. In particular, the formula for the polarization tensor in terms of $f(\vec{\boldsymbol{k}})$ remains valid :

$$\Pi^{ij}(\omega, \vec{\boldsymbol{p}}) = -g^2 T \int \frac{d^3\vec{\boldsymbol{k}}}{(2\pi)^3}\, \widehat{\boldsymbol{v}}^i_{\boldsymbol{k}}\, \frac{\partial f(\vec{\boldsymbol{k}})}{\partial \boldsymbol{k}^l} \left[\delta^{jl} - \frac{\widehat{\boldsymbol{v}}^j_{\boldsymbol{k}}\widehat{\boldsymbol{v}}^l_{\boldsymbol{k}}}{\omega - \widehat{\boldsymbol{v}}_{\boldsymbol{k}}\cdot\vec{\boldsymbol{p}} + i\epsilon}\right]$$

- Model for an anisotropic distribution : start from a generic isotropic distribution $f(\boldsymbol{k}^2)$ and squeeze it :

$$f(\boldsymbol{p}^2) \quad \longrightarrow \quad f(\boldsymbol{p}^2 + \xi(\widehat{\boldsymbol{n}}\cdot\vec{\boldsymbol{p}})^2)$$



Medium effects (anisotropic)

- Within this model, it is easy to factorize the integration over the argument of f (i.e. $\boldsymbol{p}^2 + \xi(\widehat{\boldsymbol{n}} \cdot \vec{\boldsymbol{p}})^2$) :

$$\Pi^{ij}(\omega, \vec{\boldsymbol{p}}) = m_D^2 \int \frac{d^2\widehat{\boldsymbol{v}}_{\boldsymbol{k}}}{4\pi} \, \widehat{\boldsymbol{v}}_{\boldsymbol{k}}^i \, \frac{\widehat{\boldsymbol{v}}_{\boldsymbol{k}}^l + \xi(\widehat{\boldsymbol{v}}_{\boldsymbol{k}} \cdot \widehat{\boldsymbol{n}})\widehat{\boldsymbol{n}}^l}{(1 + \xi(\widehat{\boldsymbol{v}}_{\boldsymbol{k}} \cdot \widehat{\boldsymbol{n}})^2)^2} \left[\delta^{jl} - \frac{\widehat{\boldsymbol{v}}_{\boldsymbol{k}}^j \widehat{\boldsymbol{v}}_{\boldsymbol{k}}^l}{\omega - \widehat{\boldsymbol{v}}_{\boldsymbol{k}} \cdot \vec{\boldsymbol{p}} + i\epsilon}\right]$$

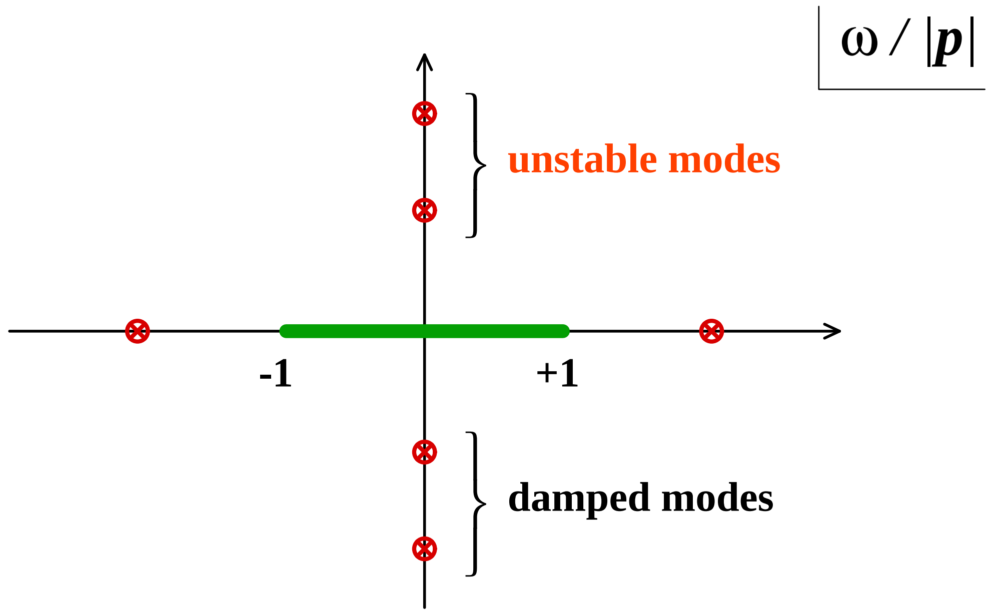
with

$$m_D^2 \equiv -\frac{g^2}{2\pi^2} \int_0^\infty dk \; k^2 \; \frac{df(k^2)}{dk^2}$$

- m_D sets the magnitude of all the medium effects on the gauge bosons
- Only the remaining integral over the unit vector $\widehat{\boldsymbol{v}}_{\boldsymbol{k}}$ is affected by the anisotropy $(\xi \neq 0)$
- The tensorial decomposition of $\Pi^{\mu\nu}$ is more complicated than in the isotropic case, because the vector $\widehat{\boldsymbol{n}}^\mu$ can be used in the construction of the basis

Singularities (anisotropic)

- In the anisotropic case, some poles of the dressed propagator have moved away from the real axis :



- Some poles have migrated to the upper half plane, and lead to instabilities
- These imaginary poles exist no matter how small the squeezing parameter ξ is (but their imaginary part goes to zero when $\xi \to 0$)

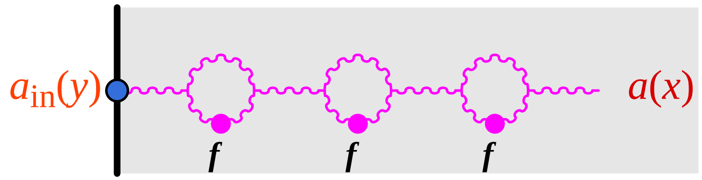
Relation to Glasma instabilities

- The existence of unstable modes lead to the indefinite growth of some fluctuations

- In-medium propagation of a fluctuation :

$$a(x) = \int d^3\vec{\boldsymbol{y}}\; G(x,y)\left[\overleftarrow{\partial_y^0} - \overrightarrow{\partial_y^0}\right] a_{\rm in}(y_0,\vec{\boldsymbol{y}})$$

- This can be represented by diagrams such as :

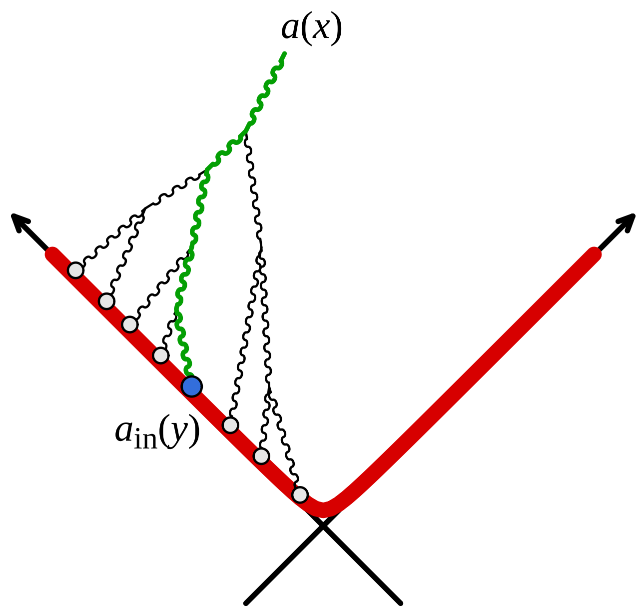


Note : the blob on one of the lines of the self-energies indicates the presence of one factor of the distribution $f(\vec{\boldsymbol{k}})$

Relation to Glasma instabilities

- Reminder : the Glasma instability also affects the propagation of small fluctuations in the forward light-cone, via diagrams such as :

$$a(x) =$$

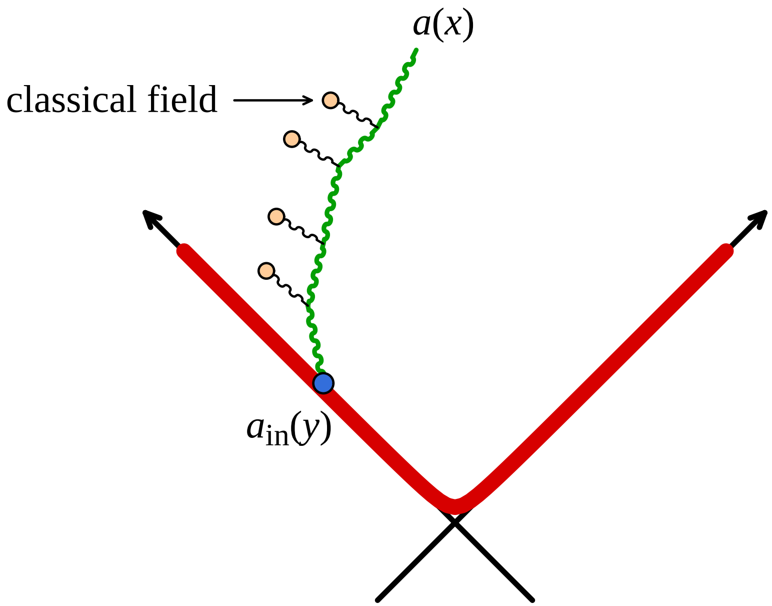


Note : each tree attached to the Green line (the propagator of the fluctuation in the background field) is an insertion of the classical field

Relation to Glasma instabilities

- Replace each such tree by a symbol denoting the background field at the point where the tree is attached :

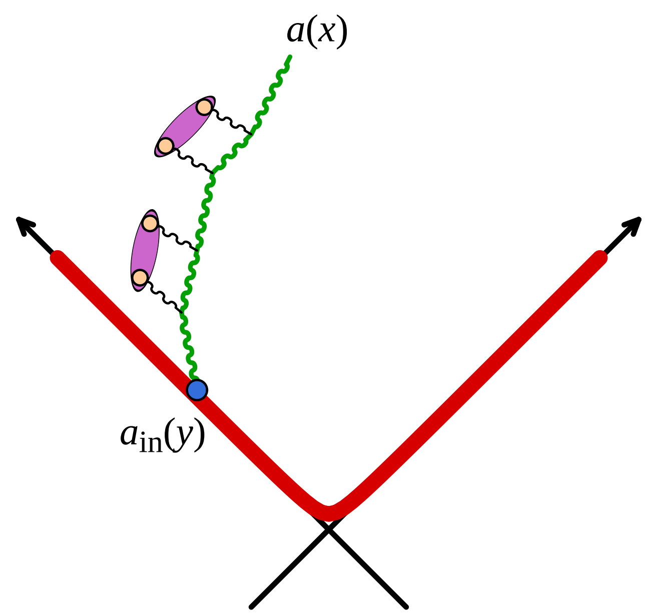
$$a(x) \quad =$$



Relation to Glasma instabilities

- The average over the sources ρ produces links between the classical fields. Some of the terms involve only connections among neighboring background field insertions :

$$a(x) \quad =$$



▷ We recover self-energy corrections very similar to the ones encountered in the study of the Weibel instability (the average of two classical fields produces an anisotropic “gluon distribution”)

Summary

Summary

- Rapidity dependent fluctuations propagating on top of the glasma fields are unstable
- The resummation of the fastest growing terms amounts to adding noise to the initial value of the classical fields
- This may lead to a very turbulent configuration of magnetic fields, possibly exhibiting a rather small “anomalous viscosity”
- The practical implementation is rendered difficult by the UV divergences (very strong in the pressure, $\sim \Lambda^4$)
- There may be a close link between the glasma instabilities and those encountered in the Hard Loop analysis of an anisotropic plasma